

Introductory Econometrics

Tutorial 3

PART A: This homework is a review of statistical concepts of mean and variance of a random variable and joint, marginal and conditional distributions of two random variables. After reviewing Week 2 lecture slides and studying Appendix B1 to B4 of the textbook, attempt Week 3 Part A quiz on Moodle. You need to submit the quiz before your tutorial and attend the tutorial to obtain 1 point for Week 3 participation.

PART B: The following questions will be covered in the tutorial. It is a good idea to attempt these questions on your own before the tutorial.

1. X and Y are random variables with mean μ_X and μ_Y respectively. The covariance between X and Y is defined as $Cov(X, Y) = E[(X - \mu_X)(Y - \mu_Y)]$. Show that:

$$Cov(X, Y) = E[(X - \mu_X)Y] = E[X(Y - \mu_Y)] = E(XY) - \mu_X\mu_Y.$$

Discuss why we cannot simplify $E(XY) - \mu_X\mu_Y$ further to $E(X)E(Y) - \mu_X\mu_Y = \mu_X\mu_Y - \mu_X\mu_Y = 0$.

2. Diversification in everyday life: [A pokie machine is called a slot machine in America and a jackpot machine in some other countries. It is a random device that operates for a price, and draws an outcome randomly, and depending on the outcome, you may win different amounts of money (including zero). The probabilities are controlled such that the expected value of the monetary prize is slightly smaller than the price]. Most pokie machines give you the option of multiplying your bet up. For example, if the machine accepts 20 cents per round for having a go at winnings given by the random variable X , you have the option of paying one dollar to scale up your winnings to $5X$. Suppose you have one dollar only. Compare the expected return and risk of using all of your money at once and betting $5X$, with using it for playing X five times (i.e. $X_1 + X_2 + X_3 + X_4 + X_5$, where $X_i, i = 1, 2, \dots, 5$ are independent and have distribution identical to X).
3. Diversification in econometrics and statistics: Suppose we are interested in estimating the mean of a random variable. We can take a sample of one observation from the random variable and use that as the estimate of the mean, or we can take a sample of 5 observations and take the average of those 5 observations as the estimate of the mean. What is the expected value of each of these estimators and which one is safer (i.e. less risky)? Discuss the similarity of this to the previous question.