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3.6 The shelf life, in days, for bottles of a certain prescribed medicine is a random variable having the density function

$$f(x) = \begin{cases} \frac{20,000}{(x+100)^3}, & x > 0, \\ 0, & \text{elsewhere.} \end{cases}$$

Find the probability that a bottle of this medicine will have a shell life of

(a) at least 200 days;

(b) anywhere from 80 to 120 days.

(a) $\int_{200}^{\infty} \frac{20000}{(X+100)^3} dx$

$= -\frac{1}{2} \frac{20000}{(X+100)^2} \Big|_{200}^{\infty}$

$= 0 - \left(-\frac{\cancel{20000}}{9\cancel{0000} \times \cancel{2}}\right) = \frac{1}{9}$ #

(b) $\int_{80}^{120} \frac{20000}{(X+100)^3} dx$

$= -\frac{1}{2} \frac{20000}{(X+100)^2} \Big|_{80}^{120}$

$= -10000 \left(\frac{1}{220^2} - \frac{1}{180^2}\right)$

$= \frac{16000}{156816} \fallingdotseq 0.102$ #

3.15 Find the cumulative distribution function of the random variable X representing the number of defectives in Exercise 3.11. Then using $F(x)$, find

(a) $P(X = 1)$;

(b) $P(0 < X \leq 2)$.

$$F(X) \begin{cases} 0, & X < 0 \\ \frac{2}{7}, & 0 \leq X < 1 \\ \frac{6}{7}, & 1 \leq X < 2 \\ 1, & 2 \geq X \end{cases}$$

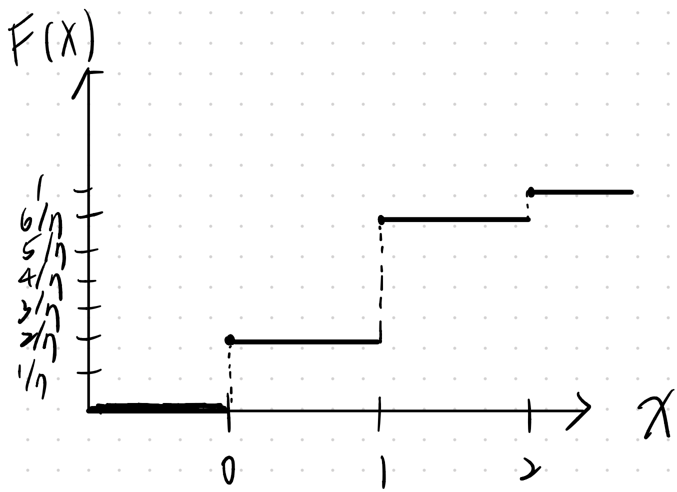
(a)

$$P(X=1) = F(X \leq 1) - F(X \leq 0) = \frac{6}{7} - \frac{2}{7} = \frac{4}{7} \#$$

(b)

$$P(0 < X \leq 2) = F(X \leq 2) - F(X \leq 0) = 1 - \frac{2}{7} = \frac{5}{7} \#$$

3.16 Construct a graph of the cumulative distribution function of Exercise 3.15.



Caculative distribution Function

3.24 Find the probability distribution for the number of comic books when 4 books are selected at random from a collection consisting of 5 comic books, 2 art books, and 3 math books. Express your results by means of a formula.

$$f(x) = \frac{\binom{5}{x}\binom{5}{4-x}}{\binom{10}{4}}, \quad \text{for } x = 0,1,2,3,4$$

3.30 Measurements of scientific systems are always subject to variation, some more than others. There are many structures for measurement error, and statisticians spend a great deal of time modeling these errors. Suppose the measurement error X of a certain physical quantity is decided by the density function

$$f(x) = \begin{cases} k(3 - x^2), & -1 \le x \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Determine k that renders $f(x)$ a valid density function.

(b) Find the probability that a random error in measurement is less than 1/2.

(c) For this particular measurement, it is undesirable if the *magnitude* of the error (i.e., $|x|$) exceeds 0.8. What is the probability that this occurs?

(a)

$$\int_{-1}^{1} k(3-x^2)\,dx = 1$$

$$3kx - \frac{1}{3}kx^3 \Big|_{-1}^{1} = 1$$

$$(3k - \frac{1}{3}k) - (-3k + \frac{1}{3}k) = 1$$

$$6k - \frac{2}{3}k = \frac{16}{3}k = 1$$

$$k = \frac{3}{16} \#$$

(b)

$$\int_{-1}^{\frac{1}{2}} \frac{3}{16}(3-x^2)\,dx = \frac{3}{16}\left(3x - \frac{x^3}{3}\Big|_{-1}^{\frac{1}{2}}\right)$$

$$= \frac{3}{16}\left(\frac{3}{2} - \frac{1}{24} + 3 - \frac{1}{3}\right)$$

$$= \frac{\cancel{3}}{16}\,\frac{99}{\cancel{24}\,8}$$

$$= \frac{99}{128} \#$$

$36 - 1 + 1 = 8$

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(c)

$$\int_{0.8}^{1} \frac{3}{16}(3-x^2) + \int_{-1}^{-0.8} \frac{3}{16}(3-x^2)$$

$$= \frac{3}{16}\left[(3x - \frac{x^3}{3})\Big|_{0.8}^{1} + (3x - \frac{x^3}{3})\Big|_{-1}^{-0.8}\right]$$

$$= \frac{3}{16}\left[(3 - \frac{1}{3} - 2.4 + \frac{0.8^3}{3}) + (-2.4 + \frac{0.8^3}{3} + 3 - \frac{1}{3})\right]$$

$$= \frac{\cancel{6}\,2}{\cancel{16}\,8}\times\frac{\cancel{1312}\;\cancel{328}\;164}{\cancel{3000}\;1000}$$

$$= 0.164 \#$$

3.40 A fast-food restaurant operates both a drive-through facility and a walk-in facility. On a randomly selected day, let X and Y, respectively, be the proportions of the time that the drive-through and walk-in facilities are in use, and suppose that the joint density function of these random variables is

$$f(x,y) = \begin{cases} \frac{2}{3}(x+2y), & 0 \le x \le 1,\ 0 \le y \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Find the marginal density of X.
(b) Find the marginal density of Y.
(c) Find the probability that the drive-through facility is busy less than one-half of the time.

(a)

$$\int_0^1 \frac{2}{3}(x+2y)\,dy = \frac{2}{3}(xy+y^2)\Big|_0^1$$
$$= \frac{2}{3}(x+1), \text{ for } 0 \le x \le 1 \ \#$$

(b)

$$\int_0^1 \frac{2}{3}(x+2y)\,dx = \frac{2}{3}\left(\frac{x^2}{2}+2xy\right)\Big|_0^1$$
$$= \frac{1}{3} + \frac{4y}{3}, \text{ for } 0 \le y \le 1 \ \#$$

(c)

$$f(x<\tfrac{1}{2}) = \int_0^{\frac{1}{2}} \frac{2}{3}(x+1)\,dx$$
$$= \frac{2}{3}\left(\frac{x^2}{2}+x\right)\Big|_0^{\frac{1}{2}}$$
$$= \frac{1}{12}+\frac{1}{3} = \frac{5}{12} \ \#$$

3.50 Suppose that X and Y have the following joint probability distribution:

$f(x,y)$		$x=2$	$x=4$
y	1	0.10	0.15
	3	0.20	0.30
	5	0.10	0.15

(a) Find the marginal distribution of X.
(b) Find the marginal distribution of Y.

(a)

x	2	4
$g(x)$	0.4	0.6

(b)

y	1	3	5
$h(y)$	0.25	0.5	0.25