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Electric Vehicle Routing Problem with Time-Dependent Waiting Times at Recharging Stations

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A B S T R A C T

In the Electric Vehicle Routing Problem with Time Windows (EVRPTW) the vehicles have a limited driving range and must recharge their battery at some points on their route. The recharging stations have a limited capacity and the newly arriving vehicles may have to queue before being recharged. In this study, we model the EVRPTW considering time-dependent queueing times at the stations. We allow but penalize late arrivals at customer locations and at the depot. We minimize the cost of vehicles, drivers, energy, and penalties for late arrivals. We formulate the problem as a mixed integer linear program and solve small instances with CPLEX. For the larger instances, we develop a matheuristic which is a combination of Adaptive Large Neighborhood Search and of the solution of a mixed integer linear program. We perform an extensive experimental study to investigate the impact of queueing at the recharging stations on the routing decisions. The results show that waiting at the stations may increase the total cost by 1%–26%, depending on the problem type and queue length. We also observe that recharges tend to shift to less crowded mid-day hours due to the time-dependent waiting times.

1. Introduction

The Electric Vehicle Routing Problem with Time Windows (EVRPTW) was introduced by Schneider et al. (2014) as an extension to the Green Vehicle Routing Problem of Erdoğan and Miller-Hooks (2012). It is a variant of the Vehicle Routing Problem with Time Windows (VRPTW) in which electric vehicles (EVs) are used instead of internal combustion engine vehicles (ICEVs). In recent years, governments have started to take actions to reduce greenhouse gas (GHG) emissions by encouraging the adoption of EVs. Many companies now use EV fleets, and their number is steadily increasing (Coplon-Newfield and Park, 2017). Several global companies are in the process of testing or implementing this technology, such as FedEx, UPS, Frito-Lay, AT&T, General Electric, and Coca-Cola (Suizo, 2013). Furthermore, some other global companies, such as Unilever, IKEA and DHL have launched a global campaign to accelerate the shift to EVs from gas and diesel powered transportation (Fairley, 2017). In parallel, the need for planning the routes of EVs has given rise to a new research area and to a new optimization problem called the Electric Vehicle Routing Problem (EVRP), which received increasing attention in recent years. An EV is equipped with a battery with limited capacity and the state of charge (SoC) of the battery goes down while the EV travels to service customers. When it becomes too low, the EV may need to visit a recharging station in order to continue its route.

The existing literature of the EVRPTW assumes that recharging at a station starts as soon as the EV arrives. However, in practice the number of chargers in a station is limited and a charger may not be available at the time of the vehicle's arrival. It may therefore have to queue before recharging, and this waiting time needs to be taken into account in the routing decisions. The waiting time may vary depending on the location of the station and on the time of the visit. Some variations in the waiting time are difficult to predict. For instance, if a traffic accident or a special event occurs near a station, there may be long queues. On the other hand, some delays are easier to foresee such as those observed during rush hour congestion. In this study, we assume that the expected queue length at all stations and at any time of the day is known in advance. Furthermore, the charging time is a non-linear concave function of the charge amount (Montoya et al., 2017; Pelletier et al., 2017).

In this paper, we extend the EVRP by considering time-dependent waiting times at the recharging stations using an M/G/1

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queueing system. We use a non-linear charging function and soft time windows at the customer locations. We approximate the random waiting times by their expected values. The planning horizon is split into a predetermined number of time intervals and an average queue length is assigned to each station for different time intervals. The routing decisions are then made according to these time-dependent waiting times at the recharging stations. We formulate the problem as a mixed integer linear program and propose a matheuristic to solve it efficiently. Our algorithm is a combination of the well-known Adaptive Large Neighborhood Search (ALNS) metaheuristic and a mixed integer linear programming.

The paper makes two main scientific contributions. It first introduces a new problem called the Electric Vehicle Routing Problem with Soft Time Windows and Time-Dependent Waiting Times at Recharging Stations, TD-EVRPSTW, using a comprehensive objective function that minimizes the total cost of fuel, drivers, vehicles, overtime, and lateness penalties at the customer locations. It formulates the problem in two ways: a first comprehensive formulation is used to compute an optimal solution from scratch, while the second formulation optimizes a single route in which the customer sequence is known in advance. Secondly, our paper proposes a matheuristic that combines ALNS with an exact solver along with problem-specific mechanisms.

The remainder of the paper is organized as follows. Section 2 provides a summary of the literature. Section 3 describes the problem and presents the mathematical model, while Section 4 describes the proposed solution approach with the single-route optimization formulation. Section 5 presents the computational study and provides the numerical results, followed by concluding remarks in Section 6.

2. Related literature

The EVRP was first studied by Conrad and Figliozzi (2011) assuming a constant recharging time. The model allowed charging fully or up to the 80% of the battery capacity at only customer locations. Erdoğan and Miller-Hooks (2012) then studied the routing of alternative fuel vehicles (AFVs) which are refueled up to the tank capacity at alternative fuel stations within a constant amount of time. Schneider et al. (2014) introduced time windows and a linear charging function assuming that the battery is fully recharged. Since then, several extensions have been proposed. The full charge assumption was first relaxed and models were developed to allow partial charging (Bruglieri et al., 2015; Keskin and Çatay, 2016). Some models take into account the fact that the stations have different chargers (Felipe et al., 2014; Keskin and Çatay, 2018; Li-ying and Yuan-bin, 2015; Sassi et al., 2014). Several studies have analyzed the heterogeneous fleet case. Some of them consider a mixed fleet composed of EVs with different characteristics (Desaulniers et al., 2016) whereas in other studies, the fleet includes different ICEVs as well (Goeke and Schneider, 2015; Hiermann et al., 2018; Kopfer and Vornhusen, 2017; Sassi et al., 2014). Some recent papers have dealt with both the location of the stations and the routing (Li-ying and Yuan-bin, 2015; Paz et al., 2018; Schiffer and Walther, 2017; Yang and Sun, 2015).

Although recharging is performed at charging stations in most studies, some papers consider battery swap stations (BSS) where the discharged battery is replaced with a fully recharged one (Liao et al., 2016; Jie et al., 2019; Masmoudi et al., 2018; Paz et al., 2018; Wang et al., 2018), and wireless charging systems (WCS) where the battery is recharged by an inductive charging system placed along the roads while the EV is traveling (Li et al., 2018). Some recent studies have considered EVs within the context of the two-echelon VRP (Breunig et al., 2018; Jie et al., 2019), technician routing (Villegas et al., 2018), reverse logistics (Zhang et al., 2018c), and pick-up and delivery problem (Grandinetti et al., 2016; Madankumar and Rajendran, 2018). Finally, the linear charging function was relaxed by Montoya et al. (2017) and Froger et al. (2017, 2019) who used a concave piecewise linear function. Afroditi et al. (2014) and Pelletier et al. (2016) reviewed EVs in goods distribution.

Regarding the unknown service times at the stations, in Sweda et al. (2017) the stations are unavailable with a certain probability. If a vehicle visits an occupied station, it should wait for some time which is random. Froger et al. (2017) solved a related problem in which the stations have limited number of chargers (one, two or three) and an EV may need to wait for service if the chargers are busy with recharging other EVs in the fleet. In this problem, the use of the chargers depends on the routing and charging decisions, whereas in our setting the queue lengths are independent of these features. Recently, Bruglieri et al. (2018) studied the waiting times at the alternative fuel stations. The AFVs are routed such that their refueling times do not overlap in the stations. The authors proposed an exact method to minimize the total distance. Kullman et al. (2018) introduced EVRP with public-private recharging strategy where the company owns a charger in its depot but the EVs may recharge at public stations as well. At the depot, the EVs start recharging as soon as they arrive, while they may queue if they recharge at public stations. The system at the stations is modeled using an $M/M/\psi_c/\infty$ queueing system. The authors showed that public-private strategies may substantially reduce the cost compared to private-only strategies.

Table 1 classifies the EVRP studies in terms of the main features of the problem. The column headings stand for recharging function (RC fcn), electricity consumption function (EC fcn), fleet composition (fleet comp), charger types in the stations, objective function (obj fcn), charge amount, solution methodology, and existence of time windows (TW) and station location decisions (Lctn) in the problem; the letters L and NL represent linear and nonlinear recharging time functions; Ho and Hc correspond to homogeneous and heterogeneous fleets; A-, G-, EVO-VNS, TS, ILS, HC, AC, DP, CG, WSM, PBA, and PSO stand for adaptive-, general-, evolutionary-variable neighborhood search, tabu search, iterated local search, heuristic concentration, ant colony, dynamic programming, column generation, weighted sum method, path based approach, and particle swarm optimization, respectively. MIP is used for the papers that only formulate the mathematical model and report results obtained by a solver. Explanations of the numbers associated with the objective function are provided in Table 2.

The time-dependent vehicle routing literature has mainly focused on time-dependent travel times due to congestion on roads. For instance, the traffic conditions at 13 h are often different from those at 18 h which is in the evening rush hour for commuters. So, a trip starting at 18 h may take a longer time than the same trip starting at 13 h. The papers can be classified as static or dynamic if the times are fixed for a given time interval, or change dynamically, respectively. Static models include deterministic (Franceschetti et al., 2017; Hashimoto et al., 2008; Jabali et al., 2012; Jung and Haghani, 2001; Malandraki and Daskin, 1992) or stochastic features (Çimen and Soysal, 2017; Huang et al., 2017; Nahum and Hadas, 2009; Taş et al., 2013; Van Woensel et al., 2008) whereas Fleischmann et al. (2004), Haghani and Jung (2005), Potvin et al. (2006), and Schilde et al. (2014) deal with dynamic models.

Malandraki and Daskin (1992) represented the time-varying travel times during the day by assigning to each arc a travel time function which is a step function of the departure time. Their model was later improved by Ichoua et al. (2003) and Fleischmann et al. (2004) who ensured that a vehicle entering an arc later than another vehicle cannot leave the same arc earlier, which is known as the non-passing or first-in-first-out (FIFO) property. Ichoua et al. (2003) assumed travel speeds that change when

Table 1
EVRP literature overview.

Papers	RC fcn.		EC fcn.		Fleet comp.		Charger type		Obj. fcn.	Charge amount		Solution method	TW	Lctn.
	L	NL	L	NL	Ho	Hc	Single	Multi.		Full	Partial			
Conrad and Figliozzi (2011)	–		✓		EVs		✓		1, 2, 3, 8	✓	✓	Iterative construction, improvement heuristics	–	–
Erdoğan and Miller-Hooks (2012)	–		✓		AFVs		✓		2	✓		Several heuristics	–	–
Schneider et al. (2014)	✓		✓		EVs		✓		1, 2	✓		Hybrid VNS and TS	✓	–
Felipe et al. (2014)	✓		✓		EVs			✓	2, 3		✓	Several heuristics and SA	✓	–
Sassi et al. (2014)	✓		✓			EVs, ICEVs		✓	1, 2, 3		✓	Several heuristics	✓	–
Li-ying and Yuan-bin (2015)	***		✓		EVs			✓	3, 5, 8	✓		Hybrid AVNS and TS	✓	✓
Goeke and Schneider (2015)	✓		*			EVs, ICEVs	✓		2, 3, 8,9,10		✓	ALNS	✓	–
Bruglieri et al. (2015)	✓		✓		EVs		✓		1, 2, 3, 4		✓	VNS branching	✓	–
Ding et al. (2015)	✓		✓		EVs		✓		2		✓	Hybrid VNS and TS	✓	–
Yang and Sun (2015)	–		✓		EVs		BSS		2, 5		–	Several heuristics	–	✓
Arslan et al. (2015)	✓		✓		PHEVs		✓		6,9,10		✓	DP	–	–
Keskin and Çatay (2016)	✓		✓		EVs		✓		1, 2		✓	ALNS	✓	–
Desaulniers et al. (2016)	✓		✓		EVs		✓		2	✓	✓	Branch-price-and-cut	✓	–
Hiermann et al. (2016)	✓		✓			EVs	✓		1, 2	✓		Branch-price, and ALNS	✓	–
Wen et al. (2016)	✓		✓		EVs		✓		1, 2		✓	ALNS	✓	–
Roberti and Wen (2016)	✓		✓		EVs		✓		2	✓	✓	GVNS, DP	✓	–
Lin et al. (2016)	✓		✓		EVs		✓		1, 2, 3	✓		MIP	–	–
Liao et al. (2016)	–		NS		EVs		BSS		8		–	DP	–	–
Grandinetti et al. (2016)	✓		✓		EVs		✓		1, 2, 15		✓	WSM	✓	–
Koç and Karaoğlan (2016)	–		✓		AFVs		✓		2	✓		Brach-and-cut	–	–
Montoya et al. (2016)	–		✓		AFVs		✓		2	✓		MSH	–	–
Hof et al. (2017)	–		✓		EVs		BSS		2, 5		–	AVNS	–	✓
Sweda et al. (2017)	**		✓		EVs			✓	2, 3, 4		✓	Several optimal and heuristic procedures	–	–
Bruglieri et al. (2017)	✓		✓		EVs		✓		1, 8		✓	VNSB based matheuristic	✓	–
Montoya et al. (2017)		✓	✓		EVs			✓	8		✓	Hybrid ILS and HC	–	–
Schiffer and Walther (2017)	✓		✓		EVs		✓		1, 2, 5	✓	✓	MIP	✓	✓
Leggieri and Haouari (2017)	✓		✓		AFVs		✓		2		✓	MIP	–	–
Froger et al. (2017)		✓	✓		EVs		✓		8		✓	Two-stage matheuristic	–	–
Froger et al. (2019)		✓	✓		EVs		✓		8		✓	MIP	–	–
Andelmin and Bartolini (2017)	–		✓		AFVs		✓		2	✓		An exact algorithm	–	–
Paz et al. (2018)	****		✓		EVs		BSS		1, 2, 5	✓	✓	MIP	✓	✓
Li et al. (2018)	✓		✓		EVs		single chargers and WCS		3, 8		✓	MIP	–	–
Kopfer and Vornhusen (2017)	✓			✓		EVs, ICEVs	✓		11		✓	MIP	✓	–
Wang et al. (2018)	–		✓		EVs		BSS		12	✓		Heuristic procedures	✓	–
Zhang et al. (2018a)	–			✓	EVs		✓		3	✓		AC and ALNS	–	–
Schiffer and Walther (2018)	✓		✓		EVs		✓		1, 5, 8		✓	ALNS, DP	✓	✓
Zhang et al. (2018b)	✓		✓		EVs		✓		1, 2, 3, 4, 8		✓	TS	✓	–
Breunig et al. (2018)	✓		✓		EVs		✓		1, 2	✓		LNS, Exact algorithm	–	–
Jie et al. (2019)	–		✓			EVs	BSS		2, 13, 14		✓	Hybrid CG and ALNS	–	–
Macrina et al. (2019)	✓		✓		EVs, ICEVs		✓		2, 3		✓	ILS	✓	–
Keskin and Çatay (2018)	✓		✓		EVs			✓	1, 3		✓	ALNS based matheuristic	✓	–
Villegas et al. (2018)		✓	✓		EVs, ICEVs			✓	1,2, 3, 6		✓	GRASP based matheuristic	✓	–
Hiermann et al. (2018)	✓		✓			EVs, ICEVs, PHEVs	✓		1,3,9		✓	GA, LNS with IP solver	✓	–
Masmoudi et al. (2018)	–		*			EVs	BSS		2		–	EVO-VNS	✓	–
Kullman et al. (2018)		✓	✓		EVs		✓		✓		✓	Several exact and heuristic methods	–	–
Bruglieri et al. (2018)	–		✓		AFVs		✓		2	✓		An exact method	–	–
Madankumar and Rajendran (2018)	–		✓		AFVs		✓		2	✓	✓	MIP	✓	–
Koç et al. (2018)		✓	✓		EVs			✓	5, 8		✓	ALNS based matheuristic	–	✓
Bruglieri et al. (2019)	–		✓		AFVs		✓		2	✓		PBA	–	–

NS: The properties of the function are not specified in the paper
* depends on vehicle mass, speed and gradient of the terrain *** linear but has different cost and rates for different chargers
** linear but has different cost components for different charge levels **** linear, constant for battery swapping and parallel with service time

Table 2
Meanings of the objective functions in Table 1.

1:	Vehicle cost	9:	Fuel cost
2:	Total travel cost	10:	Battery cost
3:	Recharging cost	11:	Charging cost
4:	Waiting cost	12:	Profit of visits
5:	Station installation cost	13:	Operational costs
6:	Stopping cost at a station	14:	Battery swapping cost
7:	Overcharging cost	15:	Unit time penalty for violated time windows
8:	Total time cost		

the boundary between two consecutive time periods is crossed. Fleischmann et al. (2004) introduced smooth travel time functions. Time windows were considered by Jung and Haghani (2001), Soler et al. (2009), Dabia et al. (2012), Wen and Eglese (2015), Kumar and Panneerselvam (2016), and Spliet et al. (2017) while Taş et al. (2014) addressed the case with stochastic and time-dependent travel times assuming soft time windows. The other VRP variants that deal with time-dependent travel times include pick-up and delivery problems (Sun et al., 2018a; Zhang et al., 2014), the orienteering problem (Black et al., 2015; Sun et al., 2018b; Verbeeck et al., 2014; Verbeeck et al., 2016), the dial-a-ride problem (Schilde et al., 2014), the inventory routing problem (Cho et al., 2014), green logistics problems (Çimen and Soysal, 2017; Franceschetti et al., 2017; Franceschetti et al., 2013; Heni et al., 2018; Jabali et al., 2012; Norouzi et al., 2017; Rabbani et al., 2018; Soysal et al., 2015; Zhang et al., 2018a), and city logistics problems (Mancini 2014; Rincon-Garcia et al., 2018).

Similar to the travel times, the service times of the customers may also vary depending on the time of the day according to several factors such as ease of access from the road and availability of parking spaces. Taş et al. (2016) considered the time-dependent service times within the context of the traveling salesman problem. A comprehensive review of the time-dependent VRP is provided by Gendreau et al. (2015).

3. Problem description and formulation

Given a homogeneous fleet of EVs, the problem is to determine a set routes that covers all customers, which have demands and soft time windows, while preserving time and energy feasibility. The EVs may visit recharging stations to charge their batteries and continue their routes. The stations are equipped with a single charger having a single charging technology. For this reason, an arriving EV may face a queue and have to wait in the queue to be serviced. This waiting time varies depending on the time of the day because some time periods are more crowded due to rush hours and the demand is higher during these periods. The energy transferred is assumed to be a concave function of the charging time (Montoya et al., 2017) and the cost of charging is proportional to the amount of energy transferred. The EVs have to pay for the energy they receive. If an EV arrives at a customer before the early service time, it has to wait until that time, but if it arrives later than the late service time, a penalty proportional to the lateness is incurred. Furthermore, the vehicles have an acquisition cost and the drivers are paid an hourly wage. If an EV arrives at the depot after the regular shift hours its driver is paid an overtime wage. The objective is to determine minimum cost routes satisfying all constraints.

3.1. Time-dependent waiting time functions

We assume that each recharging station is equipped with a single charger and the EVs arrive at the stations according to a Poisson distribution with mean λ. Hence, each station is a system involving a charger as the server and the EVs as the queueing entities. When an EV arrives at a station, if the charger is idle, it receives service immediately. Otherwise, the arriving EV has to wait until the recharging of all vehicles in the queue finishes. Since the stations are public, we do not have prior information about the queue status. Hence, it is difficult to assign a distribution function for the service time. For this reason, the service time is drawn from an unspecified distribution whose mean and standard deviation are known. Since this is a case of a single server with Poisson arrivals and generally distributed service times, we consider the M/G/1 queueing system to be appropriate.

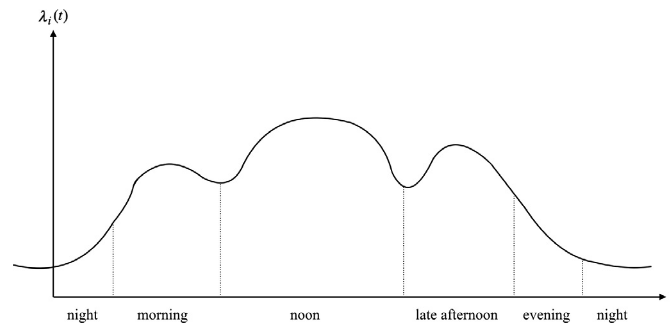

Fig. 1. Arrival rate of EVs at station i as a function of time.

In M/G/1, the letter M denotes the "memoryless property" of the exponential distribution. Since the arrivals are Poisson, the interarrival time between two consecutive EVs follows exponential distribution with parameter λ. The service times are independent and identically distributed with distribution function F_S and density f_S. Since the service times do not belong to a specific distribution, G stands for "general distribution". Finally, the number 1 in the third field indicates that there is only one server in the system. Let S be the recharging time and let the mean and standard deviation of the service time be $E[S]$ and Var(S), respectively. In order to satisfy stability of the system, the utilization rate of the charger, $\rho = \lambda E[S]$ should be less than one. The EVs receive service according to a FIFO discipline.

We handle time-dependent waiting times by discretizing the time and dividing the planning horizon into a set of time intervals $[t^m, t^{m+1}]$, $m \in M$. This approach is similar to the time-dependent travel times case where the time is split into intervals, and each interval is assigned an average speed. The starting and ending times of interval m are denoted by t^m and t^{m+1}, respectively. Since the traffic density changes with time, the arrival rate of the vehicles is also a function of time. Different stations may have different arrival rates since they are positioned at different locations. In Fig. 1, we divide the day into five time intervals and illustrate a sample pattern of the arrival rate function $\lambda_i(t)$ for station i. The noon interval is the most crowded time of the day. Late afternoon and morning are the second and third most crowded times, while the evening and night have the least traffic densities. In real life, $\lambda_i(t)$ is a smooth function as depicted in Fig. 1. However, for modeling purposes, we will discretize the time and approximate $\lambda_i(t)$ by a piecewise linear function as illustrated in Fig. 2.

In Fig. 2, the arrival rates for station i during the morning, noon, late afternoon, and evening time intervals are assumed constant and equal to λ_i^1, λ_i^2, λ_i^3 and λ_i^4, respectively. In order to preserve the FIFO property, transient periods should be introduced between the intervals in such a way that an EV which arrives at a station later than another EV cannot leave the station earlier. Then these transient zones will have variable arrival rates, contrary to other zones. These assumptions lead to an M/G/1 queueing system with Poisson arrivals having varying rates and a service time coming from a general distribution function. For an EV arriving at station i at time t, the expected waiting time $E[W_i(t)]$ in the queue can then be calculated using the steady-state equations of the M/G/1

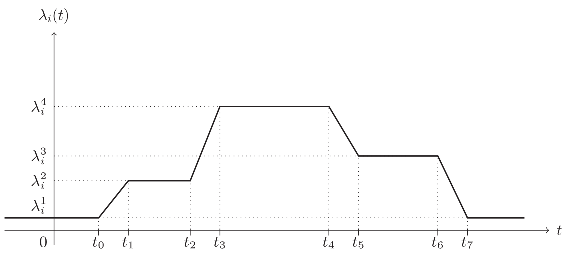

Fig. 2. Piecewise linear approximation of the arrival rate as a function of time.

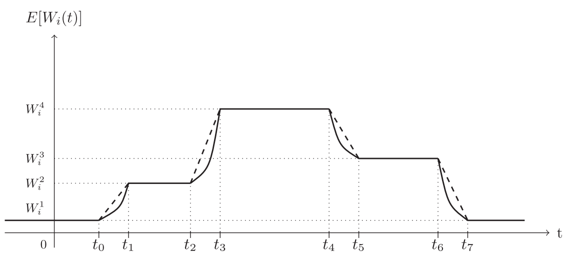

Fig. 3. Piecewise linear approximation of waiting time in the queue as a function of time.

queueing system as follows:

$$E[W_i(t)] = \frac{\rho_i}{1-\rho_i}\frac{C_{S_i}^2+1}{2}E[S_i], \quad \text{where } C_{S_i}^2 = \frac{\text{Var}(S_i)}{E^2[S_i]}. \tag{1}$$

Clearly, $E[W_i(t)]$ is non-linear in the transient zones. Hence, we approximate it by a linear function as shown in Fig. 3. Each segment of the function $E[W_i(t)]$ is characterized by an intercept W_i^m and a slope s_i^m. Hence, if a vehicle arrives at station i in the mth time interval, i.e. at time t, it will wait for a time $W_i^m + s_i^m(t - t^m)$ in the queue before being serviced. The service time for an EV is the recharging time at a station. In this study, we assume that the EVs operate between 10% and 90% of the battery SoC to minimize the degradation (Pelletier et al., 2017). Hence, all EVs enter and leave a station with an SoC within this range. To assign a mean and a variance to the service time function, we assume that the amount of energy transferred follows a uniform distribution between 0 and $0.8q$, where q is the battery capacity. Hence we can calculate the expected recharge amount, the expected time required to recharge the battery, and the variance of the recharging time using the equations of moments of the uniform distribution. Then the expected service time at a recharging station will be $E[S] = (0.8q \times r)/2 = 0.4q \times r$, where r is the recharging rate. Similarly, the variance of the service time can be computed as $\text{Var}(S) = (0.8q \times r)^2/12$.

3.2. Mathematical formulation

Let $V = \{1, \dots, n\}$ be the set of vertices representing customers to be served, and let S be the set of recharging stations. Since a station may be visited multiple times, we create copies of the stations. We denote by S' the set of recharging stations along with their copies. Let $V' = V \cup S'$ be the set of customers and stations. {0} and {N+1} denote the depot vertices where the EVs start and finish their routes, respectively. Defining the sets $V_0 = V \cup \{0\}, V_{N+1} = V \cup \{N+1\}$, $V_{0,N+1} = \{0\} \cup V_{N+1}$, $V_0' = \{0\} \cup V', V_{N+1}' = V' \cup \{N+1\}$, $V_{0,N+1}' = \{0\} \cup V_{N+1}'$, $S_0' = \{0\} \cup S'$, the problem can be represented by a complete directed graph $G = (V_{0,N+1}', A)$, where $A = \{(i,j) | i, j \in V_{0,N+1}', i \neq j\}$.

With each arc are associated a distance d_{ij} and a travel time t_{ij}. Each vehicle has a load capacity c and a battery capacity q. The energy is consumed at a rate of h per distance unit. Hence, traversing arc (i, j) requires $h \times d_{ij}$ units of energy. Each customer $i \in V$ is associated with demand d_i, service time s_i and soft time window $[e_i, l_i]$. Arrivals later than l_i are allowed but are penalized by a late arrival cost c_p multiplied by the lateness. We also assume that the depot has soft time windows where arrivals later than the depot closing time $\bar{l}$ are penalized by an overtime wage c_o. Each vehicle has an operating cost c_f which may represent maintenance and acquisition costs, and the driver is paid at a wage of c_d per time unit.

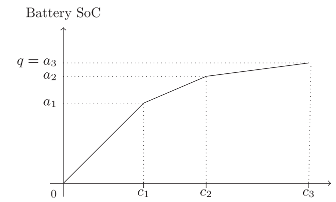

Fig. 4. Piecewise linear approximation for the charging function (Montoya et al., 2017).

The charging function is known to be concave (Pelletier et al., 2017). Hence, we adopt a non-linear charging function and approximate it by a piecewise linear function as in Montoya et al. (2017). The unit cost of energy is c_e. Fig. 4 illustrates the piecewise linear approximation of the charging function, where c_l and a_l represent the time and charge level, respectively, for the breakpoints $l \in B$, and $B = \{0, \dots, b\}$ is the set of breakpoints of the piecewise linear approximation.

Unlike what is done in Montoya et al. (2017), we assume that all stations are equipped with the same charger type; hence the vehicles are always subject to the same charging function. We denote by W_j^m the expected waiting time at station j in time interval m, while s_j^m is the slope of the piecewise linear waiting time function in time interval m. The sets, parameters and decision variables of the problem are presented in Table 3.

Given the above definitions, the mathematical model for the problem with time-dependent waiting times at the stations is formulated as follows:

$$\text{minimize} \quad c_e \sum_{i \in V_0'} \sum_{j \in V_{N+1}'} \sum_{m \in M} d_{ij} x_{ij}^m + c_p \sum_{i \in V} \upsilon_i + (c_o - c_d) \sum_{i \in V'} d_i + c_f \sum_{i \in V_0'} \sum_{j \in V^a} \sum_{m \in M} x_{ij}^m + c_d \sum_{i \in V'} f_i \tag{2}$$

subject to

$$\sum_{m \in M} \sum_{j \in V_{N+1}'} x_{ij}^m = 1 \qquad i \in V \tag{3}$$

$$\sum_{m \in M} \sum_{j \in V_{N+1}'} x_{ij}^m \leqslant 1 \qquad i \in S' \tag{4}$$

$$\sum_{m \in M} \sum_{i \in V_0'} x_{ij}^m = \sum_{m \in M} \sum_{i \in V_{N+1}'} x_{ji}^m \qquad j \in V' \tag{5}$$

$$\tau_i + (t_{ij} + s_i) \sum_{m \in M} x_{ij}^m - l_0\Big(1 - \sum_{m \in M} x_{ij}^m\Big) \leqslant \tau_j \qquad i \in V_0, j \in V' \tag{6}$$

$$\tau_i + (t_{i,N+1} + s_i) \sum_{m \in M} x_{i,N+1}^m - l_0\Big(1 - \sum_{m \in M} x_{i,N+1}^m\Big) \leqslant f_i \qquad i \in V \tag{7}$$

$$\tau_i + \Big(\sum_{l \in B} r_{il}^d c_l - \sum_{l \in B} r_{il}^a c_l\Big) + w_i(\tau_i) + t_{ij} - M\Big(1 - \sum_{m \in M} x_{ij}^m\Big) \leqslant \tau_j \qquad i \in S', j \in V \tag{8}$$

Table 3
Mathematical notation.

Sets	
V	Set of customers
S	Set of recharging stations
S'	Set of recharging stations with their copies
V'	Set of customers and recharging stations with their copies ($V \cup S'$)
S'_0	Set of departure depot and recharging stations with their copies ($\{0\} \cup S'$)
S'_{N+1}	Set of recharging stations with their copies and arrival depot ($S' \cup \{N+1\}$)
V_0	Set of departure depot and customers ($\{0\} \cup V$)
V_{N+1}	Set of customers and arrival depot ($V \cup \{N+1\}$)
$V_{0,N+1}$	Set of departure and arrival depot with customers ($\{0\} \cup V_{N+1}$)
V'_0	Set of departure depot, customers, and recharging stations with their copies ($\{0\} \cup V'$)
V'_{N+1}	Set of customers, recharging stations with their copies, and arrival depot ($V' \cup \{N+1\}$)
$V'_{0,N+1}$	Set of all vertices ($\{0\} \cup V'_{N+1}$)
$\widetilde{V}$	Set of i, j pairs such that $i \in S',\ j \in V_{N+1}$ and $i \in V_0,\ j \in V'_{N+1}$
Parameters	
d_{ij}	Distance from vertex i to vertex j
t_{ij}	Travel time from vertex i to vertex j
d_i	Demand of customer i
s_i	Service time of customer i
e_i	Early service time of customer i
l_i	Late service time of customer i
$\bar{l}$	Depot closing time
c	Cargo capacity of the vehicles
q	Battery capacity of the vehicles
c_l	Time of breakpoint l in the approximated recharging time function
a_l	SoC of breakpoint l in the approximated recharging time function
t^m	Beginning of m^{th} time interval
W_j^m	Average waiting time at station j at the beginning of mth time interval
s_j^m	Slope of the $E[W_j(t)]$ function of station j in mth time interval
h	Fuel consumption rate
c_e	Unit energy cost
c_p	Unit late service cost of customers
c_d	Driver wage per unit time
c_o	Overtime wage per unit time
c_f	Fixed vehicle cost
M	A sufficiently large number
$w_{\max}$	The longest waiting time at any station at any time interval
Decision variables	
x_{ij}^m	1 if EV departs from vertex i and arrives at vertex j in time interval m, 0 otherwise
z_{il}	1 if battery SoC at the arrival at station i is between a_{l-1} and a_l, $l \in B \backslash \{0\}$, 0 otherwise
y_{il}	1 if battery SoC at the departure from station i between a_{l-1} and a_l, $l \in B \backslash \{0\}$, 0 otherwise
τ_i	Service start time upon arrival at vertex i
f_i	Total time spent on a route that has a vertex i as last visited vertex before returning to the depot
v_i	Lateness at customer i due to the late arrival of the vehicle
d_i	Lateness of the depot where vertex i is the last visited node before the depot
u_i	Remaining cargo capacity upon arrival at vertex i
y_i^a	Battery SoC at vertex i
y_i^d	Battery SoC when departing from station i
w_i	Queue waiting time of the EV arriving at station i
r_{il}^a	Coefficient of breakpoint l in the piecewise approximation when EV arrives at station i
r_{il}^d	Coefficient of breakpoint l in the piecewise approximation when EV departs from station i

$$\tau_i + \Big(\sum_{l \in B} r_{il}^d c_l - \sum_{l \in B} r_{il}^a c_l\Big) + w_i(\tau_i) + t_{i,N+1} - M\Big(1 - \sum_{m \in M} x_{i,N+1}^m\Big) \leqslant f_i \quad i \in S' \quad (9)$$

$$t^m \sum_{i \in V_0} x_{ij}^m \leqslant \tau_j \leqslant t^{m+1} + l_0\big(1 - \sum_{i \in V_0} x_{ij}^m\big) \quad j \in V', m \in M \quad (10)$$

$$t^m \sum_{i \in S'} x_{ij}^m \leqslant \tau_j \leqslant t^{m+1} + l_0\big(1 - \sum_{i \in S'} x_{ij}^m\big) \quad j \in V, m \in M \quad (11)$$

$$t^m \times x_{i,N+1}^m \leqslant f_i \leqslant t^{m+1} + l_0(1 - x_{i,N+1}^m) \quad i \in V', m \in M \quad (12)$$

$$e_i \leqslant \tau_i \quad i \in V \quad (13)$$

$$v_i \geqslant (\tau_i - l_i) \quad i \in V \quad (14)$$

$$f_i \leqslant l_0 \quad i \in V' \quad (15)$$

$$d_i \geqslant (f_i - \bar{l}) \quad i \in V' \quad (16)$$

$$w_j(\tau_j) \geqslant W_j^m + s_j^m(\tau_j - t^m) - w_{\max}(1 - x_{ij}^m) \quad i \in V_0, j \in S', m \in M \quad (17)$$

$$y_i^a = \sum_{l \in B} r_{il}^a a_l \quad i \in S' \quad (18)$$

$$y_i^d = \sum_{l \in B} r_{il}^d a_l \quad i \in S' \quad (19)$$

$$y_j^a \leqslant y_i^d - \sum_{m \in M} (h \times d_{ij}) x_{ij}^m + 0.9q\Big(1 - \sum_{m \in M} x_{ij}^m\Big) \quad (i, j) \in \widetilde{V} \quad (20)$$

$$y_j^a \geqslant y_i^d - \sum_{m \in M} (h \times d_{ij}) x_{ij}^m - 0.9q\Big(1 - \sum_{m \in M} x_{ij}^m\Big) \quad (i, j) \in \widetilde{V} \quad (21)$$

$$y_i^a \geqslant 0.1q \sum_{m \in M} \sum_{j \in V'_0} x_{ji}^m \quad i \in V'_{N+1} \quad (22)$$

$$y_i^a \leqslant y_i^d \leqslant 0.9q \quad i \in V'_0 \quad (23)$$

$$0 \leqslant u_0 \leqslant c \tag{24}$$

$$u_j \leqslant u_i - d_i \sum_{m \in M} x_{ij}^m + c\left(1 - \sum_{m \in M} x_{ij}^m\right) \qquad i \in V_0', j \in V_{N+1}' \tag{25}$$

$$\sum_{l \in B} r_{il}^a = \sum_{l \in B \setminus \{0\}} z_{il} = \sum_{m \in M} \sum_{j \in V_{N+1}'} x_{ij}^m \qquad i \in S' \tag{26}$$

$$r_{i0}^a \leqslant z_{i1} \qquad i \in S' \tag{27}$$

$$r_{il}^a \leqslant z_{il} + z_{i,l+1} \qquad i \in S', l \in B \setminus \{0, b\} \tag{28}$$

$$r_{ib}^a \leqslant z_{ib} \qquad i \in S' \tag{29}$$

$$\sum_{l \in B} r_{il}^d = \sum_{l \in B \setminus \{0\}} y_{il} = \sum_{m \in M} \sum_{j \in V_{N+1}'} x_{ij}^m \qquad i \in S' \tag{30}$$

$$r_{i0}^d \leqslant y_{i1} \qquad i \in S' \tag{31}$$

$$r_{il}^d \leqslant y_{il} + y_{i,l+1} \qquad i \in S', l \in B \setminus \{0, b\} \tag{32}$$

$$r_{ib}^d \leqslant y_{ib} \qquad i \in S' \tag{33}$$

$$\sum_{m \in M} \sum_{j \in S'} x_{ij}^m = 0 \qquad i \in S' \tag{34}$$

$$x_{ij}^m \in \{0, 1\} \qquad i \in V_0', j \in V_{N+1}', m \in M \tag{35}$$

$$z_{ij}^m \geqslant 0 \qquad i \in S', j \in V_{N+1}, m \in M \tag{36}$$

$$w_{ib}, z_{ib} \in \{0, 1\} \qquad i \in S', b \in B \tag{37}$$

$$y_i^a, y_i^d, w_i \geqslant 0 \qquad i \in S' \tag{38}$$

$$v_i \geqslant 0 \qquad i \in V \tag{39}$$

$$\tau_i, f_i, d_i \geqslant 0 \qquad i \in V' \tag{40}$$

$$r_{il}^a, r_{il}^d \geqslant 0 \qquad i \in S', l \in B. \tag{41}$$

The objective function (2) minimizes total cost equal to the sum of five terms. The first term corresponds to the energy cost which is proportional to the distance traveled. The second and third terms are the penalties associated with customer and depot time window violations, respectively. The fourth term is total cost of vehicles, while the last term computes the driver cost. Constraints (3) ensure that each customer is visited exactly once whereas constraints (4) indicate that each station is visited at most once. The connectivity of the nodes is ensured by constraints (5). Constraints (6) and (8) keep track of arrival times after departing from a customer and a recharging station, respectively. The total time spent on a route whose last visited node is a customer and a station is tracked by constraints (7) and (9), respectively. While traveling from vertex i to vertex j, if $x_{ij}^{\bar{m}}$ is 1, the arrival time at j should be in the $\bar{m}$th time interval, which is between $[t^{\bar{m}} - t^{\bar{m}+1}]$. This in ensured by constraints (10) for departures from the depot or a customer, and by constraints (11) for departures from a station. Constraints (12) guarantee the same condition for arrivals at the depot. Constraints (13) mean that the vehicle starts its service after the early service time of that customer, while constraints (14) guarantee that lateness will have a positive value when vehicle arrives at a customer after its late service time. Constraints (15) state that the vehicles should end their trips before the maximum time limit l_0. Constraints (16) ensure that lateness at the depot will have a positive value when a vehicle arrives at the depot after its closing time. Constraints (17) link the flow variables x_{ij}^m and the waiting time $w_j(\tau_j)$ at station j. Constraints (18) and (19) determine the SoC of the vehicle at arrival at and departure from station i in terms of the breakpoints of the piecewise linear approximation. Constraints (20) and (21) track the SoC at departure from a recharging station or a customer/depot. Constraints (22) and (23) set the battery utilization between 10% and 90% of its capacity. Constraints (24) and (25) keep track of load of the vehicle and ensure that it is initially smaller than or equal to the load capacity and it never goes below 0. Constraints (26) and (30) establish the relationship between the binary variables z_{il} and y_{il} and the coefficients of the breakpoints in the piecewise linear approximation when an EV enters and leaves a station, respectively. Constraints (27)–(29) and (31)–(33) ensure that the coefficients related to the piecewise linear approximation of the charging function take correct values. Constraint (34) prevents the consecutive visits to multiple stations. Finally, (35)–(41) define the domains of the decision variables.

The value of M is selected such that constraints (8) become redundant when $\sum_{m \in M} x_{ij}^m$ is 0. Let $t_{\max}$ be the maximum travel time among all arcs and $Y_{\max}$ be the time to recharge the battery from 10% to 90% of the battery capacity. Then, $M = l_0 + Y_{\max} + w_{\max} + t_{\max}$.

4. Solution methodology

Since the problem generalizes the classical VRP, which is NP-hard, it is intractable for large-size instances. Hence, we propose a matheuristic to solve it within reasonable time. Matheuristics have been used in vehicle routing problems successfully (Archetti and Speranza, 2014). In our approach, ALNS is used to search the feasible space and determine feasible routes, while the charging decisions are optimized by solving an integer linear program exactly by CPLEX. In addition to implementing some well-known operators, we have developed a new customer removal operator and a preprocessing mechanism within the ALNS. Furthermore, to solve the fixed-sequence route optimization problem, we proposed a new mixed integer linear mathematical model.

4.1. Adaptive large neighborhood search

The ALNS metaheuristic, proposed by Ropke and Pisinger (2006a,b), is a search framework based on iterative destroy and repair phases, that has been successfully employed for solving various VRP variants (Aksen et al., 2014; Goeke and Schneider, 2015; Koç, 2016). The destroy and repair operators are selected adaptively based on their past performances. A weight ω_o and a score π_o are assigned to each operator o. Initially, all operators have the same weight and a score of 0. At each iteration, a removal and an insertion operator is selected based on a roulette wheel mechanism. The scores of the operators are increased by σ_1, σ_2 or σ_3, depending whether they yield a better or worse solution than the previous one, or an overall best solution. Every N_c iterations, the weights of the operators are updated using the formula $\omega_o \leftarrow \omega_o(1-\rho) + \rho\pi_o/\theta$ where ρ is the roulette wheel parameter and θ stands for the number of iterations during which operator o has been used since the last score update was performed. If the weight of operator o is ω_o and O denotes the set of operators, the selection probability of o is calculated by the formula $\omega_o/\Sigma_{k \in O}\omega_k$ and the scores are reinitialized. In this study, we use two types of operators which are designed for customers and stations. Candidate solutions are accepted according to a simulated annealing criterion.

4.1.1. Initial solution

At the beginning of the algorithm, all customers are inserted in a removal list in a random order. A greedy customer insertion procedure (see Section 4.1.3) is then applied to this list to construct the initial solution x_0.

4.1.2. Customer removal

The current feasible solution $x_{current}$ is destroyed by the removal operators, and insertion operators are then applied to generate a new solution. Customer removal (CR) operators remove

γ customers and add them to a removal list. The parameter γ is randomly generated at each iteration using a discrete uniform distribution with parameters $\underline{n}_c$ and $\bar{n}_c$. We use the random, worst-distance, worst-time, Shaw, proximity-based, demand-based, time-based, zone, random route and greedy route removals which have been used in related studies (Demir et al., 2012; Emeç et al., 2016; Keskin and Çatay, 2016). In addition, we introduce the following operator.

Expensive customer removal: This operator tends to remove customers whose visiting cause a high increase in the objective function. It first identifies the customers whose predecessor or successor nodes are stations, and sorts them in non-increasing order of their costs which are calculated in the following way. Let station i be the neighboring node of customer j. Then the total cost of traveling, recharging and waiting will be $c_e(d_{ji} + y_i^d - y_i^a) + c_d w_i$. The operator removes from the solution the customers having the γ largest costs.

After the removal of the selected customers, some stations may become unnecessary to visit. A route refinement procedure is applied to eliminate these unnecessary stations. It evaluates the decrease in the objective function resulting from the elimination of each station if this elimination is feasible. The station whose removal reduces the objective function the most is removed from the solution. This operation is repeated until no feasible station removal exists.

4.1.3. Customer insertion

This operator inserts the customers from the removal list back into the partial solution. We employ greedy, regret-2 and time-based insertion operators, as proposed in Keskin and Çatay (2016). However, some customers may need the insertion of a station to maintain feasibility. In such cases, greedy station insertion operator is applied to insert a station along with the customer.

4.1.4. Station insertion

We use the greedy station insertion presented in Keskin and Çatay (2016). Here we propose a preprocessing procedure that eliminates the dominated stations and speeds up the algorithm. Since the station insertion is performed to make an infeasible customer sequence feasible, the possible insertion locations are known in advance. We can therefore eliminate some stations according to their queueing time and their distances to the customers between which they are inserted. To insert a non-dominated station between customers i and k we proceed as follows: Initially, the set of stations is the same as the original set of stations, i.e. $S_{ik} = S$. Let $j \in S$ and $j' \in S$ be two candidate stations. If $d_{ij} > d_{ij'}$ and $d_{jk} > d_{j'k}$, then j' is preferred to j. However, j may have a shorter queueing time at the time of arrival. Hence, we calculate w_j and $w_{j'}$, the queueing times at stations j and j' if these are visited after customer i. If $w_j > w_{j'}$ then j' is preferred also in terms of the total time spent to visit a station. Then, j is dominated and we eliminate it from S_{ik}. By comparing all the stations pairwise in this fashion, we obtain the reduced set S_{ik}. Ties are broken in favor of the station with smaller index. This procedure is illustrated in Fig. 5. Fig. 5 depicts customers i and k and the stations j, j', j'' and j''' that can be inserted between them. It also shows the EVs waiting at each station at the time of arrival. Comparing j and j', one can see that $d_{ij'} > d_{ij}$ and $d_{j'k} > d_{jk}$. In addition, the waiting time at j' is more than at j since j' has two more EVs. Hence, station j' is dominated by j. For j and j'', clearly $d_{ij} > d_{ij''}$ and $d_{jk} > d_{j''k}$, but j has fewer EVs than j'' in its system. So, we cannot eliminate either of them. Finally, j''' cannot be eliminated since its distance to customer k is shorter than that of j and j''. In this case, the station insertion operator

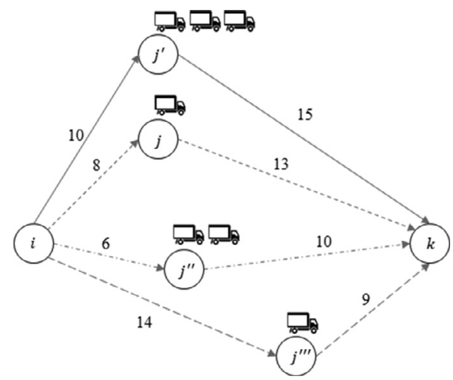

Fig. 5. Elimination of dominated stations.

will only evaluate the stations j, j'' and j''' between customers i and k.

4.2. Fixed sequence route optimization

The Fixed-Route Vehicle-Refueling Problem was introduced by Suzuki (2014) and later studied by Bruglieri et al. (2016), Montoya et al. (2017), Koç et al. (2018), and Keskin and Çatay (2018) to enhance the routing decisions. Here, we implement the same approach by developing a new mathematical model that incorporates the time-dependent waiting times at the stations. After every Δ iterations, the best known solution x_Δ is improved by solving each of its routes to optimality without changing the customer sequence. The solver optimizes the following decisions: when to visit a station, which station to visit, and how much to recharge.

4.2.1. Mathematical model

Let V be the set of customers to be served along the route. Since the customer sequence is predetermined, we can form the set of consecutive node pairs $\bar{V}$. The set definitions of S, V', V_0, V_{N+1}, $V_{0,N+1}$, V'_0, V'_{N+1} and $V'_{0,N+1}$ are the same with the new set V. Then, the problem can be represented by a complete directed graph $G = (V'_{0,N+1}, A)$ where $A = \{(i,j) | i, j \in V'_{0,N+1}, i \neq j\}$. Some decision variables are the same as in the main formulation presented in Section 4, while the new decision variables and sets are shown in Table 4.

Given the above definitions, the mathematical model for the fixed sequence route optimization problem with time dependent waiting times at stations is formulated as follows:

$$\text{minimize} \quad c_e(0.9q - y_{N+1}) + c_e \sum_{(i,k)\in\bar{V}} (y_{ik}^d - y_{ik}^a) + c_p \sum_{i\in V} v_i + c_d \tau_{N+1} + (c_o - c_d) v_{N+1} + c_f \tag{42}$$

subject to

$$\tau_i + s_i + \sum_{j\in S}\sum_{m\in M} x_{ij}^m (t_{ij} + t_{jk}) + w_{ik}(\tau_{ik}) + t_{ik}\left(1 - \sum_{j\in S}\sum_{m\in M} x_{ij}^m\right) + \left(\sum_{l\in B} r_{ikl}^d c_l - \sum_{l\in B} r_{ikl}^a c_l\right) \leqslant \tau_k \qquad (i,k)\in\bar{V} \tag{43}$$

$$t^m\left(\sum_{j\in S} x_{ij}^m\right) \leqslant \tau_i + s_i + \sum_{j\in S} x_{ij}^m t_{ij} \leqslant t^{m+1} + l_0\left(1 - \sum_{j\in S} x_{ij}^m\right) \qquad i\in V_0,\ m\in M \tag{44}$$

Table 4
Mathematical notation.

Sets	
$\bar{V}$	Set of consecutive customer pairs in the route
V_0	Set of the departure depot and customers in the route
V_{N+1}	Set of customers in the route and the arrival depot
$V_{0,N+1}$	Set of the departure depot, customers in the route and the arrival depot
Decision variables	
x_{ij}^m	1, if the vehicle departing from customer i, goes to station j and arrives there in time interval m, 0 otherwise
z_{ikl}	1, if SoC at arrival at the station between consecutive customers i and k is between a_{l-1} and a_l, 0 otherwise
w_{ikl}	1, if SoC at departure from the station between consecutive customers i and k is between a_{l-1} and a_l, 0 otherwise
y_{ik}^a	Battery SoC at a station visited between customers i and k
y_{ik}^d	Battery SoC when departing from a station visited between customers i and k
w_{ik}	Average queueing time of the EV arriving at a station visited between customers i and k
r_{ikl}^a	Coefficient of breakpoint l in the piecewise approximation when EV arrives at a station visited between customers i and k
r_{ikl}^d	Coefficient of breakpoint l in the piecewise approximation when EV departs from a station visited between customers i and k

$$w_{ik}(\tau_{ik}) \geqslant W_j^m + s_j^m(\tau_i + s_i + t_{ij} - t^m) - w_{\max}(1 - x_{ij}^m) \qquad (i,k) \in \bar{V}, j \in F, m \in M \quad (45)$$

$$w_{ik}(\tau_{ik}) \leqslant w_{\max}\Big(1 - \sum_{j\in S}\sum_{m\in M} x_{ij}^m\Big) \qquad (i,k) \in \bar{V} \quad (46)$$

$$y_{ik}^a = \sum_{l\in B} r_{ikl}^a a_l \qquad (i,k) \in \bar{V} \quad (47)$$

$$y_{ik}^d = \sum_{l\in B} r_{ikl}^d a_l \qquad (i,k) \in \bar{V} \quad (48)$$

$$y_i - h\Big[d_{ik}\Big(1 - \sum_{j\in S}\sum_{m\in M} x_{ij}^m\Big) + \sum_{j\in S}\sum_{m\in M} x_{ij}^m(d_{ij} + d_{jk})\Big] + (y_{ik}^d - y_{ik}^a) = y_k \qquad (i,k) \in \bar{V} \quad (49)$$

$$y_i - h\sum_{m\in M}\sum_{j\in S} d_{ij}x_{ij}^m \geqslant y_{ik}^a - 0.9q\Big(1 - \sum_{m\in M}\sum_{j\in S} x_{ij}^m\Big) \qquad (i,k) \in \bar{V} \quad (50)$$

$$y_i - h\sum_{m\in M}\sum_{j\in S} d_{ij}x_{ij}^m \leqslant y_{ik}^a + 0.9q\Big(1 - \sum_{m\in M}\sum_{j\in S} x_{ij}^m\Big) \qquad (i,k) \in \bar{V} \quad (51)$$

$$0.1q\sum_{m\in M}\sum_{j\in S} x_{ij}^m \leqslant y_{ik}^a \leqslant y_{ik}^d \leqslant 0.9q\sum_{m\in M}\sum_{j\in S} x_{ij}^m \qquad (i,k) \in \bar{V} \quad (52)$$

$$\sum_{l\in B} r_{ikl}^a = \sum_{l\in B\setminus\{0\}} z_{ikl} = \sum_{m\in M}\sum_{j\in S} x_{ij}^m \qquad (i,k) \in \bar{V} \quad (53)$$

$$r_{ik0}^a \leqslant z_{ik1} \qquad (i,k) \in \bar{V} \quad (54)$$

$$r_{ikl}^a \leqslant z_{ikl} + z_{ik,l+1} \qquad (i,k) \in \bar{V}, l \in B \setminus \{0, \beta\} \quad (55)$$

$$r_{ikb}^a \leqslant z_{ikb} \qquad (i,k) \in \bar{V} \quad (56)$$

$$\sum_{l\in B} r_{ikl}^d = \sum_{l\in B\setminus\{0\}} w_{ikl} = \sum_{m\in M}\sum_{j\in S} x_{ij}^m \qquad (i,k) \in \bar{V} \quad (57)$$

$$r_{ik0}^d \leqslant w_{ik1} \qquad (i,k) \in \bar{V} \quad (58)$$

$$r_{ikl}^d \leqslant w_{ikl} + w_{ik,l+1} \qquad \forall (i,k) \in \bar{V}, l \in B \setminus \{0, b\} \quad (59)$$

$$r_{ikb}^d \leqslant w_{ikb} \qquad (i,k) \in \bar{V} \quad (60)$$

$$y_i \geqslant 0.1q \qquad i \in V_{N+1} \quad (61)$$

$$y_0 = 0.90q \qquad (62)$$

$$y_i, \tau_i \geqslant 0 \qquad i \in V_{0,N+1} \quad (63)$$

$$w_{ik}, y_{ik}^a, y_{ik}^d, \geqslant 0 \qquad (i,k) \in \bar{V} \quad (64)$$

$$0 \leqslant r_{ikl}^a, r_{ikl}^d \leqslant 1 \qquad (i,k) \in \bar{V}, l \in B \quad (65)$$

$$z_{ikl}, w_{ikl} \in \{0,1\} \qquad (i,k) \in \bar{V}, l \in B \setminus \{0\} \quad (66)$$

$$(13) - (15).$$

The objective function (42) has the same components as in (2). Constraints (43) keep track of service beginning times for consecutive customers i and k. Constraints (44) ensure that if any station is visited after customer i in time period m, then the service start time should lie within time interval $[t^m - t^{m+1}]$. Constraints (45) and (46) link the flow variables x_{ij}^m and the waiting time $w_{ik}(\tau_{ik})$ at a station between consecutive customers i and k. Constraints (47) and (48) determine the arrival and departure SoC values at the station visited between consecutive customers i and k in terms of the breakpoints in the piecewise linear approximation. The battery SoC upon arrival at consecutive customers is tracked by constraints (49). If any station is not visited, i.e., all x_{ij}^ms are 0, then the SoC upon arrival at customer k will be the level at customer i, minus the energy consumed on arc (i, k). Otherwise, the recharged amount $(y_{ik}^d - y_{ik}^a)$ will be added to that value while the subtracted value will be the energy consumed on arcs (i, j) and (j, k). Similarly, the battery level upon arrival at station j after customer i is determined by constraints (50) and (51). In this case, the SoC upon arrival at the station and the departure from the station should lie between 10% and 90% of the battery capacity, which is stated by constraints (52). Constraints (53) and (57) establish the relationship between the binary variables z_{ikl} and w_{ikl} and the coefficients of the breakpoints in the piecewise approximation when an EV enters and leaves a station, respectively. Constraints (54)–(56) and (58)–(60) ensure that the coefficients related to the piecewise linear approximation of the charging function take correct values. Constraints (61) ensure that when arriving at a customer or at the depot, the SoC is at least 10% of the battery capacity. The vehicle departs from the depot with a SoC level equal to 90% the battery capacity, which is ensured by the constraint (62). Finally, (63)–(66) define the domains of the decision variables.

Having explained all components of the proposed matheuristic, its general framework is described in Algorithm 1. We denote by $f(x)$ the objective function value of solution x.

4.2.2. Preprocessing on decision variables for the fixed route formulation

To reduce the number of x_{ij}^m variables in the formulation, we introduce a preprocessing procedure that benefits from the time-dependent nature of the problem. Since we know the customer sequence of the customers in the route, we can eliminate some decision variables. First, for each customer i in the route we calculate the earliest and latest possible departure times, namely $t_{i,\text{early}}$ and $t_{i,\text{late}}$, using the fact that vehicles depart from the depot at time 0 and have to return to the depot before l_0. Therefore, departing from a customer i earlier than $t_{i,\text{early}}$ or later than $t_{i,\text{late}}$ is impossible. Using these bounds, we can calculate the earliest and latest arrival times at the stations for each customer pair and eliminate the x_{ij}^ms corresponding to the time interval outside of the limits just calculated.

Algorithm 1 General framework of the matheuristic.

Generate an initial solution x_0, $x_\Delta \leftarrow x_{\text{best}} \leftarrow x_{\text{current}} \leftarrow x_0$
Initialize the scores and probabilities of the operators, $iter \leftarrow 1$
while $iter <$ Maximum number of iterations **do**
  if $iter$ is a multiple of Δ and $f(x_\Delta) \neq f(x_{\Delta-\text{previous}})$ **then**
    for all routes in x_Δ **do**
      Remove all charging stations from the route
      Solve TD-EVRPTW for a given route, update the route in x_Δ
    end for
    $f(x_{\Delta-\text{previous}}) \leftarrow f(x_\Delta), x_{\text{current}} \leftarrow x_\Delta$
    if $f(x_\Delta) < f_{\text{best}}$ **then**
      $x_{\text{best}} \leftarrow x_\Delta, f_{\text{best}} \leftarrow f(x_\Delta)$
    end if
    $f(x_\Delta) \leftarrow \infty$
  else
    Select a Customer Removal operator and remove γ customers from $x_{current}$
    Apply *Route Refinement Procedure*
    Select a Customer Insertion operator and repair the solution
    if $f(x_{\text{current}}) < f_{\text{previous}}$ **then**
      $x_{\text{previous}} \leftarrow x_{\text{current}}, f_{\text{previous}} \leftarrow f(x_{\text{current}})$
      if $f(x_{\text{current}}) < f_{\text{best}}$ **then**
        $x_{\text{best}} \leftarrow x_{\text{current}}, f_{\text{best}} \leftarrow f(x_{\text{current}})$
      end if
    else
      Accept the solution using Simulated Annealing Criterion
      $x_{\text{previous}} \leftarrow x_{\text{current}}, f_{\text{previous}} \leftarrow f(x_{\text{current}})$
    end if
    if $f(x_{\text{current}}) < f(x_\Delta)$ **then**
      $f(x_\Delta) \leftarrow f(x_{\text{current}})$
    end if
  end if
  if $iter$ is a multiple of N_c **then**
    Update adaptive weights of CR and CI operators and calculate new selection probabilities
  end if
  $iter \leftarrow iter + 1$
end while
Return x_{best}

5. Computational experiments

This section presents the assumptions and calculations related to the waiting times and reports the results of our computational experiments. All experiments were conducted on an Intel Xeon E5 2.10 GHz processor virtual machine with 16 GB of RAM. The models described in Sections 3.2 and 4.2.1 as well as the matheuristic were coded in Java and the models were solved by CPLEX 12.6.2 with default settings. We used the EVRPTW instances of Schneider et al. (2014) with some problem-specific adaptations. These instances are based on the VRPTW instances of Solomon (1987). Small instances include five, 10, and 15 customers with varying number of stations while large instances have 100 customers and 21 stations. Each subset includes three types of instances categorized according to geographical distribution of the customers: clustered (C), randomly distributed (R), and clustered and randomly distributed together (RC). Furthermore, the instances also differ by the length of time windows and planning horizon: type 1 problems involve narrow time windows and shorter scheduling horizon whereas the time windows are wide and the scheduling horizon is longer in type 2 problems. In total, there are 56 large and $3 \times 12 = 36$ small instances.

5.1. Experimental design

Since we use a non-linear charging function and the EVRPTW data have a fixed charging rate, we need to adapt the rates. We used the piecewise linear charging function for fast chargers proposed in Montoya et al. (2017). The function has three pieces which means that we also need three different charging rates. We apply a scaling such that the recharging rate during the last piece is equal to the rate applied in Schneider et al. (2014) data. Furthermore, we assumed a battery whose SoC interval between 10% and 90% corresponds to the full capacity value used in Schneider et al. (2014).

The day is divided into five time intervals, namely morning, noon, late afternoon, evening, and night. However, since the FIFO property must hold, we need to define transition periods between each interval. It is assumed that the transition periods from a less crowded interval to a crowded interval last 30 min. Since the increase in the arrival rate is an outside factor, it is safe to assume that the increase can happen within 30 min. However, for the transitions from a crowded interval to a less crowded one, the transition period is bounded by the service time of the charger. The reason is that the number of vehicles that can be served is limited due to the charging capacity at the station. Hence, the ends of these transition periods are variable and they are determined by adding the time to serve the difference of the number of vehicles between the time periods to the end of the previous period. This means that from noon to late afternoon and from late afternoon to evening, the transition periods differ from each other and are not equal to 30 min as earlier in the day. After introducing the transition periods, the number of time intervals becomes eight. These intervals are defined as follows: [7:00,7:30), [7:30,9:30), [9:30,10:00), [10:00,15:30), [15:30,t_1), [t_1, 19:00), [19:00,t_2), [t_2, 7:00). Here, t_1 and t_2 are the ending times of the transient periods, calculated as explained above. The vehicles depart from the depot at 8:00 and must return to the depot by 20:00. Because the due date of the depot is 20:00, we do not include the night-time interval in our study. Arrivals at the depot after 18:00 are penalized with an overtime wage. Since the benchmark instances are synthetic we convert the time data proportionally to the above setting. In each data we equal due date of the depot to 12 hours and determine the time intervals accordingly. So, the time intervals are different in each data set and scaled according to the due date of the depot. In order to analyze the effect of the waiting times, we investigate different scenarios among which one does not involve waiting. For the other scenarios, we consider two types of patterns for the waiting times, namely time-independent (TI) and time-dependent (TD) waiting times. In the time-independent case, the waiting times are constant during the day. In the time-dependent case, we assume two types of transitions between time intervals, referred to as smooth and steep transitions. In the former type, the increase in the expected waiting time is smaller than that of the latter during the same transition time from an off-peak interval to a peak interval. Similarly, the decrease from a peak interval to an off-peak interval is smaller compared to the steep type. Whenever there is waiting time, we further use two scenarios where waiting times are short and long. The configurations for the time-dependent case are depicted in Fig. 6. Note that the expected waiting time doubles from night to morning and from morning to noon, then halves from noon to late afternoon and from late afternoon to evening in the smooth transition case. In the steep transition case, it quadruples from night to morning, increases by a factor of 2.5 from morning to noon and decreases by a factor of two and five from noon to late afternoon and late afternoon to evening,

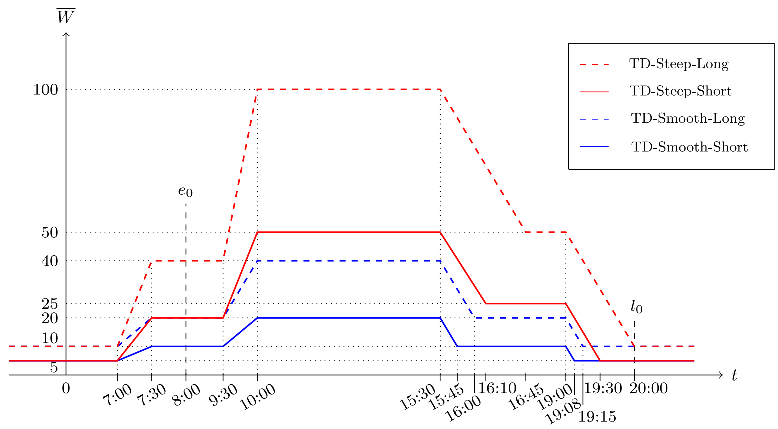

Fig. 6. Average waiting times for each scenario.

Table 5
Average waiting time ($\overline{W}$) parameters for each scenario.

Scenarios	Length of waiting times	Time Interval			
		Morning	Noon	Late afternoon	Evening
No waiting	–	0	0	0	0
Time-independent	Short	10	10	10	10
	Long	20	20	20	20
Time-dependent/	Short	10	20	10	5
Smooth transitions	Long	20	40	20	10
Time-dependent/	Short	20	50	25	5
Steep transitions	Long	40	100	50	10

respectively. Note also that the slopes of the transitions from noon to late afternoon and from late afternoon to evening are the same, which is the negative of the service rate. We assign waiting times to each interval assuming a 12-h planning period. These waiting time values in minutes are given in Table 5. For each instance the waiting times for each interval are calculated considering the due date of the depot in the data. Since the service rate μ is the same for all time intervals and scenarios, the λ values are calculated accordingly.

We adopt the objective function coefficients of Taş et al. (2013). However, we replace c_e and c_f to represent the cost figures of an EV instead of those of a diesel ICEV. In Feng and Figliozzi (2013), a conventional vehicle and an EV are compared for their fuel consumptions and purchase prices. The EV is three times more expensive than the conventional vehicle. Furthermore, using the consumption values reported in Feng and Figliozzi (2013) and the current electricity and fuel prices (U.S. Energy Information Administration, 2018), we can conclude that cost per km of the conventional vehicle is 2.5 times higher than that of the EV. Hence, the cost values for our case are determined as follows: $c_e = 0.4$, $c_p = 1$, $c_f = 1200$, $c_d = 1$, and $c_o = 11/6$. In ALNS, we use the same parameter values as reported in Keskin and Çatay (2016). The parameters of the discrete uniform distributions from which the number of customers to be removed is drawn are set as $\underline{n}_c = \min\{0.1|N|, 30\}$ and $\bar{n}_c = \min\{0.4|N|, 60\}$, similar to Emeç et al. (2016). Note that, N is the number of customers in the instance.

5.2. Results on the small instances

We first solved small instances with five, 10 and 15 customers with CPLEX with a time limit of 7200 seconds. We used the TD-Steep-Long scenario for these experiments. Table 6 compares the performance of CPLEX with that of the matheuristic. The value of %Imp is calculated as $100 \times (f_{\text{CPLEX}} - f_{\text{Math}})/f_{\text{CPLEX}}$, where f_{CPLEX} and f_{Math} stand for the solution values of CPLEX and the proposed matheuristic, respectively. The matheuristic was run 10 times and the best results are presented. The average computational times are given in seconds. We perform several iterations to obtain these results. Since the number of copies for the stations used in the optimal solution is not known, we first solve the problem with the original set of stations without using any copies. Next, we resolve the problem by creating a copy of each station as if there exist two distinct stations at the same location. If the two solutions are the same, we then stop and report the solution. If adding copies yields a better solution, then we proceed with another iteration by adding two copies of each station to the problem. This procedure is reiterated until the objective function value ceases to improve. The column $|S'|/|S|$ reports the total number of stations including their copies whereas the number of original stations in each instance is indicated in the instance name. For example, instance C101C5_S3 originally involves five customers and three stations. However, in the optimal solution at least one station is visited twice; i.e. the optimal solution is obtained by using one copy for each station, six stations in total. The optimal solution or the best upper bound is reported along with its CPU time. In the five-customer instances and some of the 10-customer instances, CPLEX was able to find optimal solutions within two hours. However, the other instances could not be solved to optimality. For the instances of which CPLEX found the optimal solution, the heuristic was also able to find the optimal solutions. For the others, it either found the same bound as CPLEX, or improved it. In two instances (RC108C15_S5 and RC204C15_S7) it was able to achieve solutions with less total cost and smaller fleet size. Furthermore, in most of the instances, the heuristic performed faster than CPLEX.

Note that in RC201C10_S4, the EV visits a station twice in the matheuristic solution, whereas the CPLEX solution was attained with one copy of the stations. When we increased the number of copies to two, CPLEX could not find the optimal solution in the time limit and gave an upper bound of 3875.37, which is worse than the optimal solution obtained with one copy.

5.3. Results on large instances

Desaulniers et al. (2016) highlighted the minor influence of wide time-window constraints on recharging decisions. Hence we decided to use instances with narrow time windows in our experimental study, and we randomly selected four instances from each data set of C1, R1, and RC1 of the Schneider et al. (2014) instances.

5.3.1. The impact of waiting on total cost and its components

We performed experiments for several scenarios of each instance and analyzed the impact of different waiting times at the stations by comparing the value of the objective function to that

Table 6
Results on small size instances.

Instance	$\lvert S'\rvert/\lvert S\rvert$	CPLEX Cost	#Veh	Time	Matheuristic Cost	#Veh	Time	% Imp.
C101C5_S3	2	4234.59	2	244	4234.59	2	3.16	–
C103C5_S2	2	2494.12	1	7.97	2494.12	1	2.08	–
C206C5_S4	1	4883.26	1	18.2	4883.26	1	3.38	–
C208C5_S3	1	3564.84	1	3.86	3564.84	1	1.92	–
R104C5_S3	1	2770.12	2	0.95	2770.12	2	1.38	–
R105C5_S3	1	2841.33	2	0.29	2841.33	2	1.02	–
R202C5_S3	1	1755.46	1	9.54	1755.46	1	2.39	–
R203C5_S4	1	2001.82	1	4.43	2001.82	1	2.36	–
RC105C5_S4	1	2954.94	2	6.57	2954.94	2	2.63	–
RC108C5_S4	1	4215.56	3	1.59	4215.56	3	1.75	–
RC204C5_S4	1	2032.12	1	15.1	2032.12	1	1.42	–
RC208C5_S3	1	1728.37	1	7.38	1728.37	1	1.83	–
C101C10_S5	3	5342.34	2	7200	5342.34	2	8.14	–
C104C10_S4	2	4181.82	2	7200	4149.23	2	22.10	0.78
C202C10_S5	1	4931.90	1	7200	4931.90	1	13.10	–
C205C10_S3	1	5959.20	1	308	5959.20	1	7.83	–
R102C10_S4	1	4374.54	3	279	4374.54	3	6.26	–
R103C10_S3	1	2887.03	2	7200	2887.03	2	7.19	–
R201C10_S4	1	2694.18	1	7200	2694.18	1	14.50	–
R203C10_S5	1	2191.54	1	7200	2162.87	1	9.62	1.31
RC102C10_S4	1	5792.04	4	7200	5792.04	4	6.33	–
RC108C10_S4	1	4458.81	3	1224	4458.81	3	5.81	–
RC201C10_S4*	1	3488.38	1	6682	3453.67	1	11.70	0.99
RC205C10_S4	1	3830.69	2	1007	3830.69	2	8.58	–
C103C15_S5	3	6679.78	3	7200	6674.73	3	23.20	0.08
C106C15_S3	1	6756.55	3	7200	6756.55	3	15.70	–
C202C15_S5	1	8258.50	2	7200	7885.32	2	35.20	4.52
C208C15_S4	1	6630.84	2	7200	6630.84	2	20.90	–
R102C15_S8	2	5854.66	4	7200	5854.66	4	15.30	–
R105C15_S6	1	5795.65	4	7200	5795.65	4	10.10	–
R202C15_S6	3	4253.51	2	7200	4009.78	2	37.10	5.73
R209C15_S5	1	4049.81	2	7200	4025.98	2	36.20	0.59
RC103C15_S5	1	5786.35	4	7200	5786.35	4	13.70	–
RC108C15_S5	1	5738.58	4	7200	4592.83	3	7.92	19.9
RC202C15_S5	3	4335.84	2	7200	4335.84	2	20.90	–
RC204C15_S7	1	3599.11	2	7200	3389.35	1	48.70	5.83

Table 7
Impact of different waiting schemes on total cost.

Instances	TI-S	TI-L	TD-Sm-S	TD-Sm-L	TD-St-S	TD-St-L
C101	1.01	1.03	1.01	1.05	1.08	1.12
C105	1.01	1.02	1.01	1.05	1.08	1.10
C107	1.01	1.02	1.02	1.03	1.04	1.10
C108	1.01	1.02	1.01	1.02	1.03	1.08
Average C	1.01	1.02	1.01	1.04	1.06	1.10
R102	1.07	1.07	1.06	1.09	1.14	1.25
R104	1.01	1.02	1.02	1.10	1.11	1.20
R109	1.08	1.09	1.08	1.10	1.10	1.26
R110	1.01	1.01	1.01	1.03	1.09	1.12
Average R	1.04	1.05	1.04	1.08	1.11	1.21
RC103	1.02	1.04	1.01	1.05	1.10	1.19
RC104	1.08	1.09	1.03	1.10	1.11	1.20
RC106	1.01	1.08	1.08	1.08	1.08	1.18
RC108	1.08	1.08	1.08	1.09	1.10	1.19
Average RC	1.05	1.07	1.05	1.08	1.10	1.19
Average All	1.03	1.05	1.04	1.07	1.09	1.17

of the base case where waiting is ignored. In Table 7, we report the ratio of the corresponding objective function values calculated as ($OFV_{\text{scenario}}/OFV_{\text{base}}$), where OFV_{scenario} and OFV_{base} are the objective function values of the best solutions over 10 runs. In the headings, “Sm” and “St” stand for Smooth and Steep transition scenarios, respectively, while the last letters “S” and “L” represent short and long waiting time cases, respectively.

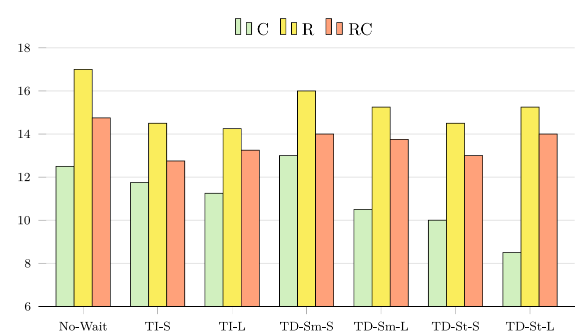

Fig. 7. Average number of recharges for each data set.

The results show that waiting at stations can increase the total cost by up to 11% and 26% for short- and long-waiting scenarios, respectively. As expected, the total cost is larger in the TD instances compared with the TI instances, when the waiting time is longer, and when the transitions are steeper. The waiting at the stations has a greater impact on the R and RC instances compared with the C instances. This is because the customers are dispersed in the random and random-and-clustered data and the vehicles need more frequent recharges (see Fig. 7). On average, the total cost increases between 3% and 17% for different waiting scenarios.

Fig. 8 illustrates the average changes in total cost and cost components compared with the base scenario. The green, yellow, and orange-colored columns represent the average results for the C, R,

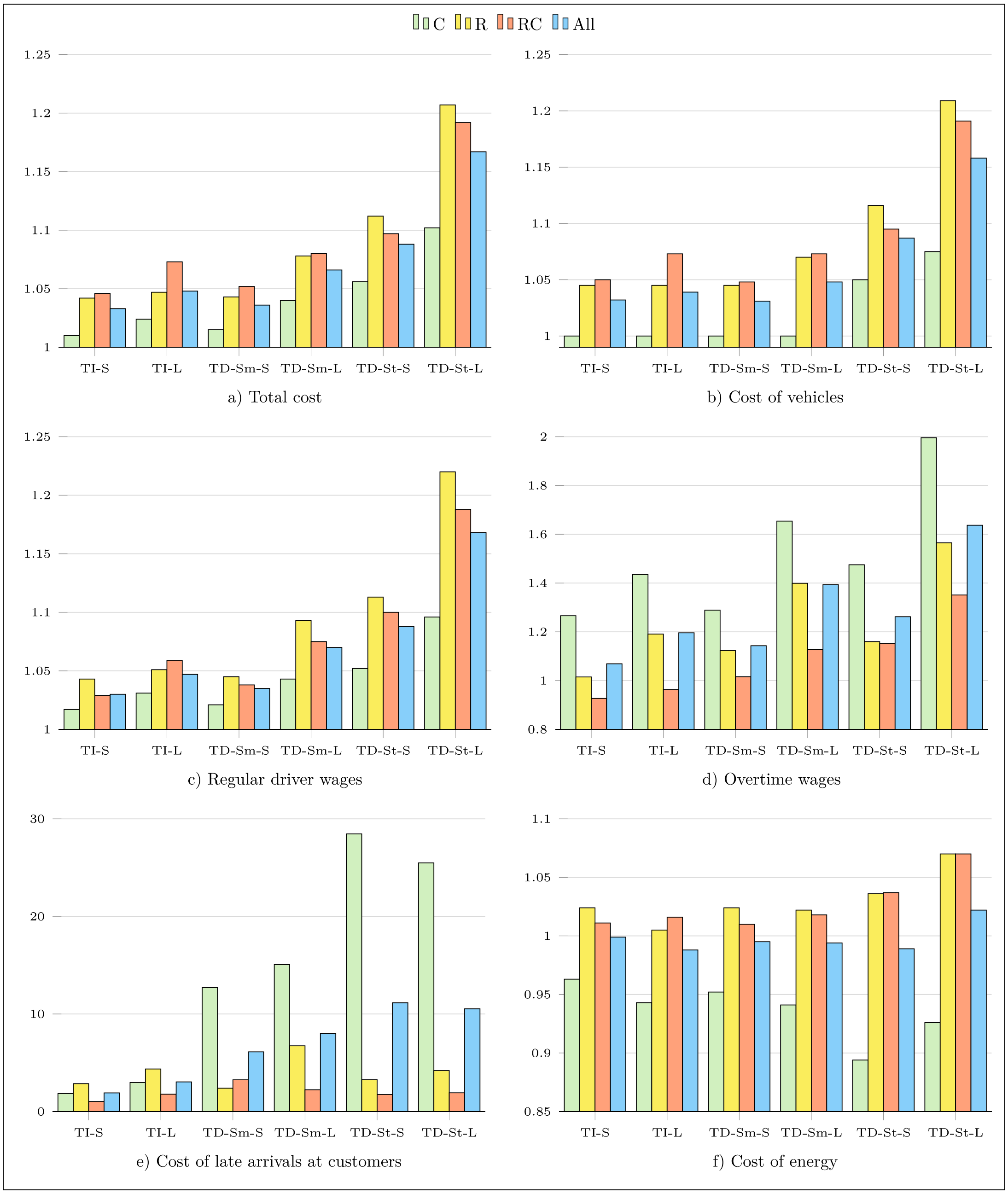

Fig. 8. Comparison of different cost components for different waiting schemes.

and RC instances, respectively, whereas the blue-colored columns show the overall averages. In Fig. 8(a)–(c) we see that waiting at the stations has a similar effect on the total cost, the cost of the vehicles, and the driver wages. On the other hand, Fig. 8(d)–(e) reveal that the overtime wages and cost of late arrivals at customers increase more significantly in the C instances compared with the other two instance types. This is because the fleet size does not increase dramatically in the C instances and the vehicles make longer tours which causes more time window violations. Finally, Fig. 8(f) shows that the cost of energy is relatively steady compared with the other cost components: the maximum increase is 7% in the worst case and it even decreases in certain scenarios. In particular, the reduced energy consumption, i.e. total traveled distance, in the C instances is rather surprising. To avoid waiting, planners avoid frequent recharges at the expense of increasing the fleet size. This, however, yields more compact tours and fewer visits between customers located in different cluster zones, hence this reduces the distance traveled.

Fig. 9 shows the percentage contribution of each cost component to the objective function for the no-waiting and the TD-St-L scenarios. We only report the results of these two extreme cases, but the distributions of the cost components exhibit similar patterns in the intermediate scenarios as well. In Fig. 10(a), we see that the vehicle cost is the major contributor to the total cost, followed by the driver wages in the R and RC instances. On the other hand, these two cost components are almost equal in the C instances. This difference is due to different time horizons in the data: the due date of the depot is about five times longer in the C instances. Since driver wages are proportional to the total travel time and the vehicles make longer tours, the total cost of wages is higher in the C instances. Fig. 9(b) depicts results similar to those of Fig. 9(a) in terms of the relationship between the cost

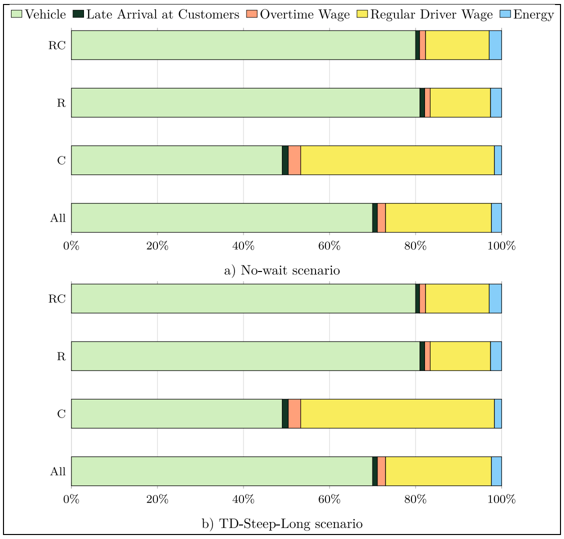

Fig. 9. Distribution of the cost components for no-wait and TD-St-L scenarios.

components. However, the shares of cost of late arrivals and overtime wages increase slightly because of the waiting times which cause more delays in arrivals.

5.3.2. The impact of waiting at the recharging stations on the decisions made in different time intervals

We now investigate how waiting at the recharging stations influences the recharging decisions made in different time intervals and the resulting cost behavior. Fig. 10 provides a temporal analysis using the average values over all instances. Fig. 10(a) shows that the vehicles do not recharge much during the morning and the evening. This is expected since they depart from the depot with full charge in the morning and they generally return to the depot without visiting any customer in the evening interval. The average number of recharges illustrated in Fig. 10(b) shows a similar behavior. However, we observe more frequent recharges in the morning when the waiting is time-dependent. When we compare the short and long waiting cases for each scenario, we see that the number of recharges and the total amount of energy charged decreases during the afternoon when the waiting times are more significant, and increases in the morning and late afternoon intervals when the waiting times are shorter. We also note that both the number of recharges and the amount of energy recharged en route are smaller in the long waiting cases than in the short cases. This unexpected result is due to the increased fleet size in the former case, as depicted in Figure 8(b). To avoid long waiting times at the stations and high costs associated with late arrivals at the customers, the algorithm tends to add more vehicles to the fleet. As a result, on average each vehicle makes fewer stops and needs less recharging en route to complete its route. These results are more apparent when we compare the results of the no-wait scenario to those of the TD-St-L scenario. Fig. 10(c) shows that substantial late arrival costs are incurred when there is more waiting at the stations. Since the vehicles rarely recharge in the morning, we observe almost no time window violations in this time interval. However, the violations are noticeable during the noon and late afternoon hours, particularly in the case of long waiting times. We also observe that some customers are served during the evening with a long delay in the TD-St-L scenario. Fig. 10(d) presents the total energy consumption (distance traveled) for each scenario. The total distance

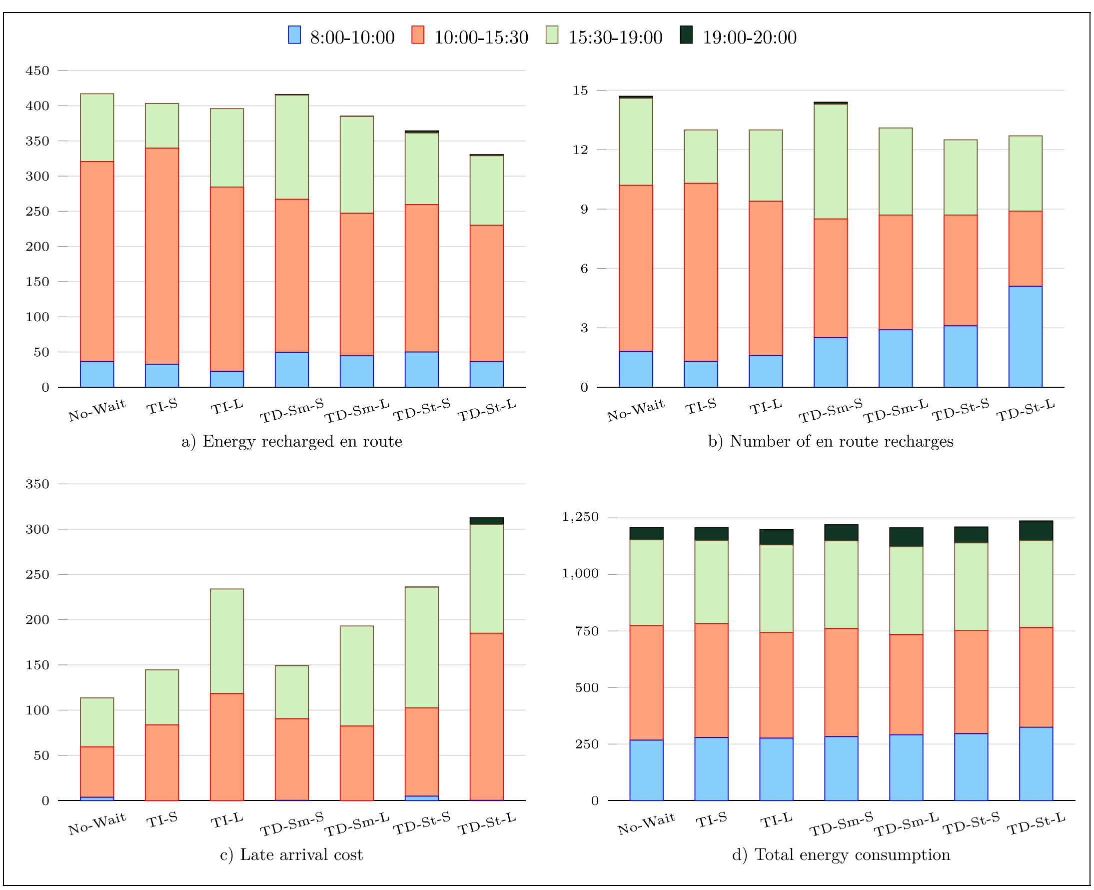

Fig. 10. Temporal analysis of recharging decisions and costs.

Table 8
Comparison of results for different late arrival penalty values.

	Penalty						
	1	5		50		100	
	Value	Value	%Δ	Value	%Δ	Value	%Δ
Total cost	24,610	25,603	4.03	26,511	7.72	26,766	8.76
Vehicle cost	12,214	13,029	6.67	13,757	12.63	13,886	13.68
# of vehicles	10.2	10.9	6.67	11.5	12.63	11.6	13.68
Late arrival cost	228	108	−52.36	28	−87.87	24	−89.5
Total lateness	228	22	−90.47	0.6	−99.76	0.2	−99.89
Overtime wage	627	564	−9.91	532	−15.15	532	−15.13
Driver wage	11,077	11,421	3.1	11,709	5.7	11,827	6.77
Energy cost	465	481	3.56	486	4.61	498	7.11

Table 9
Computation times for various waiting schemes (in minutes).

Data types	No-wait	TI-S	TI-L	TD-Sm-S	TD-Sm-L	TD-St-S	TD-St-L
C	99.2	124.5	108.0	111.1	119.2	126.7	132.8
R	95.2	84.9	84.2	104.0	93.6	102.7	91.0
RC	76.3	72.6	72.2	76.7	81.8	60.8	64.2
Average	90.2	94.0	88.1	97.3	98.2	96.7	96.0

traveled is not affected by the changing waiting times, but the solutions differ in the number of vehicles and recharge schedules.

5.3.3. Sensitivity of the solutions to the late arrival penalty

We observed in Fig. 8(e) that the cost of late arrivals at customers may increase dramatically in C-type instances. We therefore performed additional tests on these instances by increasing late arrival penalties five-, 50-, and 100-fold to investigate the sensitivity of the results. Table 8 reports the average cost values and the average percentage change in each cost component for each late arrival penalty setting. The columns "%Δ" report the percentage difference in each component compared with the base case where the unit late arrival penalty is one. While almost all cost components increase, there is a significant decrease in late arrival and overtime costs as well as in the total lateness. As the late arrival penalty increases, the fleet size also increases to avoid late arrivals at customers. As a result, overtime wages decrease as well.

5.3.4. Computational times

We now investigate the effect of different waiting scenarios on the computation times. Table 9 reports the average computational times of 10 runs for each scenario and data type, as well as the overall average of all type-1 data. The results reveal that the C instances require longer run times whereas RC instances are solved the fastest. Although the run time does not increase from short to long waiting cases, there is a noticeable rise from the no-wait to the TD-St-L scenario.

As can be seen from Table 9, the computational times are relatively long. We therefore performed a sensitivity analysis to investigate the effect of the number of iterations on solution quality. Table 10 provides the percentage deterioration in solution quality and the reduction in run time for different numbers of iterations.

Table 10
Sensitivity of solution quality and run time to number of iterations.

# of iterations	Deterioration	Relative run time
25,000	–	100.0%
20,000	0.1%	76.7%
15,000	0.5%	54.6%
10,000	2.5%	34.0%
5000	8.2%	14.8%

The results are obtained by using the average values of all type 1 data considering all scenarios. The first column gives the number of iterations which are the breakpoints for the comparison. The second column shows the percentage deterioration in the objective function value at that iteration compared with that of the final solution obtained after 25,000 iterations. For instance, the cost of best-found solution at iteration 5000 is 8.2% worse than the global best solution achieved after 25,000 iterations. The third column shows the percentage of the time spent at the corresponding breakpoint. For instance, the first 5000 iterations take 14.8% of the total run time of 25,000 iterations. These results indicate that the algorithm could have been stopped after 15,000 iterations with only a 0.5% worsening in solution quality, and saving nearly half of the computational time. However, since we aimed at obtaining best benchmark results at the expense of longer run times, we carried our detailed experimental study with 25,000 iterations.

5.3.5. Computations on R2 type instances

We used type 1 instances in our experiments since they have narrow time windows and we can better observe the effect of waiting times on routing decisions compared with the type 2 instances with wide time windows, where respecting the customer time windows is less of a concern. We also performed a limited set of experiments on R2 instances to gain some insights about the effect of the waiting times in such an environment. Table 11 presents a comparative analysis of results for R1- and R2-type data based on average cost figures across all waiting scenarios. Similar to our observations on the R1 instances, the total cost and its components in R2 instances increase when waiting is longer, both in the TI and TD settings, except for the cost of energy which does not display a discernible pattern. On the other hand, there are some fundamental differences between the two sets of results. First, the fleet size is not affected by waiting in the R2 instances and the same number of vehicles is used in all scenarios. We therefore conclude that the size of the customer time windows has a significant influence on the fleet size. Second, the costs of late arrivals and overtime wages are much smaller in the R2 instances than in the R1 instances in all scenarios. This can be considered as an expected consequence of the wider time windows. Even though late arrivals at customers and at the depot still exist, their magnitudes are negligible compared with those of the R1 instances. Finally, we see that the total travel distances are not affected by the size of the time windows as

Table 11
Comparison of results for R1 and R2 instances.

Type 1 instances	No-wait	TI-S	TI-L	TD-Sm-S	TD-Sm-L	TD-St-S	TD-St-L	Average
Total cost	18,499	19,039	19,323	19,119	19,647	20,054	21,427	19,587
# of vehicles	10.4	10.8	10.8	10.8	10.9	11.3	12.1	11
Late arrival cost	136.6	145.1	221.8	181.7	304.1	185.9	257	204.6
Overtime wage	287.1	304.4	336.5	322.8	390.8	348.6	428.4	345.5
Driver wage	5091	5205	5286	5232	5371	5440	5747	5339
Energy cost	484.8	484.2	478.7	482.8	481.2	479.1	495.2	483.7
Type 2 instances								
Total cost	6719	6738	6745	6763	6839	6775	7144	6818
# of vehicles	3	3	3	3	3	3	3	3
Late arrival cost	29.9	10.4	46.1	25.8	53.6	50.7	207.3	60.6
Overtime wage	150.1	138.8	143.5	149	155.6	156.1	225.3	159.8
Driver wage	2461	2501	2502	2530	2569	2530	2680	2539
Energy cost	478	488.6	453.6	458.8	461.2	438.4	432.1	458.7

the total energy cost figures for each scenario are similar in both data types.

6. Conclusions

We have introduced the Electric Vehicle Routing Problem with Soft Time Windows and Time-Dependent Waiting Times at Recharging Stations (TD-EVRPSTW), where recharging stations have limited capacities and EVs may queue before being serviced. We considered five time intervals in a day (morning, noon, late afternoon, evening, and night) with different queue lengths. We used the M/G/1 queueing system equations to estimate the waiting times in each time interval. The EVs are allowed to serve the customers beyond their late service time by paying a penalty proportional to the length of the delay. Similarly, the EVs may return to the depot later than the depot closing time by paying overtime wages to the drivers. The recharging time is a non-linear function of the energy transferred and is approximated using a piecewise linear function. We formulated this problem as a mixed integer linear program and developed a matheuristic that couples ALNS with an exact solver. The ALNS uses known operators with problem-specific modifications, as well as a new customer removal operator. The routes obtained by the ALNS are enhanced by optimizing the recharging-related decisions while keeping the sequence of the visited customers fixed.

To test the performance of our method we adapted benchmark instances by considering six scenarios with different waiting characteristics. On small-size instances our method outperformed CPLEX both in solution quality and computational time. Since we do not have any benchmark results for the large instances, we provided managerial insights based on the best solutions we achieved. The results showed that waiting times may be crucial in routing decisions and they should be taken into account to compute feasible and better route plans. The distribution of the cost components in the objective function remained similar from a scenario to another. When the waiting times increase, the number of vehicles also increases to avoid long routes and frequent visits to recharging stations. Similarly, station visits during the crowded time intervals decrease while the total distance traveled does not change much in all cases. The increase in unit late arrival costs results in a larger fleet size and in a higher total late arrival cost. Finally, the waiting times do not affect the solutions significantly when the customer time windows are wide.

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References

Afroditi, A., Boile, M., Theofanis, S., Sdoukopoulos, E., Margaritis, D., 2014. Electric vehicle routing problem with industry constraints: trends and insights for future research. Transp. Res. Procedia 3, 452–459.

Aksen, D., Kaya, O., Salman, F.S., Tüncel, O., 2014. An adaptive large neighborhood search algorithm for a selective and periodic inventory routing problem. Eur. J. Oper. Res. 239 (2), 413–426.

Andelmin, J., Bartolini, E., 2017. An exact algorithm for the green vehicle routing problem. Transp. Sci. 51 (4), 1288–1303.

Archetti, C., Speranza, M.G., 2014. A survey on matheuristics for routing problems. EURO J. Comput. Optim. 2 (4), 223–246.

Arslan, O., Yildiz, B., Karaşan, O.E., 2015. Minimum cost path problem for plug-in hybrid electric vehicles. Transp. Res. E Logist. Transp. Rev. 80, 123–141.

Black, D., Eglese, R., Wøhlk, S., 2015. The Time-Dependent Multiple-Vehicle Prize-Collecting Arc Routing Problem. Technical Report. Available at: http://eprints.lancs.ac.uk/73326

Breunig, U., Baldacci, R., Hartl, R.F., Vidal, T., 2018. The Electric Two-Echelon Vehicle Routing Problem. Technical Report. Available at: https://arxiv.org/pdf/1803.03628.pdf

Bruglieri, M., Mancini, S., Pezzella, F., Pisacane, O., 2016. A new mathematical programming model for the green vehicle routing problem. Electron. Notes Discrete Math. 55, 89–92.

Bruglieri, M., Mancini, S., Pezzella, F., Pisacane, O., 2019. A path-based solution approach for the green vehicle routing problem. Comput. Oper. Res. 103, 109–122.

Bruglieri, M., Mancini, S., Pezzella, F., Pisacane, O., Suraci, S., 2017. A three-phase matheuristic for the time-effective electric vehicle routing problem with partial recharges. Electron. Notes Discrete Math. 58, 95–102.

Bruglieri, M., Mancini, S., Pisacane, O., 2018. Solving the green vehicle routing problem with capacitated alternative fuel stations. In: Proceedings of 16th Cologne-Twente Workshop on Graphs and Combinatorial Optimization, Paris, France, pp. 196–199.

Bruglieri, M., Pezzella, F., Pisacane, O., Suraci, S., 2015. A variable neighborhood search branching for the electric vehicle routing problem with time windows. Electron. Notes Discrete Math. 47, 221–228.

Çimen, M., Soysal, M., 2017. Time-dependent green vehicle routing problem with stochastic vehicle speeds: an approximate dynamic programming algorithm. Transp. Res. D Transp. Environ. 54, 82–98.

Cho, D., Lee, W., Lee, Y.H., Gen M., T.Y., 2014. An adaptive genetic algorithm for the time dependent inventory routing problem. J. Intell. Manuf. 25 (5), 1025–1042.

Conrad, R.G., Figliozzi, M.A., 2011. The recharging vehicle routing problem. In: Doolen, T., Van Aken, E. (Eds.), Proceedings of the 2011 Industrial Engineering Research Conference, Reno, Nevada.

Coplon-Newfield, G., Park, S. J., 2017. Corporate fleets making the switch to electric vehicles. https://www.ecowatch.com/corporate-fleets-electric-vehicles-2499038220.html(last accessed 26 July 2018).

Dabia, S., Ropke, S., Van Woensel, T., De Kok, T., 2012. Branch and price for the time-dependent vehicle routing problem with time windows. Transportation science 47 (3), 380–396.

Demir, E., Bektaş, T., Laporte, G., 2012. An adaptive large neighborhood search heuristic for the pollution-routing problem. Eur. J. Oper. Res. 223 (2), 346–359.

Desaulniers, G., Errico, F., Irnich, S., Schneider, M., 2016. Exact algorithms for electric vehicle-routing problems with time windows. Oper. Res. 64 (6), 1388–1405.

Ding, N., Batta, R., Kwon, C., 2015. Conflict-Free Electric Vehicle Routing Problem with Capacitated Charging Stations and Partial Recharge. Technical Report. available at: http://www.acsu.buffalo.edu/~batta/Nan%20Ding.pdf

Emeç, U., Çatay, B., Bozkaya, B., 2016. An adaptive large neighborhood search heuristic for an e-grocery delivery routing problem. Comput. Oper. Res. 69, 109–125.

Erdoğan, S., Miller-Hooks, E., 2012. A green vehicle routing problem. Transp. Res. E Logist. Transp. Rev. 48 (1), 100–114.

Fairley, P., 2017. 10 Giant Companies Commit to Electric Vehicles, Sending Auto Industry a Message. Inside Climate News https://insideclimatenews.org/news/19092017/electric-cars-ev100-coalition-charging-fleet-ikea-dhl.

Felipe, A., Ortuño, M.T., Righini, G., Tirado, G., 2014. A heuristic approach for the green vehicle routing problem with multiple technologies and partial recharges. Transp. Res. E Logist. Transp. Rev. 71, 111–128.

Feng, W., Figliozzi, M., 2013. An economic and technological analysis of the key factors affecting the competitiveness of electric commercial vehicles: a case study from the USA market. Transp. Res. C Emerging Technol. 26, 135–145.

Fleischmann, B., Gnutzmann, S., Sandvoß, E., 2004. Dynamic vehicle routing based on online traffic information. Transp. Sci. 38 (4), 420–433.

Franceschetti, A., Demir, E., Honhon, D., Van Woensel, T., Laporte, G., Stobbe, M., 2017. A metaheuristic for the time-dependent pollution-routing problem. Eur. J. Oper. Res. 259 (3), 972–991.

Franceschetti, A., Honhon, D., Van Woensel, T., Bektaş, T., Laporte, G., 2013. The time-dependent pollution-routing problem. Transp. Res. B Methodol. 56, 265–293.

Froger, A., Mendoza, J.E., Jabali, O., Laporte, G., 2017. A Matheuristic for the Electric Vehicle Routing Problem with Capacitated Charging Stations. Technical Report. available at: https://hal.archives-ouvertes.fr/hal-01559524/document

Froger, A., Mendoza, J.E., Jabali, O., Laporte, G., 2019. Improved formulations and algorithmic components for the electric vehicle routing problem with nonlinear charging functions. Comput. Oper. Res. 104, 256–294.

Ichoua, S., Gendreau, M., Potvin, J.Y., 2003. Vehicle dispatching with time-dependent travel times. European journal of operational research 144 (2), 379–396.

Gendreau, M., Ghiani, G., Guerriero, E., 2015. Time-dependent routing problems: a review. Comput. Oper. Res. 64, 189–197.

Goeke, D., Schneider, M., 2015. Routing a mixed fleet of electric and conventional vehicles. Eur. J. Oper. Res. 245 (1), 81–99.

Grandinetti, L., Guerriero, F., Pezzella, F., Pisacane, O., 2016. A pick-up and delivery problem with time windows by electric vehicles. Int. J. Prod. Qual. Manage. 18 (2–3), 403–423.

Haghani, A., Jung, S.A., 2005. Dynamic vehicle routing problem with time-dependent travel times. Comput. Oper. Res. 32 (11), 2959–2986.

Hashimoto, H., Yagiura, M., Ibaraki, T., 2008. An iterated local search algorithm for the time-dependent vehicle routing problem with time windows. Discrete Optim. 5 (2), 434–456.

Heni, H., Renaud, J., Coelho, L.C., 2018. Time-Dependent Vehicle Routing Problem with Emission and Cost Minimization Considering Dynamic Paths. Technical Report. available at: http://www.cirrelt.ca/DocumentsTravail/CIRRELT-2018-14.pdf

Hiermann, G., Hartl, R.F., Puchinger, J., Vidal, T., 2018. Routing a mix of conventional, plug-in hybrid, and electric vehicles. Eur. J. Oper. Res. 272 (1), 235–248.

Hiermann, G., Puchinger, J., Ropke, S., Hartl, R.F., 2016. The electric fleet size and mix vehicle routing problem with time windows and recharging stations. Eur. J. Oper. Res. 252 (3), 995–1018.

Hof, J., Schneider, M., Goeke, D., 2017. Solving the battery swap station location-routing problem with capacitated electric vehicles using an AVNS algorithm for vehicle-routing problems with intermediate stops. Transp. Res. B Methodol. 97, 102–112.

Huang, Y., Zhao, L., Van Woensel, T., Gross, J.P., 2017. Time-dependent vehicle routing problem with path flexibility. Transp. Res. B Methodol. 95, 169–195.

Jabali, O., Van Woensel, T., de Kok, A., 2012. Analysis of travel times and CO_2emissions in time-dependent vehicle routing. Prod. Oper. Manage. 21 (6), 1060–1074.

Jie, W., Yang, J., Zhang, M., Huang, Y., 2019. The two-echelon capacitated electric vehicle routing problem with battery swapping stations: formulation and efficient methodology. Eur. J. Oper. Res. 272 (3), 879–904.

Jung, S., Haghani, A., 2001. Genetic algorithm for the time-dependent vehicle routing problem. Transp. Res. Rec. 1771, 164–171.

Keskin, M., Çatay, B., 2016. Partial recharge strategies for the electric vehicle routing problem with time windows. Transp. Res. C Emerging Technol. 65, 111–127.

Keskin, M., Çatay, B., 2018. A matheuristic method for the electric vehicle routing problem with time windows and fast chargers. Comput. Oper. Res. 100, 172–188.

Koç, C., 2016. A unified-adaptive large neighborhood search metaheuristic for periodic location-routing problems. Transp. Res. C Emerging Technol. 68, 265–284.

Koç, C., Jabali, O., Laporte, G., 2018. Long-haul vehicle routing and scheduling with idling options. J. Oper. Res. Soc. 69 (2), 235–246.

Koç, C., Karaoğlan, I., 2016. The green vehicle routing problem: a heuristic based exact solution approach. Appl. Soft Comput. 39, 154–164.

Kopfer, H., Vornhusen, B., 2017. Energy vehicle routing problem for differently sized and powered vehicles. J. Bus. Econ. doi:10.1007/s11573-018-0910-z.

Kullman, N.D., Goodson, J.C., Mendoza, J.E., 2018. Dynamic Electric Vehicle Routing: Heuristics and Dual Bounds. Technical Report. available at: https://hal.archives-ouvertes.fr/hal-01928730/document

Kumar, S.N., Panneerselvam, R., 2016. Development of an efficient genetic algorithm for the time dependent vehicle routing problem with time windows. Am. J. Oper. Res. 7, 1–25.

Leggieri, V., Haouari, M., 2017. A practical solution approach for the green vehicle routing problem. Transp. Res. E Logist. Transp. Rev. 104, 97–112.

Li, C., Ding, T., Liu, X., Huang, C., 2018. An electric vehicle routing optimization model with hybrid plug-in and wireless charging systems. IEEE Access 6, 27569–27578.

Liao, C.S., Lu, S.H., Shen, Z.J.M, 2016. The electric vehicle touring problem. Transportation Research Part B: Methodological 86, 163–180.

Li-Ying, W., Yuan-Bin, S., 2015. Multiple charging station location-routing problem with time window of electric vehicle. J. Eng. Sci. Technol. Rev. 8 (5), 190–201.

Lin, J., Zhou, W., Wolfson, O., 2016. Electric vehicle routing problem. Transp. Res. Procedia 12, 508–521.

Macrina, G., Di PugliaPugliese, L., Guerriero, F., Laporte, G., 2019. The green mixed fleet vehicle routing problem with partial battery recharging and time windows. Comput. Oper. Res. 101, 183–199.

Madankumar, S., Rajendran, C., 2018. Mathematical models for green vehicle routing problems with pickup and delivery: a case of semiconductor supply chain. Comput. Oper. Res. 89, 183–192.

Malandraki, C., Daskin, M., 1992. Time dependent vehicle routing problems: formulations, properties and heuristic algorithms. Transp. Sci. 26 (3), 185–200.

Masmoudi, M.A., Hosny, M., Demir, E., Genikomsakis, K.N., Cheikhrouhou, N., 2018. The dial-a-ride problem with electric vehicles and battery swapping stations. Transp. Res. E Logist. Transp. Rev. 118, 392–420.

Montoya, A., Guéret, C., Mendoza, J.E., Villegas, J.G., 2016. A multi-space sampling heuristic for the green vehicle routing problem. Transp. Res. C Emerging Technol. 70, 113–128.

Montoya, A., Guéret, C., Mendoza, J.E., Villegas, J.G., 2017. The electric vehicle routing problem with nonlinear charging function. Transp. Res. B Methodol. 103, 87–110.

Nahum, O.E., Hadas, Y., 2009. Developing a model for the stochastic time-dependent vehicle-routing problem. In: International Conference on Computers and Industrial Engineering, CIE 2009. Troyes, France. IEEE, pp. 118–123.

Norouzi, N., Sadegh-Amalnick, M., Tavakkoli-Moghaddam, R., 2017. Modified particle swarm optimization in a time-dependent vehicle routing problem: minimizing fuel consumption. Optim. Lett. 11 (1), 121–134.

Paz, J., Granada-Echeverri, M., Escobar, J., 2018. The multi-depot electric vehicle location routing problem with time windows. Int. J. Ind. Eng. Comput. 9 (1), 123–136.

Pelletier, S., Jabali, O., Laporte, G., 2016. Goods distribution with electric vehicles: review and research perspectives. Transp. Sci. 50 (1), 3–22.

Pelletier, S., Jabali, O., Laporte, G., Veneroni, M., 2017. Battery degradation and behaviour for electric vehicles: review and numerical analyses of several models. Transp. Res. B Methodol. 103, 158–187.

Potvin, J.Y., Xu, Y., Benyahia, I., 2006. Vehicle routing and scheduling with dynamic travel times. Comput. Oper. Res. 33 (4), 1129–1137.

Rabbani, M., Bosjin, S., Yazdanparast, R., Saravi, N., 2018. A stochastic time-dependent green capacitated vehicle routing and scheduling problem with time window, resiliency and reliability: a case study. Decis. Sci. Lett. 7 (4), 381–394.

Rincon-Garcia, N., Waterson, B., Cherrett, T.J., Salazar-Arrieta, F., 2018. A metaheuristic for the time-dependent vehicle routing problem considering driving hours regulations-an application in city logistics. Transp. Res. A Policy Pract.

Roberti, R., Wen, M., 2016. The electric traveling salesman problem with time windows. Transp. Res. E Logist. Transp. Rev. 89, 32–52.

Ropke, S., Pisinger, D., 2006a. An adaptive large neighborhood search heuristic for the pickup and delivery problem with time windows. Transp. Sci. 40 (4), 455–472.

Ropke, S., Pisinger, D., 2006b. A unified heuristic for a large class of vehicle routing problems with backhauls. Eur. J. Oper. Res. 171 (3), 750–775.

Sassi, O., Cherif, W.R., Oulamara, A., 2014. Vehicle Routing Problem with Mixed Fleet of Conventional and Heterogenous Electric Vehicles and Time Dependent Charging Costs. Technical Report. available at: https://hal.archives-ouvertes.fr/hal-01083966/

Schiffer, M., Walther, G., 2017. The electric location routing problem with time windows and partial recharging. Eur. J. Oper. Res. 260 (3), 995–1013.

Schiffer, M., Walther, G., 2018. Strategic planning of electric logistics fleet networks: a robust location-routing approach. Omega 80, 31–42.

Schilde, M., Doerner, K.F., Hartl, R.F., 2014. Integrating stochastic time-dependent travel speed in solution methods for the dynamic dial-a-ride problem. Eur. J. Oper. Res. 238 (1), 18–30.

Schneider, M., Stenger, A., Goeke, G., 2014. The electric vehicle-routing problem with time windows and recharging stations. Transp. Sci. 48 (4), 500–520.

Solomon, M.M., 1987. Algorithms for the vehicle routing and scheduling problems with time window constraints. Operations research 35 (2), 254–265.

Soler, D., Albiach, J., MartíNez, E., 2009. A way to optimally solve a time-dependent vehicle routing problem with time windows. Operations Research Letters 37 (1), 37–42.

Soysal, M., Bloemhof-Ruwaard, J.M., Bektaş, T., 2015. The time-dependent two-echelon capacitated vehicle routing problem with environmental considerations. Int. J. Prod. Econ. 164, 366–378.

Spliet, R., Dabia, S., Van Woensel, T., 2017. The time window assignment vehicle routing problem with time-dependent travel times. Transp. Sci. 52 (2), 261–276.

Suizo, G., Large & recharged: Top electric vehicles, https://www.greenfleetmagazine.com/155563/large-recharged-top-electric-fleets, 2013 (Last accessed 09.03.2019).

Sun, P., Veelenturf, L.P., Hewitt, M., Van Woensel, T., 2018a. The time-dependent pickup and delivery problem with time windows. Transp. Res. B Methodol. 116, 1–24.

Sun, P., Veelenturf, L.P., Dabia, S., Van Woensel, T., 2018b. The time-dependent capacitated profitable tour problem with time windows and precedence constraints. Eur. J. Oper. Res. 264 (3), 1058–1073.

Suzuki, Y., 2014. A variable-reduction technique for the fixed-route vehicle-refueling problem. Comput. Ind. Eng. 67, 204–215.

Sweda, T.M., Dolinskaya, I.S., Klabjan, D., 2017. Adaptive routing and recharging policies for electric vehicles. Transp. Sci. 51 (4), 1326–1348.
Taş, D., Dellaert, N., Van Woensel, T., De Kok, T., 2013. Vehicle routing problem with stochastic travel times including soft time windows and service costs. Comput. Oper. Res. 40 (1), 214–224.
Taş, D., Dellaert, N., van Woensel, T., De Kok, T., 2014. The time-dependent vehicle routing problem with soft time windows and stochastic travel times. Transp. Res. C Emerging Technol. 48, 66–83.
Taş, D., Gendreau, M., Jabali, O., Laporte, G., 2016. The traveling salesman problem with time-dependent service times. Eur. J. Oper. Res. 248 (2), 372–383.
U.S. Energy Information Administration, https://www.eia.gov (last accessed: 26 July 2018).
Van Woensel, T., Kerbache, L., Peremans, H., Vandaele, N., 2008. Vehicle routing with dynamic travel times: a queueing approach. Eur. J. Oper. Res. 186 (3), 990–1007.
Verbeeck, C., Sörensen, K., Aghezzaf, E.H., Vansteenwegen, P., 2014. A fast solution method for the time-dependent orienteering problem. Eur. J. Oper. Res. 236 (2), 419–432.
Verbeeck, C., Vansteenwegen, P., Aghezzaf, E.H., 2016. Solving the stochastic time-dependent orienteering problem with time windows. Eur. J. Oper. Res. 255 (3), 699–718.
Villegas, J., Guéret, C., Mendoza, J.E., Montoya, A., 2018. The Technician Routing and Scheduling Problem with Conventional and Electric Vehicle. Technical Report. available at: https://hal.archives-ouvertes.fr/hal-01813887/document
Wang, Y.W., Lin, C.C., Lee, T.J., 2018. Electric vehicle tour planning. Transp. Res. D Transp. Environ. 63, 121–136.
Wen, L., Eglese, R., 2015. Minimum cost VRP with time-dependent speed data and congestion charge. Comput. Oper. Res. 56, 41–50.
Wen, M., Linde, E., Ropke, S., Mirchandani, P., Larsen, A., 2016. An adaptive large neighborhood search heuristic for the electric vehicle scheduling problem. Comput. Oper. Res. 76, 73–83.
Yang, J., Sun, H., 2015. Battery swap station location-routing problem with capacitated electric vehicles. Comput. Oper. Res. 55, 217–232.
Zhang, D., Wang, X., Li, S., Ni, N., Zhang, Z., 2018a. Joint optimization of green vehicle scheduling and routing problem with time-varying speeds. PloS One 13 (2), 1–20.
Zhang, S., Gajpal, Y., Appadoo, S.S., Abdulkader, M.M.S., 2018b. Electric vehicle routing problem with recharging stations for minimizing energy consumption. Int. J. Prod. Econ. 203, 404–413.
Zhang, T., Chaovalitwongse, W.A., Zhang, Y., 2014. Integrated ant colony and tabu search approach for time dependent vehicle routing problems with simultaneous pickup and delivery. J. Combin. Optim. 28 (1), 288–309.
Zhang, X., Yao, J., Liao, Z., Li, J., 2018c. The electric vehicle routing problem with soft time windows and recharging stations in the reverse logistics. In: International Conference on Management Science and Engineering Management. Springer, Cham, pp. 171–182.