# Exam Paper

## Logic Exam Paper

\*\*Instructions:\*\*

\* Attempt all questions.

\* Clearly show all your work where required.

\* Time allowed: 45 minutes (suggested).

\* Total Marks: 25

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### Section A: Multiple Choice Questions (1 mark each)

For each question, choose the best answer.

1. Which of the following best describes a proposition in logic?

a) A command or question.

b) A statement that is always true.

c) A statement that can be either true or false.

d) A statement about a predicate.

2. Predicate logic extends propositional logic primarily by adding:

a) Only true statements.

b) Quantifiers and predicates.

c) More complex truth tables.

d) The concept of contradiction.

3. A tautology is a logical statement that is:

a) Always false, regardless of the truth values of its components.

b) Sometimes true and sometimes false.

c) Always true, regardless of the truth values of its components.

d) Used exclusively in predicate logic.

4. If a statement is a contradiction, it means it is:

a) Always true.

b) Always false.

c) Neither true nor false.

d) A proposition that is true.

5. What is the primary characteristic of propositional logic?

a) It deals with predicates and quantifiers.

b) It deals with statements that are either true or false.

c) It only deals with statements that are provably true.

d) It describes relationships between objects.

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### Section B: Short Answer Questions (5 marks each)

1. Differentiate between Propositional Logic and Predicate Logic. Your answer should highlight the key elements that each deals with.

2. Explain the concepts of a Tautology and a Contradiction. Provide a simple example for each to illustrate your explanation.

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### Section C: Essay Question (10 marks)

1. Discuss how Propositional Logic forms the foundation of logical reasoning and explain how Predicate Logic enhances this foundation. In your answer, define the core components of each type of logic and describe the significance of concepts like tautology and contradiction within these logical systems.

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## Marking Guide

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### Section A: Multiple Choice Questions (1 mark each)

1. \*\*c) A statement that can be either true or false.\*\*

\* \*Rationale:\* Directly from the definition of propositional logic.

2. \*\*b) Quantifiers and predicates.\*\*

\* \*Rationale:\* Directly from the definition of predicate logic extending propositional logic.

3. \*\*c) Always true, regardless of the truth values of its components.\*\*

\* \*Rationale:\* Direct definition of a tautology.

4. \*\*b) Always false.\*\*

\* \*Rationale:\* Direct definition of a contradiction.

5. \*\*b) It deals with statements that are either true or false.\*\*

\* \*Rationale:\* Core characteristic of propositional logic.

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### Section B: Short Answer Questions (5 marks each)

\*\*1. Differentiate between Propositional Logic and Predicate Logic. Your answer should highlight the key elements that each deals with.\*\*

\* \*\*Propositional Logic (2.5 marks):\*\*

\* Deals with simple declarative statements (propositions) that are either true or false. (1 mark)

\* Focuses on the truth values of entire propositions and how they combine using logical connectives (e.g., AND, OR, NOT, IF...THEN). (1.5 marks)

\* \*Example:\* "It is raining." is a single proposition.

\* \*\*Predicate Logic (2.5 marks):\*\*

\* Extends propositional logic. (0.5 marks)

\* Deals with the internal structure of propositions. It breaks down propositions into predicates (properties or relations) and arguments (objects or variables). (1 mark)

\* Introduces \*\*quantifiers\*\* (e.g., "for all" - universal quantifier, "there exists" - existential quantifier) to make statements about collections of objects. (1 mark)

\* \*Example:\* "All birds can fly." cannot be fully analyzed in propositional logic, but in predicate logic, "For every x, if x is a bird, then x can fly."

\*\*2. Explain the concepts of a Tautology and a Contradiction. Provide a simple example for each to illustrate your explanation.\*\*

\* \*\*Tautology (2.5 marks):\*\*

\* \*\*Explanation:\*\* A logical statement or formula that is always true, regardless of the truth values of its constituent propositions/components. (1.5 marks)

\* \*\*Example:\*\* (1 mark)

\* $P \lor \neg P$ ("P or not P").

\* \*Explanation:\* If P is true, then $T \lor F = T$. If P is false, then $F \lor T = T$. In both cases, the statement is true.

\* \*\*Contradiction (2.5 marks):\*\*

\* \*\*Explanation:\*\* A logical statement or formula that is always false, regardless of the truth values of its constituent propositions/components. It is the negation of a tautology. (1.5 marks)

\* \*\*Example:\*\* (1 mark)

\* $P \land \neg P$ ("P and not P").

\* \*Explanation:\* If P is true, then $T \land F = F$. If P is false, then $F \land T = F$. In both cases, the statement is false.

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### Section C: Essay Question (10 marks)

\*\*1. Discuss how Propositional Logic forms the foundation of logical reasoning and explain how Predicate Logic enhances this foundation. In your answer, define the core components of each type of logic and describe the significance of concepts like tautology and contradiction within these logical systems.\*\*

\*\*Expected Answer Structure and Marks Allocation:\*\*

1. \*\*Introduction (1 mark):\*\*

\* Briefly state the role of logic in formal reasoning.

\* Introduce propositional logic as fundamental and predicate logic as an extension.

2. \*\*Propositional Logic as Foundation (3 marks):\*\*

\* \*\*Definition:\*\* Explain that propositional logic deals with propositions (statements that are definitively true or false). (0.5 marks)

\* \*\*Core Components:\*\*

\* Propositions (P, Q, R, etc.). (0.5 marks)

\* Logical connectives (AND ($\land$), OR ($\lor$), NOT ($\neg$), IF...THEN ($\to$), IFF ($\leftrightarrow$)). (1 mark)

\* \*\*Foundation Role:\*\* Explain how it allows for basic truth evaluation of combined statements and establishes rules for valid inferences based purely on the truth values of whole statements. It provides a simple, yet powerful, framework for determining the truth of compound statements. (1 mark)

3. \*\*Predicate Logic Enhancement (3 marks):\*\*

\* \*\*Building upon Propositional Logic:\*\* Explain that while propositional logic handles the truth of whole statements, it cannot analyze their internal structure. Predicate logic addresses this limitation. (0.5 marks)

\* \*\*Core Components:\*\*

\* \*\*Predicates:\*\* Define as properties or relations that apply to objects/variables (e.g., $P(x)$ for "x is prime", $L(x,y)$ for "x loves y"). (1 mark)

\* \*\*Quantifiers:\*\* Define and explain the two main types:

\* Universal Quantifier ($\forall$ - "for all," "every"). (0.5 marks)

\* Existential Quantifier ($\exists$ - "there exists," "some"). (0.5 marks)

\* \*\*Variables/Objects:\*\* Mention that predicates apply to these. (0.5 marks)

\* \*\*Enhancement:\*\* Explain how it allows for reasoning about objects, their properties, and relationships, enabling more fine-grained and expressive logical statements (e.g., "All humans are mortal" or "Some students study hard"). It brings variables and functions into the logical analysis.

4. \*\*Significance of Tautology and Contradiction (2.5 marks):\*\*

\* \*\*Tautology:\*\*

\* \*\*Definition:\*\* Always true statements. (0.5 marks)

\* \*\*Significance:\*\* Represent universally valid logical truths or principles. They are fundamental rules of inference (e.g., modus ponens in its tautological form). A valid argument is one whose conditional statement from premises to conclusion is a tautology. (1 mark)

\* \*\*Contradiction:\*\*

\* \*\*Definition:\*\* Always false statements. (0.5 marks)

\* \*\*Significance:\*\* Indicate logical inconsistency or impossibility. If an argument leads to a contradiction, it signifies that at least one of its premises or its derivation is flawed. Essential for proof by contradiction. (0.5 marks)

5. \*\*Conclusion (0.5 marks):\*\*

\* Summarize how both systems are crucial, with propositional logic providing basic truth evaluation and predicate logic extending this to handle the complexity of the world's objects and relationships, both relying on concepts like tautology and contradiction for soundness.