Introductory Econometrics

Nonstationary time series

Monash Econometrics and Business Statistics

2022

Recap

The multiple regression model with time series

$$y_t = \beta_0 + \beta_1 x_{t1} + \beta_2 x_{t2} + ... + \beta_k x_{tk} + u_t, \ \ t = 1, 2, ...n.$$

- A1 model is linear in parameters: $y = X\beta + u$.
- A2 columns of X are linearly independent.
- A3 conditional mean of errors is zero: $E(u_t|x_t) = 0$ (NEW).
- A4 homoskedasticity and no serial correlation: $Var(u_t|x_t) = \sigma^2$ for all t and $E(u_t u_s|x_t, x_s) = 0$ for all $t \neq s$ (NEW).

Recap

A time series $\{y_t\}$ is called stationary if it has the properties:

P1

$$E(Y_t) = \mu \ < \infty \ \text{ for all t.}$$

(The mean is finite and time invariant)

P2

$$Var(Y_t) = E[(Y_t - \mu)^2] = \gamma_0 \ < \infty \text{ for all t.}$$

(The variance is finite and time invariant)

P3

$$Cov(Y_t, Y_{t-j}) = E[(Y_t - \mu)(Y_{t-j} - \mu)] = \gamma_j \ < \infty \text{ for all t and j.}$$

(The covariance is finite and depends only on the time interval)

- Also called covariance stationary or weakly stationary.

Nonstationary time series

Many economic and financial time series are not stationary.

- Any time series which possesses a trend of any sort is nonstationary.
- Deterministic trend: $y_t = \beta_0 + \beta_1 t + u_t$.
- Stochastic trend or unit root: $y_t = y_{t-1} + u_t$.

Outline of this lecture

- Deterministic trend
- Detrending
- Unit root
- Differencing

Deterministic trend

Deterministic trend model:

$$y_t = \beta_0 + \beta_1 t + u_t$$

- The model implies that $E(y_t) = \beta_0 + \beta_1 t$.
- Therefore, the mean of y_t is time-dependent,
- and y_t is a nonstationary time series.

If the variables do not contain unit roots,
the OLS estimator and hypothesis testing is valid.

Deterministic trend

How do we know whether y_t has a deterministic trend and a unit root?

- If y_t has a deterministic trend and no stochastic trend, the detrended series will be stationary.
- If y_t has a deterministic trend and a unit root, the detrended series will not be stationary.

Detrending

To detrend y_t, estimate the regression equation

$$y_t = \beta_0 + \beta_1 t + u_t$$

The detrended series is

$$\hat{u}_t = y_t - \hat{\beta}_0 - \hat{\beta}_1 t$$

- In case the detrended time series appears to be stationary, we conclude that y_t is a trend stationary time series.
- In case $\hat{u}_t$ is a nonstationary time series, y_t has a unit root.

Unit root

Random walk model:

$$y_t = y_{t-1} + u_t, \quad u_t \sim i.i.d(0, \sigma^2) \tag{1}$$

Has a unit root, since the coefficients of the lagged dependent variables sum to one.

- This model implies that

$$\begin{aligned} y_t &= y_{t-1} + u_t \\ &= y_{t-2} + u_{t-1} + u_t \\ &= y_0 + u_1 + u_2 + \cdots + u_{t-1} + u_t. \end{aligned}$$

- $E(y_t) = y_0$ and $Var(y_t) = \sigma^2 t$.
- The variance depends on t and increases to infinity,
- and y_t is a nonstationary time series.

Unit root

Random walk model with drift:

$$y_t = \phi_0 + y_{t-1} + u_t, \quad u_t \sim i.i.d(0, \sigma^2) \tag{2}$$

Has a deterministic trend and unit root:

- We have $E(y_t) = \phi_0 t + y_0$ and $Var(y_t) = \sigma^2 t$.
- The mean and variance depends on t.
- We can write $y_t = \phi_0 t + y_0 + u_1 + u_2 + \cdots + u_{t-1} + u_t$.

Differencing

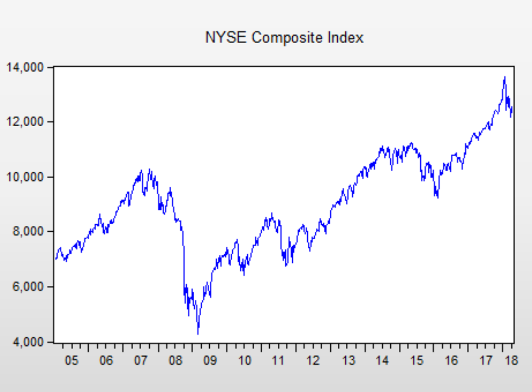
Random walk model with drift:

$$y_t = \phi_0 + y_{t-1} + u_t, \quad u_t \sim i.i.d(0, \sigma^2) \tag{3}$$

- Differencing: $\Delta y_t = y_t - y_{t-1} = \phi_0 + u_t$.
- $E(\Delta y_t) = \phi_0$, $Var(\Delta y_t) = \sigma^2$, and $Cov(\Delta y_t, \Delta y_{t-1}) = 0$.
- So Δy_t is a stationary time series.

Example 1: NYSE Composite Index

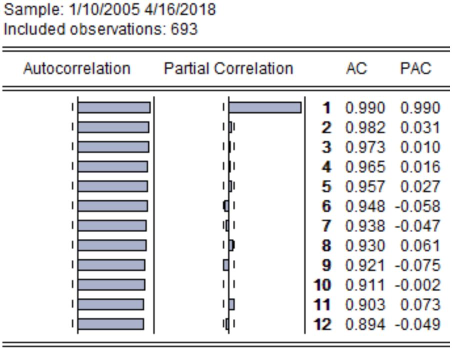
- NYSE composite index weekly data (an index made of prices at the end of the week adjusted for dividends) from 2005 to mid-2018



- Obviously this series
  - is not mean reverting, and
  - its non-stationarity is not due to a time trend

Example 1 continued

- A look at the correlogram of this series confirms that it has a unit root:



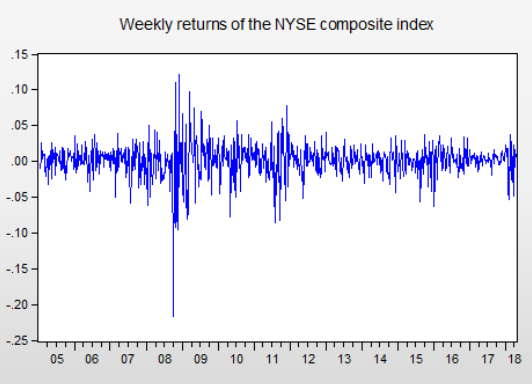
Sample: 1/10/2005 4/16/2018
Included observations: 693

Autocorrelation	Partial Correlation		AC	PAC
		1	0.990	0.990
		2	0.982	0.031
		3	0.973	0.010
		4	0.965	0.016
		5	0.957	0.027
		6	0.948	-0.058
		7	0.938	-0.047
		8	0.930	0.061
		9	0.921	-0.075
		10	0.911	-0.002
		11	0.903	0.073
		12	0.894	-0.049

- Note that
  - the first order autocorrelation is very close to 1
  - the autocorrelations decay very slowly

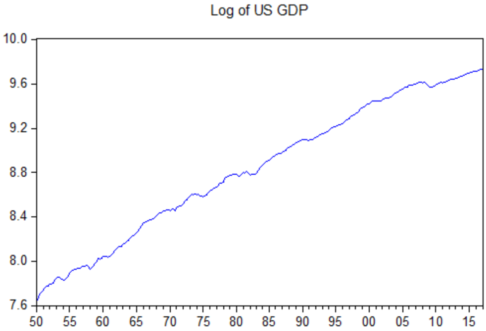
Example 1 conclusion

- Should we be flustered that we cannot use our econometric tools to model this series?
- Not really, because we are not interested in the value of the index
- We are interested in returns $\Delta \log(NYSE)$, and first differencing gets rid of the unit root!



Example 2: US real GDP

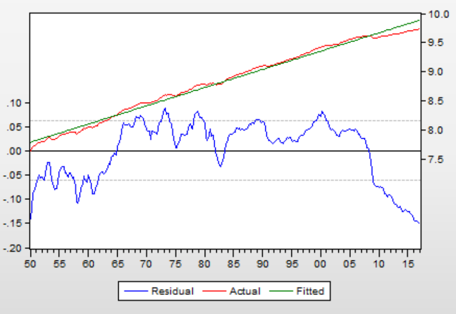
- Consider US real Gross Domestic Product (*GDP*) over the period Q1:1950 to Q1:2017 (269 observations) that we examined in last week's lecture.
- A line graph of the (log of) US GDP is given below:



- Persistence in this time series is evident, but it can just be a linear time trend

Example 2 continued

- However, after removing the linear trend, the residuals still show high persistence:



Example 2 continued

- And the correlogram of these residuals suggests a unit root:

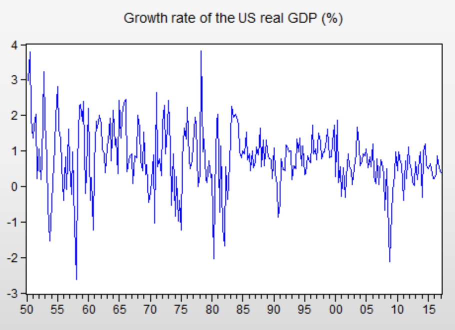
Sample: 1950Q1 2017Q1
Included observations: 269

Autocorrelation	Partial Correlation		AC	PAC
		1	0.968	0.968
		2	0.930	-0.093
		3	0.893	-0.014
		4	0.856	-0.016
		5	0.821	0.008
		6	0.790	0.042
		7	0.762	0.020
		8	0.734	-0.023
		9	0.707	0.001
		10	0.677	-0.053
		11	0.646	-0.033
		12	0.617	0.030

- So, the log of US real GDP has a unit root as well as a deterministic trend

Example 2 continued

- Should we be flustered that we cannot use our econometric tools to model log of real GDP?
- Not really, because we are interested in the growth rate of real GDP, i.e. $100\Delta \log(GDP)$, and first differencing gets rid of both deterministic trend as well as the unit root!



- We can use all tools that we have learnt to model this series.

Example 2 continued

- For example, we can find the best univariate AR model for US real GDP growth by starting with an AR(4) - since data is quarterly.
- If there is serial correlation left in the residuals, increase the lag length until residuals become white noise.
- If there isn't any serial correlation in the residuals, use t-test to indentify insignificant lags and drop them one by one, until there is no insignificant lags left, or use model selection criteria to choose between this and lower order models. When the best model is chosen, test again to make sure that no serial correlation has seeped into the residuals.
- This process will lead to the following AR model:

$$\Delta \log(GDP_t) = \underset{(0.07)}{0.49} + \underset{(0.06)}{0.36} \Delta \log(GDP_{t-1}) + \hat{u}_t$$
$$R^2 = 0.13, \;\; n = 259.$$

- There is no evidence of serial correlation in the errors of this model.

Summary

- If non-stationarity is due to a linear trend in mean, we can add a time trend to the model and proceed as usual.
- If non-stationarity is due to a unit root (perhaps as well as a linear trend), we difference the data and proceed with modelling the differenced data.
- We identify a unit root in a time series from its plot and its correlogram. The series will not be mean-reverting, will have first order autocorrelation close to 1, and its autocorrelations decay very slowly.