

Machine Learning Assignment 3

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1 Task 1

1.1 A.1 (MLE Derivation)

From the questions given, we know the following facts:

1. Since all the features follow Bernoulli distribution, we have write the following PMF for features:

$$\begin{aligned}P(x_i|y = 0, \theta) &= \alpha_i^{x_i} * (1 - \alpha_i)^{1-x_i} \\P(x_i|y = 1, \theta) &= \beta_i^{x_i} * (1 - \beta_i)^{1-x_i}\end{aligned}$$

In order to do MLE, we need to make the following assumptions:

1. Assume that there are n_0 data points from the training set that have label 0.
2. Assume that there are n_1 data points from the training set that have label 1.

$$\begin{aligned}\theta &= \underset{\theta}{\operatorname{argmax}} P(Z|\theta) \\&= \underset{\theta}{\operatorname{argmax}} P(X, Y|\theta) \\&= \underset{\theta}{\operatorname{argmax}} \prod_{i=1}^n P(x^i, y^i|\theta) \\&= \underset{\theta}{\operatorname{argmax}} \prod_{i=1}^n P(x^i|y^i, \theta) * P(y^i|\theta) \\&= \underset{\theta}{\operatorname{argmax}} \left(\prod_{i:y^i=0} P(x^i|y^i = 0, \theta) * P(y^i = 0|\theta) \right) * \left(\prod_{i:y^i=1} P(x^i|y^i = 1, \theta) * P(y^i = 1|\theta) \right) \\&= \underset{\theta}{\operatorname{argmax}} \left(\prod_{i:y^i=0} P(x^i|y^i = 0, \theta) \right) * \left(\prod_{i:y^i=1} P(x^i|y^i = 1, \theta) \right) * \left(\prod_{i:y^i=0} P(y^i = 0|\theta) * \prod_{i:y^i=1} P(y^i = 1|\theta) \right)\end{aligned}$$

We can clearly see that right now we have the following three parts from the above equation

1.

$$\prod_{i:y^i=0} P(x^i|y^i = 0, \theta)$$

2.

$$\prod_{i:y^i=1} P(x^i|y^i = 1, \theta)$$

3.

$$\prod_{i:y^i=0} P(y^i = 0|\theta) * \prod_{i:y^i=1} P(y^i = 1|\theta)$$

In order to maximize the whole equation, we can maximize each part individually. Also, the first part only relates to α . The second part only relates to β . The third part only relates to γ . Therefore, we can rewrite the above three parts as the following:

1.

$$\prod_{i:y^i=0} P(x^i|y^i=0, \alpha)$$

2.

$$\prod_{i:y^i=1} P(x^i|y^i=1, \beta)$$

3.

$$\prod_{i:y^i=0} P(y^i=0|\gamma) * \prod_{i:y^i=1} P(y^i=1|\gamma)$$

1.1.1 Find α

$$\begin{aligned}
\alpha &= \underset{\alpha}{\operatorname{argmax}} \prod_{i:y^i=0} P(x^i|y^i=0, \alpha) \\
&= \underset{\alpha}{\operatorname{argmax}} \prod_{i:y^i=0} P(x_1^i \dots x_d^i | y^i=0, \alpha) \\
&= \underset{\alpha}{\operatorname{argmax}} \prod_{i:y^i=0} \left(\prod_{j=1}^d P(x_j^i | y^i=0, \alpha_j) \right) \\
&= \underset{\alpha}{\operatorname{argmax}} \prod_{i:y^i=0} \left(\prod_{j=1}^d \alpha_j^{x_j^i} * (1 - \alpha_j)^{(1-x_j^i)} \right)
\end{aligned}$$

Since $\alpha_j (j \in [1, d])$ are independent of each other, we can choose to optimize a single α_j , then we know how to optimize all α_j s.

$$\begin{aligned}
\alpha_j &= \underset{\alpha_j}{\operatorname{argmax}} \prod_{i:y^i=0} \alpha_j^{x_j^i} * (1 - \alpha_j)^{(1-x_j^i)} \\
&= \underset{\alpha_j}{\operatorname{argmax}} \sum_{i:y^i=0} x_j^i * \log(\alpha_j) + (1 - x_j^i) * \log(1 - \alpha_j) \\
&= \underset{\alpha_j}{\operatorname{argmax}} \sum_{i:y^i=0} x_j^i * \log(\alpha_j) + \sum_{i:y^i=0} (1 - x_j^i) * \log(1 - \alpha_j) \\
&= \underset{\alpha_j}{\operatorname{argmax}} \log(\alpha_j) * \sum_{i:y^i=0} x_j^i + \log(1 - \alpha_j) * \sum_{i:y^i=0} (1 - x_j^i) \\
&= \underset{\alpha_j}{\operatorname{argmax}} \log(\alpha_j) * \sum_{i:y^i=0} x_j^i + \log(1 - \alpha_j) * (n_0 - \sum_{i:y^i=0} x_j^i)
\end{aligned}$$

Let

$$\begin{aligned}
f(\alpha_j) &= \log(\alpha_j) * \sum_{i:y^i=0} x_j^i + \log(1 - \alpha_j) * (n_0 - \sum_{i:y^i=0} x_j^i) \\
\frac{\partial f(\alpha_j)}{\partial \alpha_j} &= \frac{\sum_{i:y^i=0} x_j^i}{\alpha_j} - \frac{n_0 - \sum_{i:y^i=0} x_j^i}{1 - \alpha_j}
\end{aligned}$$

Then, let

$$\begin{aligned}
\frac{\partial f(\alpha_j)}{\partial \alpha_j} &= 0 \\
\alpha_j &= \frac{\sum_{i:y^i=0} x_j^i}{n_0}
\end{aligned}$$

1.1.2 Find β

$$\begin{aligned}
\beta &= \underset{\beta}{\operatorname{argmax}} \prod_{i:y^i=1} P(x^i|y^i=1, \beta) \\
&= \underset{\beta}{\operatorname{argmax}} \prod_{i:y^i=1} P(x_1^i \dots x_d^i | y^i=1, \beta) \\
&= \underset{\beta}{\operatorname{argmax}} \prod_{i:y^i=1} \left(\prod_{j=1}^d P(x_j^i | y^i=1, \beta_j) \right) \\
&= \underset{\beta}{\operatorname{argmax}} \prod_{i:y^i=1} \left(\prod_{j=1}^d \beta_j^{x_j^i} * (1 - \beta_j)^{(1-x_j^i)} \right)
\end{aligned}$$

Since $\beta_j (j \in [1, d])$ are independent of each other, we can choose to optimize a single β_j , then we know how to optimize all β_j s.

$$\begin{aligned}
\beta_j &= \underset{\beta_j}{\operatorname{argmax}} \prod_{i:y^i=1} \beta_j^{x_j^i} * (1 - \beta_j)^{(1-x_j^i)} \\
&= \underset{\beta_j}{\operatorname{argmax}} \sum_{i:y^i=1} x_j^i * \log(\beta_j) + (1 - x_j^i) * \log(1 - \beta_j) \\
&= \underset{\beta_j}{\operatorname{argmax}} \sum_{i:y^i=1} x_j^i * \log(\beta_j) + \sum_{i:y^i=1} (1 - x_j^i) * \log(1 - \beta_j) \\
&= \underset{\beta_j}{\operatorname{argmax}} \log(\beta_j) * \sum_{i:y^i=1} x_j^i + \log(1 - \beta_j) * \sum_{i:y^i=1} (1 - x_j^i) \\
&= \underset{\beta_j}{\operatorname{argmax}} \log(\beta_j) * \sum_{i:y^i=1} x_j^i + \log(1 - \beta_j) * (n_1 - \sum_{i:y^i=1} x_j^i)
\end{aligned}$$

Let

$$\begin{aligned}
f(\beta_j) &= \log(\beta_j) * \sum_{i:y^i=1} x_j^i + \log(1 - \beta_j) * (n_1 - \sum_{i:y^i=1} x_j^i) \\
\frac{\partial f(\beta_j)}{\partial \beta_j} &= \frac{\sum_{i:y^i=1} x_j^i}{\beta_j} - \frac{n_1 - \sum_{i:y^i=1} x_j^i}{1 - \beta_j}
\end{aligned}$$

Then, let

$$\begin{aligned}
\frac{\partial f(\beta_j)}{\partial \beta_j} &= 0 \\
\beta_j &= \frac{\sum_{i:y^i=1} x_j^i}{n_1}
\end{aligned}$$

1.1.3 Find γ

$$\begin{aligned}\gamma &= \underset{\gamma}{\operatorname{argmax}} \prod_{i:y^i=0} P(y^i = 0|\gamma) * \prod_{i:y^i=1} P(y^i = 1|\gamma) \\&= \underset{\gamma}{\operatorname{argmax}} \prod_{i:y^i=0} \gamma * \prod_{i:y^i=1} (1 - \gamma) \\&= \underset{\gamma}{\operatorname{argmax}} \gamma^{n_0} (1 - \gamma)^{n_1} \\&= \underset{\gamma}{\operatorname{argmax}} \log(\gamma^{n_0} (1 - \gamma)^{n_1}) \\&= \underset{\gamma}{\operatorname{argmax}} n_0 \log(\gamma) + n_1 \log(1 - \gamma)\end{aligned}$$

Let

$$f(\gamma) = n_0 \log(\gamma) + n_1 \log(1 - \gamma)$$

$$\frac{\partial f(\gamma)}{\partial \gamma} = \frac{n_0}{\gamma} - \frac{n_1}{1 - \gamma}$$

Let

$$\frac{\partial f(\gamma)}{\partial \gamma} = 0$$

$$\gamma = \frac{n_0}{n_0 + n_1}$$

1.2 A.2 (Parameters with MLE)

The following are printed outputs:

gamma is 0.49575551782682514

beta is [0.03198653198653199, 0.03198653198653199, 0.0016835016835016834,
0.003367003367003367, 0.0, 0.0,
0.0016835016835016834, 0.0, 0.0,
0.0016835016835016834]

alpha is [0.03424657534246575, 0.018835616438356163, 0.0,
0.0, 0.0017123287671232876, 0.0017123287671232876,
0.0, 0.0017123287671232876, 0.0017123287671232876,
0.0]

1.3 B (Naive bayes classifier)

No outputs were generated in this section, please check the code in the code file.

1.4 C (Prediction Accuracy With MLE)

Accuracy on Training Set(using MLE): 96.01018675721562 %

Accuracy on Testing Set(using MLE): 62.54777070063694 %

1.5 D.1 (MAP Derivation)

For the P.M.F of Bernoulli distribution, it was shown in section A.1. In order to do MAP, we need to make the following assumptions:

1. Assume that there are n_0 data points from the training set that have label 0.
2. Assume that there are n_1 data points from the training set that have label 1.

$$\begin{aligned}
\theta &= \underset{\theta}{\operatorname{argmax}} P(\theta|Z) \\
&= \underset{\theta}{\operatorname{argmax}} \frac{P(Z|\theta) * P(\theta)}{P(Z)} \\
&= \underset{\theta}{\operatorname{argmax}} P(Z|\theta) * P(\theta) \\
&= \underset{\theta}{\operatorname{argmax}} P(X, Y|\theta) * P(\theta) \\
&= \underset{\theta}{\operatorname{argmax}} \left(\prod_{i=1}^n P(x^i, y^i|\theta) \right) * \left(P(\gamma) * \prod_{i=1}^d P(\alpha_i)P(\beta_i) \right) \\
&= \underset{\theta}{\operatorname{argmax}} \left(\prod_{i=1}^n P(x^i|y^i, \theta) * P(y^i|\theta) \right) * \left(P(\gamma) * \prod_{i=1}^d P(\alpha_i)P(\beta_i) \right) \\
&= \underset{\theta}{\operatorname{argmax}} \left(\prod_{i:y^i=0} P(x^i|y^i = 0, \theta) * P(y^i = 0|\theta) \right) * \left(\prod_{i:y^i=1} P(x^i|y^i = 1, \theta) * P(y^i = 1|\theta) \right) * \\
&\quad \left(P(\gamma) * \prod_{i=1}^d P(\alpha_i)P(\beta_i) \right) \\
&= \underset{\theta}{\operatorname{argmax}} \left[\left(\prod_{i:y^i=0} P(x^i|y^i = 0, \theta) \right) * \left(\prod_{i=1}^d P(\alpha_i) \right) \right] * \\
&\quad \left[\left(\prod_{i:y^i=1} P(x^i|y^i = 1, \theta) \right) * \left(\prod_{i=1}^d P(\beta_i) \right) \right] * \\
&\quad \left[P(\gamma) * \left(\prod_{i:y^i=0} P(y^i = 0|\theta) \right) * \left(\prod_{i:y^i=1} P(y^i = 1|\theta) \right) \right]
\end{aligned}$$

We can clearly see that right now we have the following three parts from the above equation

1.

$$\left[\left(\prod_{i:y^i=0} P(x^i|y^i=0, \theta) \right) * \left(\prod_{i=1}^d P(\alpha_i) \right) \right]$$

2.

$$\left[\left(\prod_{i:y^i=1} P(x^i|y^i=1, \theta) \right) * \left(\prod_{i=1}^d P(\beta_i) \right) \right]$$

3.

$$\left[P(\gamma) * \left(\prod_{i:y^i=0} P(y^i=0|\theta) \right) * \left(\prod_{i:y^i=1} P(y^i=1|\theta) \right) \right]$$

In order to maximize the whole equation, we can maximize each part individually. Also, the first part only relates to α . The second part only relates β . The third part only relates to γ . Therefore, we can rewrite the above three parts as the following:

1.

$$\left[\left(\prod_{i:y^i=0} P(x^i|y^i=0, \alpha) \right) * \left(\prod_{i=1}^d P(\alpha_i) \right) \right]$$

2.

$$\left[\left(\prod_{i:y^i=1} P(x^i|y^i=1, \beta) \right) * \left(\prod_{i=1}^d P(\beta_i) \right) \right]$$

3.

$$\left[P(\gamma) * \left(\prod_{i:y^i=0} P(y^i=0|\gamma) \right) * \left(\prod_{i:y^i=1} P(y^i=1|\gamma) \right) \right]$$

1.5.1 Find α

$$\begin{aligned}
\alpha &= \underset{\alpha}{\operatorname{argmax}} \left[\left(\prod_{i:y^i=0} P(x^i|y^i=0, \alpha) \right) * \left(\prod_{i=1}^d P(\alpha_i) \right) \right] \\
&= \underset{\alpha}{\operatorname{argmax}} \left[\left(\prod_{i:y^i=0} P(x_1^i \dots x_d^i|y^i=0, \alpha) \right) * \left(\prod_{i=1}^d P(\alpha_i) \right) \right] \\
&= \underset{\alpha}{\operatorname{argmax}} \left[\left(\prod_{i:y^i=0} \left(\prod_{j=1}^d P(x_j^i|y^i=0, \alpha_j) \right) \right) * \left(\prod_{i=1}^d P(\alpha_i) \right) \right] \\
&= \underset{\alpha}{\operatorname{argmax}} \left[\left(\prod_{i:y^i=0} \left(\prod_{j=1}^d \alpha_j^{x_j^i} * (1 - \alpha_j)^{1-x_j^i} \right) \right) * \left(\prod_{i=1}^d P(\alpha_i) \right) \right]
\end{aligned}$$

Since $\alpha_j (j \in [1, d])$ are independent of each other, we can choose to optimize a single α_j , then we know how to optimize all α_j s.

$$\begin{aligned}
\alpha_j &= \underset{\alpha_j}{\operatorname{argmax}} \left(\prod_{i:y^i=0} \alpha_j^{x_j^i} * (1 - \alpha_j)^{(1-x_j^i)} \right) * P(\alpha_j) \\
&= \underset{\alpha_j}{\operatorname{argmax}} \left(\sum_{i:y^i=0} x_j^i * \log(\alpha_j) \right) + \left(\sum_{i:y^i=0} (1 - x_j^i) * \log(1 - \alpha_j) \right) + \log(P(\alpha_j)) \\
&= \underset{\alpha_j}{\operatorname{argmax}} \left(\log(\alpha_j) \sum_{i:y^i=0} x_j^i \right) + \left(\log(1 - \alpha_j) \sum_{i:y^i=0} (1 - x_j^i) \right) + \log(P(\alpha_j)) \\
&= \underset{\alpha_j}{\operatorname{argmax}} \left(\log(\alpha_j) \sum_{i:y^i=0} x_j^i \right) + \left(\log(1 - \alpha_j) \left(n_0 - \sum_{i:y^i=0} x_j^i \right) \right) + \log(P(\alpha_j))
\end{aligned}$$

Let

$$f(\alpha_j) = \log(\alpha_j) \sum_{i:y^i=0} x_j^i + \log(1 - \alpha_j) \left(n_0 - \sum_{i:y^i=0} x_j^i \right) + \log(P(\alpha_j))$$

When $\alpha_j \leq 0.5$

$$f(\alpha_j) = \log(\alpha_j) \sum_{i:y^i=0} x_j^i + \log(1 - \alpha_j) \left(n_0 - \sum_{i:y^i=0} x_j^i \right) + \log(4\alpha_j)$$

$$\frac{\partial f(\alpha_j)}{\partial \alpha_j} = \frac{(n_0 + 1)\alpha_j - \left(\sum_{i:y^i=0} x_j^i \right) - 1}{(\alpha_j - 1)\alpha_j}$$

Then, let

$$\frac{\partial f(\alpha_j)}{\partial \alpha_j} = 0$$

$$\alpha_j = \frac{\left(\sum_{i:y^i=0} x_j^i \right) + 1}{n_0 + 1}$$

When $\alpha_j > 0.5$

$$f(\alpha_j) = \log(\alpha_j) \sum_{i:y^i=0} x_j^i + \log(1 - \alpha_j) \left(n_0 - \sum_{i:y^i=0} x_j^i \right) + \log(4 - 4\alpha_j)$$

$$\frac{\partial f(\alpha_j)}{\partial \alpha_j} = \frac{(n_0 + 1)\alpha_j - \left(\sum_{i:y^i=0} x_j^i \right)}{(\alpha_j - 1)\alpha_j}$$

Then, let

$$\begin{aligned} \frac{\partial f(\alpha_j)}{\partial \alpha_j} &= 0 \\ \alpha_j &= \frac{\left(\sum_{i:y^i=0} x_j^i \right)}{n_0 + 1} \end{aligned}$$

1.5.2 Find β

$$\begin{aligned}
\beta &= \underset{\beta}{\operatorname{argmax}} \left[\left(\prod_{i:y^i=1} P(x^i|y^i=1, \beta) \right) * \left(\prod_{i=1}^d P(\beta_i) \right) \right] \\
&= \underset{\beta}{\operatorname{argmax}} \left[\left(\prod_{i:y^i=1} P(x_1^i \dots x_d^i|y^i=1, \beta) \right) * \left(\prod_{i=1}^d P(\beta_i) \right) \right] \\
&= \underset{\beta}{\operatorname{argmax}} \left[\left(\prod_{i:y^i=1} \left(\prod_{j=1}^d P(x_j^i|y^i=1, \beta_j) \right) \right) * \left(\prod_{i=1}^d P(\beta_i) \right) \right] \\
&= \underset{\beta}{\operatorname{argmax}} \left[\left(\prod_{i:y^i=1} \left(\prod_{j=1}^d \beta_j^{x_j^i} * (1 - \beta_j)^{1-x_j^i} \right) \right) * \left(\prod_{i=1}^d P(\beta_i) \right) \right]
\end{aligned}$$

Since $\beta_j (j \in [1, d])$ are independent of each other, we can choose to optimize a single β_j , then we know how to optimize all β_j s.

$$\begin{aligned}
\beta_j &= \underset{\beta_j}{\operatorname{argmax}} \left(\prod_{i:y^i=1} \beta_j^{x_j^i} * (1 - \beta_j)^{(1-x_j^i)} \right) * P(\beta_j) \\
&= \underset{\beta_j}{\operatorname{argmax}} \left(\sum_{i:y^i=1} x_j^i * \log(\beta_j) \right) + \left(\sum_{i:y^i=1} (1 - x_j^i) * \log(1 - \beta_j) \right) + \log(P(\beta_j)) \\
&= \underset{\beta_j}{\operatorname{argmax}} \left(\log(\beta_j) \sum_{i:y^i=1} x_j^i \right) + \left(\log(1 - \beta_j) \sum_{i:y^i=1} (1 - x_j^i) \right) + \log(P(\beta_j)) \\
&= \underset{\beta_j}{\operatorname{argmax}} \left(\log(\beta_j) \sum_{i:y^i=1} x_j^i \right) + \left(\log(1 - \beta_j) \left(n_1 - \sum_{i:y^i=1} x_j^i \right) \right) + \log(P(\beta_j))
\end{aligned}$$

Let

$$f(\beta_j) = \log(\beta_j) \sum_{i:y^i=1} x_j^i + \log(1 - \beta_j) \left(n_1 - \sum_{i:y^i=1} x_j^i \right) + \log(P(\beta_j))$$

When $\beta_j \leq 0.5$

$$f(\beta_j) = \log(\beta_j) \sum_{i:y^i=1} x_j^i + \log(1 - \beta_j) \left(n_1 - \sum_{i:y^i=1} x_j^i \right) + \log(4\beta_j)$$

$$\frac{\partial f(\beta_j)}{\partial \beta_j} = \frac{(n_1 + 1)\beta_j - \left(\sum_{i:y^i=1} x_j^i \right) - 1}{(\beta_j - 1)\beta_j}$$

Then, let

$$\frac{\partial f(\beta_j)}{\partial \beta_j} = 0$$

$$\beta_j = \frac{\left(\sum_{i:y^i=1} x_j^i \right) + 1}{n_1 + 1}$$

When $\beta_j > 0.5$

$$f(\beta_j) = \log(\beta_j) \sum_{i:y^i=1} x_j^i + \log(1 - \beta_j) \left(n_1 - \sum_{i:y^i=1} x_j^i \right) + \log(4 - 4\beta_j)$$

$$\frac{\partial f(\beta_j)}{\partial \beta_j} = \frac{(n_1 + 1)\beta_j - \left(\sum_{i:y^i=1} x_j^i \right)}{(\beta_j - 1)\beta_j}$$

Then, let

$$\begin{aligned} \frac{\partial f(\beta_j)}{\partial \beta_j} &= 0 \\ \beta_j &= \frac{\left(\sum_{i:y^i=1} x_j^i \right)}{n_1 + 1} \end{aligned}$$

1.5.3 Find γ

$$\begin{aligned}
\gamma &= \underset{\gamma}{\operatorname{argmax}} \left[P(\gamma) * \left(\prod_{i:y^i=0} P(y^i = 0|\theta) \right) * \left(\prod_{i:y^i=1} P(y^i = 1|\theta) \right) \right] \\
&= \underset{\gamma}{\operatorname{argmax}} P(\gamma) \gamma^{n_0} (1 - \gamma)^{n_1} \\
&= \underset{\gamma}{\operatorname{argmax}} \log(P(\gamma) \gamma^{n_0} (1 - \gamma)^{n_1}) \\
&= \underset{\gamma}{\operatorname{argmax}} n_0 \log(\gamma) + n_1 \log(1 - \gamma) + \log(P(\gamma))
\end{aligned}$$

Let

$$f(\gamma) = n_0 \log(\gamma) + n_1 \log(1 - \gamma) + \log(P(\gamma))$$

When $\gamma \leq 0.5$

$$f(\gamma) = n_0 \log(\gamma) + n_1 \log(1 - \gamma) + \log(4\gamma)$$

$$\frac{\partial f(\gamma)}{\partial \gamma} = \frac{n_0}{\gamma} + \frac{1}{\gamma} - \frac{n_1}{1 - \gamma}$$

Let

$$\begin{aligned}
\frac{\partial f(\gamma)}{\partial \gamma} &= 0 \\
\gamma &= \frac{n_0 + 1}{n_0 + n_1 + 1}
\end{aligned}$$

When $\gamma > 0.5$

$$f(\gamma) = n_0 \log(\gamma) + n_1 \log(1 - \gamma) + \log(4 - 4\gamma)$$

$$\frac{\partial f(\gamma)}{\partial \gamma} = \frac{(n_0 + n_1 + 1)\gamma - n_0}{(\gamma - 1)\gamma}$$

Let

$$\begin{aligned}
\frac{\partial f(\gamma)}{\partial \gamma} &= 0 \\
\gamma &= \frac{n_0}{n_0 + n_1 + 1}
\end{aligned}$$

1.6 D.2 (Parameters With MAP)

The following are printed outputs:

gamma is 0.4961832061068702

beta is [0.03361344537815126, 0.03361344537815126, 0.0033613445378151263,
0.005042016806722689, 0.0016806722689075631, 0.0016806722689075631,
0.0033613445378151263, 0.0016806722689075631, 0.0016806722689075631,
0.0033613445378151263]

alpha is [0.035897435897435895, 0.020512820512820513, 0.0017094017094017094,
0.0017094017094017094, 0.003418803418803419, 0.003418803418803419,
0.0017094017094017094, 0.003418803418803419, 0.003418803418803419,
0.0017094017094017094]

1.7 D.3 (Prediction Accuracy With MAP)

Accuracy on Training Set(using MAP): 88.96434634974533 %

Accuracy on Testing Set(using MAP): 75.54140127388536 %

1.8 D.4 (MLE VS. MAP)

For accuracy on training set, MLE performs better than MAP. MLE has accuracy of 96.01% on training set and MAP has accuracy of 88.96% on training set. However, when it comes to testing dataset, MAP(with accuracy of 75.54%) performed better than MLE(with accuracy of 62.55%). Clearly, MLE method caused overfitting problem.

Justification:

1. For MLE, parameters estimations only base on the dataset we have. It has no knowledge about the distribution of the parameters. As a result, MLE cannot generalize the model we are developing so that MLE caused overfitting problem.
2. By using MAP, we can avoid 0 values in α_i and β_i . If α_i and β_i calculated from MLE are 0, then 1 will be added to the numerator when calculating MAP α_i and β_i , therefore, 0 values are avoided in α_i and β_i for MAP. As a result, we can avoid 0 probability values when we do the predictions, which can make predictions more meaningful.

2 Task 2 (SVM Classifier)

2.1 A (Linear Classifier)

Accuracy on Training Set(using SVM linear kernel): 98.49357554275588 %

Accuracy on Testing Set(using SVM linear kernel): 88.08255659121171 %

2.2 B (RBF kernel effect)

Accuracy on Training Set(using SVM RBF kernel with $\gamma = 0.70$): 98.93664155959237 %

Accuracy on Testing Set(using SVM RBF kernel with $\gamma = 0.70$): 86.21837549933421 %

Accuracy on Training Set(using SVM RBF kernel with $\gamma = 0.65$): 98.84802835622509 %

Accuracy on Testing Set(using SVM RBF kernel with $\gamma = 0.65$): 86.08521970705726 %

Accuracy on Training Set(using SVM RBF kernel with $\gamma = 0.60$): 98.58218874612317 %

Accuracy on Testing Set(using SVM RBF kernel with $\gamma = 0.60$): 85.9520639147803 %

Discussion:

RBF kernel is used to measure the similarity between two points. Smaller gamma value means further influence. As a result, two points can be considered similar even if they are quite far from each other. This is not desirable for classification problem. As a consequence, smaller gamma value produces worse accuracy for classification problem.

2.3 C (IDF Importance)

Accuracy on Training Set(using SVM RBF kernel with $\gamma = 0.70$ idf=True): 99.9113867966327%

Accuracy on Testing Set(using SVM RBF kernel with $\gamma = 0.70$ idf=True): 90.0133155792277%

Accuracy on Training Set(using SVM RBF kernel with $\gamma = 0.65$ idf=True): 99.9113867966327%

Accuracy on Testing Set(using SVM RBF kernel with $\gamma = 0.65$ idf=True): 90.14647137150466%

Accuracy on Training Set(using SVM RBF kernel with $\gamma = 0.60$ idf=True): 99.9113867966327%

Accuracy on Testing Set(using SVM RBF kernel with $\gamma = 0.60$ idf=True): 90.21304926764314%

Discussion:

If we turn on use_idf, the accuracies have increased on both training set and testing for all three gamma values. Turning on use_idf can decrease the impact of frequent words such as "is", "the", etc. This can increase the impact of other more important features in disguise. As a result, accuracies on both training set and testing set have increased.