Introductory Econometrics

Modelling Dynamics

Monash Econometrics and Business Statistics

2022

Recap

Types of data structures

- Cross-sectional data: observations collected at same point in time.
  1. No natural ordering of the observations.
  2. Assume that observations are independent.
- Time series data: observations taken at different points in time.
  1. There is a natural ordering in time.
  2. Time series are characterized by temporal dependence

Recap

The multiple regression model

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + ... + \beta_k x_{ik} + u_i, \;\; i = 1, 2, ...n.$$

Toolbox:

- Dummies, logs, quadratic terms
- Hypothesis tests
- Model specification tests

All work with time series as well.

Additionally, we can also exploit ordering and temporal dependence!

Lecture Outline

- Static time series models
  - Trend
  - Seasonality
  - Structural breaks
- Dynamic time series models
  - Autoregressive models
  - Stationary time series models
  - The autoregressive distributed lag model

Time series

Since time series observations are ordered, we can model

- a trend.
- seasonality.
- a structural break.

Time series trend

Many economic time series exhibit a strong tendency to grow over time.

- Linear time trend:

$$y_t = \beta_0 + \beta_1 t + u_t,\ \ t = 1, \ldots, n,$$

where β_1 measures the change in y_t from one period to the next.

Many economic time series have a constant average growth rate, but the change in y_t varies across time.

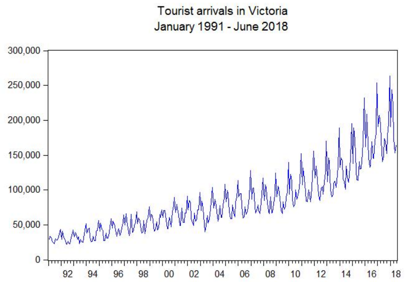
- Exponential time trend:

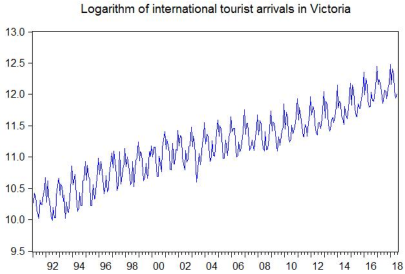
$$\log(y_t) = \beta_0 + \beta_1 t + u_t,\ \ t = 1, \ldots, n,$$

where β_1 approximates the average per period growth rate in y_t.

Example:

- International tourist arrivals in Victoria





Time series seasonality

International tourist arrivals in Victoria is highly seasonal.

- Seasonality is often removed from the data:
  seasonally adjusted time series.
- Seasonality is important for forecasting.

A set of seasonal dummy variables can account for seasonality.

Example:

- Model $\log(VIC)$ with a trend and dummies for 11 out of 12 months.

  $$\log(VIC_t) = \beta_0 + \delta_1 feb_t + \delta_2 mar_t + \delta_3 apr_t + \ldots + \delta_{11} dec_t + \beta_1\, t + u_t,$$

  where $feb_t, \ldots, dec_t$ are dummies equal to 1 if the observation is from the month indicated by their names and 0 otherwise.
- Here, January is the base month, and β_0 is the intercept for January.
- One can test for joint significance of $\delta_1, \ldots, \delta_{11}$ via an F test. Once the trend is controlled for, if the null $H_0 : \delta_1 = \ldots = \delta_{11} = 0$ cannot be rejected, then this is indication of no seasonality.

Regression output:

Dependent Variable: LOG(VIC)
Method: Least Squares
Sample: 1991M01 2018M06
Included observations: 330

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	10.34379	0.019370	534.0113	0.0000
T	0.005401	5.30E-05	101.8685	0.0000
@MONTH=2	0.182378	0.024516	7.439107	0.0000
@MONTH=3	0.109668	0.024516	4.473256	0.0000
@MONTH=4	-0.119151	0.024517	-4.860031	0.0000
@MONTH=5	-0.297504	0.024517	-12.13460	0.0000
@MONTH=6	-0.301492	0.024518	-12.29701	0.0000
@MONTH=7	-0.056995	0.024742	-2.303590	0.0219
@MONTH=8	-0.235765	0.024742	-9.528905	0.0000
@MONTH=9	-0.240985	0.024742	-9.739806	0.0000
@MONTH=10	-0.032602	0.024743	-1.317632	0.1886
@MONTH=11	0.065488	0.024743	2.646718	0.0085
@MONTH=12	0.317922	0.024743	12.84874	0.0000

R-squared	0.973804	Mean dependent var	11.18647
Adjusted R-squared	0.972813	S.D. dependent var	0.556330
S.E. of regression	0.091731	Akaike info criterion	-1.901321
Sum squared resid	2.667405	Schwarz criterion	-1.751660
Log likelihood	326.7180	Hannan-Quinn criter.	-1.841624
F-statistic	982.0226	Durbin-Watson stat	0.820242
Prob(F-statistic)	0.000000		

Regression output with a different base

Dependent Variable: LOG(VIC)
Method: Least Squares
Sample: 1991M01 2018M06
Included observations: 330

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	10.66171	0.019773	539.2010	0.0000
T	0.005401	5.30E-05	101.8685	0.0000
@MONTH=1	-0.317922	0.024743	-12.84874	0.0000
@MONTH=2	-0.135544	0.024743	-5.478088	0.0000
@MONTH=3	-0.208254	0.024743	-8.416856	0.0000
@MONTH=4	-0.437073	0.024742	-17.66506	0.0000
@MONTH=5	-0.615426	0.024742	-24.87365	0.0000
@MONTH=6	-0.619414	0.024742	-25.03489	0.0000
@MONTH=7	-0.374918	0.024967	-15.01631	0.0000
@MONTH=8	-0.553687	0.024967	-22.17689	0.0000
@MONTH=9	-0.558907	0.024966	-22.38632	0.0000
@MONTH=10	-0.350524	0.024966	-14.03994	0.0000
@MONTH=11	-0.252435	0.024966	-10.11113	0.0000

R-squared	0.973804	Mean dependent var	11.18647
Adjusted R-squared	0.972813	S.D. dependent var	0.556330
S.E. of regression	0.091731	Akaike info criterion	-1.901321
Sum squared resid	2.667405	Schwarz criterion	-1.751660
Log likelihood	326.7180	Hannan-Quinn criter.	-1.841624
F-statistic	982.0226	Durbin-Watson stat	0.820242
Prob(F-statistic)	0.000000		

- Using December as the base rather than January leads to the exact same conclusions

Time series structural breaks

Is there a structural change in a variable after a certain event happened?

- Has GFC dampened the growth rate of the Australian economy?
- Has a change in prime minister affected the growth of the economy?
- Has seatbelt legislation reduced the trend in accident fatality?

Investigate this with a dummy that is 0 before and 1 after the event.

Example:

- The effect of GFC on the growth rate of the Australian economy.
- Dummy *POSTGFC* equals 1 on and after the third quarter of 2008.
- Run a regression of the growth rate on a constant and *POSTGFC*

Dependent Variable: GDPGROWTH
Method: Least Squares
Sample (adjusted): 12/01/1959 3/01/2018
Included observations: 234 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.889201	0.073434	12.10889	0.0000
POSTGFC	-0.252745	0.179875	-1.405115	0.1613

R-squared	0.008438	Mean dependent var	0.847077
Adjusted R-squared	0.004164	S.D. dependent var	1.027588
S.E. of regression	1.025446	Akaike info criterion	2.896644
Sum squared resid	243.9574	Schwarz criterion	2.926176
Log likelihood	-336.9073	Hannan-Quinn criter.	2.908551
F-statistic	1.974347	Durbin-Watson stat	2.143309
Prob(F-statistic)	0.161325		

Static time series models

- A static time series model is

$$y_t = \beta_0 + \beta_1 x_t + u_t.$$

- There are no lags of y_t or x_t included as regressors in the model.
- A change in x in time period t only affects y in time period t.
- The effect on y of a change in x is completely contemporaneous.

Static vs dynamic models

Example

- Static model for murder rate and conviction rate:

$$mr_t = \beta_0 + \beta_1 cr_t + u_t,$$

where a change in cr_t only has an effect on mr_t.

- Dynamic model for murder rate and conviction rate:

$$mr_t = \beta_0 + \beta_1 cr_t + \beta_2 cr_{t-1} + u_t$$

where a change in cr_t has an effect on mr_t and mr_{t+1}.

Dynamic time series models

Many reasons why variables have an effect over several time periods:

- Habit persistence.
- Institutional arrangements.
- Administrative lags.
- Optimizing behavior.

Autoregressive models

The simplest dynamic time series model is the AR(p) model

$$y_t = \phi_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + ... + \phi_p y_{t-p} + u_t,$$

where it is assumed that

$$u_t \sim WN(0, \sigma^2),$$

or

$$u_t \sim i.i.d(0, \sigma^2).$$

A time series is white noise $u_t \sim WN(0, \sigma^2)$ if:

$$E(u_t) = 0 \text{ for all } t,$$
$$Var(u_t) = \sigma^2 \text{ for all } t,$$
$$Cov(u_t, u_{t-j}) = 0 \text{ for } j \neq 0.$$

Autoregressive models

The simplest AR model is the AR(1) model

$$y_t = \varphi_0 + \varphi_1 y_{t-1} + u_t.$$

- This model can produce:
  1. an uncorrelated sequence when $\varphi_1 = 0$
  2. a stationary process when $|\varphi_1| < 1$
  3. a random walk when $\varphi_0 = 0$ and $\varphi_1 = 1$
  4. a random walk with drift when $\varphi_0 \neq 0$ and $\varphi_1 = 1$
  5. an explosive process when $\varphi_1 > 1$

Stationary time series

A time series $\{y_t\}$ is called stationary if it has the properties:

P1

$$E(Y_t) = \mu \ < \infty \ \text{ for all t.}$$

(The mean is finite and time invariant)

P2

$$Var(Y_t) = E[(Y_t - \mu)^2] = \gamma_0 \ < \infty \text{ for all t.}$$

(The variance is finite and time invariant)

P3

$$Cov(Y_t, Y_{t-j}) = E[(Y_t - \mu)(Y_{t-j} - \mu)] = \gamma_j \ < \infty \text{ for all t and j.}$$

(The covariance is finite and depends only on the time interval)

The stationary AR(1) process

- If y_t is generated by an AR(1) model

$$y_t = \varphi_0 + \varphi_1 y_{t-1} + u_t, \text{ with } |\varphi_1| < 1 \text{ and } \{u_t\} \sim WN(0, \sigma^2)$$

then we have:

P1 Mean of y

$$E(y_t) = \frac{\varphi_0}{1 - \varphi_1} \text{ for all } t$$

P2 Variance of y

$$Var(y_t) = \frac{\sigma^2}{1 - \varphi_1^2} \text{ for all } t$$

P3 Autocovariances and autocorrelations of y

$$Cov(y_t, y_{t-j}) = \gamma_j = \varphi_1^j Var(y_t), \text{ for all } t \text{ and } j$$

$$Corr(y_t, y_{t-j}) = \rho_j = \frac{\gamma_j}{\gamma_0} = \varphi_1^j, \text{ for all } t \text{ and } j.$$

The autoregressive distributed lag model

The simplest dynamic time series model of the relationship between y and x is the ARDL(p,q) model

$$y_t = \varphi_0 + \varphi_1 y_{t-1} + \cdots + \varphi_p y_{t-p} + \alpha_0 x_t + \alpha_1 x_{t-1} + \cdots + \alpha_q x_{t-q} + u_t,$$

where p denotes the number of lags of y and q the number of lags of x.

- Commonly used methods to choose p and q are:
  - Information criteria.
  - The frequency of the data.
  - such that there is no autocorrelation in the error term.
- OLS and its standard errors and tests are reliable if:
  - y and x are stationary.
  - the residuals show no sign of autocorrelation.
  - the sample is large.

The autoregressive distributed lag model

Consider the ARDL(1,1) model

$$y_t = \varphi_0 + \varphi_1 y_{t-1} + \alpha_0 x_t + \alpha_1 x_{t-1} + u_t.$$

Suppose there is a one unit increase in x at time t.

- Contemporaneous change in y is α_0.
- At time $t+1$, this change is x will still affect y_{t+1} in two ways:
  - directly because x_t still appears in the equation for y_{t+1}
  - and again because y_t also appears in the equation for y_{t+1} (and y_t was already influenced by the change in x_t).
- The long-run effect on y of a one unit change in x_t equals

$$\frac{\alpha_0 + \alpha_1}{1 - \varphi_1}.$$

In the general ARDL(p,q) model

$$y_t = \varphi_0 + \varphi_1 y_{t-1} + \cdots + \varphi_p y_{t-p} + \alpha_0 x_t + \alpha_1 x_{t-1} + \cdots + \alpha_q x_{t-q} + u_t,$$

the long-run effect on y of a one unit change in x in time period t is

$$\frac{\sum_{i=0}^{q} \alpha_i}{1 - \sum_{i=1}^{p} \varphi_i} = \frac{\text{sum of the coefficients } x_t \text{ and its lags}}{1 - \text{sum of the coefficients of lags of } y_t}.$$

Example: Reserve Bank's reaction function

- How does the Reserve Bank of Australia (RBA) set the cash rate?

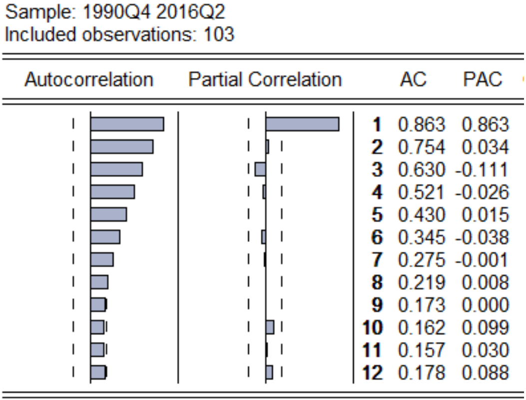
Dependent Variable: CRATE
Method: Least Squares
Sample (adjusted): 1990Q4 2016Q2
Included observations: 103 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	5.140237	0.411475	12.49221	0.0000
GDPGR	-0.102513	0.082255	-1.246288	0.2156
INFL	0.167738	0.089400	1.876276	0.0635

R-squared	0.057144	Mean dependent var	5.237379
Adjusted R-squared	0.038287	S.D. dependent var	1.974848
S.E. of regression	1.936673	Akaike info criterion	4.188514
Sum squared resid	375.0702	Schwarz criterion	4.265253
Log likelihood	-212.7085	Hannan-Quinn criter.	4.219596
F-statistic	3.030379	Durbin-Watson stat	0.129477
Prob(F-statistic)	0.052755		

Example: continued

- The residual correlogram shows:



Sample: 1990Q4 2016Q2
Included observations: 103

Autocorrelation	Partial Correlation		AC	PAC
		1	0.863	0.863
		2	0.754	0.034
		3	0.630	-0.111
		4	0.521	-0.026
		5	0.430	0.015
		6	0.345	-0.038
		7	0.275	-0.001
		8	0.219	0.008
		9	0.173	0.000
		10	0.162	0.099
		11	0.157	0.030
		12	0.178	0.088

which suggests that the t-statistics and p-values from the static model were not correct

Example: continued

- This also makes sense because the RBA only changes the cash rate very smoothly. So, we add one lag of every variable to the right hand side and we obtain:

Dependent Variable: CRATE
Method: Least Squares
Sample (adjusted): 1991Q1 2016Q2
Included observations: 102 after adjustments

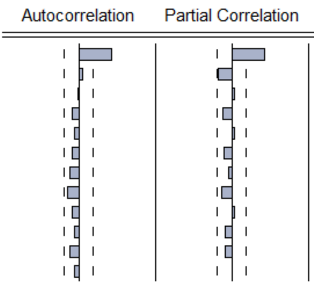
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.337620	0.151658	-2.226194	0.0283
CRATE(-1)	0.915817	0.018736	48.88083	0.0000
GDPGR	0.055108	0.015714	3.506963	0.0007
GDPGR(-1)	0.041922	0.015123	2.772032	0.0067
INFL	0.090080	0.017387	5.180861	0.0000
INFL(-1)	0.062562	0.017297	3.616874	0.0005
R-squared	0.964622	Mean dependent var		5.161242
Adjusted R-squared	0.962780	S.D. dependent var		1.826377
S.E. of regression	0.352354	Akaike info criterion		0.808664
Sum squared resid	11.91875	Schwarz criterion		0.963075

which shows parameter estimates that make better sense.

Example: continued

- However, the residuals still show sign of autocorrelation:

Sample: 1991Q1 2016Q2
Included observations: 102



	AC	PAC
1	0.453	0.453
2	0.063	-0.180
3	-0.012	0.047
4	-0.089	-0.119
5	-0.062	0.041
6	-0.085	-0.105
7	-0.112	-0.038
8	-0.154	-0.129
9	-0.092	0.040
10	-0.064	-0.088
11	-0.113	-0.086
12	-0.052	0.010

with value of the Breusch-Godfrey test statistic for testing the null of no serial correlation in errors against the alternative of first order serial correlation being 26.1, clearly rejecting the null.

Example: continued

- Adding the second lag takes care of serial correlation in errors. Dropping the insignificant lags leaves us with

Dependent Variable: CRATE
Method: Least Squares
Sample (adjusted): 1991Q2 2016Q2
Included observations: 101 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.027022	0.109588	0.246581	0.8058
CRATE(-1)	1.523767	0.076624	19.88626	0.0000
CRATE(-2)	-0.585687	0.071408	-8.201939	0.0000
GDPGR	0.048512	0.013588	3.570111	0.0006
INFL	0.042447	0.016436	2.582600	0.0113

R-squared	0.971664	Mean dependent var	5.093531
Adjusted R-squared	0.970483	S.D. dependent var	1.701967
S.E. of regression	0.292405	Akaike info criterion	0.426886
Sum squared resid	8.208084	Schwarz criterion	0.556347

with the value of $BG_{calc} = 1.2$ for the null of no serial correlation in errors against first order serial correlation, which is well below the 5% critical value of χ^2_1.

Example: continued

- Our estimated reaction function of the RBA was

$$\widehat{CRATE}_t = 0.027 + 1.524 CRATE_{t-1} - 0.586 CRATE_{t-2} + 0.049 GDPGR_t + 0.042 INFL_t$$

- According to this estimated equation:
  - the immediate impact of a 1 percentage point increase in the annualised GDP growth on the cash rate is ...
  - the immediate impact of a 1 percentage point increase in the annualised inflation rate on the cash rate is ...
  - the long-run impact of a 1 percentage point increase in the annualised GDP growth on the cash rate is ...
  - the long-run impact of a 1 percentage point increase in the annualised inflation rate on the cash rate is ...

The restrictive dynamics of regression with AR errors

- What is the difference between a regression with AR errors and a general dynamic model? Recall that

$$\begin{aligned} y_t &= \beta_0 + \beta_1 x_t + u_t \\ u_t &= \rho u_{t-1} + e_t, \\ \Rightarrow y_t &= \beta_0(1-\rho) + \beta_1 x_t - \rho\beta_1 x_{t-1} + \rho y_{t-1} + e_t \end{aligned}$$

  which is a restricted ARDL model (what is the restriction?)

- In this model a one unit increase in x changes y
  - by β_1 units immediately, and
  - by $\frac{\beta_1 - \rho\beta_1}{1-\rho} = \frac{\beta_1(1-\rho)}{1-\rho} = \beta_1$ in the long-run!
- This shows that the regression with AR errors imposes that all impact of x on y is realised immediately. All the dynamics in these models are in the error (part of y that is not explained by x)

Summary

- With time series data we can build a dynamic model by adding lags of dependent and independent variables to the list of explanatory variables.
- As long as the dependent and independent variables are stationary and errors are white noise, the OLS estimator of the parameters of a dynamic model is reliable and we can use t and F tests provided the sample size is large.
- Dynamic models give us insights about the immediate impact and the long-run effect of a change in each of the independent variables on the dependent variable, all else constant.