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A Variable Neighborhood Search Branching for the Electric Vehicle Routing Problem with Time Windows

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Abstract

E-mobility plays a key role especially in contexts where the transportation activities impact a lot on the total costs. The Electric Vehicles (EVs) are becoming an effective alternative to the internal combustion engines guaranteeing cheaper and eco-sustainable transport solutions. However, the poor battery autonomy is still an Achille's hell since the EVs require many stops for being recharged. We aim to optimally route the EVs for handling a set of customers in time considering the recharging needs during the trips. A Mixed Integer Linear Programming formulation of the problem is proposed assuming that the battery recharging level reached at each station is a decision variable in order to guarantee more flexible routes. The model aims to minimize the total travel, waiting and recharging time plus the number of the employed EVs. Finally, a Variable Neighborhood Search Branching (VNSB) is also designed for solving the problem at hand in reasonable computational times. Numerical results on benchmark instances show the performances of both the mathematical formulation and the VNSB compared to the ones of the model in which the battery level reached at each station is always equal to the

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maximum capacity.

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1 Introduction

The Electric Vehicles (EVs) play a key role for guaranteeing more eco-sustainable transportation systems. In urban contexts, they can be used also in particular zones forbidden to the Internal Combustion Engines (ICEs). The use of a such type of vehicles guarantees a reduction of the energetic costs (up to 90%); their regenerative breaking allows recuperating about 15% of the total energy; their efficiency is about 90% compared to an ICE strongly limiting the atmospheric heating. Moreover, they are less noisy than the ICEs and they do not produce harmful emissions (e.g., CO_2). However, the EVs still have a poor battery autonomy and therefore, they require many stops during the trips for being recharged. This can be a strong limitation to their diffusion if we think that it is often required to provide transportation services within precise time windows.

In this paper, we address the problem of serving a set of customers, within fixed time windows, by using EVs and considering their need to stop at the Recharging Stations (RSs) during the trips. In the literature, this problem has been already modeled as a *Vehicle Routing Problem* (VRP) *with Time Windows* (TW) and denoted as Electric-VRPTW (E-VRPTW) due to the use of EVs. The fleet is based at a common depot and each route, assigned to one EV, has to start and end at the depot itself. Due to the EV features, the E-VRPTW is different from the traditional VRPTW since the recharging needs have to be taken into account. The road network will be characterized not only by the depot and by the points which the customers are assumed to be placed in, but also by the RSs. The goal is to find a set of routes performed by EVs in order to handle all the customer requests in time, minimizing a specific cost function. The VRPTW represents a problem extensively investigated in the literature ([1] and [7]). Several scientific contributions have already addressed a VRP to control the harmful emissions (see, for example, [4] and

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[2]). In [3], the VRP with alternative fuel powered vehicles and fixed refueling times is modeled and two heuristics are also proposed: a modified Clark and Wright savings algorithm and a density-based clustering algorithm. However, neither the freight vehicle capacity nor the time windows are included. In [8], a VRPTW with EVs and RSs is formulated. Beyond the traditional VRPTW constraints, the model imposes also conditions on the battery level. The Recharging Time (RT) is the time for restoring the current battery level to the maximum capacity since it is always performed a full recharge. The authors minimize the number of used EVs and then, the total travel distance. They also describe a Variable Neighborhood Search combined with a Tabu Search. The computational results show the high quality performance of the proposed approach.

In this paper, we propose a variant of the original E-VRPTW such that the batteries are not always fully recharged, with the aim of reducing the RTs. Consequently, the partial recharges guarantee higher flexibility during the route planning. Under these considerations, a Mixed Integer Linear Programming (MILP) formulation will be proposed for minimizing the number of used EVs, the total Travel Time (TT), the total RT (RT) and the total Waiting Time (WT) (i.e., the sum of the times spent before starting the services at the customers). In the MILP, for each EV, the level of battery reached at a RS is a decision variable of the optimization process. Moreover, a Variable Neighborhood Search Branching (VNSB) is applied taking into account the general framework proposed in [6] for MILP formulations.

The rest of the paper is organized as in the following: Section 2 introduces the statement of the problem and the new formulation proposed for the E-VRPTW while Section 3 describes the VNSB applied to it. Finally, Section 4 shows numerical comparisons carried out on benchmark instances and Section 5 concludes the paper.

2 A MILP formulation of the E-VRPTW

The aim of the E-VRPTW is to handle a set of customers within their time windows through a fleet of EVs. It can be represented on a direct complete graph $G =< V, A >$ where V contains the set N of n customers, the set F of f RSs and the depot (0) plus its clone $(n + f + 1)$ while $V' = V \setminus \{0\}$ and $V'' = V \setminus \{(n + f + 1)\}$. Each route starts at 0 and ends at $(n + f + 1)$. The set A contains all the links between each couple of vertices. The fleet is homogeneous and then, each EV presents both the same freight and battery capacity, denoted by C and Q, respectively. Moreover, the average vehicle speed v is assumed to be known while r and g denote the consumption and recharging rate of the battery, respectively. In addition, $q_i \geq 0$ represents the demand associated with the customer $i \in N$. The range $[e_i, l_i]$ is the time window for serving the customer $i \in N \cup \{0\}$ where l_0 is the maximum time duration of a route. Then, the EV cannot start and finish the service before and after e_i and l_i, respectively. The service time of the customer $i \in N$ is $s_i \geq 0$. For each arc $(i, j) \in A$, the travel distance and duration are known: d_{ij} and t_{ij}, respectively. In order to mathematically formulate the problem, the following decision variables are introduced: $x_{ij}, \forall (i, j) \in A$ equal to 1 if the arc (i, j) is used, 0 otherwise; τ_i, y_i and u_i represent the arrival time, the battery level and the remaining freight capacity of the EV at the vertex $i \in V$; δ_i is the waiting time of the EV at the vertex $i \in N$ and finally, Q_i is the battery level reached at the RS $i \in F$. This means that the battery level reached at each RS is established during the optimization process. The mathematical model is detailed in the following:

$$\min \alpha \sum_{j \in N \cup F} x_{0j} + \beta \sum_{i \in V''} \sum_{j \in V' | i \neq j} t_{ij} x_{ij} + \gamma (\sum_{i \in N} \delta_i + \sum_{i \in F} g(Q_i - y_i)) \tag{1}$$

$$\sum_{j \in V | j \neq i} x_{ij} = 1 \quad \forall i \in N \tag{2}$$

$$\sum_{j \in V | j \neq i} x_{ij} \leq 1 \quad \forall i \in F \tag{3}$$

$$\sum_{i \in V'' | j \neq i} x_{ij} = \sum_{i \in V' | j \neq i} x_{ji} \quad \forall j \in N \cup F \tag{4}$$

$$\tau_i + \delta_i + (t_{ij} + s_i) x_{ij} - l_0 (1 - x_{ij}) \leq \tau_j \quad \forall i \in N \cup \{0\}, j \in V' | i \neq j \tag{5}$$

$$\tau_i + t_{ij} x_{ij} + g(Q_i - y_i) - (l_0 + gQ)(1 - x_{ij}) \leq \tau_j \quad \forall i \in F, j \in V' | i \neq j \tag{6}$$

$$e_j \leq \tau_j + \delta_j \leq l_j \quad \forall j \in V \tag{7}$$

$$0 \leq u_j \leq u_i - q_i x_{ij} + C(1 - x_{ij}) \quad \forall i \in V'', j \in V' | i \neq j \tag{8}$$

$$0 \leq u_0 \leq C \tag{9}$$

$$0 \leq y_j \leq y_i - r d_{ij} x_{ij} + Q(1 - x_{ij}) \quad \forall i \in N \cup \{0\}, j \in V' | i \neq j \tag{10}$$

$$0 \leq y_j \leq Q_i - r d_{ij} x_{ij} + Q(1 - x_{ij}) \quad \forall i \in F, j \in V' | i \neq j \tag{11}$$

$$Q_i - y_i \geq 0 \quad \forall i \in F \tag{12}$$

$$0 \leq Q_i \leq Q \quad \forall i \in F \tag{13}$$

$$y_0 \leq Q \tag{14}$$

$$x_{ij} \in \{0, 1\} \quad \forall i \in V'', j \in V' | i \neq j \tag{15}$$

The objective function (1) consists in the minimization of four different components: the EVs used, the TT, the WT and finally, the RT. The weights are introduced with the aim of monetizing the different components. In particular, α is an average daily cost for using an EV and takes into consideration, among the other things, the costs to buy the EV, its maintenance and related to the driver's salary. The parameter β represents the cost for time unit due to the battery consumption during a trip while γ denotes the cost per time unit due to the idle of the EV (i.e., due to the RTs and WTs). Each constraint (2) imposes that each customer has to be served by just one EV; the group (3) assures that a RS can or not be utilized. It is worth noting that, for feasibility reasons, the RSs are cloned ([8]). The group (4) denotes the flow conservation conditions while the group (5) determines the arrival time at $j \in V$ starting from the customer $i \in N$. The group (6) finds the arrival time at $j \in V$ starting from the RS $i \in F$. This group represents an innovative aspect of the proposed model since the battery level reached at each RS is a decision variable. The constraints (7) guarantee the respect of the time windows, while (8)-(9) assure that the freight vehicle capacity is never exceeded. The constraints (10)-(11) represent the battery consumption rate along a path that can start either from a customer $i \in N$ or from a RS $i \in F$ to reach the vertex $j \in V$. The constraints (12) guarantee that the battery level reached at each RS is never less than the current battery level. Finally, (13)-(14) assure that the battery level reached at each RS and the initial battery level at the depot do not exceed the maximum allowed capacity Q.

3 VNSB for EVRP

The proposed formulation of the E-VRPTW can be seen as a special case of 0-1 Mixed Integer Linear Program (0-1 MILP). Therefore, it is possible to adopt 0-1 MILP solution methods, e.g., math-heuristics. Among them, we chose to apply the VNSB introduced in the seminal work of [6]. It consists in adding linear constraints to the original problem for systematically changing the neighborhoods following the rules of the general VNS schema. The idea of implementing a local search by adding a linear constraint to the MILP model has been firstly presented in the work of [5], known as Local Branching Method. Therefore, the VNSB combines the VNS approach with the Local Branching one. To the best of our knowledge, it has never been applied to the VRPs. With reference to the pseudo-code described in [6], we set the parameter k_{step}, aimed to control the depth of the neighborhood explored during the shaking phase, equal to 5 while the node time limit equal to 1 second.

Beyond the classical time limit (here fixed to 100 seconds), the stopping criterion is also based on the maximum number of consecutive iterations without improvements (here fixed to 10 iterations).

4 Some numerical results

In this section, a description of the computational tests carried out on some benchmark instances described in [8] is provided. The aim is to compare the performances of the mathematical model proposed in [8] (named *Model 1*) with ours (named *Model 2*) in terms of WT, TT, RT, total Travel Distance (TD) and number of vehicles (μ). These numerical comparisons are shown in Table 1 where the best results are highlighted in bold. Moreover, in Table 2, *Model 2* is also compared with the VNSB introduced in the previous section. Both the two MILP formulations have been implemented in AMPL and solved with the state of the art solver CPLEX 12.5 on a PC Intel Xeon 2.80 GHz with 2GB RAM. For the aims of the work, some of the instances available at http://evrptw.wiwi.uni-frankfurt.de, described in [8] and designed on the well-known Solomon VRPTW instances ([9]) have been tested. In particular, 6 small instances, each one with 5 customers and 2 instances, each one with 10 customers, have been experimented. Each class of instances has been distinguished by [8] according to the geographical distribution of the customers, where the prefixes R, C and RC denote a random, a clustered and a mix distribution, respectively. Moreover, the instances have been also distinguished by [8] introducing the prefixes $R1$, $C1$ and $RC1$, with refer to a short scheduling horizon; $R2$, $C2$ and $RC2$ with refer to a long scheduling horizon.
With regard to the parameters α, β and γ, their values are set to $100, 1$ and 0.1, respectively.

Table 1: Numerical Comparisons between *Model 1* and *Model 2*

Instance	Model 1					Model 2				
	μ	TD	WT	RT	TT	μ	TD	WT	RT	TT
C206C5	1	242.56	1070.39	721.13	2484.08	1	**242.55**	**0.00**	**571.87**	**1264.42**
C208C5	1	**158.47**	1353.65	787.87	2749.99	1	158.48	**0.00**	**280.14**	**888.62**
R105C5	2	156.08	74.43	29.71	310.22	2	156.08	**0.00**	**21.46**	**227.54**
R203C5	1	**179.04**	298.76	87.74	615.54	1	179.06	**0.00**	**58.03**	**287.08**
RC108C5	2	**253.92**	15.24	78.08	397.24	2	253.93	**0.00**	**38.39**	**342.32**
RC204C5	1	176.39	512.66	41.39	780.44	1	176.39	**0.00**	**38.47**	**264.87**

Model 2 outperforms *Model 1* in terms of both RT and TT, with an average improvement of 67.99% and 111.23%, respectively (Table 1). Concerning WT, our average absolute improvement is 554.19; about TD, *Model 2* ob-

tains almost the same values found by *Model 1*, although it does not directly minimize this performance measure.

Table 2: Numerical Comparisons between *Model 2* and VNSB

Instance	Gap obj (%)	CPU-Model 2	CPU-VNSB
C206C5	0.10%	81.78	15.57
C208C5	0.00%	11.65	2.45
R105C5	0.71%	7.18	0.52
R203C5	0.00%	116.56	47.74
RC108C5	0.00%	226.12	13.06
RC204C5	0.00%	15.25	14.74

Both the two CPU times displayed in Table 2 are expressed in seconds. The numerical results clearly show how the VNSB is suitable to detect good quality solutions in total computational times that are lower than the ones required by *Model 2*. These are only preliminary numerical results but they are encouraging and therefore, our intention is to extend the experimental campaign.

5 Conclusions and Future Works

In this work, we have firstly presented an innovative mathematical formulation of the E-VRPTW. In the proposed model, the EVs are assumed not to be always fully recharged at the RSs and therefore, the battery level reached is considered a decision variable of the optimization process. Moreover, beyond the number of used EVs, the recharging, travel and waiting times have been also optimized. The performance of our model (*Model 2*) has been then compared with the already existing mathematical formulation that assumes always a full recharge of the EVs (*Model 1*). Although the numerical results are preliminary, they show that *Model 2* outperforms *Model 1* in terms of recharging, waiting and travel times by using the same number of EVs. Finally, we have also applied a VNSB to solve the E-VRPTW and the numerical results show that it is suitable to detect good quality solutions in total computational times by far lower than the ones required by *Model 2*. Future works will concern an extension of the experimental campaign on all the test instances presented in [8].

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