

Machine Learning

- computer algorithms
- applied to data to generate info
- volume, velocity, variety
- e.g. risk classification of motor insurance policyholders via in-car monitoring devices
- e.g. detection of possible fraudulent insurance claims
- e.g. cause codes of insurance claims
- e.g. chatbots for customer inquiries
- e.g. targeted ads on websites

Machine Learning

- $t = f(x_1, x_2, x_3, \dots)$

t : target value

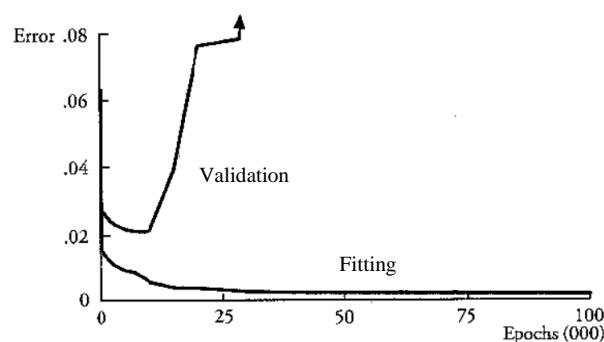
x_i : i th features / covariates / explanatory variables

f : target function

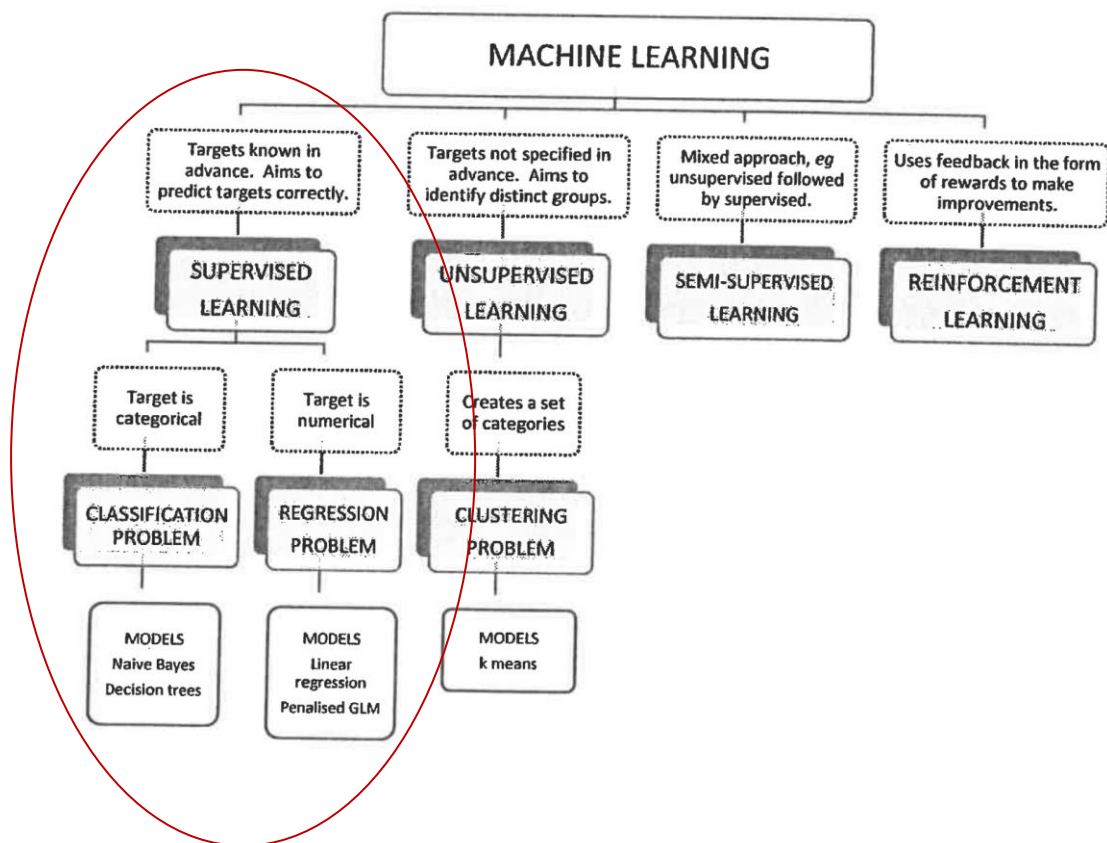
- approximation of unknown f
- mapping function which mimics f
- estimation of weights / coefficients / parameters via minimisation of error function
- iterative estimation procedure
- regression as a specific example
- machine learning on forecast performance
- statisticians on parameter significance

Training / Validation / Testing

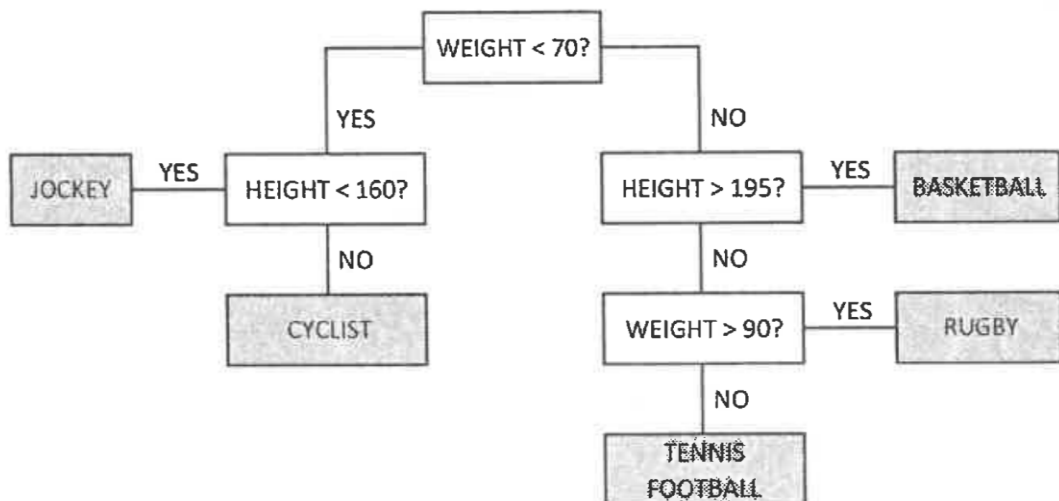
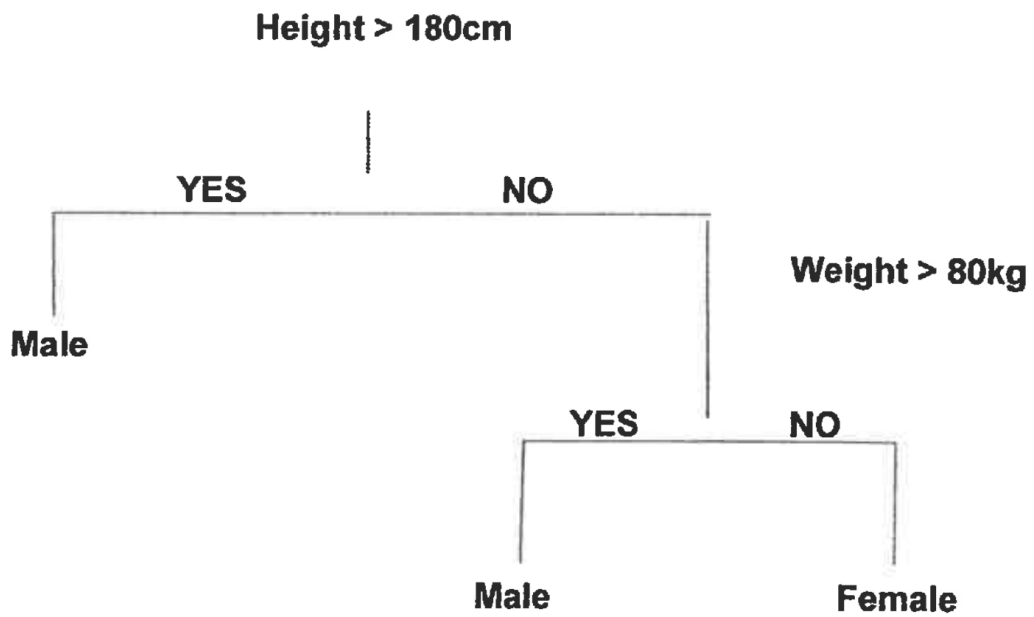
- splitting data set into *training set* and *testing set*
- further splitting training set into *fitting set* and *validation set*
- using fitting set to estimate parameters
- using validation set to adjust parameters and avoid overfitting
- AIC, BIC, L1 / L2 penalty as alternative
- using testing set to provide unbiased evaluation of forecast performance
- rule-of-thumb : 60% / 20% / 20%



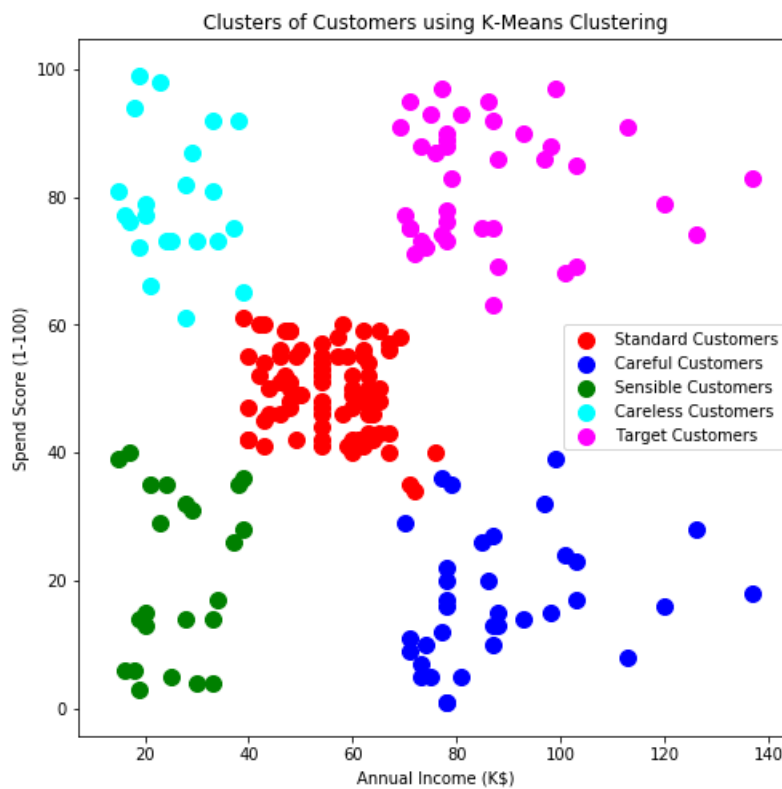
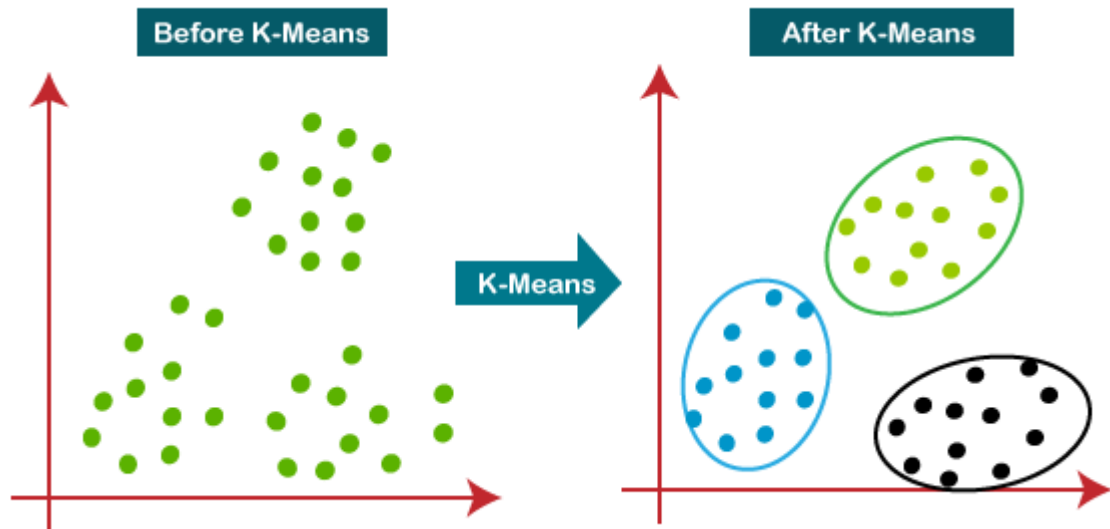
Types of Machine Learning



Decision Tree



K-Means Clustering



Types of Traditional Data

DATA TYPES				
NUMERICAL (ie numbers)		CATEGORICAL (ie not numbers)		
DISCRETE	CONTINUOUS	ATTRIBUTE (DICHOTOMOUS)	NOMINAL	ORDINAL
↓	↓	↓	↓	↓
Age last birthday Number of children Number of claims	Exact age Salary Claim amount	Alive / Dead Male / Female Claim / No claim Pass / Fail	Customer name Type of claim Occupation Marital status Country Colour of car	Date of birth (DD/MM/YY) Month (Jan, Feb, Mar, ...) Exam grade (A, B, C, ...) Size (S, M, L, XL) Agree/Don't know/Disagree

Data Preparation

- surveys, censuses, admin systems, specific databases, etc
- spreadsheet, Access, R, Python, etc
- data cleaning (errors, missing values, etc)
- exploratory data analysis
- scaling of features
- features engineering
- detailed documentation

Neural Network (NN)

- 3 layers of neurons in general
- *input* layer : features / covariates / explanatory variables
- *hidden* layer : most of learning
- *output* layer : output values
- each neuron receiving input from some neurons, and firing output to other neurons
- strengths of connections (weights / coefficients / parameters) between neurons changing epoch by epoch
- learning based on both architecture and weights of network

Neural Network (NN)

- universal approximators
- supervised learning
- FNN, RNN, LSTM, GRU, etc
- 1-layer feedforward neural network :

$$y_j = f(a_{0,j} + a_{1,j} x)$$

$$z = f(b_0 + \sum_j b_j y_j)$$

x : feature value from input layer into hidden layer

y_j : intermediate value from j th neuron in (single) hidden layer into output layer

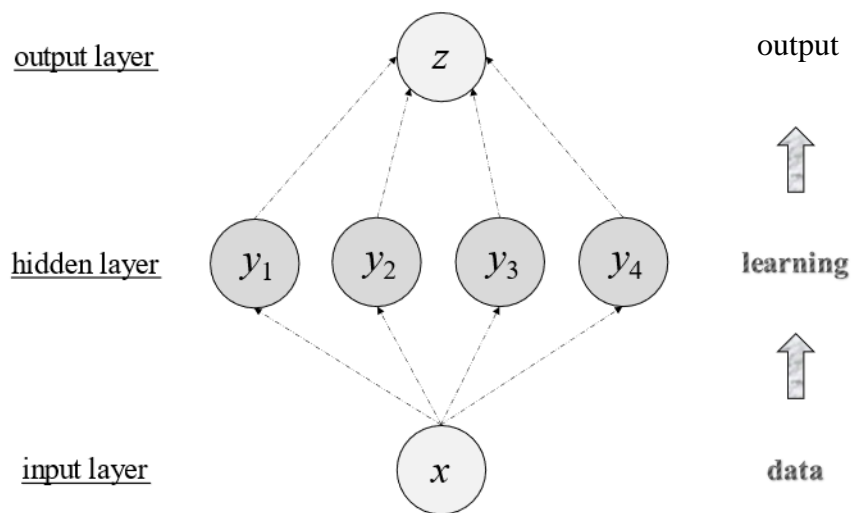
z : output value from output layer

f : activation function (e.g. logistic)

$a_{0,j}, a_{1,j}, b_0, b_j$: weights

Feedforward Neural Network (FNN)

– 1-layer FNN :



Feedforward Neural Network (FNN)

- network too small \rightarrow inadequate modelling capacity
- network too large \rightarrow overfitting and poor generalisation
- 1 or 2 hidden layers adequate for most commercial and financial applications
- rule of thumb : number of training samples around 10 times number of weights
- weights estimated by backpropagation

Backpropagation

- error function $e = 0.5 \sum_k (z_k - t_k)^2 / n$
- x_k, t_k normalised to small values
- initial values of weights in hidden layer randomised as $\text{Uniform}(-1, 1)$
- initial values of weights in output layer randomised as -1 or 1 with equal chance
- in each epoch, for each weight :

$$\varepsilon^* = \varepsilon + \lambda \quad \text{if } g \frac{\partial}{\partial w} e > 0 \quad [\text{update}$$

$$\varepsilon^* = \varepsilon \phi \quad \text{otherwise} \quad \text{learning rate}]$$

$$w^* = w - \varepsilon \frac{\partial}{\partial w} e \quad [\text{update weights}]$$

$$g^* = \theta g + (1 - \theta) \frac{\partial}{\partial w} e \quad [\text{accumulate info}]$$

$$\lambda = 0.1 \quad \phi = 0.5 \quad \theta = 0.7$$

Backpropagation

$$f(s) = \frac{\exp(s)}{1 + \exp(s)}$$

$$\frac{\partial}{\partial s} f(s) = \frac{\exp(s)}{(1 + \exp(s))^2} = f(s)(1 - f(s))$$

$$\frac{\partial e}{\partial b_0} = \frac{1}{n} \sum_k (z_k - t_k) \frac{\partial z_k}{\partial b_0} = \frac{1}{n} \sum_k (z_k - t_k) z_k (1 - z_k)$$

$$\frac{\partial e}{\partial b_j} = \frac{1}{n} \sum_k (z_k - t_k) \frac{\partial z_k}{\partial b_j} = \frac{1}{n} \sum_k (z_k - t_k) z_k (1 - z_k) y_{j,k}$$

$$\begin{aligned} \frac{\partial e}{\partial a_{0,j}} &= \frac{1}{n} \sum_k (z_k - t_k) \frac{\partial z_k}{\partial y_{j,k}} \frac{\partial y_{j,k}}{\partial a_{0,j}} \\ &= \frac{1}{n} \sum_k (z_k - t_k) z_k (1 - z_k) b_j y_{j,k} (1 - y_{j,k}) \end{aligned}$$

$$\begin{aligned} \frac{\partial e}{\partial a_{1,j}} &= \frac{1}{n} \sum_k (z_k - t_k) \frac{\partial z_k}{\partial y_{j,k}} \frac{\partial y_{j,k}}{\partial a_{1,j}} \\ &= \frac{1}{n} \sum_k (z_k - t_k) z_k (1 - z_k) b_j y_{j,k} (1 - y_{j,k}) x_k \end{aligned}$$

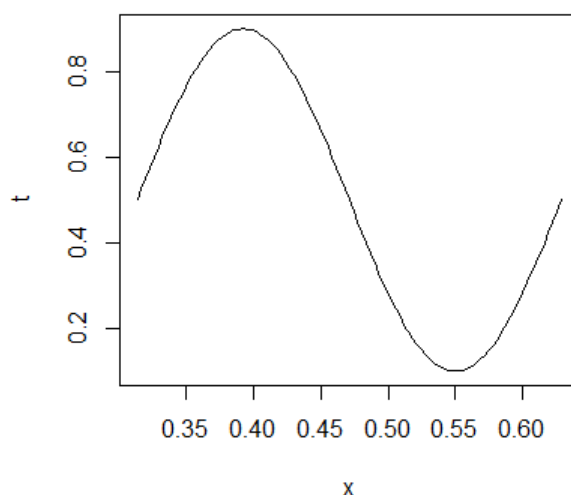
Example

```
x=seq(0.1,0.2,0.001)*pi
```

```
obs=sin(x*20)
```

```
t=(obs-(-1))/(1-(-1))*(0.9-0.1)+0.1
```

(Note: It is a toy problem without noises.)



```
n=101; hids=3; epoch=10000
```

```
y<-array(NA,c(hids,n))
```

```
z<-numeric()
```

Example

```
logistic<-function(s) {  
  if (s>700) { logistic=1 }  
  if (s<=700) { logistic=exp(s)/(1+exp(s)) }  
  logistic }
```

```
e<-numeric()  
a<-array(NA,c(2,hids))  
for (i in 1:2) { for (j in 1:hids) {  
  a[i,j]=runif(1,-1,1) } }  
b=-1+rbinom(hids+1,1,0.5)*2
```


Example

```
epsilona<-array(0.5,c(2,hids))
```

```
epsilonb=rep(0.5,hids+1)
```

```
lambda=0.1; phi=0.5; theta=0.7
```

```
for (k in 1:n) {
```

```
  for (j in 1:hids) {
```

```
    y[j,k]=logistic(a[2,j]+a[1,j]*x[k]) }
```

```
    z[k]=logistic(b[hids+1]+sum(b[1:hids]*y[1:  
hids,k])) }
```

```
for (h in 1:epoch) {
```

```
  da<-array(0,c(2,hids))
```

```
  db<-rep(0,hids+1)
```

Example

```
for (k in 1:n) {
```

```
    p=(z[k]-t[k])*z[k]*(1-z[k])/n
```

```
    for (j in 1:hids) { db[j]=db[j]+p*y[j,k] }
```

```
    db[hids+1]=db[hids+1]+p
```

```
    for (j in 1:hids) {
```

```
        q=(z[k]-t[k])*z[k]*(1-z[k])*b[j]*y[j,k]*(1-  
        y[j,k])/n
```

```
        da[1,j]=da[1,j]+q*x[k]
```

```
        da[2,j]=da[2,j]+q }
```

```
}
```

Example

```
if (h>1) {  
  for (i in 1:2) { for (j in 1:hids) {  
    if (ga[i,j]*da[i,j]>0) {  
      epsilona[i,j]=epsilona[i,j]+lambda }  
    if (ga[i,j]*da[i,j]<=0) {  
      epsilona[i,j]=epsilona[i,j]*phi }  
    }  
  }  
  for (j in 1:(hids+1)) {  
    if (gb[j]*db[j]>0) {  
      epsilonb[j]=epsilonb[j]+lambda }  
    if (gb[j]*db[j]<=0) {  
      epsilonb[j]=epsilonb[j]*phi }  
    }  
  }  
}
```

Example

$a = a - \epsilon a * da$

$b = b - \epsilon b * db$

if ($h == 1$) { $ga = da$; $gb = db$ }

if ($h > 1$) {

$ga = \theta * ga + (1 - \theta) * da$

$gb = \theta * gb + (1 - \theta) * db$ }

for (k in $1:n$) {

for (j in $1:hids$) {

$y[j,k] = \text{logistic}(a[2,j] + a[1,j] * x[k])$ }

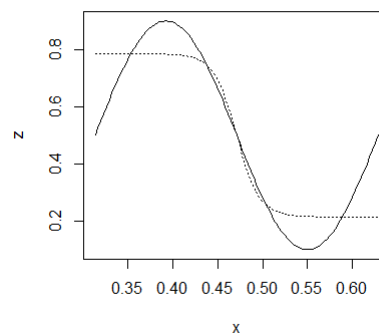
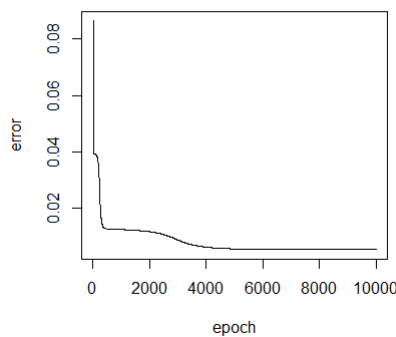
$z[k] = \text{logistic}(b[hids+1] + \text{sum}(b[1:hids] * y[1:hids,k]))$ }

Example

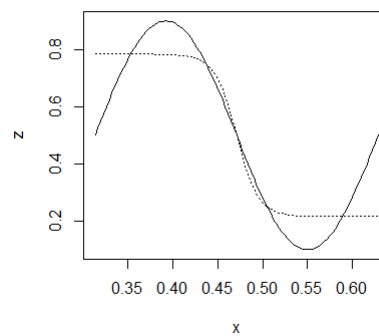
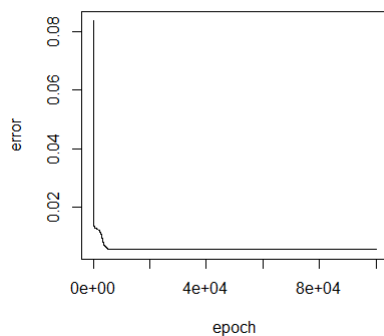
$$e[h] = 0.5 * \sum ((z-t)^2) / n$$

}

hids = 1; epoch = 10000; $t \in [0.1, 0.9]$

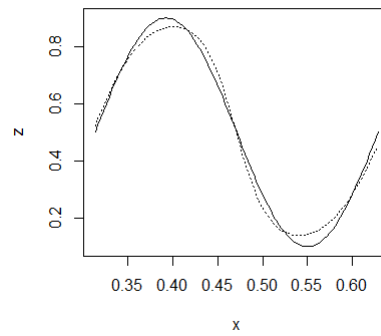
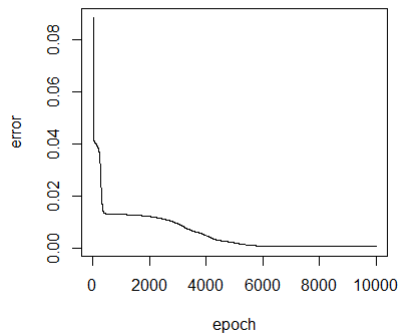


hids = 1; epoch = 100000; $t \in [0.1, 0.9]$

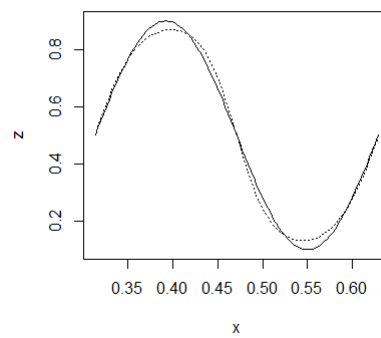
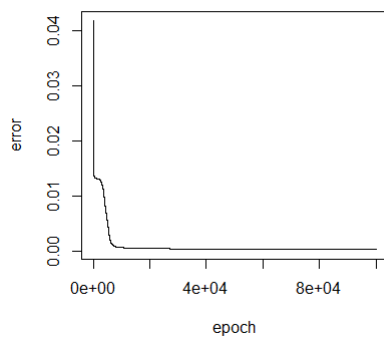


Example

$\text{hids} = 3$; $\text{epoch} = 10000$; $t \in [0.1, 0.9]$

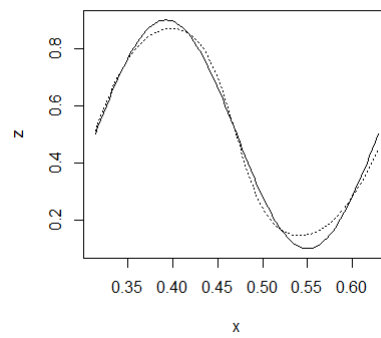
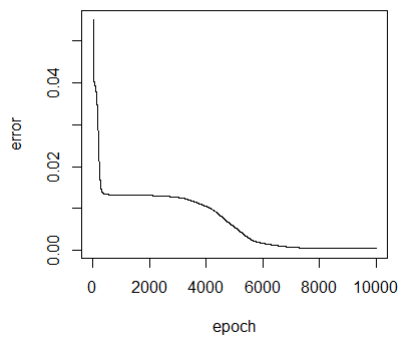


$\text{hids} = 3$; $\text{epoch} = 100000$; $t \in [0.1, 0.9]$

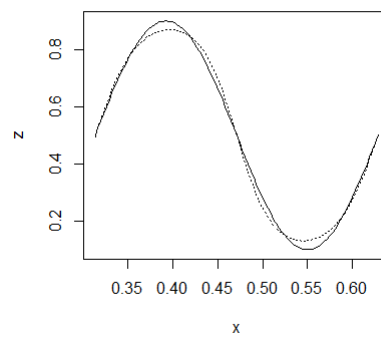
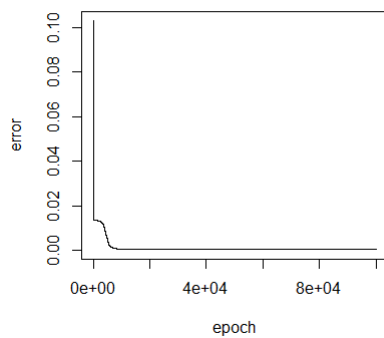


Example

$\text{hids} = 5$; $\text{epoch} = 10000$; $t \in [0.1, 0.9]$

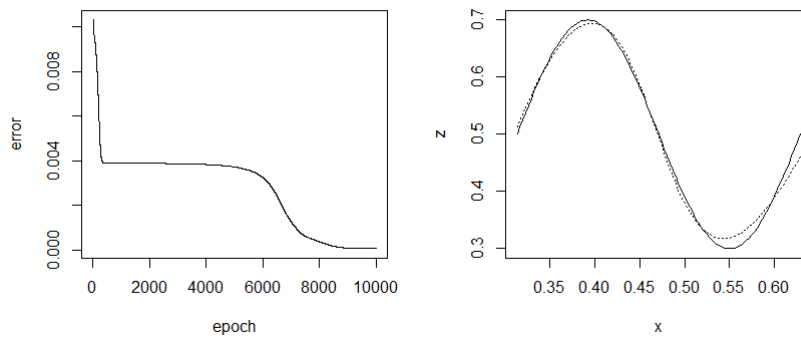


$\text{hids} = 5$; $\text{epoch} = 100000$; $t \in [0.1, 0.9]$



Example

$\text{hids} = 5$; $\text{epoch} = 10000$; $t \in [0.3, 0.7]$



$\text{hids} = 5$; $\text{epoch} = 100000$; $t \in [0.3, 0.7]$

