Tutorial 12

12.1 A mortality investigation is carried out on male actuaries who are married. A group of 1,000 actuaries is selected at random on 1 January 2014. Each person provides detailed personal information as at 1 January 2014 including name, address, date of birth, and marital status. The same information is collected as at each 1 January of 2015, 2016, 2017, and 2018. The investigation closes in 2018. Describe the ways in which the data are censored.

12.2 The following are the observed numbers of days that several machine parts of the same type are in use until failure (no asterisk) or replacement while still working (with asterisk). Use Kaplan-Meier estimation and Nelson-Aalen estimation to compute the distribution function of the time for a machine part to fail. Use Greenwood's formulae to calculate the variance of the Kaplan-Meier estimator of the distribution function and the variance of the Nelson-Aalen estimator of the integrated hazard at 16 days. Then find the 95% confidence interval for the two distribution functions at 16 days, given that the estimators are normally distributed asymptotically.

17, 13, 15*, 7*, 21, 18*, 5, 18, 6*, 22, 19*, 15, 4, 11, 14*, 18, 10, 10, 8*, 17

12.3 Six lives aged 55 are observed till dead or censored. The observations are stated below.

Life	Sex	Age at Exit	Reason for Exit
1	M	56	dead
2	F	62	censored
3	F	63	dead
4	M	66	dead
5	M	67	censored
6	M	67	censored

Use the Cox model with sex (0 for male and 1 for female) as the only covariate and write down the partial likelihood function. Then calculate the maximum likelihood parameter estimate. Is it an important covariate here?

12.4 Suppose now there are some additional data to Q12.3 as follows.

Life	Sex	Age at Exit	Reason for Exit
7	M	56	censored
8	F	62	censored

How do the extra data affect the results?

12.5 An investigation is carried out on some patients following liver transplants. For the Cox model, the covariates are $x_{i,1} = 0$ for placebo and 1 for treatment A, and $x_{i,2}$ = weight of patient in kg. The observed lifetimes are stated below. The weights in kg are given in brackets. An asterisk represents the patient is censored.

Placebo | Treatment A
3 (83) | 6* (58)
9 (68) | 11 (73)
14 (75) | 14 (68)
16 (86) | 14* (49)

What contribution is made by the deaths at time 14 to the partial likelihood function?

12.6 A mortality investigation is carried out with the following covariates:
$x_{i,1}$ = socio-economic group from 0 (low) to 4 (high)
$x_{i,2}$ = sex (0 for male and 1 for female)
$x_{i,3}$ = smoking status (0 for smoker and 1 for non-smoker)
$x_{i,4} = x_{i,1} \times x_{i,2}$
$x_{i,5} = x_{i,1} \times x_{i,3}$

Using the Cox model, calculate the ratio of the hazard functions of two lives A and B of the same age, where A is a male smoker in socio-economic group 3 and B is a male non-smoker in socio-economic group 1. How can you test whether the last two covariates have significant effects?