# Tutorial 7

7.1 $D \sim \text{Poisson}(5000 \times 0.5 \times 0.0008)$ $\qquad D \sim \text{Poisson}(2)$

$\Pr(D=0)+\Pr(D=1)+\Pr(D=2)=0.676676416$

7.2 $D \sim \text{Poisson}(50000 \times 4 \times 0.001)$ $\qquad D \sim \text{Poisson}(200)$

$D \sim \text{Normal}(200,\ 200)$ approximately

$\Pr(D>225)=0.0386$

7.3 $\hat{\mu}=\dfrac{46}{37500}=0.001226667$

$$0.001226667 \pm 1.96 \times \sqrt{\frac{0.001226667}{37500}} = (0.000872176,\ 0.001581156)$$

7.4 $D_i \sim \text{Bernoulli}({}_{b_i-a_i}q_{70+a_i})$

$L = p_{70}\ {}_{0.6}p_{70.3}\ {}_{0.5}q_{70.5}\ {}_{0.4}p_{70}\ {}_{0.9}q_{70}\ q_{70}$

under UDD

${}_{0.6}p_{70.3}=1-\dfrac{0.6q_{70}}{1-0.3q_{70}}$ $\qquad {}_{0.5}q_{70.5}=\dfrac{0.5q_{70}}{1-0.5q_{70}}$ $\qquad {}_{0.4}p_{70}=1-0.4q_{70}$ $\qquad {}_{0.9}q_{70}=0.9q_{70}$

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f<-function(q){ -(1-q)*(1-0.6*q/(1-0.3*q))*0.5*q/(1-0.5*q)*(1-0.4*q)*0.9*q*q }
nlminb(0.5,f)
```

MLE $\hat{q}_{70}=0.6582$

(Balducci + MM $\hat{q}_{70}=\dfrac{3}{1+0.6+0.5+0.4+1+1}=0.6667$)

(Poisson MLE $\hat{\mu}_{70.5}=\dfrac{3}{1+0.6+0.4+0.4+0.7+0.8}=0.769230769$)

7.5 $D_i \sim \text{Bernoulli}({}_{b_i-a_i}q_{40+a_i})$

$L = {}_{0.88}p_{40.12}\ {}_{0.62}p_{40.23}\ {}_{0.72}p_{40.28}\ {}_{0.44}q_{40.31}\ {}_{0.53}p_{40.39}\ {}_{0.59}p_{40.41}\ {}_{0.28}p_{40.52}\ {}_{0.22}q_{40.65}\ {}_{0.29}p_{40.71}\ {}_{0.18}p_{40.82}$

under UDD

${}_{0.88}p_{40.12}=1-\dfrac{0.88q_{40}}{1-0.12q_{40}}$ $\qquad {}_{0.62}p_{40.23}=1-\dfrac{0.62q_{40}}{1-0.23q_{40}}$ $\qquad {}_{0.72}p_{40.28}=1-\dfrac{0.72q_{40}}{1-0.28q_{40}}$

${}_{0.44}q_{40.31}=\dfrac{0.44q_{40}}{1-0.31q_{40}}$ $\qquad {}_{0.53}p_{40.39}=1-\dfrac{0.53q_{40}}{1-0.39q_{40}}$ $\qquad {}_{0.59}p_{40.41}=1-\dfrac{0.59q_{40}}{1-0.41q_{40}}$

${}_{0.28}p_{40.52}=1-\dfrac{0.28q_{40}}{1-0.52q_{40}}$ $\qquad {}_{0.22}q_{40.65}=\dfrac{0.22q_{40}}{1-0.65q_{40}}$ $\qquad {}_{0.29}p_{40.71}=1-\dfrac{0.29q_{40}}{1-0.71q_{40}}$

${}_{0.18}p_{40.82}=1-\dfrac{0.18q_{40}}{1-0.82q_{40}}$

```
f<-function(q){ -(1-0.88*q/(1-0.12*q))*(1-0.62*q/(1-0.23*q))*(1-0.72*q/(1-0.28*q))*0.44*q/(1-0.31*q)*(1-0.53*q/(1-0.39*q))*(1-0.59*q/(1-0.41*q))*(1-0.28*q/(1-0.52*q))*0.22*q/(1-0.65*q)*(1-0.29*q/(1-0.71*q))*(1-0.18*q/(1-0.82*q))   }
nlminb(0.5,f)
```

MLE $\hat{q}_{40} = 0.3458$

(Balducci + MM $\hat{q}_{40} = \dfrac{2}{0.88+0.62+0.72+0.69+0.53+0.59+0.28+0.35+0.29+0.18} = 0.389863547$ )

(under Balducci

$_{0.88}p_{40.12} = 1-0.88q_{40}$ $\qquad$ $_{0.62}p_{40.23} = \dfrac{_{0.85}p_{40}}{_{0.23}p_{40}} = \dfrac{1-0.77q_{40}}{1-0.15q_{40}}$ $\qquad$ $_{0.72}p_{40.28} = 1-0.72q_{40}$

$_{0.44}q_{40.31} = 1-\dfrac{_{0.75}p_{40}}{_{0.31}p_{40}} = \dfrac{0.44q_{40}}{1-0.25q_{40}}$ $\qquad$ $_{0.53}p_{40.39} = \dfrac{_{0.92}p_{40}}{_{0.39}p_{40}} = \dfrac{1-0.61q_{40}}{1-0.08q_{40}}$ $\qquad$ $_{0.59}p_{40.41} = 1-0.59q_{40}$

$_{0.28}p_{40.52} = \dfrac{_{0.8}p_{40}}{_{0.52}p_{40}} = \dfrac{1-0.48q_{40}}{1-0.2q_{40}}$ $\qquad$ $_{0.22}q_{40.65} = 1-\dfrac{_{0.87}p_{40}}{_{0.65}p_{40}} = \dfrac{0.22q_{40}}{1-0.13q_{40}}$ $\qquad$ $_{0.29}p_{40.71} = 1-0.29q_{40}$

$_{0.18}p_{40.82} = 1-0.18q_{40}$

```
f<-function(q){ -(1-0.88*q)*(1-0.77*q)/(1-0.15*q)*(1-0.72*q)*0.44*q/(1-0.25*q)*(1-0.61*q)/(1-0.08*q)*(1-0.59*q)*(1-0.48*q)/(1-0.2*q)*0.22*q/(1-0.13*q)*(1-0.29*q)*(1-0.18*q) }
nlminb(0.5,f)
```

MLE $\hat{q}_{40} = 0.3837$ )

7.6 Life 1: $E^c_{64}$ = 1 month $E_{64}$ = 3 months

Life 2: $E^c_{64}$ = 2 months $E_{64}$ = 12 months

Life 3: $E^c_{64}$ = 2 months $E_{64}$ = 2 months

Total: $E^c_{64}$ = 5 months $E_{64}$ = 17 months

7.7 Life 1:

$E^c_{30}$ = 30+28+31+30+31+30+31+31+30+31+11 = 314 days $E^c_{31}$ = 366 days

$E^c_{32}$ = 365 days $E^c_{33}$ = 365 days $E^c_{34}$ = 19+29 = 48 days ($E_x = E^c_x$)

Life 2:

$E^c_{24}$ = 2 days $E^c_{25}$ = 365 days $E^c_{26}$ = 365 days

$E^c_{27}$ =29+31+30+31+1 = 122 days ($E_x = E^c_x$)

Life 3:

$E^c_{24}$ = 30+10 = 40 days $E^c_{25}$ = 18+31+30+31+21 = 131 days ($E_{25}$ = 365 days)

Life 4:

$E^c_{23}$ = 21+30+31+30+31+31+8 = 182 days $E^c_{24}$ = 365 days

$E^c_{25}$ = 20+31+30+31+30+31+31+30+31+30+31+1 = 327 days ($E_x = E^c_x$)

7.8 Life 1: 1

Life 2: 0

Life 3: 310/365.25

Life 4: 1

Life 5: 1

Life 6: 238/365.25

Life 7: 1

Life 8: 1

Life 9: 1

Life 10: 254/365.25

$q_{30}$ = 2/(6+(310+238+254)/365.25) = 0.244028729

0.244028729 +/- 1.96[0.244028729×(1-0.244028729)/(6+(310+238+254)/365.25)]$^{0.5}$

(0, 0.538087865) note: lower bound cannot be negative

7.9 $q_{49.5}$ = 30/(0.5×2100+2150+2100+0.5×2100+0.5×30) = 0.004713276

$q_{50.5}$ = 32/(0.5×2150+2150+2150+0.5×2100+0.5×32) = 0.004968173

$q_{50}$ = 0.004840724 (on average)

$\mu_{50}$ = 30/(0.5×2100+2150+2100+0.5×2100) = 0.004724409