(a) $\Theta(\log(\log(n)))$ $T(n) = \log(\log n)$

As i is approaching towards n by squaring itself, thus in the end $2^{2^k} = n$ $\log n = 2^k$ $\log(\log(n)) = k$ where k is the number of times the loop is run.

(b) the inner loop runs at big-Θ of $\Theta(k^3)$, and i is changing as a multiplier of $\sqrt{n}$, $i = c\sqrt{n}$, with in each if statement

$$T(n) = \sum_{i=1}^{n} (\Theta(1) + \sum_{j=1}^{n} \sum_{k=0}^{j^3} \Theta(1))$$

increase at rate of $\sqrt{n}$ ++

$$= \Theta(n) + \sum_{j=1}^{\sqrt{n}} j^3 n^{\frac{3}{2}}$$

$$= \Theta(n) + n^{\frac{3}{2}} \sum_{j=1}^{\sqrt{n}} j^3$$

$$= \Theta(n) + n^{\frac{3}{2}} (\sqrt{n})^4$$

$$= \Theta(n) + \Theta(n^{\frac{7}{2}})$$

$$= \Theta(n^{\frac{7}{2}})$$

there are $\sqrt{n}$ # of $\sqrt{n}$ within n. so the if statement trigger $\sqrt{n}$ times

$i^3 = (c\sqrt{n})^3 = c^3 n^{\frac{3}{2}}$

which c range from 1 to $\sqrt{n}$

(c) the worst case is that all element get hit by the for loop. which is n times. and with each time the run time for the inner for loop is $\Theta(\log(n))$, then there are $n^2 - n$ times where the inner loop is not hit. so the total runtime is $n\log n + (n^2 - n)\Theta(1)$ which is $\Theta(n^2)$

$$\sum_{i=1}^{n} \sum_{k=1}^{n} (\Theta(1) + \Theta(\log n))$$

$$= \sum_{i=1}^{n} \sum_{k=1}^{n} \Theta(1) + \sum_{i=1}^{n} \Theta(\log(n))$$

$$= (n^2 - n)\Theta(1) + \Theta(n\log n)$$

$$= \Theta(n^2) + \Theta(n\log n)$$

$$= \Theta(n^2)$$

(d) $$\sum_{i=0}^{n-1} (\Theta(1) + \sum_{i} \Theta(1))$$

$$= \Theta(n) + \sum_{k=1}^{\log_{\frac{3}{2}} \frac{n}{10}} 10 (\tfrac{3}{2})^k$$

$$= \Theta(n) + 10 \sum_{k=1}^{\log_{\frac{3}{2}} \frac{n}{10}} (\tfrac{3}{2})^k$$

$$= \Theta(n) + 10\,\Theta\left(\tfrac{3}{2}^{\log_{\frac{3}{2}} \frac{n}{10}}\right)$$

$$= \Theta(n) + 10\,\Theta(\tfrac{n}{10})$$

$$= \Theta(n) + \Theta(n)$$

$$= \Theta(n)$$

k	size	loop	size
1	10	$10(\frac{3}{2})^1$	15
2	15	$10(\frac{3}{2})^2$	22
3	22	$10(\frac{3}{2})^3$	33

$10(\frac{3}{2})^k = n$ $k = \log_{\frac{3}{2}} \frac{n}{10}$