Question 1

When mortality is constant over time (Week 6 lecture), we can simply use

$$\overset{\circ}{e}_x \approx \frac{1}{2} + e_x = \frac{1}{2} + \sum_{k=1}^{\infty} {}_k p_x \,,$$

where ${}_k p_x \approx \exp(-m_x)\exp(-m_{x+1})...\exp(-m_{x+k-1})$ or ${}_k p_x = (1-q_x)(1-q_{x+1})...(1-q_{x+k-1})$.

But when mortality declines over time (Week 9 lecture), we have to use

$$\overset{\circ}{e}_x \approx \frac{1}{2} + e_x = \frac{1}{2} + \sum_{k=1}^{\infty} {}_k p_x^* \,,$$

where for *future* years $t = 1, 2, 3, \dots$, ${}_k p_x^* \approx \exp(-m_{x,1})\exp(-m_{x+1,2})...\exp(-m_{x+k-1,k})$ or ${}_k p_x^* = (1-q_{x,1})(1-q_{x+1,2})...(1-q_{x+k-1,k})$. It is called *cohort* life expectancy and it always refers to the *same* cohort or year of birth. That is, we use *m* or *q* of a person aged *x* in future year 1, who then becomes age $x + 1$ in future year 2, age $x + 2$ in future year 3, and so on. We allow for how the mortality within the same cohort declines over time.

Question 2

Suppose the data range covers the period of 1980-2018. The fitting set can include years 1980, 1990, 2000, the validation set can include year 2010, and the testing set can include year 2018. When normalising the *year* feature, we can use 1/39, 11/39, 21/39, 31/39, 39/39 = 1 to represent years 1980, 1990, 2000, 2010, 2018.

The major steps of tackling this question include:

1. Fit the one-layer FNN to the log central death rates in years 1980, 1990, 2000 (fitting set) using the R code in the lecture slides / tutorial solutions. At the end of every single epoch, use your own code to apply the FNN in that epoch to year 2010 and to calculate the error of year 2010.
2. Find the optimal number of epochs such that the error of year 2010 (validation set) is minimum, as in Page 3 of the lecture slides.
3. Fit the FNN to the log central death rates in years 1980, 1990, 2000 again using the same initial values of *a* and *b* in Step 1 and the optimal number of epochs from Step 2.
4. Apply the calibrated FNN from Step 3 to year 2018 and calculate the error of year 2018 (testing set) with your own code.
5. Plot the observed and fitted (1980, 1990, 2000) / projected (2010, 2018) log central death rates over age for different years. Demonstrate and explain the model performance.