Tutorial 7

7.1 A large company always maintains a workforce of exactly 5,000 young workers. Assume all workers experience a constant force of mortality of 0.0008 per annum. Use the Poisson model to estimate the probability that there will be fewer than three deaths during any six-month period.

7.2 The population of a small town remains constant at 50,000 over the next few years. Assume all inhabitants experience a constant force of mortality of 0.001 per annum. Use the Poisson model to estimate the probability that there will be more than 225 deaths in the town during the next four years.

7.3 In a mortality investigation of 5 years there are 46 deaths and the population remains approximately constant at 7,500. Use the Poisson model to estimate the force of mortality and the corresponding 95% confidence interval.

7.4 In a mortality investigation six homogenous and independent lives are observed. The i^{th} life is observed between ages $70 + a_i$ and $70 + b_i$ for $0 \le a_i < b_i \le 1$. Let D_i be an indicator random variable for the death of the i^{th} life and d_i be its observed value. Let $70 + t_i$ be the age of death of the i^{th} life for $a_i < t_i < b_i$ if the life dies during its observation period. The following data are recorded:

Life i	a_i	b_i	d_i	t_i
1	0	1	0	NA
2	0.3	0.9	0	NA
3	0.5	1	1	0.9
4	0	0.4	0	NA
5	0	0.9	1	0.7
6	0	1	1	0.8

Use the binomial model to write down the likelihood function of these observations with the UDD assumption. Then use R to calculate the maximum likelihood estimate of the mortality rate at age 70. Compare this result with the moment matching estimate.

By contrast use the Poisson model to calculate the maximum likelihood estimate of the force of mortality at age 70.5.

7.5 In a mortality investigation ten homogenous and independent lives are observed. The i^th^ life is observed between ages $40 + a_i$ and $40 + b_i$ for $0 \leq a_i < b_i \leq 1$. Let D_i be an indicator random variable for the death of the i^th^ life and d_i be its observed value. Let $40 + t_i$ be the age of death of the i^th^ life for $a_i < t_i < b_i$ if the life dies during its observation period. The following data are recorded:

Life i	a_i	b_i	d_i	t_i
1	0.12	1	0	NA
2	0.23	0.85	0	NA
3	0.28	1	0	NA
4	0.31	0.75	1	0.6
5	0.39	0.92	0	NA
6	0.41	1	0	NA
7	0.52	0.8	0	NA
8	0.65	0.87	1	0.7
9	0.71	1	0	NA
10	0.82	1	0	NA

Use the binomial model to write down the likelihood function of these observations with the UDD assumption. Then use R to calculate the maximum likelihood estimate of the mortality rate at age 40. Compare this result with the moment matching estimate.

Repeat the above with the Balducci assumption instead.

7.6 A mortality investigation covers the period of 1 July 2000 to 1 July 2002. The following data are recorded:

Life	Date of Birth	Date of Death
1	1 Oct 1935	1 Aug 2000
2	1 Sep 1937	1 Nov 2001
3	1 May 1938	NA

Calculate the total central exposed to risk and also the total initial exposed to risk at age 64 last birthday. Give your answers in months.

7.7 An investigation of resignation covers the period of 1 January 1999 to 1 January 2003. The age label is 'aged x last birthday'. The following data are recorded:

Life	Date of Birth	Date of Entry	Date of Exit	Mode of Exit
1	11 Nov 1968	24 Mar 1997	29 Dec 2002	death
2	1 Sep 1975	30 Aug 2000	NA	NA
3	10 Feb 1974	10 Oct 1998	21 Jun 1999	resignation
4	8 Feb 1977	10 Aug 2000	NA	NA

Calculate the central exposed to risk and also the initial exposed to risk for each life at each age.

7.8 A mortality investigation covers the period of 1 January 1996 to 1 January 2001. The following data are recorded:

Life	Date of Birth	Date of Entry	Date of Exit	Mode of Exit
1	17 Mar 1967	20 Jun 1995	NA	NA
2	6 May 1967	6 Aug 1995	12 Jun 1996	death
3	12 Aug 1967	18 Dec 1995	18 Jun 1998	withdrawal
4	27 Oct 1967	4 Jan 1996	NA	NA
5	4 Jan 1968	28 Apr 1996	29 Aug 1999	death
6	18 Apr 1968	16 Jun 1996	12 Dec 1998	withdrawal
7	20 May 1968	29 Oct 1996	21 Apr 1999	death
8	4 Jul 1968	16 Feb 1997	NA	NA
9	16 Sep 1968	22 Aug 1997	22 Feb 2000	withdrawal
10	11 Dec 1968	1 Apr 1999	17 Jul 1999	death

Estimate q_{30} and find its 95% confidence interval.

7.9 A mortality investigation covers the period of 1 January 1997 to 1 January 2000. The number of deaths is 30 at age 50 nearest birthday and is 32 at age 51 nearest birthday. Estimate q_{50} and μ_{50} using the following data.

Date	Population aged 49 last birthday	Population aged 50 last birthday	Population aged 51 last birthday
1/1/1997	2,000	2,200	2,100
1/1/1998	2,100	2,200	2,100
1/1/1999	2,100	2,100	2,200
1/1/2000	2,000	2,200	2,000