MONASH University

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Semester One 2019
Examination Period

Faculty of Business & Economics

EXAM CODES: **ETC3550**

TITLE OF PAPER: **Applied Forecasting for Business and Economics**

EXAM DURATION: 2 hours writing time

READING TIME: 10 minutes

THIS PAPER IS FOR STUDENTS STUDYING AT: (tick where applicable)

☐ Caulfield ✓ Clayton ☐ Parkville ☐ Peninsula
☐ Monash Extension ☐ Off Campus Learning ☐ Malaysia ☐ South Africa
☐ Other (specify)

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<u>AUTHORISED MATERIALS</u>

OPEN BOOK ☐ YES ✓ NO

CALCULATORS ✓ YES ☐ NO
Only HP 10bII+ or Casio FX82 (any suffix) calculator permitted

SPECIFICALLY PERMITTED ITEMS ☐ YES ✓ NO
if yes, items permitted are:

Candidates must complete this section if required to write answers within this paper

STUDENT ID: ___ ___ ___ ___ ___ ___ ___ ___ DESK NUMBER: ___ ___ ___ ___ ___ ___

The exam contains FIVE questions. ALL questions must be answered. The exam is worth 100 marks in total.

QUESTION 1

Write about a quarter of a page each on any FOUR of the following topics. (Clearly state if you agree or disagree with each statement. No marks will be given without any justification.)

(a) The trouble with statistical methods of forecasting is that they assume the patterns in the past will continue in the future.

(b) A time series decomposition into trend, seasonal and remainder terms is only useful when there are no cycles in the data.

(c) With STL decompositions and ETS models, we always need to transform our data before estimating the components.

(d) Some ETS models are not always suitable and should be avoided.

(e) The combination of AR and MA components guide long-run ARIMA forecasts.

(f) Linear regression models are simplistic because the real world is nonlinear.

Total: 20 marks

— END OF QUESTION 1 —

QUESTION 2

Figure 1 shows the number of employees (in thousands) in child day care services in New York City over the period the period January 1990–February 2019.

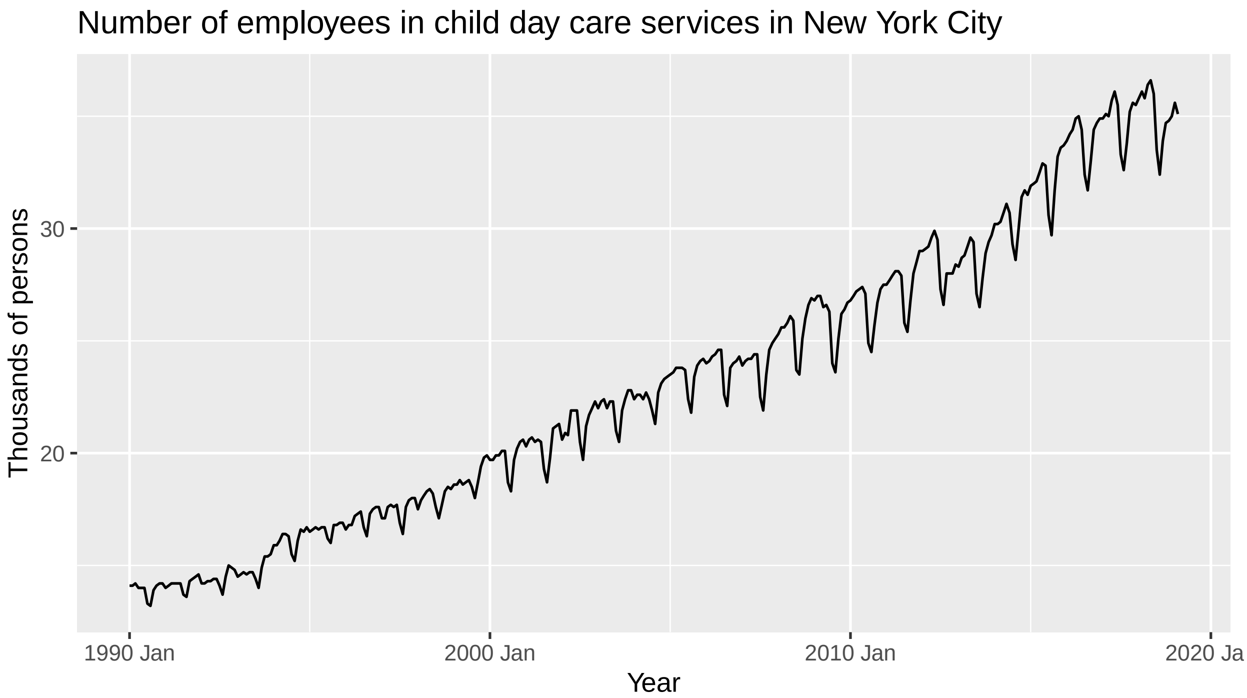


Figure 1:

(a) The following code has been used to produce Figures 1, 2 and 3.

```
daycare %>%
  autoplot(Count) +
  ggtitle("Number of employees in child day care services in New York City") +
  xlab("Year") + ylab("Thousands of persons")

daycare %>%
  gg_season(Count, labels='both') +
  ggtitle("Number of employees in child day care services in New York City") +
  ylab("Thousand of persons")

daycare %>%
  gg_subseries(Count) +
  ggtitle("Number of employees in child day care services in New York City") +
  ylab("Thousand of persons")
```

Using Figures 1, 2 and 3, describe the `daycare` series.

4 marks

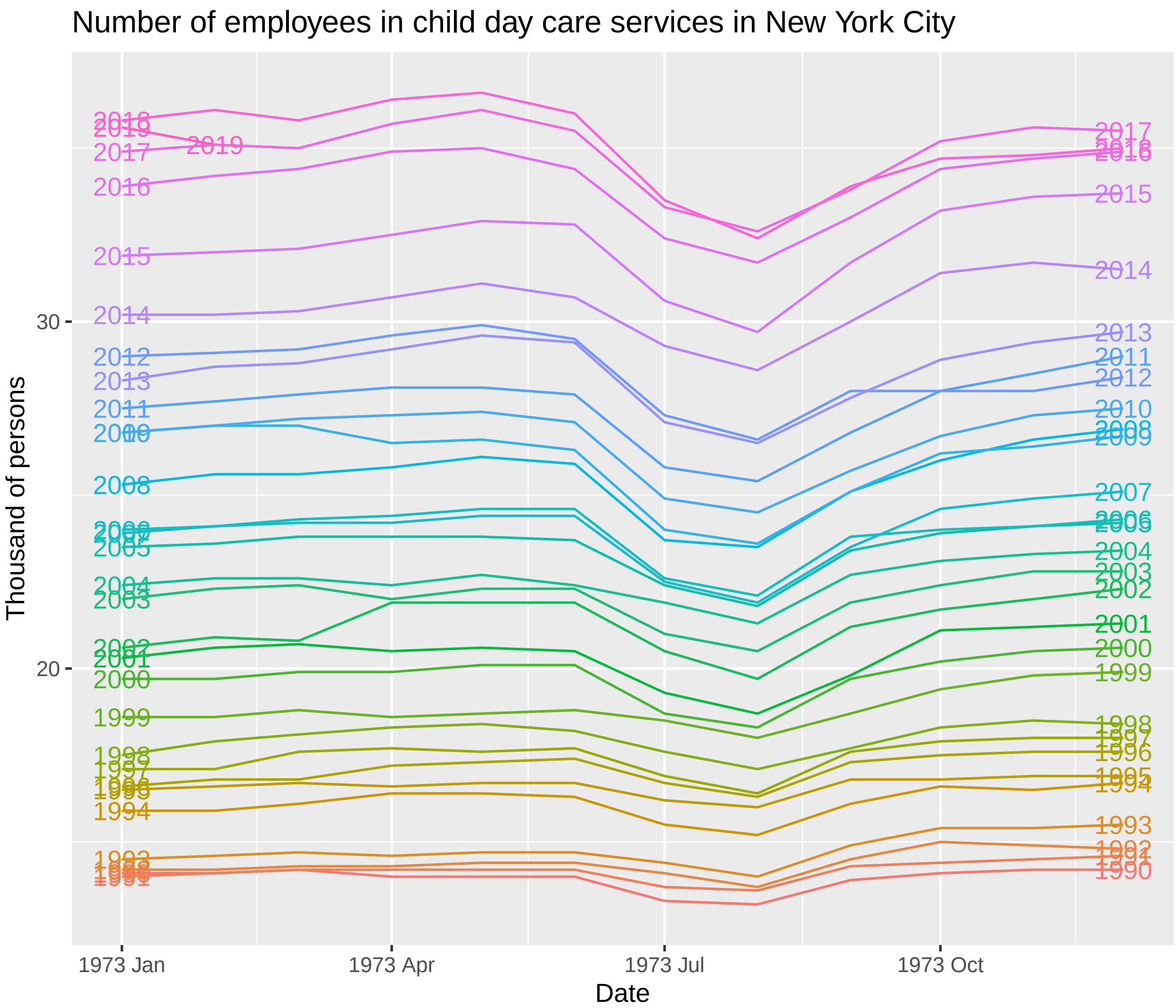


Figure 2:

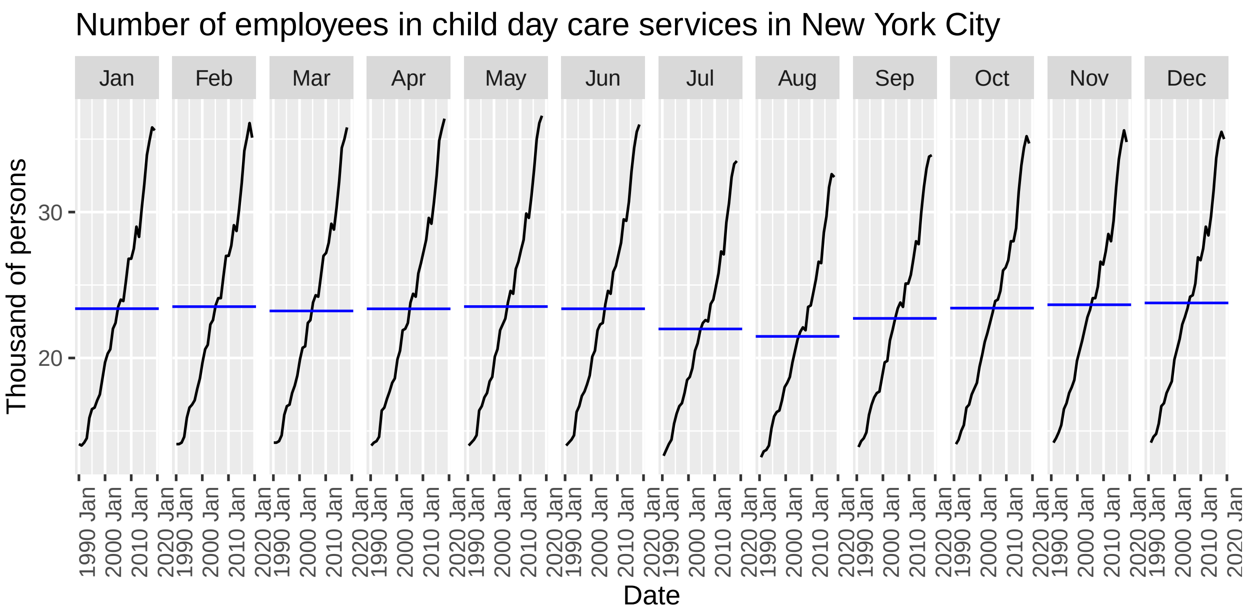


Figure 3:

(b) Using the code below, describe what is plotted in Figure 4. Comment on the selection of `window`.

6 marks

```
daycare %>%
  model(STL(log(Count) ~ season(window = 21))) %>%
  components() %>%
  autoplot() +
    ggtitle("Number of employees in child day care services in New York City")
```

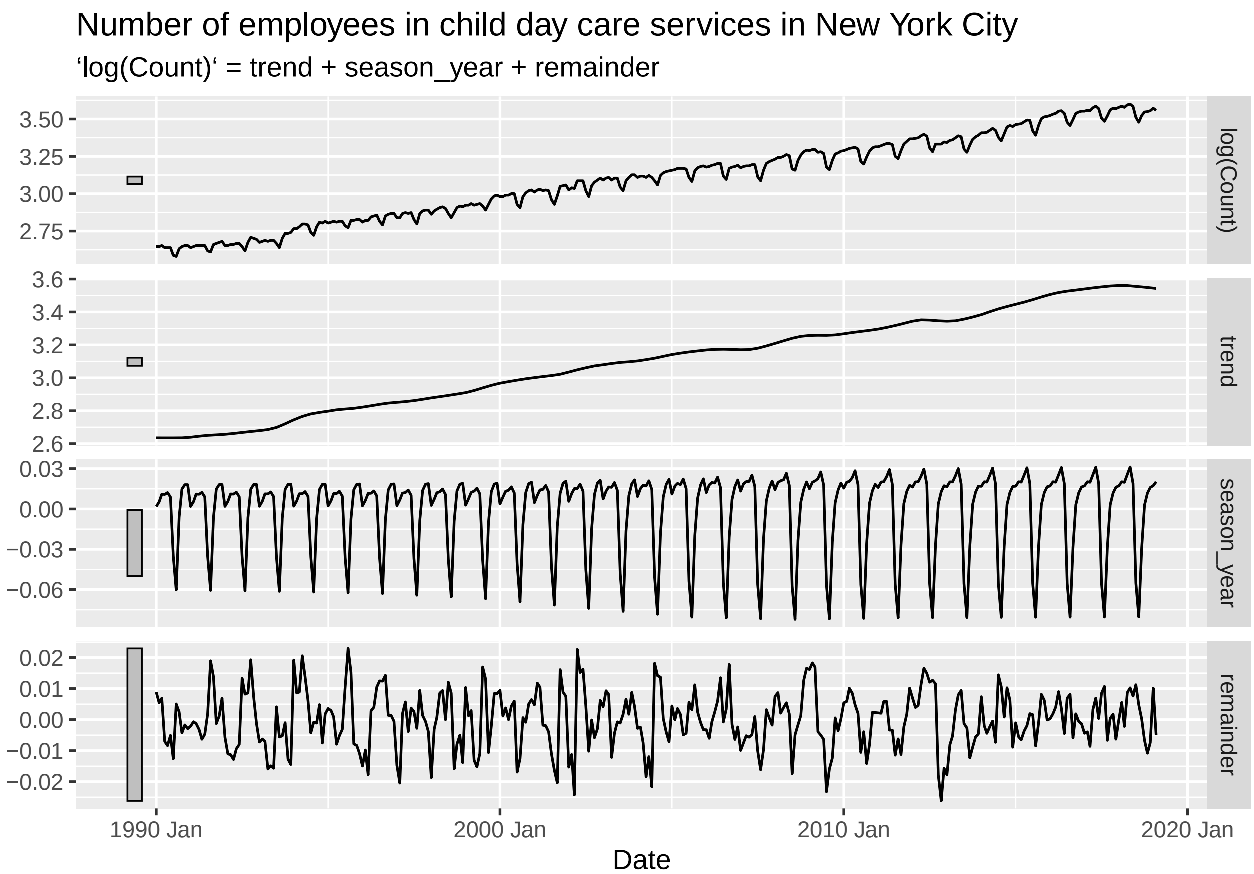


Figure 4:

(c) You are asked to provide forecasts for the next two years for the `daycare` series shown in Figure 1. Consider applying each of the methods and models below. Comment, in a few words each, on whether each one is appropriate for forecasting the data. No marks will be given for simply guessing whether a method or a model is appropriate without justifying your choice.

A. Seasonal naïve method.
B. Drift method plus seasonal dummies.
C. Holt-Winters additive damped trend method.
D. Holt-Winters multiplicative damped trend method.
E. ETS(A,N,M).
F. ETS(M,A_d,M).
G. ARIMA(1,1,4).
H. ARIMA(3,1,2)$(1,1,0)_{12}$.
I. ARIMA(0,1,1)$(2,0,0)_{12}$.
J. Regression model with time and Fourier terms.

10 marks

Total: 20 marks

— END OF QUESTION 2 —

QUESTION 3

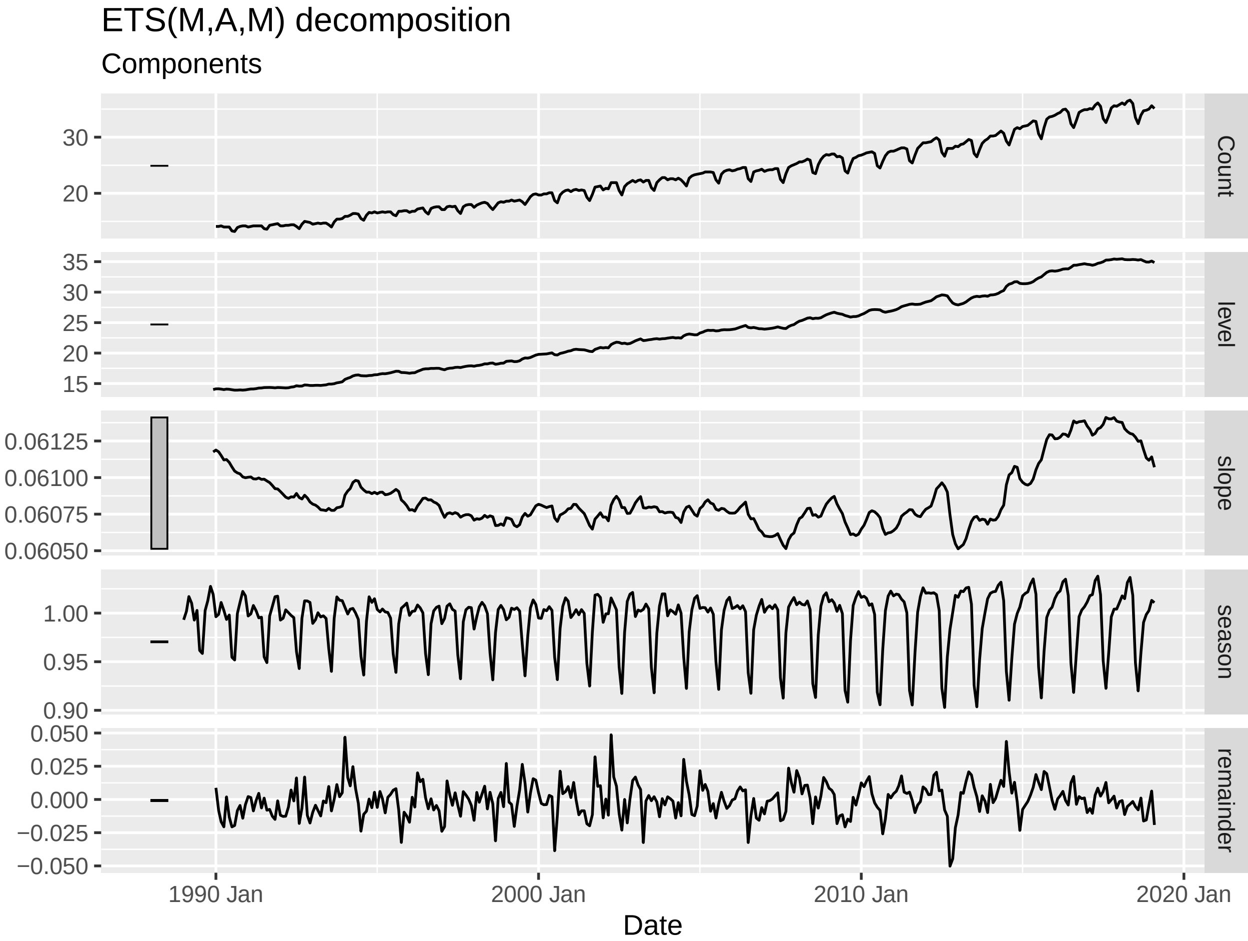
The following R code and output concerns two models for the `daycare` series plotted in Figure 1. The estimated components of the models are plotted in Figure 5.

```
fit_ets <- daycare %>%
  model(
    trend = ETS(Count ~ trend(method = "A")),
    damped = ETS(Count ~ trend(method = "Ad"))
  )
fit_ets %>% select(trend) %>% report()
```

```
## Series: Count
## Model: ETS(M,A,M)
##   Smoothing parameters:
##     alpha = 0.46736
##     beta  = 0.00010498
##     gamma = 0.35681
##
##   Initial states:
##       l        b     s1     s2     s3     s4      s5      s6     s7     s8
##  14.013 0.061175 1.0192 1.0275 1.0127 1.0027 0.95847 0.96163 1.0028 0.9928
##      s9     s10     s11      s12
##  1.0102 1.0171 1.0017 0.99314
##
##   sigma^2:  2e-04
##
##    AIC   AICc    BIC
## 1198.8 1200.6 1264.4
```

```
fit_ets %>% select(damped) %>% report()
```

```
## Series: Count
## Model: ETS(M,Ad,M)
##   Smoothing parameters:
##     alpha = 0.89049
##     beta  = 0.015802
##     gamma = 0.069298
##     phi   = 0.98
##
##   Initial states:
##       l        b    s1     s2     s3      s4      s5      s6     s7     s8
##  13.545 0.045311 1.022 1.0193 1.0126 0.98462 0.92759 0.94846 1.0018 1.0164
##      s9    s10   s11    s12
##  1.0204 1.0211 1.014 1.0118
##
##   sigma^2:  1e-04
##
##    AIC   AICc    BIC
## 1147.3 1149.4 1216.8
```


ETS(M,A,M) decomposition
Components
Count
level
slope
season
remainder
30
20
35
30
25
20
15
0.06125
0.06100
0.06075
0.06050
1.00
0.95
0.90
0.050
0.025
0.000
−0.025
−0.050
1990 Jan
2000 Jan
2010 Jan
2020 Jan
Date

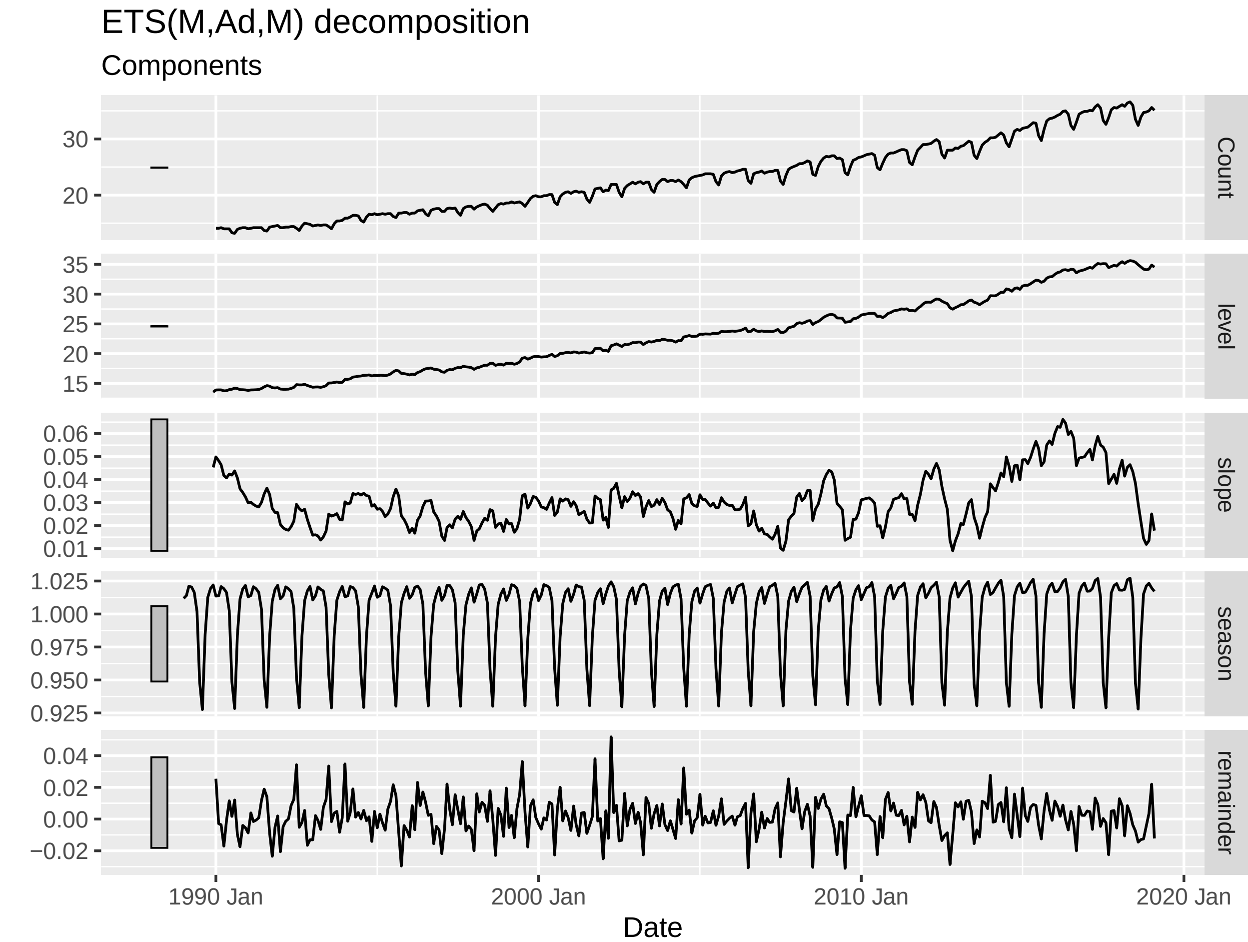

ETS(M,Ad,M) decomposition
Components
Count
level
slope
season
remainder
30
20
35
30
25
20
15
0.06
0.05
0.04
0.03
0.02
0.01
1.025
1.000
0.975
0.950
0.925
0.04
0.02
0.00
−0.02
1990 Jan
2000 Jan
2010 Jan
2020 Jan
Date

Figure 5:

(a) Comment on the differences between the two model specifications.

2 marks

(b) Comment on Figure 5 and how this relates to the estimated parameters of the models.

6 marks

(c) Figures 6 and 7 and the R-output below these relate to the residuals from the two estimated models. Comment on these in relation to the fit of the models. Give as many details as you can. What do your conclusions here imply about using these models for forecasting?

4 marks

```
fit_ets %>%
  select(trend) %>%
  gg_tsresiduals()
```

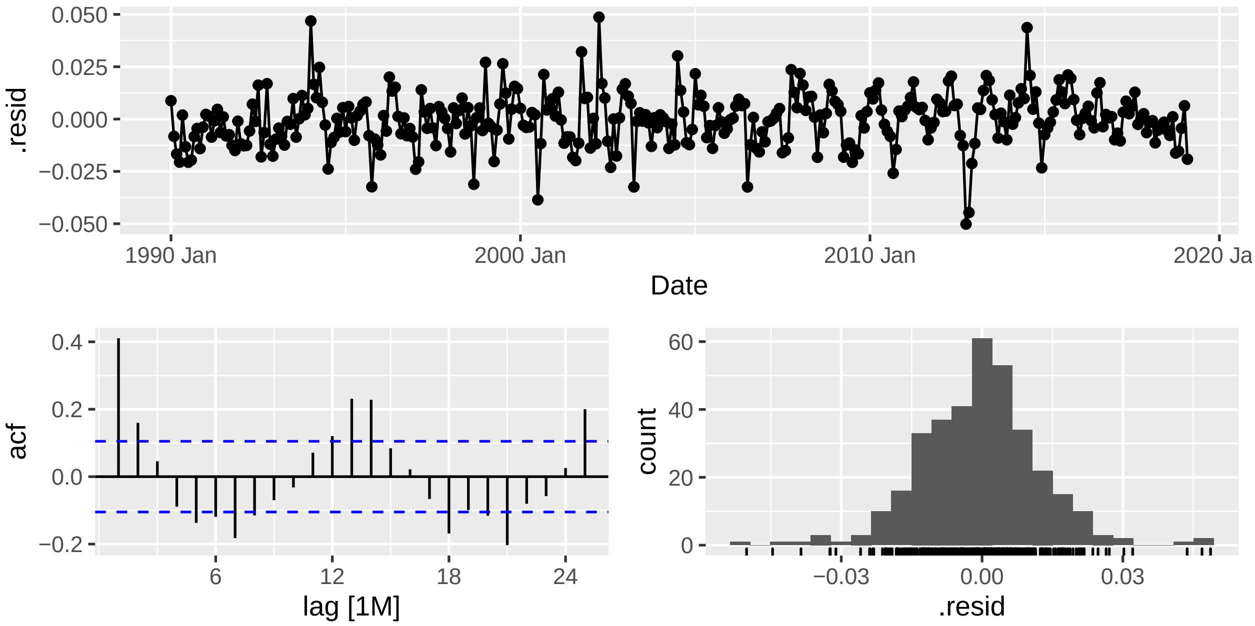


Figure 6:

```
fit_ets %>%
  select(trend) %>%
  augment() %>%
  features(.resid, ljung_box, lag = 24, dof = 16)
```

```
## # A tibble: 1 x 3
##   .model lb_stat lb_pvalue
##   <chr>    <dbl>     <dbl>
## 1 trend     191.         0
```

```r
fit_ets %>%
  select(damped) %>%
  gg_tsresiduals()
```

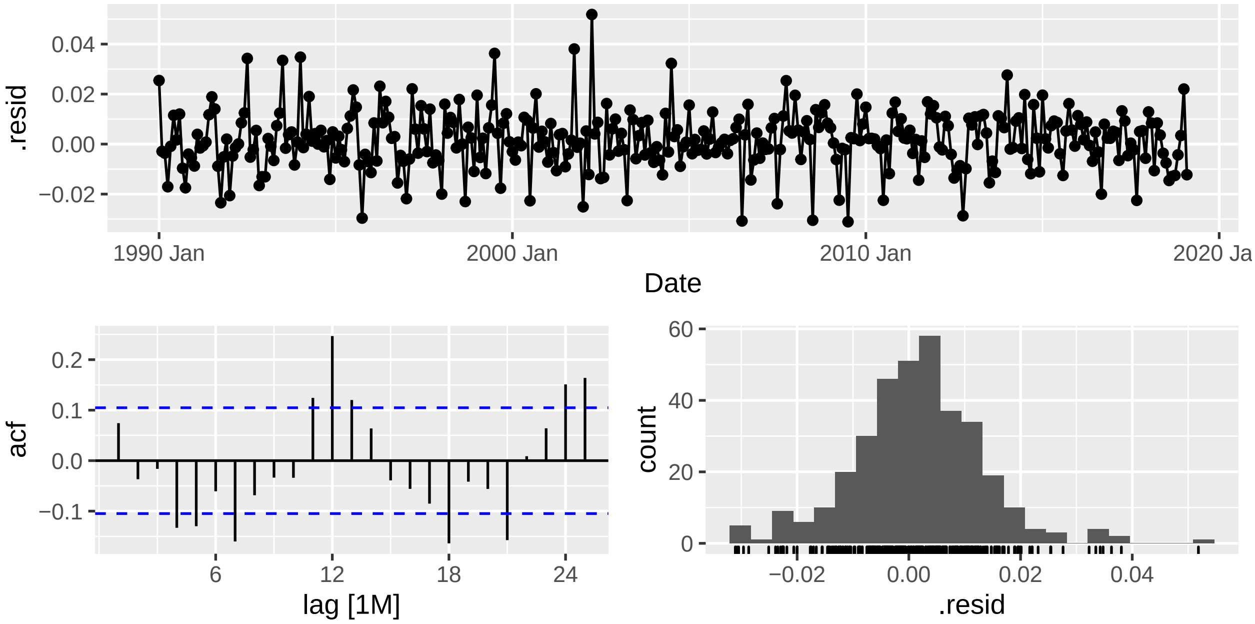


Figure 7:

```r
fit_ets %>%
  select(damped) %>%
  augment() %>%
  features(.resid, ljung_box, lag = 24, dof = 17)
```

```
## # A tibble: 1 x 3
##   .model lb_stat lb_pvalue
##   <chr>    <dbl>     <dbl>
## 1 damped    98.1         0
```

(d) Considering all the analysis so far, which model would you choose for forecasting and why?

2 marks

(e) Figure 8 shows forecasts from the two models (in order to improve visualization only data from 2000 onwards is included in the plots). Comment on the two sets of forecasts. Based on these would you change your decision as to which model you would choose for forecasting.

4 marks

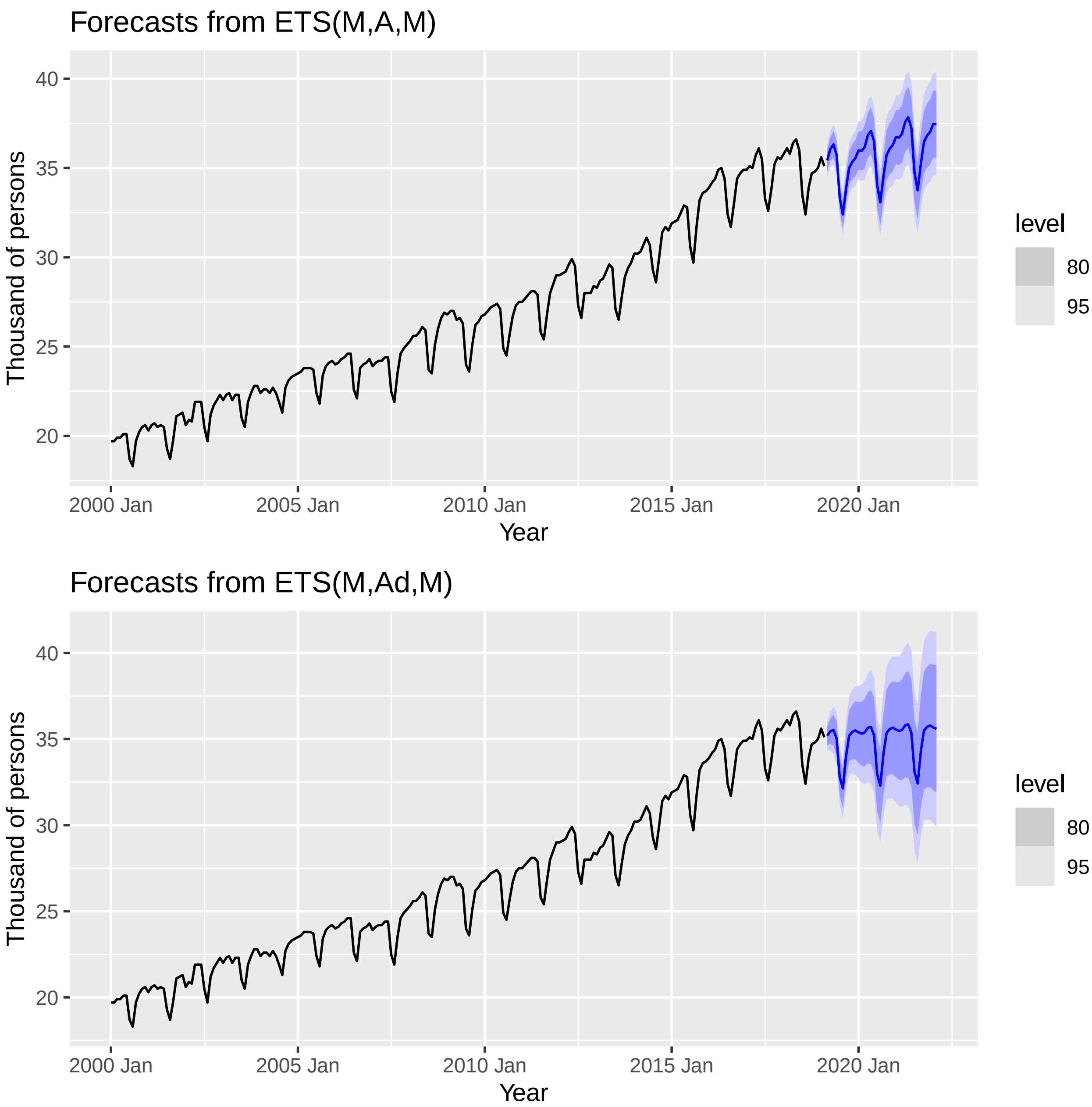


Figure 8:

(f) Write down in full your selected estimated model.

2 marks

Total: 20 marks

— END OF QUESTION 3 —

QUESTION 4

(a) Figures 9 and 10 show time plots, ACFs and PACFs related to the daycare series. The variables plotted were constructed as follows.

```
daycare %>%
  mutate(
    log_count = log(Count),
    diff_log_count = difference(log(Count)),
    sdiff_log_count = difference(log(Count),12),
    diff_sdiff_log_count = difference(difference(log(Count),12))
  )
```

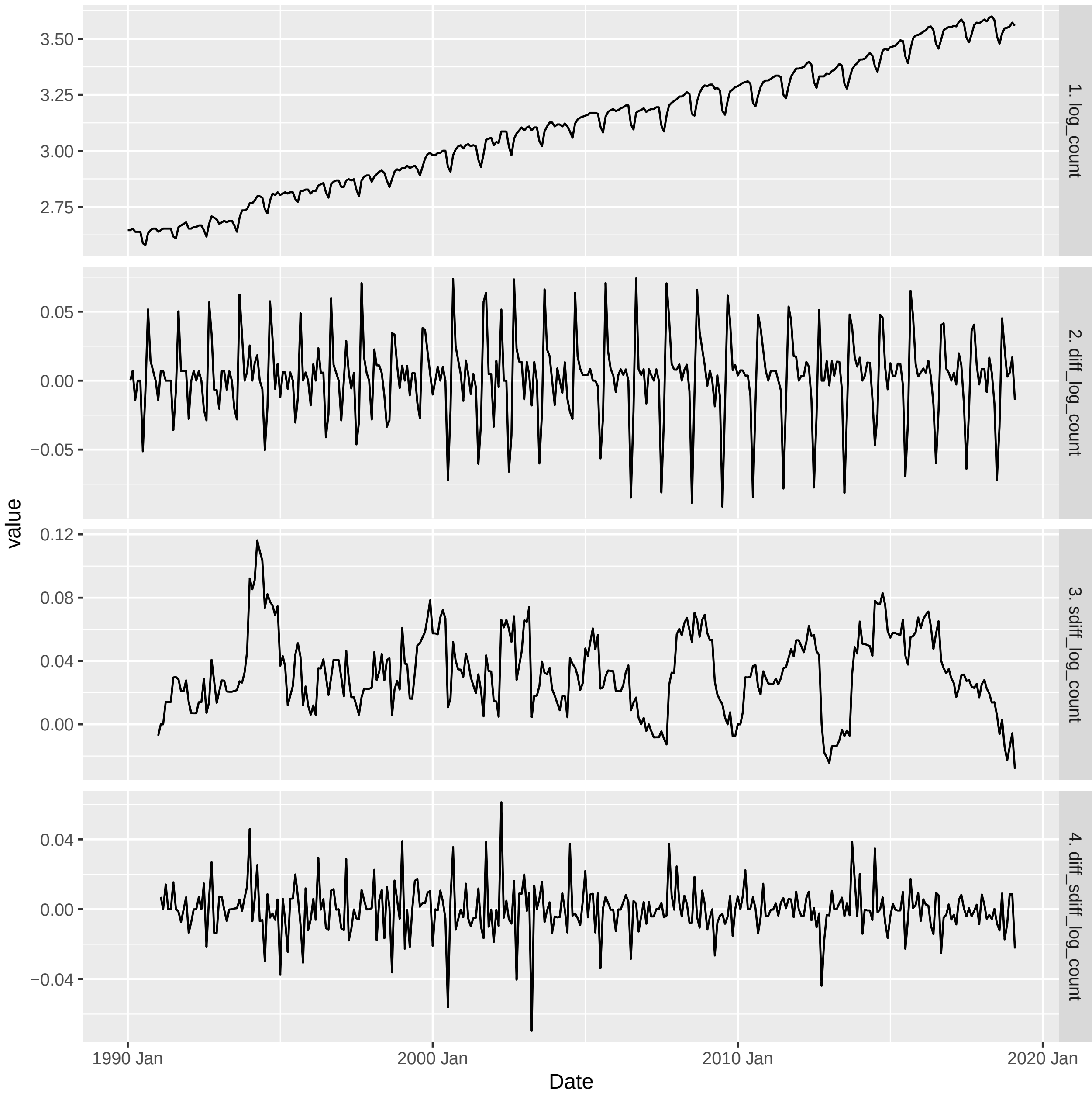


Figure 9:

Explain what each of the ACFs and PACFs show about the stationarity, seasonality and other features of the time series.

4 marks

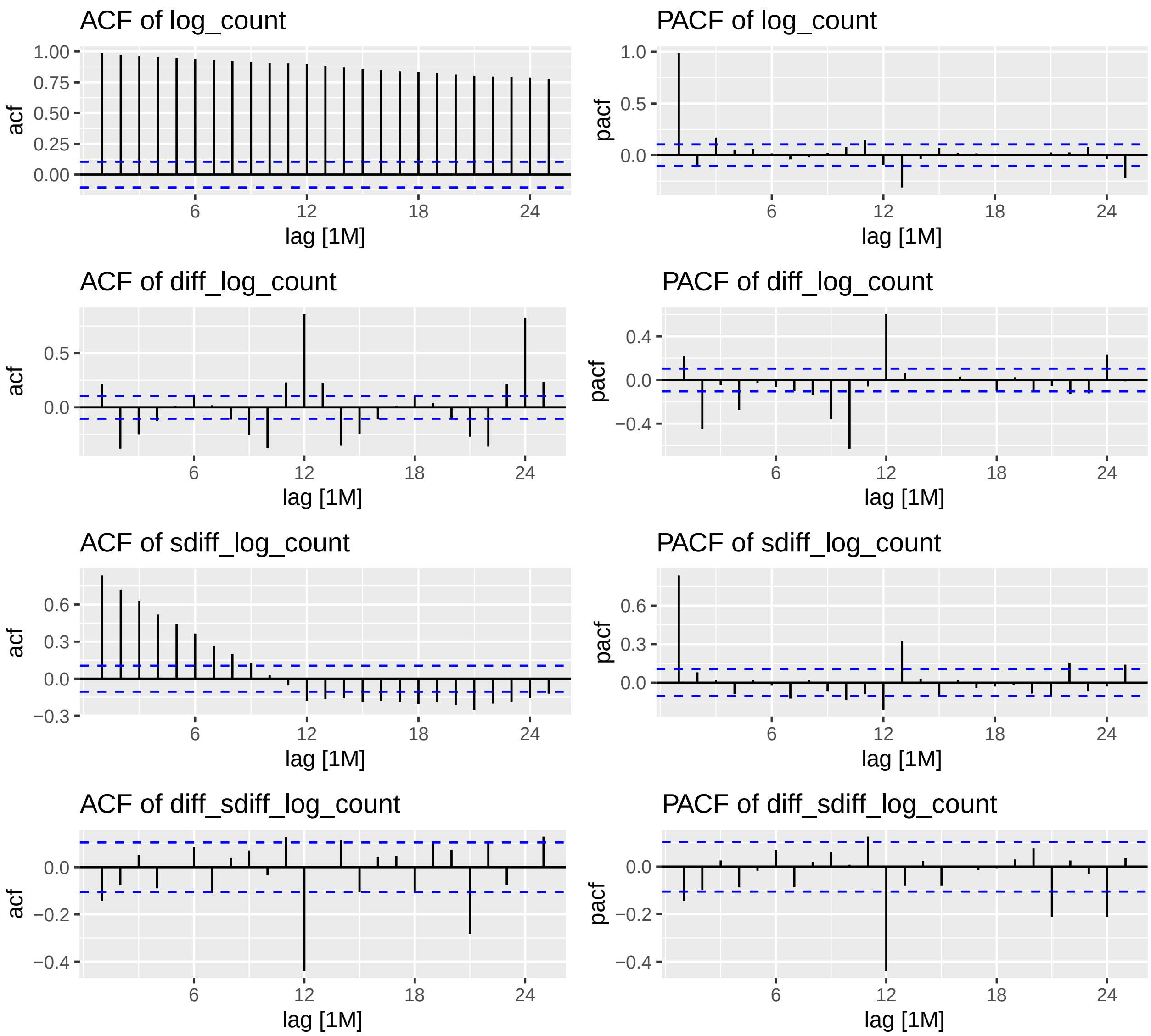


Figure 10:

(b) The following R code and output concerns two models estimated for the `daycare` series. Relate the estimated models to the relevant ACFs and PACFs from Figure 10 and describe what dynamics they capture.

6 marks

```
fit.arima <- daycare %>%
  model(
    arima1 = ARIMA(log(Count)),
    arima2 = ARIMA(log(Count) ~ pdq(d=1))
  )
fit.arima %>% select(arima1) %>% report()
```

```
## Series: Count
## Model: ARIMA(1,0,1)(0,1,1)[12] w/ drift
## Transformation: log(.x)
##
## Coefficients:
##          ar1      ma1     sma1   constant
##       0.9191  -0.0920  -0.7295     0.0026
```

```
## s.e.   0.0257   0.0621   0.0403    0.0002
##
## sigma^2 estimated as 0.0001186:  log likelihood=1045.4
## AIC=-2080.8   AICc=-2080.6   BIC=-2061.7
```

```
fit.arima %>% select(arima2) %>% report()
```

```
## Series: Count
## Model: ARIMA(0,1,1)(0,1,1)[12]
## Transformation: log(.x)
##
## Coefficients:
##           ma1     sma1
##       -0.1420  -0.7254
## s.e.   0.0576   0.0399
##
## sigma^2 estimated as 0.0001224:  log likelihood=1036.6
## AIC=-2067.2   AICc=-2067.1   BIC=-2055.7
```

(c) Write down the estimated model of `arima2` using backshift notation and expand this to the point where it can be used to generate point forecasts.

4 marks

(d) Some diagnostics for the residuals of the model associated with `arima2` are presented below. Briefly comment as to whether these are satisfactory (comment on the significant spikes in the ACF).

2 marks

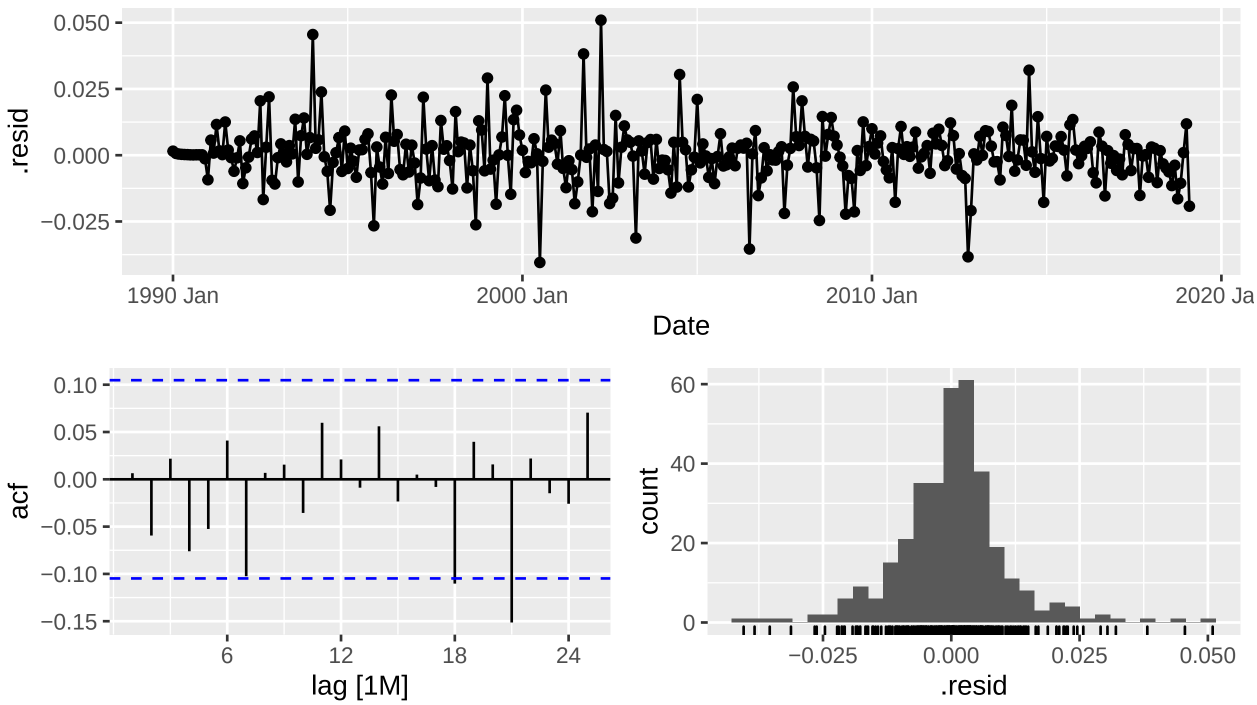


Figure 11:

```
## # A tibble: 1 x 3
##   .model lb_stat lb_pvalue
##   <chr>    <dbl>     <dbl>
## 1 arima2    26.6     0.228
```

(e) Figure 12 plots forecasts from the two ARIMA models estimated in part (b). Comment on the difference in the point and interval forecasts and in particular the role the differencing plays. *4 marks*

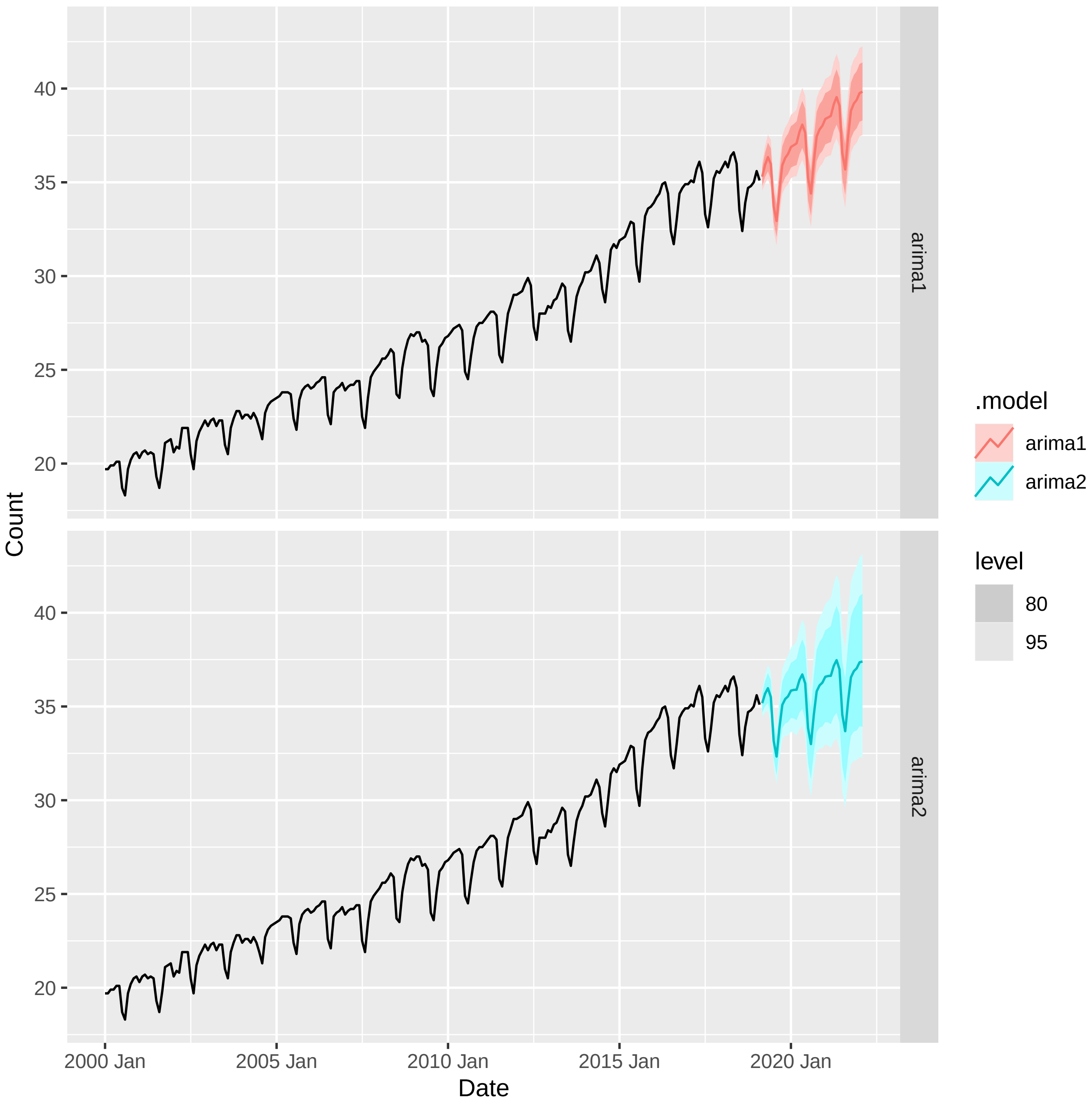


Figure 12:

Total: 20 marks

— END OF QUESTION 4 —

QUESTION 5

In the following code, a series of dynamic harmonic regression models are fitted to the day care data shown in Figure 1.

```
dhr <- daycare %>%
  model(
    fit1 = ARIMA(log(Count) ~ trend() + fourier(12,1)),
    fit2 = ARIMA(log(Count) ~ trend() + fourier(12,2)),
    fit3 = ARIMA(log(Count) ~ trend() + fourier(12,3)),
    fit4 = ARIMA(log(Count) ~ trend() + fourier(12,4)),
    fit5 = ARIMA(log(Count) ~ trend() + fourier(12,5)),
    fit6 = ARIMA(log(Count) ~ trend() + fourier(12,6)),
    #fit7 = ARIMA(log(Count) ~ trend() + fourier(12,7))
  )
glance(dhr)
```

```
## # A tibble: 6 x 8
##   .model   sigma2 log_lik    AIC   AICc    BIC ar_roots   ma_roots
##   <chr>     <dbl>   <dbl>  <dbl>  <dbl>  <dbl> <list>     <list>
## 1 fit1   0.000163   1024. -2034. -2034. -2007. <cpl [13]> <cpl [0]>
## 2 fit2   0.000136   1060. -2096. -2095. -2049. <cpl [25]> <cpl [2]>
## 3 fit3   0.000130   1071. -2116. -2115. -2066. <cpl [25]> <cpl [1]>
## 4 fit4   0.000120   1089. -2148. -2146. -2090. <cpl [25]> <cpl [1]>
## 5 fit5   0.000115   1097. -2160. -2158. -2094. <cpl [13]> <cpl [13]>
## 6 fit6   0.000115   1097. -2158. -2156. -2088. <cpl [13]> <cpl [13]>
```

(a) The seventh model (commented out) would cause an error if the code was run. Why?

2 marks

(b) Which model would you select from the six models fitted? Why?

1 marks

(c) One of the models has the following output:

```
## Series: Count
## Model: LM w/ ARIMA(1,0,1)(1,0,1)[12] errors
## Transformation: log(.x)
##
## Coefficients:
##          ar1      ma1    sar1     sma1  trend()  fourier(12, 5)C1_12
##       0.9016  -0.0615  0.8082  -0.5787   0.0027               0.0238
## s.e.  0.0271   0.0622  0.0784   0.1056   0.0001               0.0030
##       fourier(12, 5)S1_12  fourier(12, 5)C2_12  fourier(12, 5)S2_12
##                    0.0139              -0.0155              -0.0184
## s.e.               0.0030               0.0016               0.0016
##       fourier(12, 5)C3_12  fourier(12, 5)S3_12  fourier(12, 5)C4_12
##                    0.0049               0.0117              -0.0021
## s.e.               0.0012               0.0012               0.0010
##       fourier(12, 5)S4_12  fourier(12, 5)C5_12  fourier(12, 5)S5_12  intercept
##                   -0.0072              -0.0011               0.0025     2.6195
## s.e.               0.0010               0.0009               0.0009     0.0184
##
## sigma^2 estimated as 0.000115:  log likelihood=1097
## AIC=-2160   AICc=-2158   BIC=-2094
```

Write down the form of the model using equations. Explain how each of the coefficients contributes to the forecast function.

8 marks

(d) Using the model above, the point forecast for the next observation is 35.34. Give a 95% prediction interval for this month.

3 marks

(e) Why is the selected ARIMA model stationary when the data are clearly nonstationary?

2 marks

(f) For the model shown above, the residuals can be tested to assess the model assumptions.

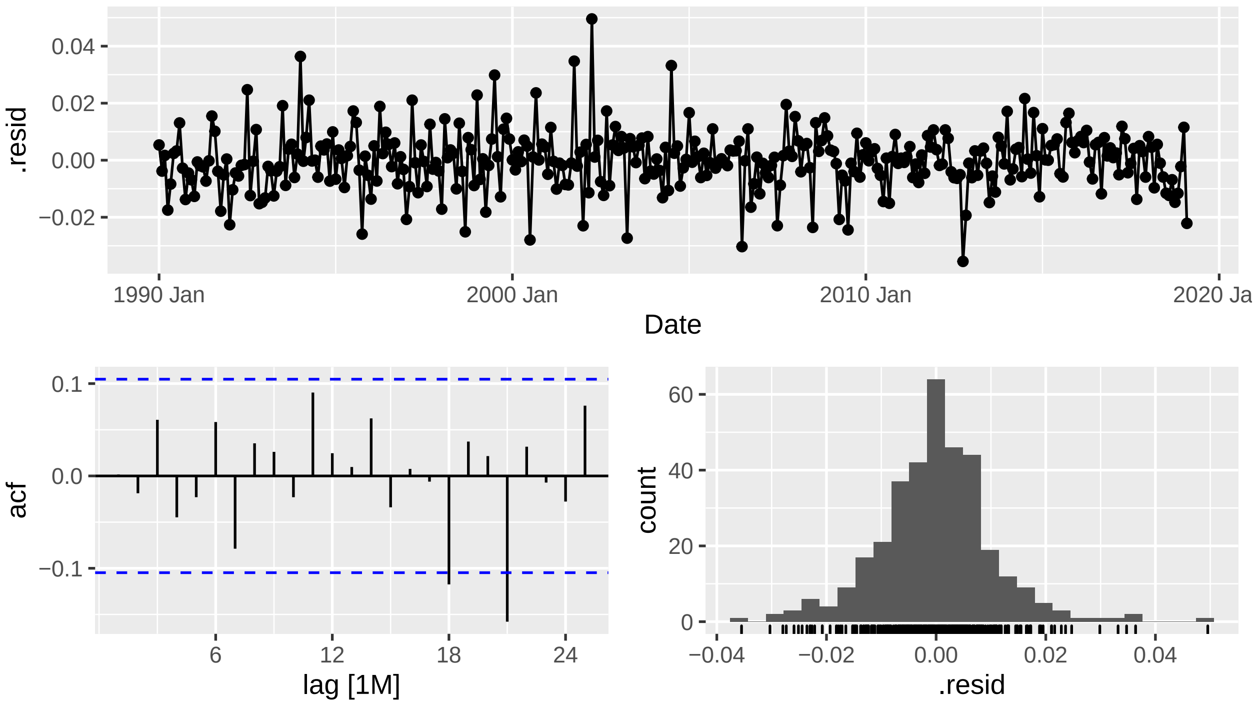


Figure 13:

```
## # A tibble: 1 x 2
##   lb_stat lb_pvalue
##     <dbl>     <dbl>
## 1    27.6   0.00111
```

Comment on what this tells you about the model assumptions. Should you trust the point forecasts and forecast intervals that are produced from the model?

4 marks

Total: 20 marks

— END OF QUESTION 5 —

Table 1: *State space equations for each of the models in the ETS framework.*

ADDITIVE ERROR MODELS

Trend	**Seasonal** **N**	**A**	**M**
N	$y_t = \ell_{t-1} + \varepsilon_t$ $\ell_t = \ell_{t-1} + \alpha\varepsilon_t$	$y_t = \ell_{t-1} + s_{t-m} + \varepsilon_t$ $\ell_t = \ell_{t-1} + \alpha\varepsilon_t$ $s_t = s_{t-m} + \gamma\varepsilon_t$	$y_t = \ell_{t-1}s_{t-m} + \varepsilon_t$ $\ell_t = \ell_{t-1} + \alpha\varepsilon_t/s_{t-m}$ $s_t = s_{t-m} + \gamma\varepsilon_t/\ell_{t-1}$
A	$y_t = \ell_{t-1} + b_{t-1} + \varepsilon_t$ $\ell_t = \ell_{t-1} + b_{t-1} + \alpha\varepsilon_t$ $b_t = b_{t-1} + \beta\varepsilon_t$	$y_t = \ell_{t-1} + b_{t-1} + s_{t-m} + \varepsilon_t$ $\ell_t = \ell_{t-1} + b_{t-1} + \alpha\varepsilon_t$ $b_t = b_{t-1} + \beta\varepsilon_t$ $s_t = s_{t-m} + \gamma\varepsilon_t$	$y_t = (\ell_{t-1} + b_{t-1})s_{t-m} + \varepsilon_t$ $\ell_t = \ell_{t-1} + b_{t-1} + \alpha\varepsilon_t/s_{t-m}$ $b_t = b_{t-1} + \beta\varepsilon_t/s_{t-m}$ $s_t = s_{t-m} + \gamma\varepsilon_t/(\ell_{t-1} + b_{t-1})$
$\mathbf{A_d}$	$y_t = \ell_{t-1} + \phi b_{t-1} + \varepsilon_t$ $\ell_t = \ell_{t-1} + \phi b_{t-1} + \alpha\varepsilon_t$ $b_t = \phi b_{t-1} + \beta\varepsilon_t$	$y_t = \ell_{t-1} + \phi b_{t-1} + s_{t-m} + \varepsilon_t$ $\ell_t = \ell_{t-1} + \phi b_{t-1} + \alpha\varepsilon_t$ $b_t = \phi b_{t-1} + \beta\varepsilon_t$ $s_t = s_{t-m} + \gamma\varepsilon_t$	$y_t = (\ell_{t-1} + \phi b_{t-1})s_{t-m} + \varepsilon_t$ $\ell_t = \ell_{t-1} + \phi b_{t-1} + \alpha\varepsilon_t/s_{t-m}$ $b_t = \phi b_{t-1} + \beta\varepsilon_t/s_{t-m}$ $s_t = s_{t-m} + \gamma\varepsilon_t/(\ell_{t-1} + \phi b_{t-1})$

MULTIPLICATIVE ERROR MODELS

Trend	**Seasonal** **N**	**A**	**M**
N	$y_t = \ell_{t-1}(1 + \varepsilon_t)$ $\ell_t = \ell_{t-1}(1 + \alpha\varepsilon_t)$	$y_t = (\ell_{t-1} + s_{t-m})(1 + \varepsilon_t)$ $\ell_t = \ell_{t-1} + \alpha(\ell_{t-1} + s_{t-m})\varepsilon_t$ $s_t = s_{t-m} + \gamma(\ell_{t-1} + s_{t-m})\varepsilon_t$	$y_t = \ell_{t-1}s_{t-m}(1 + \varepsilon_t)$ $\ell_t = \ell_{t-1}(1 + \alpha\varepsilon_t)$ $s_t = s_{t-m}(1 + \gamma\varepsilon_t)$
A	$y_t = (\ell_{t-1} + b_{t-1})(1 + \varepsilon_t)$ $\ell_t = (\ell_{t-1} + b_{t-1})(1 + \alpha\varepsilon_t)$ $b_t = b_{t-1} + \beta(\ell_{t-1} + b_{t-1})\varepsilon_t$	$y_t = (\ell_{t-1} + b_{t-1} + s_{t-m})(1 + \varepsilon_t)$ $\ell_t = \ell_{t-1} + b_{t-1} + \alpha(\ell_{t-1} + b_{t-1} + s_{t-m})\varepsilon_t$ $b_t = b_{t-1} + \beta(\ell_{t-1} + b_{t-1} + s_{t-m})\varepsilon_t$ $s_t = s_{t-m} + \gamma(\ell_{t-1} + b_{t-1} + s_{t-m})\varepsilon_t$	$y_t = (\ell_{t-1} + b_{t-1})s_{t-m}(1 + \varepsilon_t)$ $\ell_t = (\ell_{t-1} + b_{t-1})(1 + \alpha\varepsilon_t)$ $b_t = b_{t-1} + \beta(\ell_{t-1} + b_{t-1})\varepsilon_t$ $s_t = s_{t-m}(1 + \gamma\varepsilon_t)$
$\mathbf{A_d}$	$y_t = (\ell_{t-1} + \phi b_{t-1})(1 + \varepsilon_t)$ $\ell_t = (\ell_{t-1} + \phi b_{t-1})(1 + \alpha\varepsilon_t)$ $b_t = \phi b_{t-1} + \beta(\ell_{t-1} + \phi b_{t-1})\varepsilon_t$	$y_t = (\ell_{t-1} + \phi b_{t-1} + s_{t-m})(1 + \varepsilon_t)$ $\ell_t = \ell_{t-1} + \phi b_{t-1} + \alpha(\ell_{t-1} + \phi b_{t-1} + s_{t-m})\varepsilon_t$ $b_t = \phi b_{t-1} + \beta(\ell_{t-1} + \phi b_{t-1} + s_{t-m})\varepsilon_t$ $s_t = s_{t-m} + \gamma(\ell_{t-1} + \phi b_{t-1} + s_{t-m})\varepsilon_t$	$y_t = (\ell_{t-1} + \phi b_{t-1})s_{t-m}(1 + \varepsilon_t)$ $\ell_t = (\ell_{t-1} + \phi b_{t-1})(1 + \alpha\varepsilon_t)$ $b_t = \phi b_{t-1} + \beta(\ell_{t-1} + \phi b_{t-1})\varepsilon_t$ $s_t = s_{t-m}(1 + \gamma\varepsilon_t)$