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A Fuel Efficient Green Vehicle Routing Problem with varying speed constraint (F-GVRP)

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ABSTRACT

A bi-objective Fuel efficient Green Vehicle Routing Problem (F-GVRP) with varying speed constraint is discussed in this paper as an extension of Green Vehicle Routing Problem (G-VRP). F-GVRP is modelled to minimize both route cost and fuel consumption using goal programming. The problem is solved using Particle Swarm Optimization with Greedy Mutation Operator and Time varying acceleration coefficient (TVa-PSOGMO). The objective of this paper is to study the behaviour of F-GVRP under varying speed environment and its impact on the route cost and fuel consumption. Experiments are conducted with constant and varying speed constraints and it is observed that better routing plan with minimum fuel consumption can be achieved under varying speed environment.

1. Introduction

Vehicle Routing Problem (VRP) is an important combinatorial optimization problem in distribution logistics. Green logistics is routing vehicles with a concern towards environment. Recently, several routing and scheduling of VRP emulate that are of economic, social, and environmental importance. Research on green routing is gaining importance due to its impact on the environment and on the society. This motivated us to model a VRP with environmental concern that aims to reduce fuel consumption which is an important parameter in Green House Gas (GHG) emission.

Green Vehicle Routing Problem (G-VRP) was proposed by Erdogan and Miller-Hooks (2012). The objective of the problem is to devise low cost route for a set of homogenous vehicles stationed at a depot. Each vehicle takes a tour serving a set of geographically distributed customers with limited fuel capacity and time. A vehicle can refuel in a set of refuelling stations and can continue the trip within the time limit. To have environment friendly routing, the refuelling stations are replaced by Artificial Fuelling Station (AFS).

The proposed model addresses the green objective of environment friendly routing as minimal fuel consumption leads to minimal GHG emission, carbon-di-oxide in particular. Generally, vehicles travel within a maximum and a minimum speed rather than travelling at a constant speed. Hence, the behaviour of F-GVRP is studied under varying speed environment and is simulated using triangular distribution. To the best of our knowledge, the impact of varying speed on fuel consumption minimization for G-VRP is not available in literature.

There are some considerable differences between G-VRP and the proposed F-GVRP. G-VRP is a single objective optimization problem that aims to reduce the overall route cost but F-GVRP is a bi objective optimization problem that aims to minimize both route cost and fuel consumption. In G-VRP, vehicle speed and fuel consumption rate are kept constant, but in the proposed method, speed is a varying entity and hence, fluctuations arise in determining the fuel consumption rate which is not constant. G-VRP is solved using Modified Clarke and Wright Savings Algorithm (MCWS) and Density Based Clustering Algorithm (DBCA) and F-GVRP is solved using Particle Swarm Optimization with Greedy Mutation Operator along with Time Varying acceleration (TVa-PSOGMO).

2. Literature review

Many models under VRP are studied since its inception by Dantzig and Ramser (1959). Literature on VRP mostly span around some traditional variants like Capacitated VRP (CVRP), VRP with Pickup and Delivery (VRPPD), VRP with Time Windows (VRPTW), Stochastic VRP (SVRP) etc., The literature on VRP is rich with many exact and heuristic approaches for solving variants on VRP.

VRP continues to be an active area of research because of its practical relevance and considerable difficulty. Apart from traditional VRP, several variants exist with the introduction of practical constraints associated with real world problems. Although many

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variants on VRP exist, this literature reviews various models on green VRP.

Literature on vehicle routing problems that halts for refuelling at fuel stations or recharging their batteries at re-charging stations are limited in comparison with other traditional variants. VRP with a limitation on the capacity of vehicle was addressed by Bard, Huang, Jaillet, and Dror (1998) where the vehicle stops at satellite facilities to reload their capacity and continue the tour. Erdogan and Miller Hooks (2012) formulated a Green Vehicle Routing Problem (G-VRP) where the refuelling stations are modelled with environment friendly fuel like biogas and the vehicle halts for re fuelling with the idea conceived from Bard et al. (1998). They solved the problem using Density Based Clustering Algorithm (DBCA) and Modified Clarke and Wright Savings (MCWS) algorithm. Schneider, Stenger, and Goeke (2014) extended G-VRP and discussed Electric Vehicle Routing Problem with Time Windows (E-VRPTW) and recharging stations. They solved the problem using Tabu search with variable neighbourhood algorithm and got better results than Erdogan and Miller Hooks (2012). Later, Schneider, Stenger, and Hof (2015) developed Vehicle Routing Problem with Intermediate Stops (VRP-IS) and solved the problem using adaptive variable neighbourhood search algorithm. Felipe, Ortuño, Righini, and Tirado (2014) have studied a variation of Electric VRP (E-VRP) which includes partial recharges with several recharging technologies. They determined the amount of energy recharged and solved the problem using constructive and local search heuristics within a simulated annealing framework. Poonthalir, Nadarajan, and Geetha (2015) discussed a bi objective vehicle routing problem with limited refuelling halts, where the number of halts made by vehicles at refuelling station is minimized along with route cost. They solved the problem using Particle Swarm Optimization with Greedy Mutation Operator (PSO-GMO).

A simulated annealing based exact solution approach using branch and cut technique (Koc & Karaoglan, 2016) was used to solve G-VRP. An electric vehicle routing problem that considers vehicle load on battery consumption was studied by Lin, Zhou, and Wolfson (2016). A multi space sampling heuristic for G-VRP was solved by Montoya, Guéret, Mendoza, and Villegas (2016). They used a two phase approach that builds the routes using randomized travelling salesman heuristic and in the second phase, they solved it by formulating a set partitioning problem of the routes. The model concentrated in reducing the length of route and CO2 emission. The formulation of G-VRP as proposed by Erdogan and Miller Hooks (2012) includes multiple copies of AFS, to enable each visit to the refuelling station. This increases the complexity of the problem. Bruglieri, Mancini, Pezzella, and Pisacane (2016) gave an alternate formulation which prevents the creation of multiple copies of AFS which eventually reduces the use of number of variables. Their model includes a pre computation of AFS to be included between each pair of customers.

An exact algorithm to solve G-VRP (Andelmin & Bartolini, 2017) was formulated as a set partitioning problem where columns represented the feasible routes like simple circuits in multiple graphs and several valid inequalities were added. Computational experiments were carried out on G-VRP test instances. Leggieri and Haouari (2017) gave a non linear formulation for time and energy consumption constraints of G-VRP. An MILP formulation of the problem is proposed by linearizing the constraints using Reformulation-Linearization technique. Their approach reduced the use of number of variables and constraints and included a set of pre processing conditions which made the problem to be efficiently solvable using commercial solvers. A path based Mixed Integer Linear Programming formulation was done for G-VRP by Pisacane, Bruglieri, Mancini, and Pezzella (2017). They generated all feasible routes and eliminated all dominated paths from the feasible set which is given as an input to a set partitioning formulation that gives solution to the problem. When vehicle equipped with artificial fuel need to take long distance tour, it has to halt for refuelling in alternate fuelling station. When AFS are limited, a vehicle has to take up a route forcefully to refuel in the limited number of AFS which can increase the route cost. This problem can be tackled if vehicle can switch between battery and fuel depending on the requirements as proposed by Mancini (2017) in hybrid VRP. An MILP formulation of the problem is carried out and is solved using a large neighbourhood search based matheuristic.

VRP that concentrates in minimizing GHG emission and their modelling is also discussed under Green VRP in literature. A fuel emission minimization model that concentrated in reducing emission for solid waste collection trucks was done by Apaydin and Gonullu (2008). Emission minimization VRP was proposed by Figliozzi (2010) that concentrated in routing vehicles with less emission where emission minimization is taken as an additional objective. Bektas and Laporte (2011) developed a Pollution Routing Problem (PRP) which is an extension of VRPTW that considers both load and speed as major factors in emission and proposed a non linear mixed integer programming model to solve the problem. Later, Demir, Bektaş, and Laporte (2012) solved the PRP using an Adaptive Large Neighbourhood Search (ALNS) heuristic to minimize fuel consumption, emission and driver cost. The algorithm for solving PRP iterates between solving VRPTW and speed optimization. Initial routes are formed for VRPTW using fixed speed and are improved using a speed optimization algorithm. They further extended the work to solve PRP as a bi objective optimization problem (Demir, Bektaş, & Laporte, 2014a) that concentrated in arriving at a trade off between the conflicting objectives of fuel consumption and total driving time. The problem is modelled with four different *a posteriori* methods using an enhanced ALNS procedure where speed optimization is done at each iteration. They have used a comprehensive emission model to estimate the fuel consumption.

Pitera, Sandoval, and Goodchild (2011) formulated an emission minimization routing decision for urban pick up system with heterogeneous fleet. Their study demonstrated a significant improvement in emission reduction. A bi objective GVRP using NSGA II was formulated by Jemai, Zekri, and Mellouli (2012) with objectives to minimize distance and CO2 emission. A green vehicle distribution model for public transport network was developed as a non linear optimization problem by Jovanović, Pamučar, and Pejčić-Tarle (2014). They designed an Adaptive Neuro Fuzzy Inference System (ANFIS) model with environmental parameters, passenger costs, and their impact on green routing to solve the problem. Ćirović, Pamučar, and Božanić (2014) concentrated in routing light delivery vehicles using adaptive neural network that concentrated in reducing air pollution, noise level, and logistics operating costs. They concentrated in routing limited Environmentally Friendly Vehicles (EFV) and Unfriendly Vehicles (EUV) separately. The input parameters for neural network were logistics operating costs and environmental parameters which were used to assess the performance of network links for calculating routes and then a Clarke and Wright Savings algorithm was used.

Xiao and Konak (2015) presented a time dependent heterogeneous green vehicle and scheduling problem, with the objectives to minimize CO2 emission and weighted tardiness. Travel schedules of vehicles were determined using travelled distance in different time periods and solved using dynamic programming approach. A Satisfactory-GVRP was developed by Afshar-Bakeshloo, Mehrabi, Safari, Maleki, and Jolai (2016) as an extension of pollution routing problem that takes into account the economic, environmental, and customer satisfaction as objectives. A periodic G-VRP with time dependent urban traffic and time window was studied by Mirmohammadi, Babaee Tirkolaee, Goli, and Dehnavi-Arani (2017). They modelled the problem to minimize carbon emission, earliness and lateness penalties and demonstrated their results for some generated test problem instances. A detailed review on various green VRP can be referred from Lin, Choy, Ho, Chung, and Lam (2014).

Fuel consumption act as an important parameter for CO2 emission and several models were proposed to calculate the fuel consumption and its minimization in vehicles as the amount of fuel consumption is directly proportional to the amount of CO2 emission. As stated in Demir et al. (2014a), fuel consumption estimation can be done based on the various estimation models available in literature or using the factors that affects the fuel consumption like vehicle load, speed etc., Bektas and Laporte (2011), Demir et al. (2012), Demir et al. (2014b) used comprehensive emission model for fuel consumption estimation, Scott, Urquhart, and Hart (2010) and Jovici et al. (2010) used COmputer Program to calculate Emission from Road Transport (COPERT) to estimate fuel consumption, Jabali, Woensel, and De Kok (2012) and Pan, Ballot, and Fontane (2013) have used Methodology for calculating Transport Emissions and Energy consumption (MEET) to derive an estimation and so on. A detailed report on various fuel consumption models can be found in Demir et al. (2014b) and Bektas, Demir, and Laporte (2016).

Fuel consumption estimation is also done based on several influential parameters of vehicles like vehicle load, speed or travel time. Kuo (2010) used a string based Simulated Annealing to minimize fuel consumption for time dependent vehicle routing problem. They solved the problem for retail stores. Their calculation consists in formulating a route, calculating the transportation time and finding the total fuel consumption for a time dependent VRP. This work is based on the fuel consumption estimation used by Kuo (2010). Suzuki (2011) studied a time constrained, multi stop truck routing problem to minimize fuel consumption and pollutant emission. Xiao, Zhao, Kaku, and Xu (2012) proposed a fuel consumption model which produces better routes with less fuel consumption for Time Dependent VRP (TDVRP). They used a regression model to estimate the fuel consumption. Their results demonstrated an improvement of 24.61% in fuel consumption with better routing plan, lower fuel consumption but longer transportation time and distance.

Kara, Kara, and Yetis (2007) solved an energy minimizing vehicle routing problem which concentrated in minimizing the weighted load function which minimizes the fuel consumption. Li (2012) concentrated in developing a model to minimize fuel consumption in VRPTW. They solved the problem using Tabu search with random variable neighbourhood decent procedure. Teng and Zhang (2016) solved a green vehicle routing problem with load factor that concentrated in minimizing total fuel cost and driving distance and solved using simulated annealing. Jabir, Panicker, and Sridharan (2017) developed a hybrid ant colony optimization for capacitated multi depot green vehicle routing problem with an objective to reduce emission cost. Kazemian and Aref (2017) proposed a capacitated time dependent model on VRP which aims to minimize both fuel consumption and fuel emission. This work is based on the fuel consumption estimation used by Kuo (2010).

3. Problem definition and formulation

This section describes the modelling of F-GVRP as a bi objective optimization problem using goal programming and the simulation of varying speed constraint using triangular distribution.

3.1. Multi objective optimization

A general multi objective optimization deals with more than one objective where a single objective is not possible to solve the problem with many constraints. Mathematically it is defined as in Eq. (1),

$$\begin{aligned} &\min(f_1(x), f_2(x), \ldots, f_k(x)) \\ &\text{such that} \\ &x \in X \end{aligned} \tag{1}$$

Where $k \geq 2$ defines the number of objectives. X represents a set of feasible decision vectors. Since, single objective function value cannot satisfy all the objective function, the concept of Pareto dominance function is used.

A feasible solution x is said to be Pareto dominance on another solution y when $f_i(x) \leq f_i(y)\ \forall i$, i ∈ {1, 2,..., K} and $f_j(x) < f_j(y)$ for at least one index j ∈ {1, 2,..., K}. A solution $x^* \in X$ and the corresponding $f(x^*)$ is called Pareto optimal, if another solution dominating it does not exist.

3.2. Goal programming

F-GVRP is a bi objective optimization problem and is modelled using Goal Programming (GP) approach. GP is a powerful method for solving multi objective optimization problem. It is one of the oldest multi criteria decision making techniques to optimize several goals and at the same time minimize the deviation for each of the objectives from the desired target. It is a branch of multi criteria decision analysis. It aims to solve problems with multiple objectives. Each of the objectives is designed with a goal to achieve and aims to seek a solution that is close to the goal as possible. GP was first used by Charnes and Cooper (1977). It is an analytical framework where a solution that optimizes the multiple objectives is attained. The procedure used in GP strives to produce solutions that satisfies the given constraints and are in the satisfactory level. The goal of GP is to obtain a solution with minimum deviation and with hierarchical satisfaction of the objectives. The modelling approach of goal programming is intended to maximize or minimize the desired goal but seek to minimize the deviations between the obtained value and the desired goals. F-GVRP is modelled on two goals where Goal1 ($G1$) is to minimize route cost and Goal2 ($G2$) is used to minimize fuel consumption.

Goal Programming in VRP was used by Ghoseiri and Ghannadpour (2010) for minimizing both distance and total number of vehicles in VRP with Time Windows (VRPTW), Calvete, Galé, Oliveros, and Sánchez-Valverde (2007) applied goal programming to solve single objective VRP with soft Time Windows.

3.3. Triangular distribution

Triangular distribution is used to design the varying speed constraint. A triangular distribution is a continuous probability distribution with the lowest possible value a, the highest possible value b and most likely value c where $a < b$ and $a \leq c \leq b$. It's probability distribution function is shaped like a triangle. It is a useful tool when a variable is to be estimated subjectively. The expected value of a triangular distribution is one third of the sum of the three parameters. It is represented in Fig. 1.

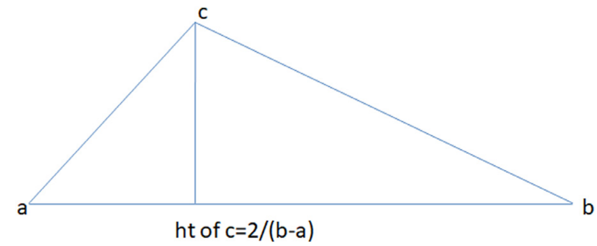


Fig. 1. Triangular distribution.

For the triangular distribution, the probability density function (*pdf*) for values less than a and greater than b is 0. For the values between a and b, it is a piecewise linear function rising from 0 to $2/(b-a)$ and dropping down to 0 at b from c.

The *pdf* $f(x)$ is defined in Eq. (2).

$$f(x) = \begin{cases} 0 & x < a \\ \frac{2(x-a)}{(b-a)(c-a)} & a \le x \le c \\ \frac{2(b-x)}{(b-a)(b-c)} & c \le x \le b \\ 0 & x > b \end{cases} \quad (2)$$

Triangular distribution is similar to normal distribution as both the distributions raises to some point and falls. But, triangular distribution is more flexible and intuitive than normal distribution. A triangular distribution is also skewed if the value of c is closer to either a or b. It is mostly used for stochastic modelling rather than statistical modelling.

In real time, a vehicle is not allowed to travel at a constant speed and is likely to travel within a maximum and minimum speed limit ϑ_{max} and ϑ_{min} respectively. This is better represented using triangular distribution as vehicle speed fluctuates between a minimum and a maximum speed and may travel with some most likely speed.

To know the probability distribution of an area, Eq. (3) is used which returns the expected value of the speed.

$$E(X) = \int_p^q x.f(x)dx \quad (3)$$

Assume that the speed of the vehicle fluctuates between 30 and 60, and the speed is taken in an interval of 10 from 30 to 60 as [30 40 50 60] where the expected speed within each interval needs to be calculated and the average of these expected speed is the speed taken by the vehicle within the interval 30–60. To calculate the expected speed of the vehicle between the speed range 30 and 40, let a to be the initial speed limit 30, b be the maximum speed limit 40 and let 37 be the most likely speed limit which is taken at random between 30 and 40 or it can be a mean value. Then, the expected speed $E(X)$ is calculated as,

$$\begin{aligned} E(X) &= \int_a^b x f(x) d(x) \\ &= \int_{30}^{40} x f(x) d(x) \\ &= \int_{30}^{37} x \frac{2(x-30)}{(40-30)(37-30)} dx + \int_{37}^{40} x \frac{2(40-x)}{(40-30)(40-37)} dx \\ &= 32 \text{ miles/gallon} \end{aligned}$$

and the corresponding triangular distribution is given in Fig. 2. Similarly, the expected speed is calculated for the remaining speed intervals and the average expected speed is taken.

3.3.1. Fuel consumption calculation for varying speed in F-GVRP using triangular distribution

This section describes the fuel consumption calculation for the vehicles that travel from node i to node j. In real life, it is not possible for vehicles to travel with constant speed. Given a minimum speed limit ϑ_{min} and maximum speed limit ϑ_{max}, a vehicle may travel with varying speed within ϑ_{min} and ϑ_{max}. To portray this, a triangular distribution is used. If a vehicle travels with constant speed, fuel consumption will be a constant. Since varying speed is used, the fuel consumption is calculated as explained below and is based on the work of Kuo (2010).

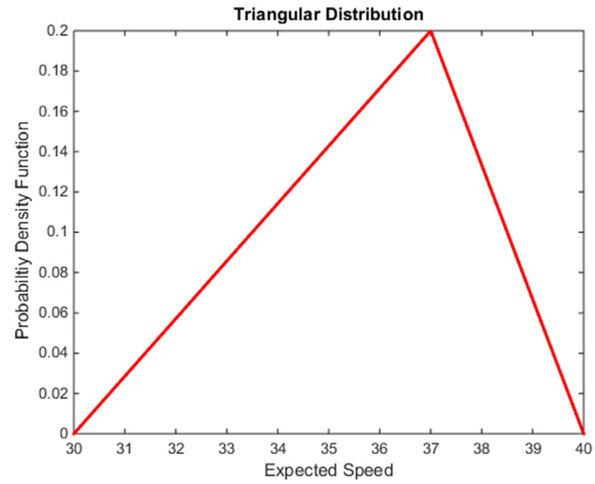


Fig. 2. Triangular distribution.

Let the average expected speed between two nodes i and j is defined as as_{ij}. The total Distance travelled Per Gallon is given by DPG_{ij}, then the fuel consumption in gallons per hour *(fgph)* is calculated as in Eq. (4),

$$fgph_{ij} = as_{ij}/DPG_{ij} \quad (4)$$

If the total distance travelled is d_{ij} with average expected speed as_{ij}, then the fuel consumption FC_{ij} between the nodes i and j is as in Eq. (5),

$$FC_{ij} = fgph_{ij}{}^{*}d_{ij}/as_{ij} \quad (5)$$

3.3.2. Fuel consumption calculation with an example

The fuel consumption calculation defined above is explained with an example.

Average expected speed between i and j = 38 miles/h
Total distance travelled = 160 miles.
Total miles travelled per gallon = 6 miles/gallon
Fuel consumption in gallons per hour = 38/6 = 6.33 gallons/h
Total time travelled = total distance/average expected speed

$$\begin{aligned} &= d_{ij}/as_{ij} \\ &= 160/38 = 4.21 \text{ h} \end{aligned}$$

Total fuel consumption = 6.3*4.21 = 26.523 gallons

The expected speed between any two nodes whose distance is 160 miles is calculated using triangular distribution. The total fuel consumed varies for each part of the route and hence, the total fuel consumption will not be uniform, which predicts the behaviour of vehicles in real time.

3.4. Bi objective optimization model of F-GVRP using goal programming

F-GVRP is defined on an undirected connected graph $G=(V, E)$ where the vertex set V has a set of N customer nodes $C=\{C_1,C_2,..., C_N\}$, M Refuelling Stations (*RFS*) $R=\{R_1,R_2,..., R_M\}$ and depot (V_0). The vertex set V is defined as a combination of $V=C\cup\{V_0\}$. There is a limited number of *RFS* available with unlimited capacity of fuel. A set R' is used to represent several visits to each *RFS* to enable multiple visits to refuelling station R. Hence, the set of vertices is represented by $V=V\cup R'$. The depot V_0 has a set of homogenous vehicles K, with fuel capacity$Q$ gallons. Each vertex $i \in V$ has a service time ρ_i which is the time a vehicle spends at customer location or at refuelling station. The depot can also be served as a refuelling station. Each vehicle is refuelled to its maximum capacity Q. T is the maximum time allocated for each vehicle.

An edge corresponds to an arc between i and j where $i, j \in V$ and $i \neq j$. Each edge has a measure of distance d_{ij} and time t_{ij}. Each vehicle is allowed to serve a set of customers and a customer is served once by one vehicle only. It is not possible for vehicles to travel with uniform speed for the entire distance. Due to a range of speculation about traffic, road repair or rain, vehicles may have varying speed within a specified upper bound.

To formulate the problem, the following notations are used and are defined as follows,

d_{ij}	distance between node i and node j
t_i	travel time till node i
μ_{ij}	travel time between nodes i and j
T	maximum time for each vehicle
ρ_i	service time at node i where $i \in C \cup R'$
r	fuel consumption rate in miles per gallon (mpg)
as_{ij}	expected average speed between nodes i and j
f_i	remaining fuel at node i
$fgph_{ij}$	fuel consumption in gallons per hour between nodes i and j

The objective is to minimize

$$\min \sum_{i,j \in V, k \in K, i \neq j} d_{ij} x_{ijk} \leq G1 \tag{6}$$

$$\min \sum_{i,j \in V, i \neq j} \frac{d_{ij}}{as_{ij}} * fgph_{ij} * y_{ij} \leq G2 \tag{7}$$

Where

$$x_{ijk} = \begin{cases} 1 & \text{if vehicle k travels from i to j} \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

$$y_{ij} = \begin{cases} 1 & \text{if route exists between i and j} \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

The objective function defined in (6) is to minimize route cost with target defined as $G1$ and the objective function defined in (7) aims to reduce fuel consumption with target as $G2$. Eqs. (8) and (9) specify the decision variables. As goal programming does not maximize or minimize the objective function directly as in linear programming problem, it seeks to minimize the deviation between the goals and the obtained solution. Hence, the objective functions (6) and (7) are written as,

$$\sum_{i,j \in V, k \in K, i \neq j} d_{ij} x_{ijk} - w1 + v1 = G1$$
$$\sum_{i,j \in V, i \neq j} \frac{d_{ij}}{as_{ij}} * fgph_{ij} * y_{ij} - w2 + v2 = G2$$

Where $w1$, $w2$, $v1$ and $v2$ are non negative deviational variables and $w1$, $w2$, $v1$, $v2$ $\geq$ 0, and are used to measure the deviation between the set goals. Then, the objective is to minimize the deviation given as,

$$Min\ \mathrm{w1} \tag{10}$$

$$Min\ \mathrm{w2} \tag{11}$$

Subject to,

$$\sum_{i,j \in V, k \in K, i \neq j} d_{ij} x_{ijk} - w1 + v1 = G1 \tag{12}$$

$$\sum_{i,j \in V, i \neq j} \frac{d_{ij}}{as_{ij}} * fgph_{ij} * y_{ij} - w2 + v2 = G2 \tag{13}$$

$$\sum_{i \in C, i \neq j} x_{ijk} = 1 \ \forall \mathrm{j} \in V \ \mathrm{k} \in \mathrm{K} \tag{14}$$

$$\sum_{j \in V, i \neq j} x_{ijk} \leq 1 \ \forall \mathrm{i} \in \mathrm{R}' \ \mathrm{k} \in \mathrm{K} \tag{15}$$

$$\sum_{i \in V, l \neq i} x_{ilk} - \sum_{j \in V, l \neq j} x_{ljk} = 0 \ \ \forall l \in V, \ \ \mathrm{k} \in \mathrm{K} \tag{16}$$

$$\sum_{i \in V \backslash \{V_0\}} x_{V_0 ik} \leq K \tag{17}$$

$$\sum_{i \in V \backslash \{V_0\}} x_{iV_0 k} \leq K \tag{18}$$

$$t_i + (\mu_{ij} + p_j) x_{ijk} - T(1 - x_{ijk}) \leq t_j \ \mathrm{i} \in \mathrm{V}, \ \mathrm{j} \in \mathrm{V} \backslash \{\mathrm{V}_0\}, i \neq j \tag{19}$$

$$t_j \leq T - (\mu_{jV_0} + \rho_j) \ \forall \mathrm{j} \in \mathrm{V} \backslash \{\mathrm{V}_0\} \tag{20}$$

$$f_j \leq f_i - \left(\frac{d_{ij}}{as_{ij}}\right)\left(\frac{as_{ij}}{mpg}\right) x_{ijk} + Q\left(1 - x_{ijk}\right) \ \ \forall j \in C, i \in V, k \in K \tag{21}$$

$$f_j = Q \ \forall \mathrm{j} \in \mathrm{R} \cup \mathrm{V}_0 \tag{22}$$

$$f_j \geq \min \left\{ \left(\frac{d_{jV_0}}{mpg}\right)^* r, \left(\frac{d_{ji}}{mpg}\right)^* r + d_{iV_0} \right\} \text{ where } \mathrm{j} \in \mathrm{C}, \mathrm{i} \in \mathrm{R}' \tag{23}$$

The objective function is redefined to include only the deviation necessary for a minimization problem and is given in Eqs. (10) and (11). Constraints (12) and (13) specify the difference between the goals and the attained solution. Constraint (14) specifies that from a customer vertex, any vertex can be visited. Constraint (15) shows that refuelling station can have customer /refuelling station/depot as its successor. Flow conservation is ensured using (16). Constraints (17) and (18) ensure that, the number of vehicles entering and leaving the depot is at most K. The arrival time at each vertex is tracked using (19). Constraint (20) ensures that each vehicle returns to depot within the maximum time. The remaining fuel level is tracked using constraint (21), based on the distance between i and j and the average expected speed between i and j. Constraint (22) ensures that the fuel level reaches its maximum capacity Q in refuelling station and depot. Constraint (23) is used to check that there is enough fuel to reach either refuelling station or depot and no vehicle is left stranded.

4. Proposed TVa-PSOGMO for solving F-GVRP

PSO is a stochastic search technique. It mimics the behaviour of fish schooling or birds flocking. The highlight of PSO is group behaviour and the interaction among individuals that motivates to make a bigger move. PSO is a population based Meta heuristic. Each individual solution is called a particle in PSO. Each particle has d dimension and is represented as a vector as, $\vec{X_i} = (x_{i1}, x_{i2}, \ldots, x_{id},)$ which indicates the position of the ith particle in the swarm. Each particle is updated by a velocity $\vec{V_i} = (v_{i1}, v_{i2}, \ldots, v_{id},)$ which guides the PSO to achieve global best position. Each particle has its personal best solution p_{best} that gets updated with iteration once better position is obtained than the current position. Each swarm has a global best solution g_{best} which identifies the best move over all iterations. Each particle in the swarm tries to achieve the global best solution. To prevent PSO from local or premature convergence, a Greedy Mutation Operator (GMO) proposed by Poonthalir et al. (2015) is used with time varying acceleration which guides the search for better exploration.

Multi objective PSO has an archive α which is used to store the set of all global non dominated solutions and also stores the best solution attained by each particle as suggested by Coello, Pulido, and Lechuga (2004). A detailed study on multi objective PSO can be found in the works of Hu and Eberhart (2002), Tripathi, Bandyopadhyay, and Pal (2007) and Parsopoulos and Vrahatis (2002).

4.1. Sequential steps of TVa-PSOGMO

This section describes the steps carried out in TVa-PSOGMO.

4.1.1. Initial population

An initial swarm of particles are generated. Each particle has only customer positions $\{C_1, C_2, ..., C_N\}$. To generate initial swarm of particles Nearest Neighbour Heuristic (NNH) is used with a random selected customer. Almost, 75% of particles are generated using NNH and the remaining 25% of the particles are generated randomly. Let $(x_{i1}, x_{i2}, ..., x_{iN})$ be a representation of the ith particle with N customers $\{C_1, C_2, ..., C_N\}$ with swarm sizes S.

4.1.2. Solution encoding

Each generated particle represents the sequence of customers to be served. These particles are converted to feasible routes by inserting refuelling stations and depot. Let $(V_0, x_{i1}, x_{i2}, RFS_1, ..., x_{iL}, V_0, ... x_{ij}, ..., V_0)$ be the solution encoding of the particle $(x_{i1}, x_{i2}, ..., x_{iN})$ with depot and refuelling station. Hence there are two representations used, particle represents the customer sequence and solution represents the route formed.

4.1.3. Fitness function

To evaluate the strength of each particle, an evaluation method called fitness function is defined. It is calculated based on the distance between two nodes k and l where $k, l \in V$. As already mentioned, there are two encoding used, one is customer encoding which is stored as particle which is subjected to velocity update and the other is solution encoding which takes the particle encoding and converts it to route by inserting refuelling station and depot. Fitness/cost of the route are calculated using Eq. (24). Eq. (25) gives the total route cost of $k \in K$ vehicles. The second objective which aims for calculating fuel consumption is done as defined in Eqs. (4) and (5)

$$Route\_Cost_b = Cost_{V_0,depot} + \sum_{i,j \in V/\{V_0\}} Cost_{i,j} + Cost_{j,V_0} \quad \forall i \in V/\{V_0\} \tag{24}$$

$$Total\_Route\_Cost = \sum_{k \in K} Route\_Cost_b \tag{25}$$

4.1.4. p_{best} and g_{best}

The initial fitness of the particle constitutes the initial p_{best} value and the best of all p_{best} value is the g_{best} value for the swarm. Each particle's p_{best} is calculated using Eq. (26) and g_{best} using Eq. (27).

$$p_{best} = \begin{cases} fitness_t & \text{t} = 1 \\ \min\left(p_{best}, \text{fitness}_t\right) & \text{t} > 1 \end{cases} \tag{26}$$

$$g_{best} = \min\left(p_{best_t}\right) \forall t = 1 \ldots \max\_iter \tag{27}$$

where $fitness_t$ is the $Total\_Route\_Cost$ at iteration t and is updated with the best of p_{best}

4.1.5. Velocity updation

Each particle $(x_{i1}, x_{i2}, ..., x_{iN})$ is updated by a velocity as given in Eq. (28)

$$\begin{aligned} v_{i+1,j} &= \omega . v_{ij} + c_1 r_1 (p_{best} - x_{ij}) + c_2 r_2 (g_{best} - x_{ij}) \\ x_{i+1,j} &= x_{ij} + v_{i+1,j} \end{aligned} \tag{28}$$

p_{best} is particle's personal best and g_{best} is particle's global best position. Each particle's position is updated using velocity based on the particle's individual performance which is captured in p_{best} and the particle's global performance which is captured in g_{best} that are influential in guiding the particle in the search space. In each iteration, both p_{best} and g_{best} are updated based on individual best position and global best position of the particles. Here, c_1 and c_2 are called cognition and social parameters respectively which are used to enhance the individual and social performance of the particle. c_1 is responsible for exploration and c_2 for exploitation of particles. r_1 and r_2 are random numbers defined within the interval [0, 1].

4.1.6. Time varying inertia

Generally, to improve the convergence of the particles inertia weight (ω) is used. Since the search space of the multi objective optimization is complex, a time variant inertia weight is used. Initially, particles are subjected to high inertia weight which leads to high global exploration and as iteration progresses the particles are subjected to low inertia weight to guide local exploitation. Hence, varying inertia weight (ω_{curr_iter}) is used to optimize the behaviour of the search with global exploration and local exploitation as specified in Eq. (29).

$$\omega_{curr\_iter} = (\omega_{\max} - \omega_{\min})^* \frac{\max\_iter - curr\_iter}{\max\_iter} + \omega_{\min} \tag{29}$$

Where $\omega_{\max}$ and $\omega_{\min}$ are the maximum and minimum inertia weights respectively and $\max\_iter$, $curr\_iter$ are the maximum number of iterations and current iteration number, respectively.

4.1.7. Time varying acceleration

Time varying acceleration concentrates in enhancing global search in the early part of the search and convergence in the latter part of the search. c_1 and c_2 are used to achieve this, where c_1 is the cognitive acceleration coefficient and c_2 is the social acceleration coefficient. In the initial stage of the iteration c_1 is reduced from a specified maximum value $c_{1\max}$ to a minimum value $c_{1\min}$ and cognitive acceleration coefficient c_2 is increased from $c_{2\min}$ to $c_{2\max}$ As explained by Ratnaweera, Halgamuge, and Watson (2004), a small cognitive component and a large social component allows particle to converge to global optimum in the latter part of the optimization process and is given in Eqs. (30) and (31).

$$c_1 = (c_{1\max} - c_{1\min})^* \frac{curr\_iter}{\max\_iter} + c_{1\min} \tag{30}$$

$$c_2 = (c_{2\max} - c_{2\min})^* \frac{curr\_iter}{\max\_iter} + c_{2\min} \tag{31}$$

Time variant acceleration coefficients are employed to have a better compromise between exploration and exploitation for multi objective optimization. The performance of PSO improves with the introduction of acceleration and inertia coefficients varying it over the iterations.

4.1.8. Particle encoding and decoding

Each particle is converted to a feasible route and the dimension of each particle will be different based on the inclusion of refuelling station and depot. To update velocity, routes are converted to particle positions using Rank of Value (ROV) as given by Eq. (32),

$$x_{ij} = x_{\min} + \frac{(x_{\max} - x_{\min})}{N}(y_{ij} - 1 + r_1) \tag{32}$$

Algorithm 1 Archive updation.

Let $current\_solution \in \alpha$ and $new\_solution \notin \alpha$
If $new\_solution < current\_solution$ then
  $\alpha = \alpha - current\_solution \cup new\_solution$
else if $new\_solution > current\_solution$ then
  α not updated

where y_{ij} represents ith particle with dimension j, r_1 is a random variable in the interval [0, 1]. $x_{\min}$ and $x_{\max}$ are boundary values of particle positions and N represents the number of customers.

4.1.9. Greedy mutation operator (GMO)

The problem with PSO is pre mature convergence and local minima. To avoid this, better exploration is required and hence PSO is equipped with GMO as suggested in Poonthalir et al. (2015).

4.1.10. Archive

After each iteration, the Pareto optimal solutions are maintained in an archive (α) and is updated with the best solutions. Updating the archive is an important step for any multi objective optimization problem. A total of δ non dominated solutions are available in the archive and gets updated with better solution as the iteration progresses. The process of archive updation is described in the following algorithm. Algorithm 1

4.2. TVa-PSOGMO algorithm for solving F-GVRP

This section presents the TVa-PSOGMO algorithm for F-GVRP. Algorithm 2

4.3. Theoretical analysis

This section is used to justify the claim made by the proposed TVa-PSOGMO on F-GVRP

Lemma 1. All routes obtained using TVa-PSOGMO are feasible.

Proof. Consider a random particle X_i with customer positions given as$X_i = (x_{i1}, x_{i2},..., x_{ik}, x_{il}, x_{im},..., x_{iN})$. To justify whether the route produced is feasible is to verify that the constraints defined from Eqs. (12) to (23) are satisfied. To convert each particle X_i to *Route*, depot (V_0) and *RFS* are inserted. From a customer, say, x_{ik} to reach x_{il}, several factors are checked. Apart from the check made on time and fuel to reach x_{il}, constraint (23) should be satisfied i.e. no vehicle is stranded in the middle. To check that, there should be sufficient fuel and time left to reach say, x_{im} after serving x_{il} in the sequence X_i or to reach depot V_0 or to reach any of the nearest *RFS*. Consider a partial *Route* as $Route = (V_0, x_{ik}, ...x_{il})$, to check whether next customer x_{im} is to be inserted into *Route*, any one of the following conditions may happen. If $t_m > T$ then, $Route = Route \cup \{V_0\}$ or f_k is not sufficient to reach x_{im}, then $Route = Route \cup \{RFS\}$ which makes the route feasible, or if $t_m < T$ and f_k is sufficient to reach x_{im}, $Route = Route \cup \{x_{im}\}$ and hence, constraints (19)–(21) are satisfied. The process gets repeated for each x_{ij} in the sequence. Hence, all routes produced by TVa-PSOGMO are feasible. Schneider et al. (2014) solution for G-VRP requires a penalization mechanism with in-feasible routes. This requires an extra operator to be used for the insertion/removal of refuelling stations. So, an additional operator for checking in-feasible solution is not required with TVa-PSOGMO as the route built satisfies all feasibility conditions of the route. □

Lemma 2. The fuel consumption with varying speed is less, compared to the fuel consumption with constant speed.

Proof. Consider a random particle X_i with customer positions as$X_i = (x_{i1}, x_{i2},..., x_{ik}, x_{il}, x_{im},..., x_{iN})$. To calculate the fuel consumption between any two customers, say, x_{il} and x_{im}, the speed travelled

Algorithm 2 TVa-PSOGMO for F-GVRP.

Initialize swarm S
Initialize particles in swarm S using *NNH* and *random*
Initialize p_{best} and g_{best}
Let X_b be the particle in S
$X_b(j)$represents position of jth customer in X_b
m is an index for number of iterations and $m = 1$
r_c is an index for *Route* and $r\_c = 1$
b is an index for a particle with size N, where N is the number of customers
$\max\_iter$ is the maximum number of iterations
$fuel, rem\_fuel$ represents fuel consumed and remaining fuel respectively
$time, rem\_time$ represents the time consumed and remaining time respectively
While $m \le \max\_iter$
 $b = 1$
 while $b \le S$
  Let $X_b = \{x_{b1}, x_{b2},..., x_{bN}\}$
  Let $Route = \{V_o\}$
  Let $r\_c = 1$
  for $j = 1$ to N // *j is an index for* X_b
   Calculate $fuel = FC_{Route(r\_c), X_b(j)}$, $time = t_{Route(r\_c), X_b(j)}$
   Calculate rem_fuel, rem_time
   if $fuel \le Q$ and $time \le T$
    include j in route and check for fuel/time availability to visit k
    where $k \in V$ and $k = X_b(j+1)/V_0/RFS$
    $flag = 1$
    $Route = Route \cup \{j\}$
    $j = j + 1$
    $r\_c = r\_c + 1$
   else
    if $fuel > Q$ or $time > T$ then
     if $j = 1$ then
      $X_b(j)$ not served; $j = j + 1$
     else
      if $fuel > Q$ then
       $Route = Route \cup \{RFS\}$ // *nearby refueling station*
       $r\_c = r\_c + 1$
      end
      If $time > T$ then
       $Route = Route \cup \{V_0\}$ // *introduce new vehicle*
       $r\_c = r\_c + 1$
     end
  end while
   Calculate fitness for the route
   Update p_{best} and g_{best}
   Store the best g_{best} in archive α
   Remove *RFS* and V_0 from *Route*
   Convert *Route* to particle using *ROV*
   Update velocity, inertia and acceleration for each particle
 End while

between the distance ($dist$) x_{il} and x_{im} is to be determined. To carry out this, the speed intervals are used. Assume that there are τ intervals with some defined difference σ in the speed. Then, the average expected speed as_{lm} over the interval $1...\tau$ with difference σ is estimated using triangular distribution. Let $s_{z_i z_j}$ be the speed defined over the interval $[z_i, z_j]$. Let the τ intervals and the respective speed be represented as $s_{z_0 z_1}, s_{z_1 z_2}, \ldots, s_{z_{\tau-1} z_\tau}$. Then, the average expected speed is determined using $as_{lm} = \frac{\sum_{i=0}^{\tau-1} s_{z_i z_{i+1}}}{\tau}$. Since, the speed travelled between x_{il} and x_{im} covers the distance with speed between a defined minimum and maximum speed $\vartheta_{\min}$ and $\vartheta_{\max}$, the distance is covered with both $\vartheta_{\min}$ and $\vartheta_{\max}$, maintaining an expected average speed as_{lm}. So, the fuel consumption also varies which is computed using Eqs. (4) and (5). On the other hand, constant speed covers the entire distance with a given constant speed and constant fuel consumption rate. Hence, the fuel consumption with varying speed is less than the fuel consumption with constant speed. □

5. Results and discussion

The proposed algorithm is tested on the G-VRP data set of Erdogan and Miller Hooks (2012). Their results are improved by many researchers and the Best Known Solution (BKS) from

Table 1
Fuel consumption and route cost of uniformly distributed customers.

Data set	Best known solution			F-GVRP using TVa-PSOGMO with varying speed			Percentage deviation
	BKS	Fuel consumption	Speed	Route cost	Expected fuel consumption	Expected speed	
20c3sU1	1797.49	359.49	40	1797.49	**284.84**	40.05	26.21
20c3sU2	1574.78	314.95	40	1574.78	**277.66**	39.91	13.43
20c3sU3	1708.48	341.69	40	1708.48	**262.47**	39.99	30.18
20c3sU4	1482	296.4	40	1482.00	**302.48**	40.65	−2.01
20c3sU5	1689.37	337.87	40	1689.37	**231.72**	44.5	45.81
20c3sU6	1618.65	323.73	40	1618.65	**324.74**	39.88	−0.31
20c3sU7	1713.66	342.73	40	1713.66	**246.52**	40.02	39.03
20c3sU8	1706.5	341.3	40	1706.50	**294.58**	39.55	15.86
20c3sU9	1708.82	341.76	40	1708.82	**202.54**	40.89	68.74
20c3sU10	1181.31	236.26	40	1181.31	**284.84**	40.05	−17.05

Montoya et al. (2016) are displayed against each data set. All the data set have 10 data instances. There are 4 small data sets with 20 customers each. There are 3 refuelling stations. Each vehicle has a maximum time limit of 11 h and each vehicle travels at a constant speed of 40 miles/h with a constant fuel consumption rate of 0.2 gallons/mile. The capacity of the fuel tank is 60 gallons. The service time at customer and refuelling station is 30 and 15 min, respectively. The locations of customer and refuelling station are specified with latitude and longitude values and are converted to Euclidean distance co-ordinates with the radius of the earth taken as 4182.4449. It is implemented on Intel core I3-3220 processor with 3.30 GHz and 4 GB RAM using MATLAB R2014a.

TVa-PSOGMO works well for all the data sets and the obtained route cost and fuel consumption details are projected in Tables 2–5. The first goal $G1$ is the route cost of BKS and the second goal $G2$ is the fuel consumption which is initially taken as a large value and is updated with a better solution over the iterations.

The parameter setting of TVa-PSOGMO is based on Tripathi et al. (2007). The swarm size is 30. The cognitive and social acceleration coefficients are taken as $c_{1\max}=0.5$ and $c_{1\min}=2.5$, $c_{2\min}=0.5$ to $c_{2\max}=2.5$ A linearly decreasing inertia coefficient is used with initial and final inertia weight as $\omega_{\max}=0.4$ and $\omega_{\min}=0.9$ respectively. Particle positions are taken as $x_{\min}=2.0$ and $x_{\max}=0.0$ respectively. The cross over and mutation probabilities are taken as 0.9 and 0.4 respectively. The important advantage of using varying acceleration coefficient is to have better compromise between exploration and exploitation as specified in Ratnaweera et al. (2004).

Comparison of fuel consumption with varying and constant speed

The route cost obtained with the proposed method is competent with the BKS. To compare the fuel consumption of F-GVRP, the fuel consumption of G-VRP should be calculated. Since G-VRP use constant speed with a constant fuel consumption rate of 0.2 gallons/mile, Eq. (33) is used to determine the total fuel consumption of G-VRP.

$$fuel\_consumption = route\_cost \times fuel\_consumption\_rate \tag{33}$$

To validate the performance of the proposed algorithm, the total increase or decrease in the fuel consumption is calculated using Eq. (34).

$$\frac{optimal\ cost - obtained\ cost}{optimal\ cost} \times 100 \tag{34}$$

The results obtained for all 4 data sets contribute to a decrease in the fuel consumption when varying speed is used. Tables 1–4 project the average expected speed of the entire route. A 20.48% decrease in the fuel consumption is realized using varying speed. Since the entire distance is not travelled by the vehicle with uniform speed there is a drop in the use of fuel. Hence, up to 20% in the cost of the fuel can be saved which indirectly contribute to lesser carbon footprint.

The results specify the total fuel consumption under constant and varying speed environment with speed varying between a minimum and maximum speed limit of 10 and 70 respectively taken in an interval of 10 as [10 20 30 40 50 60 70]. To derive the comparison between constant and varying speed fuel consumption rate, the algorithm is run with the goal $G1$ taken as the BKS's route cost and the corresponding non dominated solution obtained for the fuel consumption is displayed. It is observed that, as fuel consumption is less, it is possible to serve more customers with less number of vehicles and a substantial decrease in the route cost can be achieved, if goal $G1$ is relaxed. A positive percentage deviation is obtained for almost all data instances which represents the efficiency of the varying speed constraint.

The results project that TVa-PSOGMO performs well and is competent with BKS. It is observed that, even though the route cost obtained using varying and constant speed is the same, the fuel consumption need not be the same. It is dependent on the road condition, traffic conditions, the speed of the vehicle, and several other factors which are influential in the fuel consumption calculation. For example, if the route 0-2-8-5-0 costs 14.2 and the route cost of 0-8-5-2-0 is also 14.2, it is not necessary that both should travel with same fuel consumption as it may vary with the road condition, speed, traffic congestion, and many other factors. For data instances S2_4i6s, S2_4i8s, S2_4i10s, the cost of the route is the same and the constant fuel rate gives the same fuel consumption for all the three instances. But, in varying speed environment, the fuel consumption varies and is dependent on the speed.

As observed from Tables 1–4, though the route cost is the same, the routes obtained for F-GVRP using TVa-PSOGMO is different from the route obtained using G-VRP. To show the structural difference between the routes obtained with fuel consumption as an additional objective, the route obtained for the data set 20c3sU1 is plotted and is depicted in Fig. 3. It is seen that the routes obtained for constant and varying speed is different and the vehicle has taken a different route because of lesser fuel consumption. The route obtained for F-GVRP is served by one vehicle less than the route obtained for G-VRP because the vehicle can travel for longer duration as the vehicle spends less time in the refuelling station, since fuel consumption is less. Almost in half of the data instances, there is a decrease in the use of the number of vehicles, attributed by the fuel consumption.

Pareto Optimal Front

The problem is modelled as a bi-objective optimization problem to minimize both route cost and fuel consumption. The challenge is to determine the route with the BKS's route cost and to estimate the fuel consumption for the obtained route cost. Goal $G1$ is set as BKS and the algorithm is run for a specified maximum number of iterations or till the best route is obtained. The best routes are stored in the archive. The route that archives $G1$ with compromising fuel consumption is taken as the non dominated solution and is displayed in Tables 1–4. As already noted, if $G1$ is relaxed,

Table 2
Fuel consumption and route cost of clustered customers.

Best known solution				F-GVRP using TVa-PSOGMO with varying speed			Percentage deviation
Data set	BKS	Fuel consumption	Speed	Route cost	Expected fuel consumption	Expected speed	
20c3sc1	1173.57	234.71	40	1173.57	**200.69**	40.22	14.49
20c3sc2	1539.97	307.99	40	1539.97	**215.52**	41.53	30.02
20c3sc3	880.29	176.06	40	880.29	**151.61**	38.57	13.89
20c3sc4	1059.35	211.87	40	1059.35	**169.7**	42.6	19.9
20c3sc5	2156.01	431.2	40	2156.01	**288.56**	40.12	33.08
20c3sc6	2758.17	551.63	40	2758.17	**441.55**	40.3	19.96
20c3sc7	1393.99	278.8	40	1393.99	**254.81**	40.5	8.6
20c3sc8	3139.72	627.94	40	3139.72	**512.24**	40.2	18.43
20c3sc9	1799.94	359.99	40	1799.94	**285.23**	38.5	20.77
20c3sc10	2583.42	516.68	40	2583.42	**257.83**	37.6	50.1

Table 3
Fuel consumption and route cost of both uniform and clustered customers.

Best known solution				F-GVRP using TVa-PSOGMO with varying speed			Percentage deviation
Data set	BKS	Fuel consumption	Speed	Route cost	Expected fuel consumption	Expected speed	
S1_2i6s	1578.12	315.62	40	1578.12	**278.13**	40.43	11.88
S1_4i6s	1397.27	279.45	40	1397.27	**253.80**	40.01	9.18
S1_6i6s	1560.49	312.09	40	1560.49	**278.91**	39.96	10.63
S1_8i6s	1692.32	338.46	40	1692.32	**261.12**	40.01	22.85
S1_10i6s	1173.48	234.69	40	1173.48	**201.43**	40.02	14.17
S2_2i6s	1633.10	326.62	40	1633.10	**298.48**	40.95	8.62
S2_4i6s	1505.07	301.01	40	1505.07	**325.48**	40.28	−8.13
S2_6i6s	2431.33	486.26	40	2431.33	**388.15**	39.56	20.18
S2_8i6s	2158.35	431.67	40	2158.35	**304.84**	37.55	29.38
S2_10i6s	1585.46	317.09	40	1585.46	**266.42**	41.28	15.0

Table 4
Fuel consumption and route cost of both uniform and clustered customers.

Best known solution				F-GVRP using TVa-PSOGMO with varying speed			Percentage deviation
Data set	BKS	Fuel consumption	Speed	Route cost	Expected fuel consumption	Expected speed	
S1_4i2s	1582.21	316.442	40	1582.21	**283.71**	39.98	11.54
S1_4i4s	1460.09	292.018	40	1460.09	**137.83**	40.2	63.29
S1_4i6s	1397.27	279.454	40	1397.27	**236.88**	41.35	17.97
S1_4i8s	1397.27	279.454	40	1397.27	**242.59**	39.62	15.2
S1_4i10s	1396.02	279.204	40	1396.02	**124.57**	40.8	59.94
S2_4i2s	1059.35	211.87	40	1059.35	**184.14**	38.90	8.27
S2_4i4s	1446.08	289.216	40	1446.08	**267.12**	41.52	15.06
S2_4i6s	1434.14	286.828	40	1434.14	**245.84**	39.65	16.67
S2_4i8s	1434.14	286.828	40	1434.14	**253.25**	40.25	13.26
S2_4i10s	1434.14	286.828	40	1434.14	**248.91**	39.99	15.23

Table 5
Impact of fuel consumption on uniform customers for different speed intervals.

BKS		Category 2		Category 3		Category 4	
Data set	Route cost	Expected fuel consumption	Expected speed	Expected fuel consumption	Expected speed	Expected fuel consumption	Expected speed
20c3sU1	1797.49	327.81	40.10	335.56	40.52	342.78	46.77
20c3sU2	1574.78	284.20	39.9	274.01	40.64	300.29	46.85
20c3sU3	1708.48	277.65	40.06	255.48	41.25	298.35	46.92
20c3sU4	1482	262.36	40.08	271.57	41.55	284.46	45.71
20c3sU5	1689.37	302.48	39.97	345.25	42.25	378.57	45.67
20c3sU6	1618.65	235.64	40.12	248.99	41.06	265.89	46.89
20c3sU7	1713.66	324.81	39.95	338.09	41.37	364.89	47.94
20c3sU8	1706.5	236.24	40.56	246.16	40.16	292.45	46.65
20c3sU9	1708.82	282.17	39.97	281.77	41.38	297.12	46.21
20c3sU10	1181.31	176.23	39.87	201.85	41.25	227.46	45.82
Sum		2709.59	400.58	2798.73	411.43	3052.26	465.43
Average		270.96	40.06	279.87	41.14	305.22	46.54

then most of the instances may give a better route cost when compared with best solutions. But, there are no published results to compare the performance of the proposed TVa-PSOGMO, hence to derive the comparison between the route cost of G-VRP, F-GVRP's goal $G1$ is based on the BKS's route cost and the corresponding fuel consumption is obtained. A depiction of the Pareto dominant solution front obtained for various data sets is displayed in Fig. 4.

Impact of speed intervals on fuel consumption

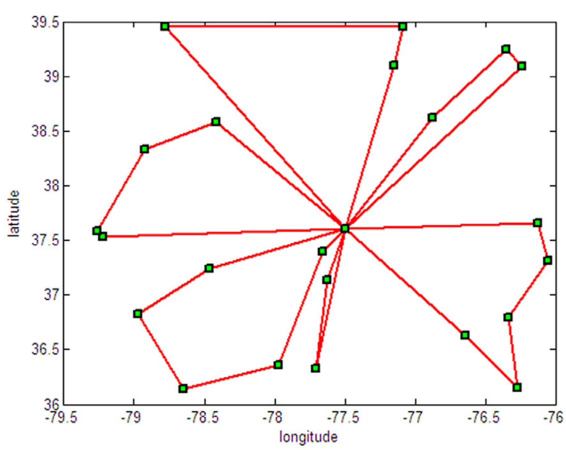
The speed variation result in substantial decrease in the fuel consumption. It is important to analyze the impact of various speed intervals on fuel consumption intervals. Experiments are

Table 6
Impact of fuel consumption on clustered customers for different speed intervals.

BKS		Category 2		Category 3		Category 4	
Data set	Route cost	Expected fuel consumption	Expected speed	Expected fuel consumption	Expected speed	Expected fuel consumption	Expected speed
20c3sc1	1173.57	230.27	39.91	221.00	41.31	217.55	46.56
20c3sc2	1539.97	249.31	40.06	247.36	41.46	263.31	46.69
20c3sc3	880.29	158.67	40.01	157.67	41.05	142.49	47.34
20c3sc4	1059.35	153.10	40.04	152.73	41.30	159.97	46.93
20c3sc5	2156.01	234.37	40.10	274.77	41.33	283.28	46.95
20c3sc6	2758.17	472.83	40.02	485.21	41.56	498.63	46.74
20c3sc7	1393.99	130.24	40.05	132.26	41.51	128.02	44.29
20c3sc8	3139.72	502.50	39.98	518.27	41.33	558.96	46.20
20c3sc9	1799.94	319.66	40.09	294.18	41.22	287.23	46.89
20c3sc10	2583.42	448.23	40.02	447.99	40.48	381.85	47.00
Sum		2899.18	400.28	2931.44	412.55	2921.21	465.59
Average		289.91	40.03	293.14	41.26	292.12	46.56

Table 7
Impact of fuel consumption on uniform and clustered customers for different speed intervals.

BKS		Category 2		Category 3		Category 4	
Data set	Route cost	Expected fuel consumption	Expected speed	Expected fuel consumption	Expected speed	Expected fuel consumption	Expected speed
S1_2i6s	1578.12	275.22	39.99	233.34	41.29	301.25	47.89
S1_4i6s	1397.27	254	40	254.37	42.17	252.91	46.27
S1_6i6s	1560.49	258.78	40.04	258.26	40.14	271.13	45.24
S1_8i6s	1692.32	293.65	40.52	301.27	41.25	320.55	46.72
S1_10i6s	1173.48	201.34	40.22	215.56	40.58	227.89	45.81
S2_2i6s	1633.10	248.23	41.55	287.47	41.15	295.87	44.25
S2_4i6s	1505.07	251.88	39.87	325.48	41.22	341.18	45.57
S2_6i6s	2431.33	352.63	39.95	365.74	41.12	355.93	45.81
S2_8i6s	2158.35	338.57	40.05	345.89	40.25	346.88	46.61
S2_10i6s	1585.46	302.51	39.98	314.13	40.07	286.58	46.76
Sum		2776.81	402.17	2901.51	409. 24	3000.17	460.93
Average		277.68	40.22	290.15	40.92	300.17	46.09



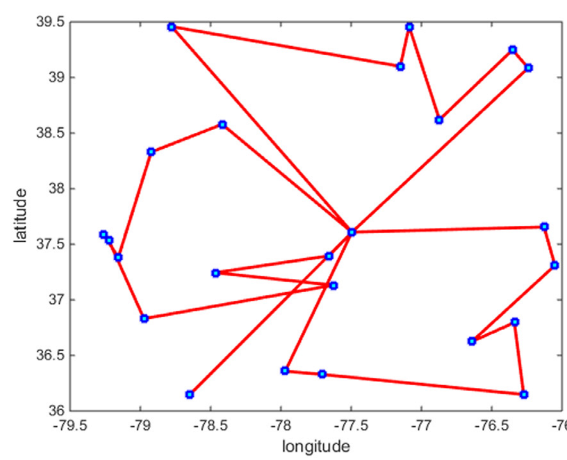


Fig. 3. Routes obtained for constant and varying speed for 20c3sU1.

conducted to find the relationship between various speed intervals and fuel consumption.

A snap shot of triangular distribution for various categories of speed intervals is derived and is projected in Fig. 5.

To study the behaviour of speed under various speed intervals, speed is categorised with 4 different speed intervals. Category 1 has speed in the range [10 20 30 40 50 60 70] with interval 10, category 2 [5 10 15 20 25 30 35 40 45 50 55 60 65 70 75] has speed interval as 5, category 3 [1 20 40 60 80] is taken with interval 20 and category 4 [10 30 60] with interval 30. The maximum speed is taken not to exceed 80. The Tables from 1 to 4 are run with category 1 where the mean speed for each travel from node i to node j, where $i, j \in V$ is taken at random and the impact of the varying speed on each instance of the data set is studied and the results are recorded. To study the fuel consumption details under varying speed in different categories, the algorithm is run to get the same BKS for the goal $G1$ in the route cost. The results depicted are the best obtained from 10 runs of each instance of the data set.

The speed obtained for various categories are displayed in Tables 5–8. It is noted that, there is a positive correlation between speed and the fuel consumption. The expected average speed increases when the length of the interval increases. The expected average speed is 40.10 in category 2 where the speed has an interval of 5miles/h, 41.19 in category 3 with the interval being 20 miles/h and in category 4 it is 46.48 with an interval of 30 miles/h.

Prominent deviations are not observed with the speed intervals of 10 or 5 miles difference. When speed is taken in intervals of

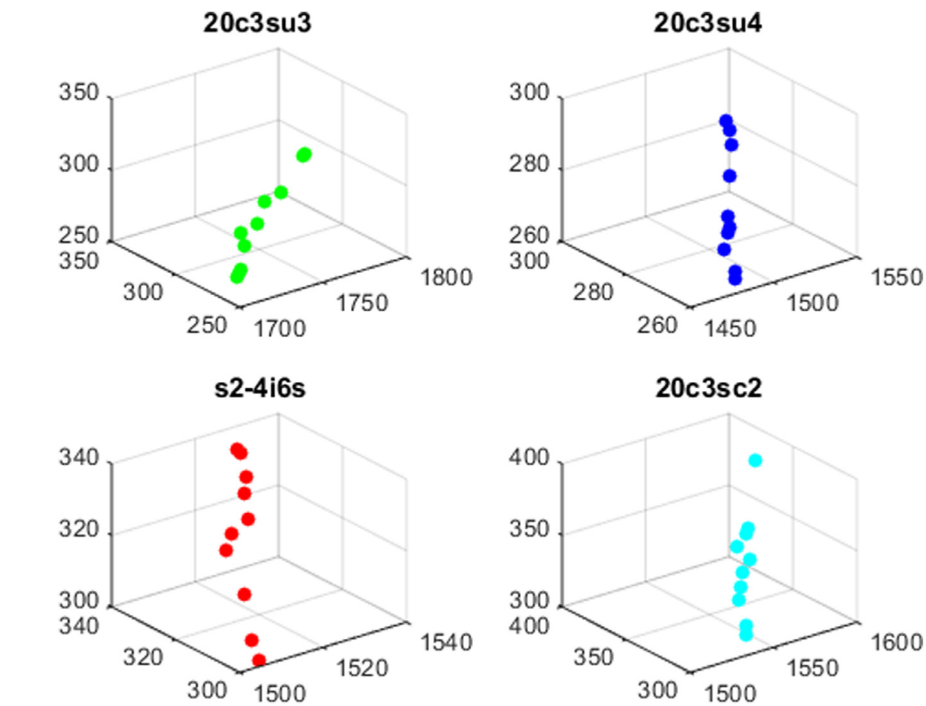


Fig. 4. Pareto dominance of various data instances using TVa-PSOGMO.

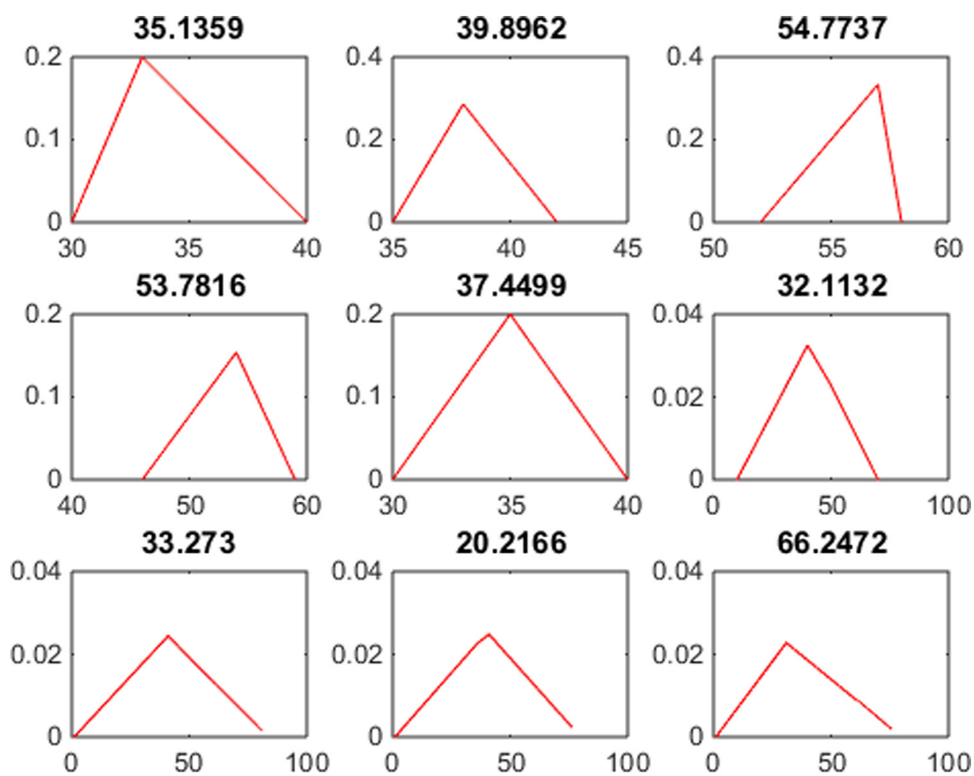


Fig. 5. Triangular distributions for various speed intervals.

30 as in category 4, an increase in the speed with high fuel consumption is observed for most of the data instances, though for few instances it gives better fuel consumption rate. It shows that, though varying speed reduces fuel consumption, the speed interval should not be more to realize the benefits from the fuel consumption. Also, it is hard to get the BKS at this rate as there is a frequent requirement for fuel or the time is insufficient for the vehicle to serve successive customers. Hence, the algorithm performs poor when the interval is taken with 30miles/h difference. Though better fuel consumption can be realized with varying speed, to derive a better routing plan, proper selection of speed interval is important. As seen from the results, category 1 and 2 are able to get better fuel consumption minimization for most of the data instances.

Table 8
Impact of fuel consumption on uniform and clustered customers for different speed intervals.

BKS		Category 2		Category 3		Category 4	
Data set	Route cost	Expected fuel consumption	Expected speed	Expected fuel consumption	Expected speed	Expected fuel consumption	Expected speed
S1_4i2s	1582.21	276.65	40.03	279.23	41.70	261.96	46.44
S1_4i4s	1460.09	268.24	39.99	274.56	40.87	289.76	46.96
S1_4i6s	1397.27	299.02	39.91	297.84	41.42	275.81	47.32
S1_4i8s	1397.27	290.11	40.05	289.19	40.93	234.51	45.75
S1_4i10s	1396.02	256.63	40.08	255.79	40.66	246.52	46.22
S2_4i2s	1059.35	184.44	39.90	183.39	41.5	200.53	46.76
S2_4i4s	1446.08	278.91	40.86	279.71	42.52	298.63	47.75
S2_4i6s	1434.14	241.49	40.04	224.15	41.49	241.36	46.83
S2_4i8s	1434.14	280.12	40.03	286.81	41.62	270.80	47.09
S2_4i10s	1434.14	302.47	39.90	304.92	41.72	284.87	45.96
Sum		2678.08	400.79	2675.59	414.43	2604.75	467.08
Average		267.808	40.079	267.559	41.443	260.475	46.708

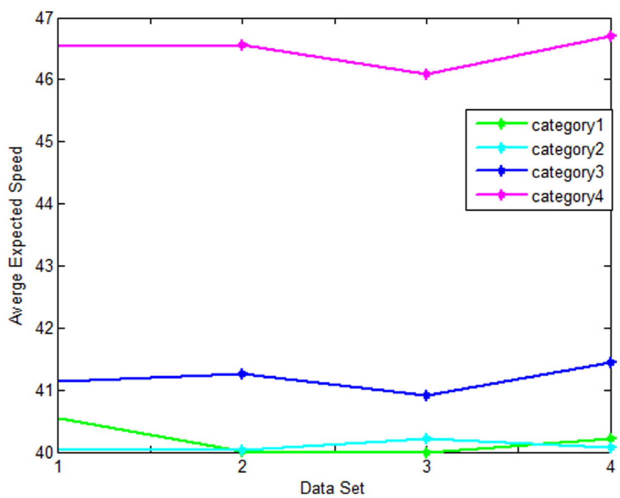


Fig. 6. Expected speed for various speed categories.

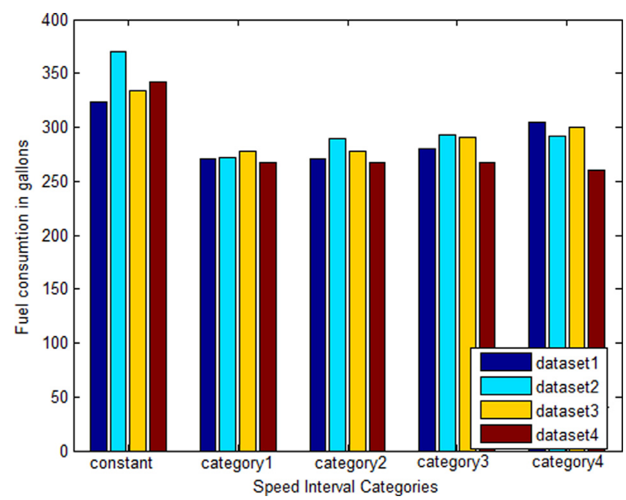


Fig. 7. Fuel consumption for various speed categories.

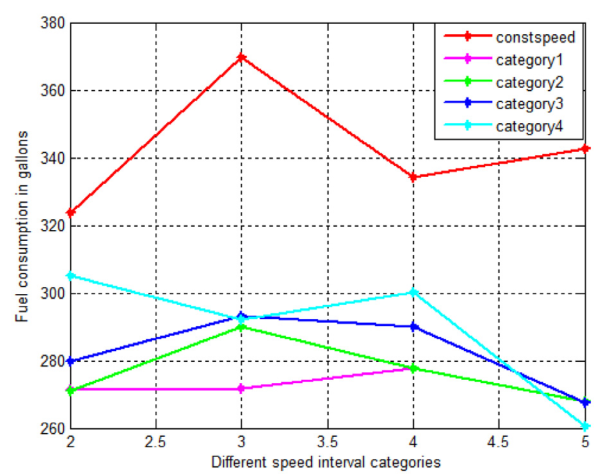


Fig. 8. Comparison of fuel consumption with constant and varying speed.

It is useful to observe from Tables 5 to 8 that the average expected fuel consumption increases when there is an increase in the average expected speed and it is also noted that the interval of increase/decrease in speed has an impact on the fuel consumption rate. But, when a comparison is derived between constant and varying speed, regardless of the speed intervals, varying speed performs better than constant speed in fuel conservation.

Table 8 shows that the fuel consumption is almost the same in all 3 different categories of speed intervals. The data instances of Table 8 are designed to study the impact of refuelling stations on the route cost. For data instance S1_4i8s and S2_4i10s, speed category 4 is able to serve all customers with one vehicle less. Hence, there is a significant decrease in fuel consumption. This is due to the inclusion of more refuelling station. It should also be observed that for data instances S2_4i6s, S2_4i8s, S2_4i10s, though the route cost is same, fuel consumption is different. But, it is very difficult to quantify the results as they are depicted here as the result of manual findings with many rigorous experiments.

Based on the observations from Tables 5–8, it is seen that the average expected speed in category 1 and 2 is less than the average expected speed in category 3 and 4. Fig. 6 shows the comparison in the obtained average speed between various speed categories, the expected average speed in category 4 is more than any other speed categories.

Figs. 7 and 8 show the comparison derived for fuel consumption on various data sets with different speed intervals. It shows the comparison within various speed intervals and also between constant and varying speed. It is seen that the constant speed consumes more fuel compared to varying speed. Also, under varying speed, category 1 performs better for all 4 data sets. Both categories 1 and 2 show a considerable decrease in the fuel consumption than categories 3 and 4.

It is also observed through experiments that the use of the number of vehicles is reduced in most of the cases, since speed is taken as a varying quantity and, the vehicle is able to serve more customers with less time. A total of 222 vehicles were used for G-VRP as specified in Montoya et al. (2016), but TVa-PSOGMO algorithm uses a total of 209 vehicles where a considerable decrease in the number of vehicles is realized as the algorithm is able to serve more customers with less vehicles.

The impact of this research is to ascertain that when route cost minimization is taken as a primary objective in solving VRP, an additional objective that takes care of environmental issues need to be addressed to have a green environment and parameters like vehicle speed can be influential in fuel consumption reduction.

6. Conclusion and future work

F-GVRP is introduced as an extension of G-VRP in this paper. The problem is modelled as a bi objective optimization problem to minimize both route cost and fuel consumption. This paper addresses speed as a varying constraint and proposes a model to calculate the route cost and fuel consumption for vehicles that travel with varying speed. Experiments are conducted with constant and varying speed constraint to study the impact of route cost and fuel consumption where a substantial reduction in fuel consumption is realized using varying speed constraint. The proposed method suggests a routing plan with less fuel consumption, where the carbon emission can be greatly reduced.

The contribution of this research is three fold. First, a bi objective optimization model F-GVRP that intends to minimize both route cost and fuel consumption is introduced and is modelled using goal programming approach. Second, speed is taken as a varying constraint to study the behaviour of F-GVRP which is simulated using triangular distribution. Experiments are conducted with four different categories of speed intervals and the corresponding route cost and fuel consumption are estimated and analyzed. Significant improvement in fuel consumption is realized under varying speed environment. Better routing plan is achieved with less number of vehicles. Third, a particle swarm optimization with greedy mutation operator and time varying acceleration is used to solve the problem and the results are compared with benchmark data sets which prove the efficiency of the algorithm.

The impact of this research is to address the environmental concern associated with the planning and routing of delivery vehicles and the influence of speed on the fuel consumption rate which has a direct consequence on fuel emission minimization. Organization can plan their vehicles to opt for a better speed interval plan to realize fuel consumption minimization.

Some insights are observed during the study, the use of number of vehicles get reduced as vehicles can serve more customers since fuel consumption is minimized. The average expected speed increases with an increase in the speed interval. Also, speed interval is important in determining the fuel consumption minimization. Better routing plan can be derived with cautious selection of speed. It is also observed that regardless of the speed intervals, constant speed consumes more fuel than varying speed.

An immediate extension of this study is to model F-GVRP with fuel emission as an additional objective and to study the behaviour with constant and varying speed environment. Also, this research can be further extended to study the impact on route cost and fuel consumption in electric VRP where speed variation can occur with partial and fully charged batteries. The model can also be used to study the impact of speed and fuel consumption with hybrid vehicles, when there is a switch over between battery and fuel which has an effect on the speed. It can also be extended to study the behaviour of heterogeneous vehicles under varying speed environment. Additional constraints like road conditions, vehicle load can be included with varying speed environment in F-GVRP and their impact on route cost and fuel consumption can be studied.

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References

Afshar-Bakeshloo, M., Mehrabi, A., Safari, H., Maleki, M., & Jolai, F. (2016). A green vehicle routing problem with customer satisfaction criteria. *Journal of Industrial Engineering International, 12*(4), 529–544.

Andelmin, J., & Bartolini, E. (2017). An Exact algorithm for the green vehicle routing problem. *Transportation Science*. doi:10.1287/trsc.2016.0734.

Apaydin, O., & Gonullu, M. T. (2008). Emission control with route optimization in solid waste collection process: A case study. *Sadhana, 33*(2), 71–82.

Bard, J. F., Huang, L., Jaillet, P., & Dror, M. (1998). A decomposition approach to the inventory routing problem with satellite facilities. *Transportation science, 32*(2), 189–203.

Bektaş, T., & Laporte, G. (2011). The pollution-routing problem. *Transportation Research Part B: Methodological, 45*(8), 1232–1250.

Bektas, T., Demir, E., & Laporte, G. (2016). Green vehicle routing. In *Green transportation logistics* (pp. 243–265). Springer International Publishing.

Bruglieri, M., Mancini, S., Pezzella, F., & Pisacane, O. (2016). A new mathematical programming model for the green vehicle routing problem. *Electronic Notes in Discrete Mathematics, 55*, 89–92.

Calvete, H. I., Galé, C., Oliveros, M. J., & Sánchez-Valverde, B. (2007). A goal programming approach to vehicle routing problems with soft time windows. *European Journal of Operational Research, 177*(3), 1720–1733.

Charnes, A., & Cooper, W. W. (1977). Goal programming and multiple objective optimizations: Part 1. *European Journal of Operational Research, 1*(1), 39–54.

Coello, C. A. C., Pulido, G. T., & Lechuga, M. S. (2004). Handling multiple objectives with particle swarm optimization. *IEEE Transactions on Evolutionary Computation, 8*(3), 256–279.

Ćirović, G., Pamučar, D., & Božanić, D. (2014). Green logistic vehicle routing problem: Routing light delivery vehicles in urban areas using a neuro-fuzzy model. *Expert Systems with Applications, 41*(9), 4245–4258.

Dantzig, G. B., & Ramser, J. H. (1959). The truck dispatching problem. *Management Science, 6*(1), 80–91.

Erdoğan, S., & Miller-Hooks, E. (2012). A green vehicle routing problem. *Transportation Research Part E: Logistics and Transportation Review, 48*(1), 100–114.

Demir, E., Bektaş, T., & Laporte, G. (2012). An adaptive large neighborhood search heuristic for the pollution-routing problem. *European Journal of Operational Research, 223*(2), 346–359.

Demir, E., Bektaş, T., & Laporte, G. (2014a). The bi-objective pollution-routing problem. *European Journal of Operational Research, 232*(3), 464–478.

Demir, E., Bektaş, T., & Laporte, G. (2014b). A review of recent research on green road freight transportation. *European Journal of Operational Research, 237*(3), 775–793.

Felipe, Á., Ortuño, M. T., Righini, G., & Tirado, G. (2014). A heuristic approach for the green vehicle routing problem with multiple technologies and partial recharges. *Transportation Research Part E: Logistics and Transportation Review, 71*, 111–128.

Figliozzi, M. (2010). Vehicle routing problem for emissions minimization. *Transportation Research Record: Journal of the Transportation Research Board, 2197*, 1–7.

Ghoseiri, K., & Ghannadpour, S. F. (2010). Multi-objective vehicle routing problem with time windows using goal programming and genetic algorithm. *Applied Soft Computing, 10*(4), 1096–1107.

Hu, X., & Eberhart, R. (2002). Multiobjective optimization using dynamic neighborhood particle swarm optimization. In *Proceedings of the congress on evolutionary computation: 2* (pp. 1677–1681).

Jabali, O., Woensel, T., & De Kok, A. G. (2012). Analysis of travel times and CO2 emissions in time dependent vehicle routing. *Production and Operations Management, 21*(6), 1060–1074.

Jabir, E., Panicker, V. V., & Sridharan, R. (2017). Design and development of a hybrid ant colony-variable neighbourhood search algorithm for a multi-depot green vehicle routing problem. *Transportation Research Part D: Transport and Environment, 57*, 422–457.

Jemai, J., Zekri, M., & Mellouli, K. (2012 April). An NSGA-II algorithm for the green vehicle routing problem. In *European conference on Evolutionary Computation in Combinatorial Optimization* (pp. 37–48). Berlin, Heidelberg: Springer.

Jovanović, A. D., Pamučar, D. S., & Pejčić-Tarle, S. (2014). Green vehicle routing in urban zones – A neuro-fuzzy approach. *Expert Systems with Applications, 41*(7), 3189–3203.

Jovičić, N. M., Bošković, G. B., Vujić, G. V., Jovičić, G. R., Despotović, M. Z., Milovanović, D. M., et al. (2010). Route optimization to increase energy efficiency and reduce fuel consumption of communal vehicles. *Thermal Science, 14*(suppl.), 67–78.

Kara, I., Kara, B., & Yetis, M. K. (2007 August). Energy minimizing vehicle routing problem. In *international conference on Combinatorial Optimization and Applications* (pp. 62–71). Berlin, Heidelberg: Springer.

Kazemian, I., & Aref, S. (2017). A green perspective on capacitated time-dependent vehicle routing problem with time windows. *International Journal of Supply Chain and Inventory Management, 2*(1), 20–38.

Koç, Ç., & Karaoglan, I. (2016). The green vehicle routing problem: A heuristic based exact solution approach. *Applied Soft Computing, 39*, 154–164.

Kuo, Y. (2010). Using simulated annealing to minimize fuel consumption for the time-dependent vehicle routing problem. *Computers & Industrial Engineering, 59*(1), 157–165.

Leggieri, V., & Haouari, M. (2017). A practical solution approach for the green vehicle routing problem. *Transportation Research Part E, 104*, 97–112.

Li, J. (2012). Vehicle routing problem with time windows for reducing fuel consumption. *Journal of Computers, 7*(12), 3020–3027.

Lin, C., Choy, K. L., Ho, G. T., Chung, S. H., & Lam, H. Y. (2014). Survey of green vehicle routing problem: Past and future trends. *Expert Systems with Applications, 41*(4), 1118–1138.

Lin, J., Zhou, W., & Wolfson, O. (2016). Electric vehicle routing problem. *Transportation Research Procedia, 12*, 508–521.

Mancini, S. (2017). The hybrid vehicle routing problem. *Transportation Research Part C, 78*, 1–12.

Mirmohammadi, S. H., Babaee Tirkolaee, E., Goli, A., & Dehnavi-Arani, S. (2017). The periodic green vehicle routing problem with considering of time-dependent urban traffic and time windows. *Iran University of Science & Technology, 7*(1), 143–156.

Montoya, A., Guéret, C., Mendoza, J. E., & Villegas, J. G. (2016). A multi-space sampling heuristic for the green vehicle routing problem. *Transportation Research Part C: Emerging Technologies, 70*, 113–128.

Pan, S., Ballot, E., & Fontane, F. (2013). The reduction of greenhouse gas emissions from freight transport by pooling supply chains. *International Journal of Production Economics, 143*(1), 86–94.

Parsopoulos, K. E., & Vrahatis, M. N. (2002, March). Particle swarm optimization method in multiobjective problems. In *Proceedings of the ACM symposium on applied computing* (pp. 603–607).

Pisacane, O., Bruglieri, M., Mancini, S., & Pezzella, F. (2017). A path-based mixed integer linear programming formulation for the green vehicle routing problem. In *Proceedings of the annual workshop of the EURO working group on vehicle routing and logistics (VeRoLog)* 10–12 July, Amsterdam.

Pitera, K., Sandoval, F., & Goodchild, A. (2011). Evaluation of emissions reduction in urban pickup systems. *Transportation Research Record: Journal of the Transportation Research Board, 2224*(1), 8–16.

Poonthalir, G., Nadarajan, R., & Geetha, S. (2015). Vehicle routing problem with limited refueling halts using particle swarm optimization with greedy mutation operator. *RAIRO-Operations Research, 49*(4), 689–716.

Ratnaweera, A., Halgamuge, S. K., & Watson, H. C. (2004). Self-organizing hierarchical particle swarm optimizer with time-varying acceleration coefficients. *IEEE Transactions on evolutionary computation, 8*(3), 240–255.

Schneider, M., Stenger, A., & Goeke, D. (2014). The electric vehicle-routing problem with time windows and recharging stations. *Transportation Science, 48*(4), 500–520.

Schneider, M., Stenger, A., & Hof, J. (2015). An adaptive VNS algorithm for vehicle routing problems with intermediate stops. *OR Spectrum, 37*(2), 353–387.

Scott, C., Urquhart, N., & Hart, E. (2010, April). Influence of topology and payload on CO_2 optimised vehicle routing. *In the European conference on Applications of Evolutionary Computation* (pp. 141–150). Berlin, Heidelberg: Springer.

Suzuki, Y. (2011). A new truck-routing approach for reducing fuel consumption and pollutants emission. *Transportation Research Part D: Transport and Environment, 16*(1), 73–77.

Teng, L., & Zhang, Z. (2016). Green vehicle routing problem with load factor. *Advances in Transportation Studies, 3*, 75–82.

Tripathi, P. K., Bandyopadhyay, S., & Pal, S. K. (2007). Multi-objective particle swarm optimization with time variant inertia and acceleration coefficients. *Information Sciences, 177*(22), 5033–5049.

Xiao, Y., Zhao, Q., Kaku, I., & Xu, Y. (2012). Development of a fuel consumption optimization model for the capacitated vehicle routing problem. *Computers & Operations Research, 39*(7), 1419–1431.

Xiao, Y., & Konak, A. (2015). Green vehicle routing problem with time-varying traffic congestion. In *Proceedings of the fourteenth INFORMS computing society conference* (pp. 134–148).