Introductory Econometrics
Tutorial 9 (suggested answers)

Part B: This part will be covered in the tutorial. It is still a good idea to attempt these questions before the tutorial.

1. The general model is:

$$profits_i = \beta_0 + \delta_0 mno_i + \beta_1 assets_i + \delta_1 (mno_i \times assets_i) + u_i \tag{1}$$

(a)

$$H_0 \quad : \quad \delta_0 = \delta_1 = 0$$
$$H_1 \quad : \quad \text{at least one of } \delta_0 \text{ or } \delta_1 \text{ is not zero}$$

(b)

$$\widehat{profits} = \underset{(2.78)}{1.56} + \underset{(4.09)}{8.23} mno + \underset{(0.006)}{0.05} assets - \underset{(0.009)}{0.05} mno * assets$$
$$n = 69, \quad R^2 = 0.52$$

i. Breusch-Pagan test when the alternative hypothesis is $Var(u_i \mid mno_i, assets_i) = \alpha_0 + \alpha_1 mno_i + \alpha_2 assets_i$

$$H_0 \quad : \quad Var(u_i \mid mno_i, assets_i) = \sigma^2 \text{ for all } i$$
$$H_1 \quad : \quad Var(u_i \mid mno_i, assets_i) = \alpha_0 + \alpha_1 mno_i + \alpha_2 assets_i$$

Estimated auxiliary regression:

$$\hat{u}^2 = 12.84 + 0.81 assets - 157.47 mno$$
$$n = 69, \quad R_{\hat{u}}^2 = 0.183$$
$$BP = nR_{\hat{u}}^2 \sim \chi_2^2 \quad \text{under } H_0, \ BP_{crit} = 5.99 \text{ at the 5\% l.o.s.}$$
$$BP_{calc} = 69 \times 0.183 = 12.63 \text{ (may not be exactly the same as EViews due to rounding)}$$
$$BP_{calc} > BP_{crit} \Rightarrow \text{We reject the null and conclude that errors are HTSK}$$

Notation sometimes bugs students. For example, here under the null I have denoted the variance by σ^2, but under the alternative, if α_1 and α_2 are zero, the variance will be the same for all observations, but it is denoted by α_0. Of course notation is arbitrary, but it can confuse some. If that is the case, you may want to denote the variance under the null α_0.

ii. White test.

$$H_0 \quad : \quad Var(u_i \mid mno_i, assets_i) = \sigma^2 \text{ for all } i$$
$$H_1 \quad : \quad Var(u_i \mid mno_i, assets_i) \text{ is a smooth function of } mno_i \text{ and } assets_i$$

Estimated auxiliary regression:

$$\hat{u}^2 = -470.11 + 536.17 mno + 3.33 assets - 0.001 assets^2$$
$$\qquad - 3.29 mno * assets + 0.001 mno * assets^2$$
$$n = 69, \quad R_{\hat{u}}^2 = 0.372$$
$$W = nR_{\hat{u}}^2 \sim \chi_5^2 \quad \text{under } H_0, \ W_{crit} = 11.07 \text{ at the 5\% l.o.s.}$$
$$W_{calc} = 69 \times 0.372 = 25.67 \text{ (may not be exactly the same as EViews due to rounding)}$$
$$W_{calc} > W_{crit} \Rightarrow \text{We reject the null and conclude that errors are HTSK}$$

Note that mno^2 is the same as mno, so we do not include it twice (otherwise we get exact multicollinearity).

iii. The special form of the White test that uses the predicted value of *profits* and its square as predictors of variance.

$$H_0 \quad : \quad Var\left(u_i \mid mno_i, assets_i\right) = \sigma^2 \quad \text{for all } i$$
$$H_1 \quad : \quad Var\left(u_i \mid mno_i, assets_i\right) \text{ is a smooth function of } mno_i \text{ and } assets_i$$

Note that the null and the alternative is the same as what they were for the regular White test. Estimated auxiliary regression:

$$
\begin{aligned}
\hat{u}^2 &= -529.70 + 65.45\widehat{profits} - 0.43\widehat{profits}^2 \\
\widehat{profits} &= profits - \hat{u} \text{ is an easy way to compute } \widehat{profits} \\
n &= 69, \quad R_{\hat{u}}^2 = 0.369 \\
W &= nR_{\hat{u}}^2 \sim \chi_2^2 \quad \text{under } H_0, \; W_{crit} = 5.99 \text{ at the 5\% l.o.s.} \\
W_{calc} &= 69 \times 0.369 = 25.46 \\
W_{calc} &> W_{crit} \Rightarrow \text{We reject the null and conclude that errors are HTSK}
\end{aligned}
$$

(c) No. *profits* can be negative, so logarithmic transformation is not an option.

(d) It is good to go over why these weighting cures HTSK at least for one of them.

    i. $Var\left(u_i \mid mno_i, assets_i\right) = \sigma^2 \times assets_i \rightarrow w_i = \frac{1}{\sqrt{assets_i}}$

    ii. $Var\left(u_i \mid mno_i, assets_i\right) = \sigma^2 \times assets_i^2 \rightarrow w_i = \frac{1}{assets_i}$

    iii. $Var\left(u_i \mid mno_i, assets_i\right) = \sigma^2 \log(assets_i) \rightarrow w_i = \frac{1}{\sqrt{\log(assets_i)}}$