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4.24 Referring to the random variables whose joint probability distribution is given in Exercise 3.39 on page 125,

(a) find $E(X^2Y - 2XY)$;

(b) find $\mu_X - \mu_Y$.

3.39 From a sack of fruit containing 3 oranges, 2 apples, and 3 bananas, a random sample of 4 pieces of fruit is selected. If X is the number of oranges and Y is the number of apples in the sample, find

$$f(x,y) = \frac{\binom{3}{x}\binom{2}{y}\binom{3}{4-x-y}}{\binom{8}{4}}$$

for $x = 0,1,2,3$

$y = 0,1,2$

$1 \le x+y \le 4$

(a)

$$\sum_{x=0}^{3}\sum_{y=0}^{2} f(x,y)(x^2y-2xy)$$

$2 \quad 2^{20} \quad 21 \quad 4-8$

$$= \cancel{(0-0)}\frac{2}{70} + \cancel{(0-0)}\frac{3}{70} + \cancel{(0-0)}\frac{3}{70} + (1-2)\frac{18}{70} + (2-4)\frac{9}{70} + \cancel{(0-0)}\frac{9}{70}$$

$$+ \cancel{(4-4)}\frac{18}{70} + \cancel{(8-8)}\frac{3}{70} + \cancel{(0-0)}\frac{3}{70} + (9-6)\frac{2}{70}$$

$$= \frac{-18-18+6}{70} = -\frac{3}{7} \#$$

(b)

$E(X) - E(Y)$

Marginal distribution of X

x	0	1	2	3
$g(x)$	$\frac{5}{70}$	$\frac{30}{70}$	$\frac{30}{70}$	$\frac{5}{70}$

Marginal distribution of Y

y	0	1	2
$h(y)$	$\frac{15}{70}$	$\frac{40}{70}$	$\frac{15}{70}$

$E(X) = \frac{3}{2}$

$E(Y) = 1$

$\mu_X - \mu_Y = \frac{1}{2} \#$

4.44 Find the covariance of the random variables X and Y of Exercise 3.39 on page 125.

$$\sigma_{XY} = E(XY) - E(X)E(Y)$$

$$= \sum_{x=0}^{3}\sum_{y=0}^{2} xy\, f(x,y) - \frac{5}{2}$$

$$= \left(\frac{18}{70} + 2\cdot\frac{9}{70} + 2\cdot\frac{18}{70} + 4\cdot\frac{3}{70} + 3\cdot\frac{2}{70}\right) - \frac{3}{2}$$

$$= \frac{18+18+36+12+6}{70} - \frac{105}{70}$$

$$= \frac{-15}{70} = \frac{-3}{14} \quad \#$$

4.60 Suppose that X and Y are independent random variables having the joint probability distribution

y \ x	2	4
1	0.15	0.10
3	0.25	0.25
5	0.15	0.10

Find
(a) $E(2X - 3Y)$;
(b) $E(XY)$.

$E(X) = 2 \cdot 0.55 + 4 \cdot 0.45$
$= 1.1 + 1.8 = 2.9$

$E(Y) = 0.25 + 1.5 + 1.25$
$= 3$

(a)
$2E(X) - 3E(Y)$
$= 5.8 - 9 = -3.2$ #

(b)
$E(XY) = E(X) \cdot E(Y) = 8.7$ #

4.78 Compute $P(\mu - 2\sigma < X < \mu + 2\sigma)$, where X has the density function

$$f(x) = \begin{cases} 30x^2(1-x)^2, & 0 < x < 1, \\ 0, & \text{elsewhere} \end{cases}$$

and compare with the result given in Chebyshev's theorem.

$30x^6 - 60x^5 + 30x^4$

$\mu = \int_0^1 x \cdot 30x^2 \cdot (1-x)^2 \, dx$
$= \int_0^1 30x^5 - 60x^4 + 30x^3 \, dx$
$= 5x^6 - 12x^5 + \frac{15}{2}x^4 \Big|_0^1 = 0.5$

$\sigma^2 = \int_0^1 x^2 \cdot 30x^2 \cdot (1-x^2) dx - 0.25$
$= \frac{30}{7}x^7 - 10x^6 + 6x^5 \Big|_0^1 - 0.25$
$= \frac{1}{28}$
$\sigma \doteq 0.189$

$\int_{0.122}^{0.878} 30x^2 \cdot (1-x)^2 dx$
$= 6x^5 - 15x^4 + 10x^3 \Big|_{0.122}^{0.878} = 0.985 - 0.015 = 0.97$ #

By Chebyshev's theorem $k = 2$, $(1 - \frac{1}{k^2}) = 0.75$

$0.97 \geq 0.75$, $\Rightarrow$ most of the values are grouped around the mean #

4.98 A convenience store has two separate locations where customers can be checked out as they leave. These locations each have two cash registers and two employees who check out customers. Let X be the number of cash registers being used at a particular time for location 1 and Y the number being used at the same time for location 2. The joint probability function is given by

	y		
x	0	1	2
0	0.12	0.04	0.04
1	0.08	0.19	0.05
2	0.06	0.12	0.30

(a) Give the marginal density of both X and Y as well as the probability distribution of X given $Y = 2$.

(b) Give $E(X)$ and $\mathrm{Var}(X)$.

(c) Give $E(X \mid Y = 2)$ and $\mathrm{Var}(X \mid Y = 2)$.

(a) Marginal density of Y :

Y	0	1	2
$h(y)$	0.26	0.35	0.39

probability distribution of X given Y=2 :

x	0	1	2
$P(X\mid Y=2)$	$\frac{4}{39}$	$\frac{5}{39}$	$\frac{30}{39}$

#

(b)

Marginal density of x :

x	0	1	2
$g(x)$	0.2	0.32	0.48

$E(X) = 0.32 + 0.96 = 1.28$ #

$\mathrm{Var}(x) = (0.32 + 1.92) - 1.28^2 = 0.6016$ #

(c)

$E(X \mid Y=2) = \frac{5}{39} + \frac{60}{39} = \frac{65}{39}$ #

$\mathrm{Var}(X \mid Y=2) = \left(\frac{5}{39} + \frac{120}{39}\right) - \left(\frac{65}{39}\right)^2 = \frac{650}{1521} = \frac{50}{117}$ #