



# Introduction (Fast Fourier Transform – FFT)

• The vibration of a machine is a physical motion.

• Vibration transducers convert this motion into an electrical signal. The electrical signal is then passed on to data collectors or analyzers. The analyzers then process this signal to give the FFTs and other parameters.



## FFT (continue)

- To achieve the final relevant output, the signal is processed with the following steps:
  - Analog signal input
  - Anti-alias filter
  - A/D converter
  - Overlap
  - Windows
  - FFT
  - Averaging
  - Display/storage.



## Fourier Transform

- A vibration or a system response can be represented by displacement, velocity and acceleration amplitudes in both time and frequency domains (Figure 4.1).
- Time domain consists of amplitude that varies with time. This is commonly referred to as filter-out or overall reading.
- Frequency domain is the domain where amplitudes are shown as series of sine and cosine waves. These waves have a magnitude and a phase, which vary with frequency.
- The measured vibrations are always in analog form (time domain), and need to be transformed to the frequency domain. This is the purpose of the fast Fourier transform (FFT). The FFT is thus a calculation on a sampled signal.



## Fourier Transform

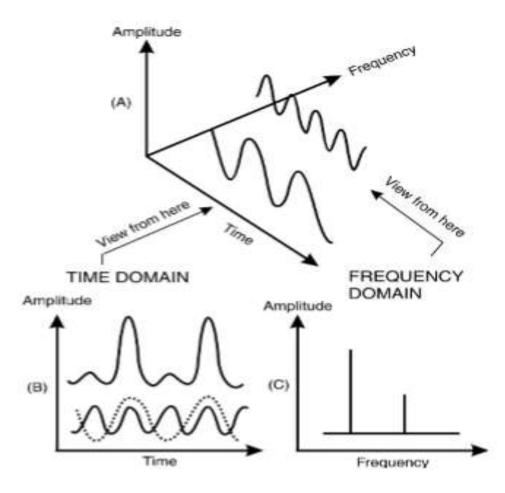


Figure 4.1 Fourier transform



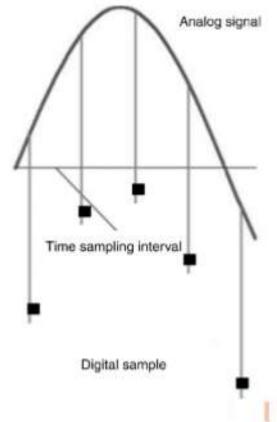
# Analog to Digital Converter

- The vibration waves collected by transducers are analog signals.
- Analog signals must be converted to digital values for further processing. This conversion from an analog signal to a digital signal is done by an Analog to Digital (A/D) converter.
- The A/D conversion is essentially done by microprocessors. Like any digital processor, A/D conversion works in the powers of two (called binary numbers). A 12-bit A/D converter provides 4096 intervals whereas a 16-bit A/D converter would provide 65 536 discrete intervals (Figure 4.3).

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# Analog to Digital Converter



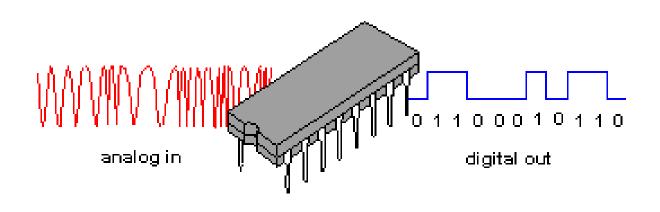


Figure 4.3

Analog to digital converters

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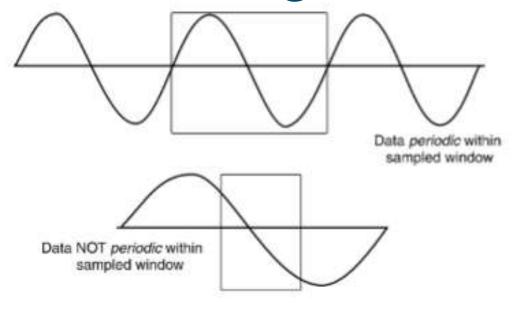
#### WINDOWING

- After the signal was digitized using an A/D converter, the next step in the process (before it can be subjected to the FFT algorithm) is called windowing
- Windowing is the equivalent of multiplying the signal sample by a window function of the same length
- A 'window' must be applied to the data to minimize signal 'leakage' effects

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# The Principle of Windowing





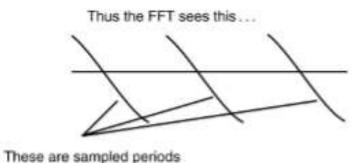
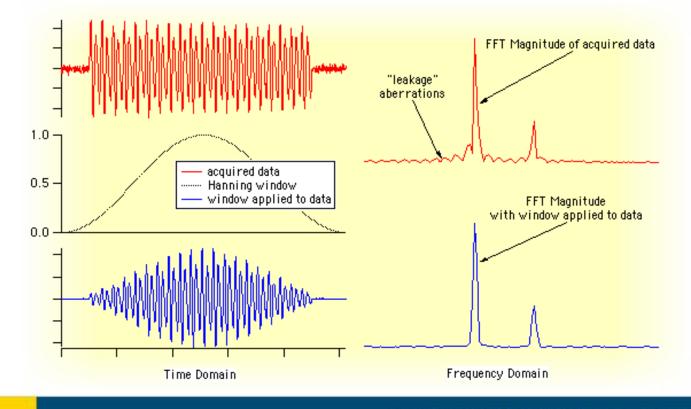


Figure 4.4
The principle of windowing

## What is Leakage



- <u>Leakage</u> is a discontinuous signal that can cause a noise in FFT diagram
- FFT algorithm reads a discontinuous signal as a sidebands, so that the analyst will be more difficult in understanding the diagram.





- There are many window functions. Some used in vibration signal processing are:
  - 1. Rectangular (basically no window)
  - 2. Flat top
  - 3. Hann
  - 4. Hamming
  - 5. Kaiser Bessel
  - 6. Blackman
  - 7. Barlett

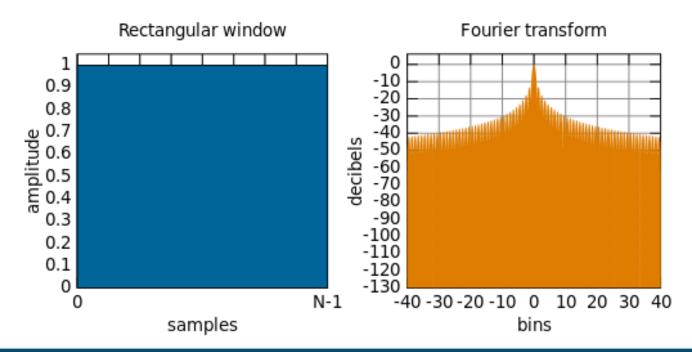
Generally, only the first three window functions mentioned above are available in most analyzers.



#### 1. Rectangular

The rectangular window (sometimes known as the **boxcar** or **Dirichlet window**) is the simplest window, equivalent to replacing all but *N* values of a data sequence by zeros, making it appear as though the waveform suddenly turns on and off:

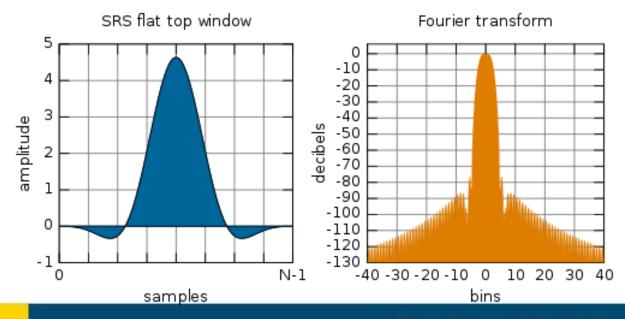
$$w(n) = 1.$$





#### 2. Flat Top

A flat top window is a partially negative-valued window that has minimal scalloping loss in the frequency domain. Such windows have been made available in spectrum analyzers for the measurement of amplitudes of sinusoidal frequency components. Drawbacks of the broad bandwidth are poor frequency resolution and high noise bandwidth.

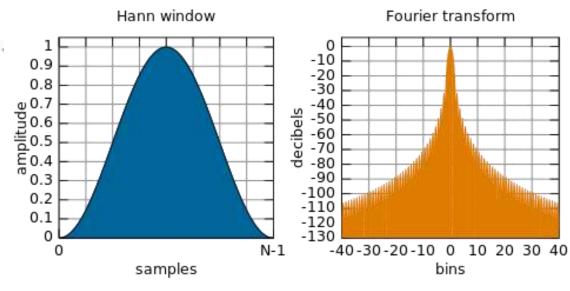




#### 3. Hann Windows

The Hann window, named after Julius von Hann, is sometimes referred to as *Hanning*, presumably due to its linguistic and formulaic similarities to Hamming window. It is also known as *raised cosine*, because the zero-phase version, is one lobe of an elevated cosine function. On the interval the Hann window function is:

$$w(n) = 0.5 \left[1-\cos\!\left(rac{2\pi n}{N-1}
ight)
ight] = \sin^2\!\left(rac{\pi n}{N-1}
ight).$$

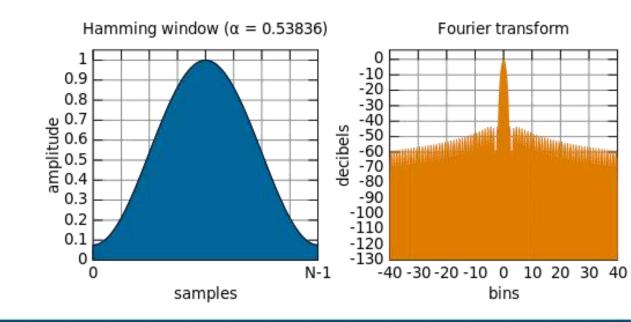




#### 4. Hamming Windows

The window with these particular coefficients was proposed by Richard W. Hamming. The window is optimized to minimize the maximum (nearest) side lobe, giving it a height of about one-fifth that of the Hann window.

$$w(n)=lpha-eta\,\cos\!\left(rac{2\pi n}{N-1}
ight)$$
 with  $lpha=0.54,\;eta=1-lpha=0.46.$ 





#### 5. Kaiser Bessel Windows

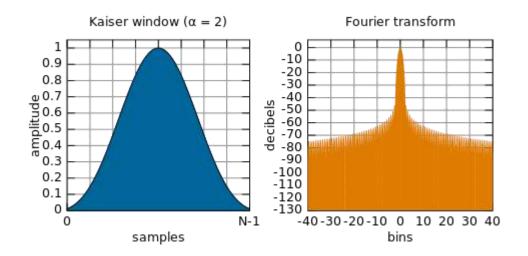
The Kaiser, or Kaiser-Bessel, window is a simple approximation of the DPSS window using Bessel functions, discovered by James Kaiser.

$$w(n)=rac{I_0\left(\pilpha\sqrt{1-(rac{2n}{N-1}-1)^2}
ight)}{I_0(\pilpha)}$$

where  $I_0$  is the zero-th order modified Bessel function of the first kind. Variable parameter  $\alpha$  determines the tradeoff between main lobe width and side lobe levels of the spectral leakage pattern. The main lobe width, in between the nulls, is given by  $2\sqrt{1+\alpha^2}$ , in units of DFT bins, and a typical value of  $\alpha$  is 3.



#### Kaiser Bessel Windows



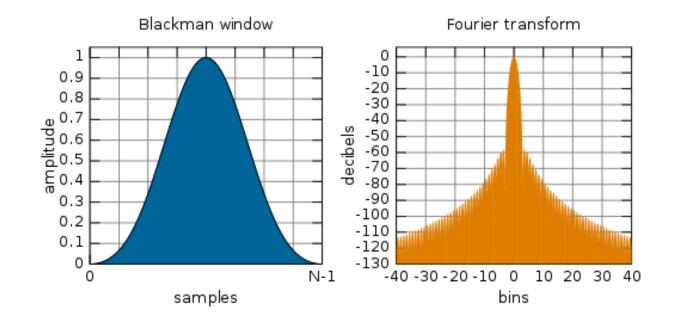


#### 5. Blackmann Windows

Blackman windows are defined as:

$$w(n) = a_0 - a_1 \cos igg(rac{2\pi n}{N-1}igg) + a_2 \cos igg(rac{4\pi n}{N-1}igg)$$

$$a_0=rac{1-lpha}{2};\quad a_1=rac{1}{2};\quad a_2=rac{lpha}{2}$$

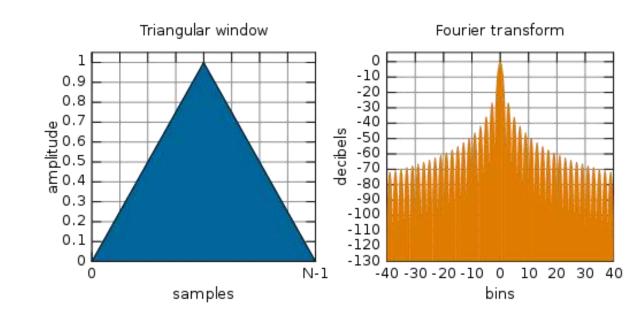




#### 5. Barlett Windows

Barlett or Triangular Windows defined as:

$$w(n)=1-\left|rac{n-rac{N-1}{2}}{rac{L}{2}}
ight|.$$





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