Tutorial 11

11.1 Perform the statistical tests to check the graduated estimates' adherence to data for the following information. Analyse your results.

Age x	Initial Exposed to Risk E_x	Observed Number of Deaths d_x	Graduated Estimates $\mathring{q}_x$
14	12,800	3	0.00038
15	15,300	8	0.00043
16	12,500	5	0.00048
17	15,000	14	0.00053
18	16,500	17	0.00059
19	10,100	9	0.00066
20	12,800	15	0.00074
21	13,700	10	0.00083
22	11,900	10	0.00093

11.2 Perform the statistical tests to check the graduated estimates' adherence to data for the following information. Analyse your results.

Age x	Initial Exposed to Risk E_x	Observed Number of Deaths d_x	Graduated Estimates $\mathring{q}_x$
60	3,023	38	0.01368
61	3,214	48	0.01493
62	2,845	32	0.01628
63	2,772	61	0.01767
64	2,476	50	0.01911
65	2,697	42	0.02059
66	2,477	69	0.02216
67	2,432	52	0.02389
68	2,535	51	0.02585
69	2,944	83	0.02806
70	2,670	97	0.03052

11.3 Perform the statistical tests to check whether the experience in the standard table differs significantly from the actual data for the following information. Analyse your results.

Age	Exposed to Risk	Observed Deaths	ELT14 Males	Expected Deaths	Variance	Standardised Deviation	Contribution to chi squared
x	E_x	θ_x	q_x	$E_x q_x$	$E_x q_x p_x$	$\frac{\theta_x - E_x q_x}{\sqrt{E_x q_x p_x}}$	$\frac{(\theta_x - E_x q_x)^2}{E_x q_x p_x}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
35	1051	2	0.00113	1.19	1.19	0.75	0.56
36	940	2	0.00123	1.16	1.15	0.79	0.62
37	1048	3	0.00134	1.40	1.40	1.35	1.82
38	716	3	0.00148	1.06	1.06	1.89	3.56
39	719	3	0.00165	1.19	1.18	1.67	2.78
40	1051	1	0.00184	1.93	1.93	-0.67	0.45
41	1042	1	0.00206	2.15	2.14	-0.78	0.61
42	1804	3	0.00231	4.17	4.16	-0.57	0.33
43	1468	6	0.0026	3.82	3.81	1.12	1.25
44	1576	4	0.00293	4.62	4.60	-0.29	0.08
45	1647	8	0.00332	5.47	5.45	1.08	1.18
46	1861	5	0.00376	7.00	6.97	-0.76	0.57
47	1669	8	0.00425	7.09	7.06	0.34	0.12
48	1624	4	0.00481	7.81	7.77	-1.37	1.87
49	1157	5	0.00545	6.31	6.27	-0.52	0.27
50	2193	14	0.00615	13.49	13.40	0.14	0.02
51	1803	17	0.00694	12.51	12.43	1.27	1.62
52	2402	25	0.00781	18.76	18.61	1.45	2.09
53	2120	20	0.00877	18.59	18.43	0.33	0.11
54	2406	32	0.00982	23.63	23.39	1.73	3.00
55	1975	21	0.01098	21.69	21.45	-0.15	0.02
56	2564	31	0.01224	31.38	31.00	-0.07	0.00
57	1798	26	0.01361	24.47	24.14	0.31	0.10
58	2536	48	0.01509	38.27	37.69	1.59	2.51
59	2511	46	0.0167	41.93	41.23	0.63	0.40
60	1858	33	0.01843	34.24	33.61	-0.21	0.05
61	1835	49	0.02028	37.21	36.46	1.95	3.81
62	1393	30	0.02229	31.05	30.36	-0.19	0.04
63	1462	32	0.02448	35.79	34.91	-0.64	0.41
64	1245	34	0.02687	33.45	32.55	0.10	0.01
65	1064	26	0.02949	31.38	30.45	-0.97	0.95
66	1502	42	0.03238	48.63	47.06	-0.97	0.94
67	875	29	0.03555	31.11	30.00	-0.38	0.15
68	927	34	0.03903	36.18	34.77	-0.37	0.14
69	497	27	0.04285	21.30	20.38	1.26	1.60
70	983	57	0.04703	46.23	44.06	1.62	2.63
TOTAL							36.64

11.4 Comment on the smoothness of the following graduated estimates.

Age	Graduated Estimates
35	0.002050
36	0.002731
37	0.003359
38	0.003952
39	0.004525
40	0.005050
41	0.005561
42	0.006050
43	0.006536
44	0.007034
45	0.007565

11.5 Suggest potential dangers of using undergraduated rates or overgraduated rates for a life insurance company.

11.6 Suggest how you would choose an appropriate standard table for graduation by reference to standard table.

11.7 Suggest how you would use weighted least squares to fit the mathematical formula $\mathring{\mu}_x = A + Hx + Bc^x$ to certain observed data.

11.8 Suggest how you would use weighted least squares and maximum likelihood to fit the mathematical formula $\mathring{q}_x \Big/ \left(1 - \mathring{q}_x\right) = A + Bc^x + Hx + Jx^2$ to certain observed data.

11.9 A graduation is performed on some crude estimates of mortality rate at ages 35 to 71 using cubic spline. Knots are placed at ages 35, 47, 59, and 71. Given that $\mathring{q}_{35} = 0.00223$, $\mathring{q}_{47} = 0.00897$, $b_1 = 1.388 \times 10^{-7}$, and $b_2 = 3.194 \times 10^{-6}$, calculate $\mathring{q}_{65}$.

11.10 Obtain the graduated estimates of force of mortality for the following data, using Gompertz' Law and maximum likelihood. The age label of the data is 'aged x nearest birthday'.

Age x	Central Exposed to Risk E^C_x	Observed Number of Deaths d_x
70	1,000	80
71	1,005	90
72	1,010	95
73	1,008	105
74	1,006	115
75	998	125

11.11 Obtain the graduated estimates of mortality rate for the following data, using $\ln\left(\dot{q}_x/\left(1-\dot{q}_x\right)\right)=a+b\,x$ and weighted least squares. The age label of the data is 'aged x last birthday'.

Age x	Initial Exposed to Risk E_x	Observed Number of Deaths d_x
30	70,000	39
31	66,672	43
32	68,375	34
33	65,420	31
34	61,779	23
35	66,091	50
36	68,514	48
37	69,560	43
38	65,000	48
39	66,279	47
40	67,300	62
41	65,368	63
42	65,391	84
43	62,917	86
44	66,537	120
45	62,302	121
46	62,145	122
47	63,856	162
48	61,097	151
49	61,110	184

11.12 Obtain the graduated estimates of mortality rate for the following data, using standard table figures (with $\dot{q}_x = a + b\, q_x^S$) and weighted least squares. The age label of the data is ‘aged *x* last birthday’.

Age x	Initial Exposed to Risk E_x	Observed Number of Deaths d_x	Standard Table Figures $q^S{}_x$
47	166	2	0.00505
48	187	2	0.00570
49	218	4	0.00644
50	243	6	0.00728
51	276	2	0.00826
52	302	4	0.00930
53	347	7	0.01051
54	390	3	0.01184
55	430	9	0.01331
56	494	9	0.01492
57	558	8	0.01668
58	628	11	0.01859
59	701	14	0.02065
60	813	18	0.02287
61	917	18	0.02525
62	1,040	24	0.02778
63	1,182	30	0.03049
64	1,299	43	0.03339
65	1,432	41	0.03648
66	1,596	54	0.03978
67	1,752	64	0.04332