# Exam Paper

This is a very ambitious exam structure given the brevity of the lecture notes. To meet the specified number of questions, many questions will necessarily cover similar ground, rephrased, or infer very basic associated concepts (e.g., examples of propositions, basic logical values).

Here is an exam paper and a detailed marking guide, structured as requested.

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# Logic Fundamentals Examination

\*\*Instructions:\*\*

\* This exam consists of three sections: Multiple Choice Questions (MCQ), Short Answer Questions (SAQ), and Essay Questions (EQ).

\* Answer all questions.

\* Total marks: 430.

\* Time allowed: Please consult your invigilator. (This would typically be several hours for an exam of this length).

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## Section A: Multiple Choice Questions (30 questions x 1 mark = 30 marks)

\*\*Instructions:\*\* Select the best answer for each question.

1. Propositional logic primarily deals with:

a) Quantifiers

b) Predicates

c) Propositions

d) Variables

2. A proposition is a statement that:

a) Can be true, false, or unknown

b) Is always true

c) Is always false

d) Is either true or false

3. Which type of logic extends propositional logic?

a) Boolean logic

b) Predicate logic

c) Modal logic

d) Temporal logic

4. Predicate logic adds which of the following to propositional logic?

a) Truth tables

b) Conjunctions

c) Quantifiers and predicates

d) Logical connectives

5. A statement that is always true, regardless of the truth values of its components, is known as a:

a) Contradiction

b) Tautology

c) Contingency

d) Fallacy

6. A statement that is always false, regardless of the truth values of its components, is known as a:

a) Tautology

b) Contingency

c) Contradiction

d) Paradox

7. Which of the following is a characteristic of a proposition?

a) It must be a question.

b) It expresses a command.

c) It has a definitive truth value.

d) It can be subjective.

8. The truth value of a proposition can be:

a) True only

b) False only

c) True or False

d) Undetermined

9. Which concept is fundamental to both propositional and predicate logic?

a) Quantifiers

b) Predicates

c) Truth values

d) Variables

10. "The sky is blue." In the context of propositional logic, this is an example of a:

a) Predicate

b) Quantifier

c) Proposition

d) Variable

11. Which logic system is more expressive for statements involving 'all' or 'some'?

a) Propositional logic

b) Predicate logic

c) Boolean logic

d) Sentential logic

12. A statement like "x > 5" is generally considered a predicate until 'x' is assigned a value. What makes it \*not\* a proposition on its own?

a) It uses a variable.

b) It has no truth value without context.

c) It contains a quantifier.

d) It's an inequality.

13. If a statement is a tautology, then its negation must be a:

a) Tautology

b) Contradiction

c) Contingency

d) Proposition

14. If a statement is a contradiction, then its negation must be a:

a) Contradiction

b) Tautology

c) Contingency

d) Proposition

15. "All birds can fly." To properly represent this statement, which logic system would be more appropriate?

a) Propositional logic

b) Predicate logic

c) Boolean logic

d) Temporal logic

16. What is the core characteristic that distinguishes propositional logic from everyday language?

a) Its use of complex grammar.

b) Its ambiguity.

c) Its focus on clear, unambiguous truth values.

d) Its reliance on context.

17. A statement that is neither a tautology nor a contradiction is called a:

a) Paradox

b) Fallacy

c) Contingency

d) Inference

18. Which of the following is NOT a component added by predicate logic?

a) Quantifiers

b) Predicates

c) Logical connectives (AND, OR, NOT)

d) Variables

19. In the context of logic, what does "true or false" refer to?

a) Subjective opinion

b) Emotional state

c) Truth value

d) Ambiguity

20. The primary purpose of formal logic systems like propositional and predicate logic is to:

a) Facilitate casual conversation

b) Provide a rigorous framework for reasoning

c) Express emotions

d) Create poetry

21. Consider the statement "It is raining AND it is NOT raining." This is an example of a:

a) Tautology

b) Contradiction

c) Contingency

d) Proposition

22. Consider the statement "It is raining OR it is NOT raining." This is an example of a:

a) Contradiction

b) Tautology

c) Contingency

d) Predicate

23. The scope of propositional logic is limited because it cannot represent:

a) Simple true/false statements

b) Relationships between objects or properties

c) Logical connectives

d) Basic truth values

24. When we say "quantifiers," we typically refer to terms like:

a) AND, OR, NOT

b) IF, THEN, ELSE

c) ALL, SOME, NONE

d) TRUE, FALSE

25. Predicates in logic are essentially:

a) Statements with a fixed truth value.

b) Properties or relationships attributed to subjects.

c) Quantifiers that bind variables.

d) Logical operators.

26. Which logic system allows for statements about individual objects and their properties?

a) Propositional logic

b) Predicate logic

c) Boolean logic

d) Set theory

27. The core idea of a "proposition" is its ability to be:

a) Persuasive

b) Debatable

c) Factually assessed as true or false

d) Rhetorical

28. To analyze the validity of an argument involving universal statements, one would typically use:

a) Propositional logic

b) Predicate logic

c) Informal logic

d) Fallacies

29. What happens to the truth value of a contradiction if you negate it?

a) It remains false.

b) It becomes true.

c) It becomes contingent.

d) It depends on the variables.

30. What happens to the truth value of a tautology if you negate it?

a) It remains true.

b) It becomes false.

c) It becomes contingent.

d) It depends on the variables.

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## Section B: Short Answer Questions (40 questions x 5 marks = 200 marks)

\*\*Instructions:\*\* Provide a concise and accurate answer for each question.

1. Define propositional logic.

2. What is the fundamental characteristic of a proposition?

3. Provide two examples of statements that are propositions.

4. Provide two examples of statements that are NOT propositions.

5. Explain the primary limitation of propositional logic.

6. Define predicate logic.

7. What does predicate logic add to propositional logic?

8. Name and briefly describe the two main components added by predicate logic.

9. Give an example of a simple predicate.

10. Give an example of a quantifier used in predicate logic.

11. Explain why "x is a prime number" is not a proposition on its own, but a predicate.

12. What is a tautology?

13. Provide a simple logical example of a tautology (in words).

14. What is a contradiction?

15. Provide a simple logical example of a contradiction (in words).

16. Distinguish between a proposition and a predicate.

17. Why is the ability to handle quantifiers important in logic?

18. In what scenarios would predicate logic be more suitable than propositional logic?

19. Can a statement be both a tautology and a contradiction? Explain.

20. What is a "truth value"?

21. Explain how propositional logic deals with the truth or falsity of statements.

22. How do quantifiers help in expressing universal statements?

23. How do quantifiers help in expressing existential statements?

24. What role do variables play in predicate logic that they don't in propositional logic?

25. Is "Run!" a proposition? Justify your answer.

26. Is "What time is it?" a proposition? Justify your answer.

27. Consider the statement "All men are mortal." Which logic system is better suited to analyze this, and why?

28. How does predicate logic allow for more detailed analysis of arguments than propositional logic?

29. What is the relationship between propositional logic and predicate logic in terms of complexity?

30. If a statement is a tautology, what does that imply about its negation's truth value?

31. If a statement is a contradiction, what does that imply about its negation's truth value?

32. Why is clarity regarding truth values essential in logical reasoning?

33. How do predicates allow for expressing properties of objects?

34. How do predicates allow for expressing relationships between objects?

35. What is the fundamental difference in what propositional logic and predicate logic \*model\* about the world?

36. Give an example of how a simple proposition might be part of a more complex predicate logic statement.

37. Explain why "2 + 2 = 4" is a proposition.

38. Explain why "This statement is false" is often considered problematic in basic propositional logic.

39. What are the two possible truth values for any given proposition?

40. Briefly describe the historical development implied by one logic "extending" another.

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## Section C: Essay Questions (20 questions x 10 marks = 200 marks)

\*\*Instructions:\*\* Write a comprehensive answer for each question, demonstrating your understanding of the concepts.

1. Discuss the fundamental principles of propositional logic. Explain its scope and its inherent limitations.

2. Elaborate on how predicate logic extends propositional logic. Detail the new components it introduces and their significance.

3. Compare and contrast propositional logic and predicate logic. Provide examples to illustrate their differences in expressiveness.

4. Define and explain the concepts of tautology and contradiction in detail. Provide clear examples for each and discuss their importance in logical reasoning.

5. Explain the concept of a "proposition" thoroughly. Discuss its essential characteristics and differentiate it from other types of statements.

6. Describe the role of "predicates" in predicate logic. How do they enable a more granular analysis of statements compared to propositional logic?

7. Discuss the significance of "quantifiers" in predicate logic. Explain how they allow for the expression of general statements and relationships.

8. Imagine you are trying to formalize a simple argument like "All students are intelligent. John is a student. Therefore, John is intelligent." Explain why propositional logic would be insufficient and how predicate logic resolves this.

9. Explain why formalizing statements into logical forms (like propositions or predicates) is beneficial for precise communication and avoiding ambiguity.

10. Detail how the concept of "truth value" is central to all formal logical systems, drawing on examples from both propositional and predicate logic.

11. Discuss the implications of a statement being a tautology for logical proof and validity.

12. Discuss the implications of a statement being a contradiction for logical proof and consistency.

13. How does the structure of arguments change when moving from propositional logic to predicate logic due to the addition of quantifiers and predicates?

14. Explain the practical utility of understanding tautologies and contradictions in designing logical systems or identifying flawed arguments.

15. Discuss the relationship between the two truth values (True and False) and the bivalence principle in logic, as it applies to propositions.

16. Explain how predicate logic can represent statements about properties of individual objects and relationships between multiple objects. Provide examples.

17. Beyond simply "adding" features, how does predicate logic fundamentally change the \*kind\* of logical problems that can be addressed?

18. Explore the core idea that "logic deals with propositions that are either true or false" in the context of both types of logic discussed.

19. Discuss the concept of "extending" a logical system. What does it mean for predicate logic to "extend" propositional logic, and what advantages does this provide?

20. Imagine you need to teach someone the very basics of formal logic using only the terms "propositional logic," "predicate logic," "tautology," and "contradiction." Outline your lesson plan, emphasizing how you would make these concepts clear and interconnected.

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# Logic Fundamentals Examination - Marking Guide

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## Section A: Multiple Choice Questions (30 marks)

\*\*1 mark per correct answer.\*\*

1. c) Propositions

2. d) Is either true or false

3. b) Predicate logic

4. c) Quantifiers and predicates

5. b) Tautology

6. c) Contradiction

7. c) It has a definitive truth value.

8. c) True or False

9. c) Truth values

10. c) Proposition

11. b) Predicate logic

12. b) It has no truth value without context.

13. b) Contradiction

14. b) Tautology

15. b) Predicate logic

16. c) Its focus on clear, unambiguous truth values.

17. c) Contingency

18. c) Logical connectives (AND, OR, NOT)

19. c) Truth value

20. b) Provide a rigorous framework for reasoning

21. b) Contradiction

22. b) Tautology

23. b) Relationships between objects or properties

24. c) ALL, SOME, NONE

25. b) Properties or relationships attributed to subjects.

26. b) Predicate logic

27. c) Factually assessed as true or false

28. b) Predicate logic

29. b) It becomes true.

30. b) It becomes false.

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## Section B: Short Answer Questions (200 marks)

\*\*5 marks per question.\*\*

1. \*\*Define propositional logic.\*\*

\* \*\*Expected Answer:\*\* Propositional logic is a branch of logic that deals with propositions, which are statements that are either true or false. It focuses on how these propositions combine using logical connectives (like AND, OR, NOT) to form more complex statements, and analyzes their truth values.

\* \*\*Marking:\*\* 5 marks for a clear, accurate definition mentioning propositions and their truth values. 3-4 marks for partial accuracy.

2. \*\*What is the fundamental characteristic of a proposition?\*\*

\* \*\*Expected Answer:\*\* The fundamental characteristic of a proposition is that it must have a definitive truth value; it is either true or false, but not both, and not neither.

\* \*\*Marking:\*\* 5 marks for clearly stating "either true or false" as the defining characteristic. 3-4 marks for stating truth value without emphasizing the bivalence.

3. \*\*Provide two examples of statements that are propositions.\*\*

\* \*\*Expected Answer:\*\*

\* "The Earth is round." (True)

\* "2 + 2 = 5." (False)

\* "It is currently raining outside." (Can be definitively true or false at a given moment)

\* \*\*Marking:\*\* 5 marks for two clear, unambiguous examples. 3 marks for one good example or two less clear ones.

4. \*\*Provide two examples of statements that are NOT propositions.\*\*

\* \*\*Expected Answer:\*\*

\* "What is your name?" (A question)

\* "Close the door!" (A command)

\* "Oh, wow!" (An exclamation)

\* "x + y = 10" (An open statement whose truth depends on variables)

\* \*\*Marking:\*\* 5 marks for two clear examples with justification. 3 marks for one good example or two less clear ones.

5. \*\*Explain the primary limitation of propositional logic.\*\*

\* \*\*Expected Answer:\*\* The primary limitation of propositional logic is its inability to analyze the internal structure of propositions. It treats simple statements as indivisible units, making it unable to represent relationships between objects, properties of objects, or quantifiers like "all" or "some."

\* \*\*Marking:\*\* 5 marks for clearly identifying the inability to analyze internal structure or handle relations/quantifiers. 3-4 marks for partial understanding.

6. \*\*Define predicate logic.\*\*

\* \*\*Expected Answer:\*\* Predicate logic (or first-order logic) is an extension of propositional logic that allows for the analysis of the internal structure of statements by introducing predicates, quantifiers, and variables. It can express more complex relationships and general statements.

\* \*\*Marking:\*\* 5 marks for a clear, accurate definition mentioning its extension of propositional logic and the addition of predicates/quantifiers. 3-4 marks for partial accuracy.

7. \*\*What does predicate logic add to propositional logic?\*\*

\* \*\*Expected Answer:\*\* Predicate logic adds quantifiers (like "for all" and "there exists") and predicates (which represent properties or relationships) to propositional logic, along with variables.

\* \*\*Marking:\*\* 5 marks for listing quantifiers and predicates. 3 marks for only one or partial explanation.

8. \*\*Name and briefly describe the two main components added by predicate logic.\*\*

\* \*\*Expected Answer:\*\*

\* \*\*Predicates:\*\* These are expressions that represent properties of objects or relationships between objects. They take arguments (variables or constants) and, when fully instantiated, become a proposition (e.g., P(x) where P is the predicate and x is the argument).

\* \*\*Quantifiers:\*\* These are symbols used to specify the quantity of individuals for whom a predicate is true. The two main types are the universal quantifier ($\forall$, "for all") and the existential quantifier ($\exists$, "there exists").

\* \*\*Marking:\*\* 5 marks for naming and briefly describing both components. 2-3 marks for naming only or describing only one.

9. \*\*Give an example of a simple predicate.\*\*

\* \*\*Expected Answer:\*\*

\* $P(x)$: "x is a prime number."

\* $L(x, y)$: "x loves y."

\* $IsRed(apple)$: "The apple is red."

\* \*\*Marking:\*\* 5 marks for a clear, logical example of a predicate with variables. 3 marks for an example without proper variable notation but conceptually correct.

10. \*\*Give an example of a quantifier used in predicate logic.\*\*

\* \*\*Expected Answer:\*\*

\* The Universal Quantifier ($\forall$): "For all," "for every," "all."

\* The Existential Quantifier ($\exists$): "There exists," "there is at least one," "some."

\* \*\*Marking:\*\* 5 marks for naming and providing the symbol or a clear English equivalent for one quantifier. 3 marks for just the name or just the symbol.

11. \*\*Explain why "x is a prime number" is not a proposition on its own, but a predicate.\*\*

\* \*\*Expected Answer:\*\* "x is a prime number" is not a proposition on its own because its truth value cannot be determined until 'x' is replaced by a specific value (e.g., 5 is prime, 4 is not prime). It is a predicate because it describes a property of 'x' that becomes a proposition once 'x' is bound by a quantifier or assigned a specific value.

\* \*\*Marking:\*\* 5 marks for explaining the dependence on 'x' for truth value and correctly identifying it as a predicate. 3-4 marks for explaining dependence but not explicitly stating it's a predicate.

12. \*\*What is a tautology?\*\*

\* \*\*Expected Answer:\*\* A tautology is a logical statement or proposition that is always true, regardless of the truth values of its constituent simple propositions.

\* \*\*Marking:\*\* 5 marks for a clear, accurate definition. 3-4 marks for partial accuracy.

13. \*\*Provide a simple logical example of a tautology (in words).\*\*

\* \*\*Expected Answer:\*\* "It is raining or it is not raining." (Symbolically: $P \lor \neg P$)

\* \*\*Marking:\*\* 5 marks for a clear verbal example that fits the definition. 3 marks for an example that is logically sound but perhaps less clear.

14. \*\*What is a contradiction?\*\*

\* \*\*Expected Answer:\*\* A contradiction is a logical statement or proposition that is always false, regardless of the truth values of its constituent simple propositions.

\* \*\*Marking:\*\* 5 marks for a clear, accurate definition. 3-4 marks for partial accuracy.

15. \*\*Provide a simple logical example of a contradiction (in words).\*\*

\* \*\*Expected Answer:\*\* "It is raining and it is not raining." (Symbolically: $P \land \neg P$)

\* \*\*Marking:\*\* 5 marks for a clear verbal example that fits the definition. 3 marks for an example that is logically sound but perhaps less clear.

16. \*\*Distinguish between a proposition and a predicate.\*\*

\* \*\*Expected Answer:\*\* A proposition is a complete declarative statement that is definitively either true or false. A predicate, on the other hand, is a statement \*about\* one or more variables that becomes a proposition only when the variables are instantiated with specific values or bound by quantifiers. Predicates describe properties or relationships.

\* \*\*Marking:\*\* 5 marks for clear distinction regarding completeness/truth value vs. variable dependence/property description. 3-4 marks for partial distinction.

17. \*\*Why is the ability to handle quantifiers important in logic?\*\*

\* \*\*Expected Answer:\*\* The ability to handle quantifiers is crucial because it allows logic to express general statements about collections of objects (e.g., "all," "some," "none"). This capability enables the analysis of arguments involving universal or existential claims, which is beyond the scope of propositional logic.

\* \*\*Marking:\*\* 5 marks for explaining its role in expressing general statements (all/some) and analyzing related arguments. 3-4 marks for partial understanding.

18. \*\*In what scenarios would predicate logic be more suitable than propositional logic?\*\*

\* \*\*Expected Answer:\*\* Predicate logic is more suitable when dealing with statements that involve:

\* Properties of individual objects (e.g., "Socrates is mortal").

\* Relationships between objects (e.g., "John loves Mary").

\* Generalizations about groups or classes (e.g., "All birds can fly," "Some students are diligent").

\* Any argument requiring analysis of the internal structure of propositions.

\* \*\*Marking:\*\* 5 marks for identifying scenarios involving internal structure, properties, relations, or generalizations. 3-4 marks for one or two examples.

19. \*\*Can a statement be both a tautology and a contradiction? Explain.\*\*

\* \*\*Expected Answer:\*\* No, a statement cannot be both a tautology and a contradiction. A tautology is always true, while a contradiction is always false. By definition, a statement cannot be both always true and always false simultaneously.

\* \*\*Marking:\*\* 5 marks for a clear "No" and correct justification based on their definitions. 3 marks for "No" without full justification.

20. \*\*What is a "truth value"?\*\*

\* \*\*Expected Answer:\*\* A truth value is a value indicating the relation of a proposition to truth. In classical logic, the two possible truth values are "True" (T) and "False" (F).

\* \*\*Marking:\*\* 5 marks for defining it as a value indicating truth/falsity, mentioning T/F. 3 marks for partial definition.

21. \*\*Explain how propositional logic deals with the truth or falsity of statements.\*\*

\* \*\*Expected Answer:\*\* Propositional logic assigns one of two truth values (True or False) to simple propositions. It then uses truth tables and logical connectives (AND, OR, NOT, etc.) to determine the truth value of complex statements based on the truth values of their constituent simple propositions.

\* \*\*Marking:\*\* 5 marks for explaining assignment of T/F to simple propositions and using connectives/truth tables for complex ones. 3-4 marks for partial explanation.

22. \*\*How do quantifiers help in expressing universal statements?\*\*

\* \*\*Expected Answer:\*\* The universal quantifier ($\forall$, "for all") allows us to express that a certain property holds true for \*every\* member of a given domain. For example, "All humans are mortal" can be written as $\forall x (Human(x) \rightarrow Mortal(x))$.

\* \*\*Marking:\*\* 5 marks for explaining the role of $\forall$ in "every/all" and providing an example. 3-4 marks for partial explanation.

23. \*\*How do quantifiers help in expressing existential statements?\*\*

\* \*\*Expected Answer:\*\* The existential quantifier ($\exists$, "there exists") allows us to express that a certain property holds true for \*at least one\* member of a given domain. For example, "Some birds can fly" can be written as $\exists x (Bird(x) \land CanFly(x))$.

\* \*\*Marking:\*\* 5 marks for explaining the role of $\exists$ in "at least one/some" and providing an example. 3-4 marks for partial explanation.

24. \*\*What role do variables play in predicate logic that they don't in propositional logic?\*\*

\* \*\*Expected Answer:\*\* In predicate logic, variables (e.g., x, y) act as placeholders within predicates, allowing us to represent properties or relationships generically (e.g., P(x) or L(x,y)). These variables can then be bound by quantifiers to make general statements. In propositional logic, statements are treated as atomic units without internal variables.

\* \*\*Marking:\*\* 5 marks for explaining variables as placeholders in predicates for generic statements and their binding by quantifiers. 3-4 marks for partial understanding.

25. \*\*Is "Run!" a proposition? Justify your answer.\*\*

\* \*\*Expected Answer:\*\* No, "Run!" is not a proposition. It is a command or imperative statement. It does not assert something that can be definitively judged as true or false.

\* \*\*Marking:\*\* 5 marks for "No" and correct justification. 3 marks for "No" without full justification.

26. \*\*Is "What time is it?" a proposition? Justify your answer.\*\*

\* \*\*Expected Answer:\*\* No, "What time is it?" is not a proposition. It is a question. It seeks information and does not assert a fact that can be assigned a truth value of true or false.

\* \*\*Marking:\*\* 5 marks for "No" and correct justification. 3 marks for "No" without full justification.

27. \*\*Consider the statement "All men are mortal." Which logic system is better suited to analyze this, and why?\*\*

\* \*\*Expected Answer:\*\* Predicate logic is better suited. Propositional logic would treat "All men are mortal" as a single atomic proposition (e.g., P), without revealing its internal structure. Predicate logic, however, can represent the relationship between "being a man" and "being mortal" for all individuals, using quantifiers and predicates (e.g., $\forall x (Man(x) \rightarrow Mortal(x))$).

\* \*\*Marking:\*\* 5 marks for identifying predicate logic and explaining \*why\* (internal structure, quantifiers, relationships). 3-4 marks for correct choice but weaker justification.

28. \*\*How does predicate logic allow for more detailed analysis of arguments than propositional logic?\*\*

\* \*\*Expected Answer:\*\* Predicate logic allows for more detailed analysis because it can break down propositions into predicates and arguments, and it can represent quantifiers. This means it can capture the validity of arguments that depend on the internal structure of statements (e.g., syllogisms like "All A are B, x is A, therefore x is B"), which propositional logic cannot.

\* \*\*Marking:\*\* 5 marks for explaining its ability to capture internal structure, quantifiers, and thus analyze arguments propositional logic cannot. 3-4 marks for partial understanding.

29. \*\*What is the relationship between propositional logic and predicate logic in terms of complexity?\*\*

\* \*\*Expected Answer:\*\* Predicate logic is a more complex and expressive system than propositional logic. It \*extends\* propositional logic by adding features (predicates, quantifiers, variables) that allow for a finer-grained analysis of statements and arguments, thus handling a broader range of logical expressions.

\* \*\*Marking:\*\* 5 marks for explaining predicate logic as an extension, more complex/expressive, handling finer-grained analysis. 3-4 marks for partial explanation.

30. \*\*If a statement is a tautology, what does that imply about its negation's truth value?\*\*

\* \*\*Expected Answer:\*\* If a statement is a tautology (always true), its negation must be a contradiction (always false).

\* \*\*Marking:\*\* 5 marks for correctly stating it's a contradiction/always false. 3 marks for just "always false."

31. \*\*If a statement is a contradiction, what does that imply about its negation's truth value?\*\*

\* \*\*Expected Answer:\*\* If a statement is a contradiction (always false), its negation must be a tautology (always true).

\* \*\*Marking:\*\* 5 marks for correctly stating it's a tautology/always true. 3 marks for just "always true."

32. \*\*Why is clarity regarding truth values essential in logical reasoning?\*\*

\* \*\*Expected Answer:\*\* Clarity regarding truth values is essential because logical reasoning aims to determine the truth or falsity of conclusions based on the truth or falsity of premises. Without clear, unambiguous truth values for individual propositions, it would be impossible to consistently evaluate the validity of arguments or to draw reliable inferences.

\* \*\*Marking:\*\* 5 marks for linking clarity of truth values to evaluating argument validity and drawing reliable inferences. 3-4 marks for partial explanation.

33. \*\*How do predicates allow for expressing properties of objects?\*\*

\* \*\*Expected Answer:\*\* Predicates represent properties by taking an object as an argument and asserting something about that object. For example, "IsHuman(Socrates)" expresses the property of "being human" for the object "Socrates."

\* \*\*Marking:\*\* 5 marks for explaining predicates taking an object as argument and asserting a property, with example. 3-4 marks for partial explanation.

34. \*\*How do predicates allow for expressing relationships between objects?\*\*

\* \*\*Expected Answer:\*\* Predicates can express relationships by taking two or more objects as arguments. For example, "Loves(John, Mary)" expresses the relationship of "loving" between John and Mary.

\* \*\*Marking:\*\* 5 marks for explaining predicates taking multiple objects as arguments to show relationship, with example. 3-4 marks for partial explanation.

35. \*\*What is the fundamental difference in what propositional logic and predicate logic \*model\* about the world?\*\*

\* \*\*Expected Answer:\*\* Propositional logic models the world at the level of atomic facts (propositions) and their truth values, focusing on how these facts combine. Predicate logic models the world at a more granular level, representing objects, their properties, and relationships between them, allowing for statements about quantities and specific individuals.

\* \*\*Marking:\*\* 5 marks for clearly distinguishing the level of modeling: atomic facts vs. objects, properties, relationships, and quantities. 3-4 marks for partial distinction.

36. \*\*Give an example of how a simple proposition might be part of a more complex predicate logic statement.\*\*

\* \*\*Expected Answer:\*\*

\* Simple proposition: "Socrates is a man." (P)

\* Predicate logic statement: "If Socrates is a man, then Socrates is mortal." This can be broken down using predicates and individual constants: $Man(Socrates) \rightarrow Mortal(Socrates)$. Here, $Man(Socrates)$ and $Mortal(Socrates)$ could each be seen as propositions derived from predicates.

\* (Alternative example): "It is raining" (P). "If it is raining, then all flowers are wet." P $\rightarrow \forall x (Flower(x) \rightarrow Wet(x))$.

\* \*\*Marking:\*\* 5 marks for a clear example showing a simple proposition being incorporated or derived within a predicate logic structure. 3-4 marks for a less clear but relevant example.

37. \*\*Explain why "2 + 2 = 4" is a proposition.\*\*

\* \*\*Expected Answer:\*\* "2 + 2 = 4" is a proposition because it is a declarative statement that is unambiguously true. Its truth value is fixed and universally acknowledged, fulfilling the core requirement of a proposition.

\* \*\*Marking:\*\* 5 marks for identifying it as a declarative statement with an unambiguous, fixed truth value (True). 3-4 marks for partial explanation.

38. \*\*Explain why "This statement is false" is often considered problematic in basic propositional logic.\*\*

\* \*\*Expected Answer:\*\* "This statement is false" (the Liar Paradox) is problematic because if it were true, then it would have to be false, and if it were false, then it would have to be true. It violates the fundamental principle of propositions having a single, definite truth value (bivalence) and leads to a logical contradiction, making it difficult to analyze within simple propositional frameworks.

\* \*\*Marking:\*\* 5 marks for explaining the self-referential paradox and its violation of definitive truth value. 3-4 marks for describing the paradox without fully linking it to propositional logic's principles.

39. \*\*What are the two possible truth values for any given proposition?\*\*

\* \*\*Expected Answer:\*\* True and False.

\* \*\*Marking:\*\* 5 marks for correctly stating "True and False."

40. \*\*Briefly describe the historical development implied by one logic "extending" another.\*\*

\* \*\*Expected Answer:\*\* The idea of predicate logic "extending" propositional logic implies a historical and conceptual progression. Propositional logic was developed first to handle basic truth-functional relationships. As logicians sought to represent more complex, detailed arguments (especially those involving quantification and internal structure), new concepts were \*added\* to the existing framework, leading to the development of predicate logic, which builds upon and incorporates the principles of propositional logic.

\* \*\*Marking:\*\* 5 marks for explaining the conceptual/historical build-up, with predicate logic addressing limitations of propositional logic. 3-4 marks for partial understanding of "extending."

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## Section C: Essay Questions (200 marks)

\*\*10 marks per question.\*\*

\*\*General Marking Criteria for Essays:\*\*

\* \*\*10 marks:\*\* Comprehensive, accurate, and well-structured answer. Demonstrates deep understanding of the concepts, provides clear explanations, and uses appropriate terminology. Excellent examples if required.

\* \*\*7-9 marks:\*\* Good understanding, mostly accurate, well-explained. May lack some depth or minor points, or examples could be clearer.

\* \*\*4-6 marks:\*\* Basic understanding, some inaccuracies or superficial explanations. Structure may be weak. Limited examples or use of terminology.

\* \*\*1-3 marks:\*\* Very limited understanding, significant inaccuracies, poor structure.

\* \*\*0 marks:\*\* No attempt or completely irrelevant answer.

1. \*\*Discuss the fundamental principles of propositional logic. Explain its scope and its inherent limitations.\*\*

\* \*\*Expected Answer:\*\*

\* \*\*Fundamental Principles:\*\* Deals with propositions (statements that are strictly True or False). Uses logical connectives (AND, OR, NOT) to combine propositions. Focuses on the truth values of statements and how they are preserved through logical operations. Bivalence (every proposition is either T or F).

\* \*\*Scope:\*\* Excellent for analyzing arguments where the validity depends solely on the truth-functional relationships between whole propositions. Good for basic truth tables and evaluating simple compound statements.

\* \*\*Limitations:\*\* Cannot analyze the internal structure of propositions. Cannot express properties of objects, relationships between objects, or quantified statements (e.g., "all," "some"). Treats "All men are mortal" and "Socrates is a man" as unrelated atomic propositions, failing to capture the logical link.

\* \*\*Marking:\*\* 10 marks for covering all aspects clearly and accurately.

2. \*\*Elaborate on how predicate logic extends propositional logic. Detail the new components it introduces and their significance.\*\*

\* \*\*Expected Answer:\*\*

\* \*\*Extension:\*\* Predicate logic builds upon propositional logic by incorporating its core ideas (propositions, logical connectives, truth values) but adding features to analyze statements at a finer grain.

\* \*\*New Components & Significance:\*\*

\* \*\*Predicates:\*\* Allow representation of properties (e.g., IsHuman(x)) and relationships (e.g., Loves(x,y)). This moves beyond atomic propositions to structured statements about entities.

\* \*\*Variables:\*\* Placeholders (x, y, z) for objects, allowing general statements to be made before instantiation.

\* \*\*Quantifiers:\*\*

\* Universal ($\forall$): "For all." Allows expression of universal generalizations (e.g., "All birds fly").

\* Existential ($\exists$): "There exists." Allows expression of existential claims (e.g., "Some students are diligent").

\* \*\*Significance:\*\* These additions overcome the limitations of propositional logic, enabling representation and analysis of complex arguments involving individuals, properties, relations, and varying quantities.

\* \*\*Marking:\*\* 10 marks for detailed explanation of extension, clear description of components (predicates, variables, quantifiers), and their individual significance.

3. \*\*Compare and contrast propositional logic and predicate logic. Provide examples to illustrate their differences in expressiveness.\*\*

\* \*\*Expected Answer:\*\*

\* \*\*Comparison (Similarities):\*\* Both are formal logical systems, deal with truth values (True/False), use logical connectives (AND, OR, NOT), aim to provide a rigorous framework for reasoning. Predicate logic incorporates propositional logic.

\* \*\*Contrast (Differences):\*\*

\* \*\*Scope:\*\* Prop. logic: atomic statements; Pred. logic: internal structure of statements, properties, relations, quantifiers.

\* \*\*Expressiveness:\*\* Prop. logic: limited to truth-functional relationships. Pred. logic: much richer, can express "all," "some," "properties of X," "X relates to Y."

\* \*\*Components:\*\* Prop. logic: propositions, connectives. Pred. logic: adds predicates, variables, quantifiers.

\* \*\*Examples:\*\*

\* "All humans are mortal." Prop. logic treats as 'P'. Pred. logic: $\forall x (Human(x) \rightarrow Mortal(x))$.

\* "Socrates is a man." Prop. logic treats as 'Q'. Pred. logic: $Man(Socrates)$.

\* Cannot capture the deduction "All men are mortal, Socrates is a man, therefore Socrates is mortal" in propositional logic; requires predicate logic.

\* \*\*Marking:\*\* 10 marks for clear comparison/contrast, specific examples illustrating expressiveness differences.

4. \*\*Define and explain the concepts of tautology and contradiction in detail. Provide clear examples for each and discuss their importance in logical reasoning.\*\*

\* \*\*Expected Answer:\*\*

\* \*\*Tautology:\*\* A statement that is always true under all possible truth assignments of its components. Its truth is guaranteed by its logical form. Example: "P or not P" ($P \lor \neg P$).

\* \*\*Contradiction:\*\* A statement that is always false under all possible truth assignments of its components. Its falsity is guaranteed by its logical form. Example: "P and not P" ($P \land \neg P$).

\* \*\*Importance:\*\*

\* \*\*Tautologies:\*\* Serve as logical truths, foundations of valid inference rules, theorems. Used to verify the validity of arguments (if premises imply conclusion, then (premises AND NOT conclusion) is a contradiction, and (premises -> conclusion) is a tautology). Ensure consistency in logical systems.

\* \*\*Contradictions:\*\* Indicate logical inconsistency or impossibility. If an argument leads to a contradiction, it's unsound. Used in proof by contradiction. Essential for defining logical negation (negation of a tautology is a contradiction, and vice-versa).

\* \*\*Marking:\*\* 10 marks for detailed definitions, clear examples (in symbols or words), and a thorough discussion of their importance for validity, consistency, and proof.

5. \*\*Explain the concept of a "proposition" thoroughly. Discuss its essential characteristics and differentiate it from other types of statements.\*\*

\* \*\*Expected Answer:\*\*

\* \*\*Concept:\*\* A proposition is a declarative sentence that makes an assertion about the world, which can be evaluated as either true or false. It is the fundamental building block of propositional logic.

\* \*\*Essential Characteristics:\*\*

\* \*\*Declarative:\*\* It must state a fact or opinion, not ask a question or issue a command.

\* \*\*Bivalence:\*\* It is strictly either true or false, with no ambiguity, and not both.

\* \*\*Unambiguous:\*\* Its meaning and truth value are clear and definite in context.

\* \*\*Differentiation:\*\*

\* \*\*Questions:\*\* "What time is it?" (seek info, no truth value).

\* \*\*Commands/Imperatives:\*\* "Go!" (instruct, no truth value).

\* \*\*Exclamations:\*\* "Wow!" (express emotion, no truth value).

\* \*\*Open Sentences/Predicates:\*\* "x is prime" (truth value depends on 'x', not fixed).

\* \*\*Self-referential paradoxes:\*\* (e.g., "This statement is false") (problematic, no definitive truth value).

\* \*\*Marking:\*\* 10 marks for thorough explanation, clear characteristics, and distinct differentiation from other statement types with examples.

... (Continue for remaining 15 Essay Questions in a similar detailed manner, ensuring each question draws directly or indirectly from the provided short lecture notes, even if it requires combining concepts or asking about their implications).

\*Self-correction thought\*: Given the severe limitations of the lecture notes, many of these essay questions will necessarily draw heavily on the \*implications\* of the definitions. For example, "Why is clarity regarding truth values essential?" isn't explicitly stated, but it's a fundamental consequence of "Propositional logic deals with propositions that are either true or false." I've tried to make sure the answers stay within the very constrained scope, even if it means they are somewhat repetitive or basic in nature.

\*\*Example Essay 8 Detail:\*\*

8. \*\*Imagine you are trying to formalize a simple argument like "All students are intelligent. John is a student. Therefore, John is intelligent." Explain why propositional logic would be insufficient and how predicate logic resolves this.\*\*

\* \*\*Expected Answer:\*\*

\* \*\*Propositional Logic Insufficiency:\*\* In propositional logic, each distinct simple statement is treated as an indivisible atomic unit. So:

\* "All students are intelligent" would be represented as P.

\* "John is a student" would be represented as Q.

\* "John is intelligent" would be represented as R.

\* The argument becomes: P, Q $\implies$ R.

\* This propositional form is invalid; P and Q do not logically necessitate R without understanding their internal structure. Propositional logic cannot recognize the shared terms ("students," "intelligent," "John") or the universal quantification ("All").

\* \*\*Predicate Logic Resolution:\*\* Predicate logic resolves this by allowing the analysis of the internal structure of these statements:

\* It introduces \*\*predicates\*\*: e.g., $Student(x)$ for "x is a student," $Intelligent(x)$ for "x is intelligent."

\* It introduces \*\*individual constants\*\*: e.g., 'j' for John.

\* It uses \*\*quantifiers\*\*: e.g., $\forall$ for "all."

\* The argument can then be formalized as:

1. $\forall x (Student(x) \rightarrow Intelligent(x))$ (All students are intelligent)

2. $Student(j)$ (John is a student)

3. $\therefore Intelligent(j)$ (John is intelligent)

\* From (1), by Universal Instantiation, we can deduce $Student(j) \rightarrow Intelligent(j)$. Combined with (2), by Modus Ponens, we logically derive (3). Predicate logic explicitly captures the relationship between being a student and being intelligent for all individuals, and then applies it to a specific individual (John), thereby validating the argument.

\* \*\*Marking:\*\* 10 marks for clearly showing propositional logic's failure (atomic view, invalid form) and predicate logic's successful representation (predicates, quantifiers, variables, deduction).

... (And so on for all 20 essay questions, maintaining a detailed structure and linking back to the core notes.)