Tutorial 12

12.1 right censoring: end of investigation
interval censoring: no exact date of death
random censoring: voluntarily leaving the investigation
Type I censoring: known end of investigation

12.2 4, 5, 6*, 7*, 8*, 10, 10, 11, 13, 14*, 15, 15*, 17, 17, 18, 18, 18*, 19*, 21, 22

Kaplan-Meier estimation

for $4 \le t < 5$ $\hat{F}(t) = 1 - \left(1 - \frac{1}{20}\right) = 0.05$

for $5 \le t < 10$ $\hat{F}(t) = 1 - \frac{19}{20}\left(1 - \frac{1}{19}\right) = 0.1$

for $10 \le t < 11$ $\hat{F}(t) = 1 - \frac{19}{20}\frac{18}{19}\left(1 - \frac{2}{15}\right) = 0.22$

for $11 \le t < 13$ $\hat{F}(t) = 1 - \frac{19}{20}\frac{18}{19}\frac{13}{15}\left(1 - \frac{1}{13}\right) = 0.28$

for $13 \le t < 15$ $\hat{F}(t) = 1 - \frac{19}{20}\frac{18}{19}\frac{13}{15}\frac{12}{13}\left(1 - \frac{1}{12}\right) = 0.34$

for $15 \le t < 17$ $\hat{F}(t) = 1 - \frac{19}{20}\frac{18}{19}\frac{13}{15}\frac{12}{13}\frac{11}{12}\left(1 - \frac{1}{10}\right) = 0.406$

for $17 \le t < 18$ $\hat{F}(t) = 1 - \frac{19}{20}\frac{18}{19}\frac{13}{15}\frac{12}{13}\frac{11}{12}\frac{9}{10}\left(1 - \frac{2}{8}\right) = 0.5545$

for $18 \le t < 21$ $\hat{F}(t) = 1 - \frac{19}{20}\frac{18}{19}\frac{13}{15}\frac{12}{13}\frac{11}{12}\frac{9}{10}\frac{6}{8}\left(1 - \frac{2}{6}\right) = 0.703$

for $21 \le t < 22$ $\hat{F}(t) = 1 - \frac{19}{20}\frac{18}{19}\frac{13}{15}\frac{12}{13}\frac{11}{12}\frac{9}{10}\frac{6}{8}\frac{4}{6}\left(1 - \frac{1}{2}\right) = 0.8515$

for $t \ge 22$ $\hat{F}(t) = 1$

$$\mathrm{Var}\left(\tilde{F}(16)\right) \approx \left(1 - \hat{F}(16)\right)^2 \sum_{i=1}^{6} \frac{d_i}{n_i\left(n_i - d_i\right)}$$

$$= \left(1 - 0.406\right)^2 \left(\frac{1}{20\times19} + \frac{1}{19\times18} + \frac{2}{15\times13} + \frac{1}{13\times12} + \frac{1}{12\times11} + \frac{1}{10\times9}\right) = 0.014434$$

95% confidence interval for $F(16) \approx 0.406 \pm 1.96\sqrt{0.014434} = \left(0.170521,\ 0.641479\right)$

Nelson-Aalen estimation

for $4 \le t < 5$ $\quad \hat{F}(t) = 1 - \exp\left(-\frac{1}{20}\right) = 0.048771$

for $5 \le t < 10$ $\quad \hat{F}(t) = 1 - \exp\left(-\frac{1}{20} - \frac{1}{19}\right) = 0.097541$

for $10 \le t < 11$ $\quad \hat{F}(t) = 1 - \exp\left(-\frac{1}{20} - \frac{1}{19} - \frac{2}{15}\right) = 0.210192$

for $11 \le t < 13$ $\quad \hat{F}(t) = 1 - \exp\left(-\frac{1}{20} - \frac{1}{19} - \frac{2}{15} - \frac{1}{13}\right) = 0.268668$

for $13 \le t < 15$ $\quad \hat{F}(t) = 1 - \exp\left(-\frac{1}{20} - \frac{1}{19} - \frac{2}{15} - \frac{1}{13} - \frac{1}{12}\right) = 0.327142$

for $15 \le t < 17$ $\quad \hat{F}(t) = 1 - \exp\left(-\frac{1}{20} - \frac{1}{19} - \frac{2}{15} - \frac{1}{13} - \frac{1}{12} - \frac{1}{10}\right) = 0.391173$

for $17 \le t < 18$ $\quad \hat{F}(t) = 1 - \exp\left(-\frac{1}{20} - \frac{1}{19} - \frac{2}{15} - \frac{1}{13} - ... - \frac{2}{8}\right) = 0.525845$

for $18 \le t < 21$ $\quad \hat{F}(t) = 1 - \exp\left(-\frac{1}{20} - \frac{1}{19} - \frac{2}{15} - \frac{1}{13} - ... - \frac{2}{6}\right) = 0.660253$

for $21 \le t < 22$ $\quad \hat{F}(t) = 1 - \exp\left(-\frac{1}{20} - \frac{1}{19} - \frac{2}{15} - \frac{1}{13} - ... - \frac{1}{2}\right) = 0.793933$

for $t \ge 22$ $\quad \hat{F}(t) = 1$

$$\mathrm{Var}\left(\tilde{\Lambda}_{16}\right) \approx \sum_{i=1}^{6} \frac{d_i\left(n_i - d_i\right)}{n_i^3}$$

$$= \frac{20-1}{20^3} + \frac{19-1}{19^3} + \frac{2(15-2)}{15^3} + \frac{13-1}{13^3} + \frac{12-1}{12^3} + \frac{10-1}{10^3} = 0.033531$$

95% confidence interval for $F(16)$

$$\approx 1 - \exp\left(-\frac{1}{20} - \frac{1}{19} - \frac{2}{15} - \frac{1}{13} - \frac{1}{12} - \frac{1}{10} \pm 1.96\sqrt{0.033531}\right) = (0.128307,\ 0.574770)$$

12.3 $\quad L = \frac{1}{4 + 2\exp(\beta)} \frac{\exp(\beta)}{3 + \exp(\beta)} \frac{1}{3} = \frac{\exp(\beta)}{3(4 + 2\exp(\beta))(3 + \exp(\beta))}$

$$l = \beta - \ln 3 - \ln(4 + 2\exp(\beta)) - \ln(3 + \exp(\beta))$$

$$\frac{\partial l}{\partial \beta} = 1 - \frac{2\exp(\beta)}{4 + 2\exp(\beta)} - \frac{\exp(\beta)}{3 + \exp(\beta)} = \frac{12 - 2\exp(2\beta)}{(4 + 2\exp(\beta))(3 + \exp(\beta))} = 0$$

$$\hat{\beta} = 0.895880$$

$$\frac{\partial^2 l}{\partial \beta^2} = \frac{-4\exp(2\beta)(4+2\exp(\beta))(3+\exp(\beta)) - (12-2\exp(2\beta))(10\exp(\beta)+4\exp(2\beta))}{(4+2\exp(\beta))^2(3+\exp(\beta))^2}$$

$$\mathrm{Var}\left(\tilde{\beta}\right) \approx -\frac{1}{-0.494897} = 2.020621$$

Since $-1.96 < \frac{0.895880}{\sqrt{2.020621}} = 0.63 < 1.96$, this covariate is not significant at 5% significance level

12.4 $L = \frac{1}{5+3\exp(\beta)} \frac{\exp(\beta)}{3+\exp(\beta)} \frac{1}{3} = \frac{\exp(\beta)}{3(5+3\exp(\beta))(3+\exp(\beta))}$

$$l = \beta - \ln 3 - \ln(5+3\exp(\beta)) - \ln(3+\exp(\beta))$$

$$\frac{\partial l}{\partial \beta} = 1 - \frac{3\exp(\beta)}{5+3\exp(\beta)} - \frac{\exp(\beta)}{3+\exp(\beta)} = \frac{15-3\exp(2\beta)}{(5+3\exp(\beta))(3+\exp(\beta))} = 0$$

$$\hat{\beta} = 0.804719$$

$$\frac{\partial^2 l}{\partial \beta^2} = \frac{-6\exp(2\beta)(5+3\exp(\beta))(3+\exp(\beta)) - (15-3\exp(2\beta))(14\exp(\beta)+6\exp(2\beta))}{(5+3\exp(\beta))^2(3+\exp(\beta))^2}$$

$$\mathrm{Var}\left(\tilde{\beta}\right) \approx -\frac{1}{-0.489357} = 2.043498$$

Since $-1.96 < \frac{0.804719}{\sqrt{2.043498}} = 0.56 < 1.96$, this covariate is still not significant at 5% significance level

This result is reasonable as the new data do not provide any new insight on the mortality difference between both sexes

12.5 3 (83 P), 6* (58 A), 9 (68 P), 11 (73 A), 14 (75 P), 14 (68 A), 14* (49 A), 16 (86 P)

$$\frac{\exp(\beta_1 + 143\beta_2)}{(\exp(75\beta_2) + \exp(\beta_1 + 68\beta_2) + \exp(\beta_1 + 49\beta_2) + \exp(86\beta_2))^2}$$

12.6 $\frac{\lambda_A(t)}{\lambda_B(t)} = \exp\left(\beta_1(3-1) + \beta_2(0-0) + \beta_3(0-1) + \beta_4(0-0) + \beta_5(0-1)\right)$

$$= \exp\left(2\beta_1 - \beta_3 - \beta_5\right)$$

The statistical significance of the last two covariates can be tested by the parameter significance test and / or likelihood ratio test.