Tutorial 6

6.1 Let $S_0(t) = (1-(t/110))^{1/3}$ for $0 < t < 110$. Calculate

(a) the probability that a newborn life survives to age 50
(b) the probability that a newborn life dies between ages 60 and 70
(c) the probability that a life aged 40 survives for the next 10 years
(d) the probability that a life aged 50 dies between ages 60 and 80
(e) the age to which a life aged 60 has a 60% chance of surviving
(f) the force of mortality at age 30

6.2 Let $S_0(t) = \exp(-\lambda t)$ for $\lambda, t > 0$. Prove that $S_0(t) = S_x(t)$ for $x > 0$.

6.3 If p_{55} = 0.99403, p_{56} = 0.99336, p_{57} = 0.99261, p_{58} = 0.99177, and p_{59} = 0.99082, calculate (i) the probability that a life aged 55 survives for the next 3 years, (ii) the probability that a life aged 56 dies before age 60, and (iii) the probability that a life aged 55 dies aged 57 or 58.

6.4 Find the distribution function, density function, mean, and variance of the future lifetime T_x when $\mu_x = 3/(1+x)$ for $x > 0$.

6.5 If μ_x is 0.01 for $60 < x \le 70$, 0.015 for $70 < x \le 80$, and 0.025 for $x > 80$, calculate the probability that a life aged 65 dies between ages 80 and 83.

6.6 Both Tim and Tom are aged 40. Tim is subject to a force of mortality that is k% greater than that for Tom at all ages. The probability that Tim survives to age 50 is 0.97247 and the probability that Tom survives to age 50 is 0.973. Find the value of k.

6.7 A life aged 50 is subject to an extra hazard over the year of age 50 to 51. The extra hazard results an addition to the force of mortality which is 0.002 at age 50 and decreases linearly to 0.0018 at age 51. If this hazard did not exist, the probability that the life would survive to age 51 would be 0.993. Calculate the probability that the life survives to age 51.

6.8 Find $\mathring{e}_x$ when (i) $F_0(t) = t/100$ for $0 < t < 100$ and (ii) $F_x(t) = 1 - \exp(-0.01t)$ for $t > 0$.

6.9 If q_{105} = 33/82, q_{106} = 23/49, q_{107} = 14/26, q_{108} = 8/12, q_{109} = 1, find the probability mass function, mean, and variance of the curtate future lifetime K_{105}.

6.10 If p_{40} = 0.9977, calculate ${}_{0.5}p_{40}$ and ${}_{0.25}p_{40.5}$ using (i) the UDD assumption and (ii) the Balducci assumption.

6.11 Mortality Table 1's force of morality is twice of Mortality Table 2's. Mortality Table 2 follows Gompertz' Law. If the probability that a life aged x survives for the next n years under Mortality Table 1 is the same as the probability that a life aged $x + a$ survives for the next n years under Mortality Table 2, find a in terms of c.

6.12 If $\mu_{50} = 0.0045$, $\mu_{70} = 0.0380$, and μ_x follows Gompertz' Law, calculate ${}_{20}p_{30}$.

6.13 If $\mu_{20} = 0.003667$, $\mu_{40} = 0.009891$, $\mu_{60} = 0.031835$, and μ_x follows Makeham's Law, calculate A, B, and c.

6.14 If ${}_{10}p_{30} = 0.94481$, ${}_{10}p_{40} = 0.91050$, ${}_{10}p_{50} = 0.85214$, and μ_x follows Makeham's Law, calculate μ_{70}.