Probability and Statistics – online Midterm (5/6/2022)

Note:
1. When applying any probability distribution, please specify clearly the "name of the probability distribution and its parameters" that you use to calculate probability values.
2. Please find out probability "values" using textbook appendix tables whenever possible.
3. Please explain the reason if you use any approximation of probability distributions.
4. For problems 1 ~ 4, please solve them manually with help of a calculator (主要手動推導計算，複雜數字可用計算機) . For problem 5, please write Matlab code to get answers (按照題目引導寫程式找出答案).

1. The lengths of power failures, in minutes, are recorded in the following table.

22, 18, 135, 83, 55, 28, 70, 66, 74,
40, 98, 87, 50, 96, 118, 15, 90, 78,
121, 106, 13, 89, 103, 24, 132, 115, 21,
158, 74, 78, 69, 22, 47, 28, 83, 98.

1.1 Construct a stem-and-leaf plot for the lengths of power failures in minutes. [10%]
1.2 Construct a box-and-whisker plot for the lengths of power failures in minutes. [10%]

2. In a battery factory, three robotic systems are used to assemble Lithium ion battery modules used for Tesla vehicles with production capacity of 40% for system A, 35% for system B, and 25% for system C. It is known that defective rates for systems A, B, C are 0.02, 0.03, 0.02, separately.
2.1 Now, if a Lithium ion battery from the assembly line is found to be defective, what are the probability values that the battery is produced by systems A, B, C, separately? [10%]
2.2 Given that battery defects occur independently, a quality control engineer randomly picks a battery to check at a time and put it back. Please give the probability distribution of X, where X represents the number of defectives among n randomly selected batteries. [10%]

3. A box contains ten smartphones. Among them, five are Apple smartphones, three are Samsung smartphones, and two are Xiaomi smartphones. Now, three smartphones are randomly selected simultaneously from the box with X representing number of Apple smartphones selected, Y representing number of Samsung smartphones selected, and Z representing number of Xiaomi smartphones selected.

3.1 Please list probability values of probability distribution $f(x, y, z)$ using several tables similar to the example below. [10%]

(z = 0)

y \ x	0	1	2
0			
1			
2			

3.2 Please list probability values of the probability distribution $g(x + y \mid x = 1)$ using a table as in 3.1. [10%]

4.

A LCD manufacturer wish to examine the defective rate of its new production line. It is conjectured that the defective rate of this new production line is 40%. Let *X* be the number of defective LCDs among 14 randomly selected LCDs. Now, if 14 LCDs are randomly selected and 10 are found to be defective, please find $P(X \geq 10)$ and use the result to comment on the conjecture on the defective rate. Please give detailed explanation of your conclusion. [10%]

5. Matlab simulation

5.1 In statistics, skewness of a random variable can be quantified by moment coefficient of skewness $\tilde{\mu}_{3\_X}$. For example, the skewness of a binomial random variable X is $\tilde{\mu}_{3\_X} = \frac{1-2p}{\sqrt{npq}}$.

To understand how $\tilde{\mu}_{3\_X}$ is affected by shapes of binomial distributions, please perform a simulation by Matlab using n = 10^2 and p = 0.05, 0.1, 0.15 … 0.95. Please plot $\tilde{\mu}_{3\_X}$ versus p to show your results. In addition, using Matlab built-in probability distribution functions, please plot five binomial distributions using bar graphs with n = 10^2 and p = 0.1, 0.3, 0.5, 0.7, 0.9, separately. Please explain your findings by observing shapes of bar graphs and values of $\tilde{\mu}_{3\_X}$. A total of 6 plots should be presented in the PDF file that you turn in. [15%]

5.2 Please use Matlab built-in functions for Binomial and Poisson distributions to compare plots of probability distributions under the following four conditions of Binomial distributions:

$(n, p) = (10^4, 0.01), (10^4, 0.1), (10^4, 0.2), (10^4, 0.5)$. Calculate parameters of Poisson distributions by using approximation from Binomial distributions. Generate a figure for each (n, p) condition. In each figure, overlay the corresponding Binomial distribution and the Poisson distribution with different colors. Comment on the accuracy of approximation under each (n, p) condition and explain why your observations make sense. A total of four plots should be presented in the PDF file that you turn in. [15%]

Thank you for your hard work in this course! Please stay healthy!

(end of the midterm exam)