# ETC3430

# Financial Mathematics under Uncertainty

# Assignment 1, Semester 1, 2022

## Submission

This assignment comprises 10% of the assessment for ETC3430. **ENSURE that you**

- Name your assignment: *Surname-Initials_A1.docx*, e.g., *Einstein-A_A1.docx*.
- Upload this file to Moodle as follows: Go to the "Assignments" section. Click on the "Submit your Assignment 1 here" link to upload. The following message will appear momentarily, "File uploaded successfully."

  [To later confirm your upload was successful, go to the "Assignments" section and click on the Assignment 1 uploading link. The uploaded file's name will be shown.]

  *NB, DO NOT submit any Excel files. You may upload only ONE file.*
- Submit the electronic copy before the due, otherwise your submission will NOT be accepted.
- Retain your marked assignment until after the publication of final results for this unit.

## Further Information

- A maximum penalty of 10% of the total mark allocated to this assessment will be deducted for each day that it is late, up to 4 days. An assignment may not be submitted if it is late by more than 4 days.
- Extensions beyond the due date will only be allowed in special circumstances. You may visit

  https://www.monash.edu/exams/changes/special-consideration

  for the university policy and application procedure for special consideration.

- If you don't understand what the questions are asking,
  - study the unit's content prior to attempting the tutorial and assignment questions. This should enhance your ability to understand the questions.
  - ask a staff member to clarify the questions for you. A staff consultation roster is on Moodle.

## Avoid Plagiarism!

*Intentional plagiarism amounts to cheating. See the Monash Policy.*

Plagiarism: Plagiarism means to take and use another person's ideas and or manner of expressing them and to pass these off as one's own by failing to give appropriate acknowledgement. This includes material from any source, staff, students or the internet-published and unpublished works.

Collusion: Collusion is unauthorised collaboration with another person or persons. Where there are reasonable grounds for believing that intentional plagiarism or collusion has occurred, this will be reported to the Chief Examiner, who may disallow the work concerned by prohibiting assessment or refer the matter to the Faculty Manager.

**For each question, you should include all intermediary steps in which you derived the final solution.**

**Question 1.** Perform the following simulation exercise in Matlab or R, using your student number as the seed( rng( your student no.)). Consider a three state Markov Chain("sunny", "rainy" and "overcast")to model the daily weather in Melbourne.

(a) Without using data, suggest a reasonable estimate of the transition matrix. The later questions should be considered based on your estimates.

(b) Given each starting state respectively, what is the most likely state of the weather in the next five days. Present your answer in a table.

(c) Label the three states as $\{1, 2, 3\}$, i.e., $X_t = 1$ if the weather is sunny, $X_t = 2$ if the weather is rainy and $X_t = 2$ if the weather is overcast. Given each starting state respectively, what is $\mathbb{E}[X_t]$ in the next five days. Present your answer in a table.

(d) Compare and Contrast your answers from (c) and (d)

(e) generate histograms of $X_3$ with an initial distribution of $X_0$ generated with a seed given by your postcode for sample sizes $n = 100, 1000, 10^4$ and $10^5$, compare them with the associated probability density function of $X_3$, then comment on your findings;

(f) use samples with sizes $n = 100, 1000, 10^4$ and $10^5$ to estimate $\mathbb{P}(X_3 > 1|X_1 = 1)$, comment on the Monte-Carlo estimation with the theoretical answer as $n$ increases;

(g) use samples with sizes $n = 100, 1000, 10^4$ and $10^5$ to estimate $\mathbb{P}(X_3 > 1|X_1 = 1, X_2 = 1)$, comment on the Monte-Carlo estimation with the theoretical answer as $n$ increases.

(h) compare and contrast the results from (f) and (g).

(40 Marks)

**Question 2.** *Perform the following simulation exercise in Matlab or R, using your student number as the seed( rng( your student no.)). In the excel file "Data.Sheet1", you are given two data sets of Markov Chains. Here, $X_t$ denote the state that occupied by time t.*

a *Use your chosen appropriate method of estimation, suggest a point estimate of the transition matrix for Dataset 1 given the initial state is 1.*

b *Use your chosen appropriate method of estimation, suggest a point estimate of the transition matrix for Dataset 2 given the initial state is 1.*

c *Given your estimated transition matrices, project both datasets 10 periods further into the future.*

d *Comments on the long term behaviour of both series.*

(30 Marks)

**Question 3.** *Perform the following simulation exercise in Matlab or R, using your student number as the seed( rng( your student no.)). In the excel file "Data.Sheet2", you are given two data sets of continuous time Markov Process with two states. Here, $W_n$ is the waiting time between the $n-1$th and $n$th jump of the process*

*a Use your chosen appropriate method of estimation, suggest a point estimate of the transition matrix for Dataset 1 given the initial state is 1.*

*b Use your chosen appropriate method of estimation, suggest a point estimate of the transition matrix for Dataset 2 given the initial state is 1.*

*c Given your estimated transition matrices, project both datasets 10 jumps further into the future.*

*d Comments on the long term behaviour of both series.*

*(30 Marks)*