MiniQuiz - Week 3

1. **01 - Week3**

   MULTI | 1.0 point | 0.10 penalty | Single | Shuffle

   X is a discrete random variable with possible values $\{-3, -1, 0, 1, 5\}$. The expected value of X is?

   (a) 0 → *We need the probability of each outcome to be able to compute $E(X)$*
   (b) 0.4 → *We are not told that the outcomes are equally likely*
   (c) We cannot compute $E(X)$ because we do not know the probability of each outcome. (100%) → *Excellent*

2. **02 - Week 3**

   MULTI | 1.0 point | 0.10 penalty | Single | Shuffle

   X and Y are random variables. If $E(X) = 3$ and $E(Y) = 4$ what is $E(2X - Y)$?

   (a) $2^2 \times 3 - 4 = 8$
   (b) We cannot compute $E(2X - Y)$ because we do not know different possible values of $2X - Y$ and probability of each of those values.
   (c) $2 \times 3 - 4 = 2$ (100%) → *Excellent*

   $E(2X - Y) = 2E(X) - E(Y) = 2 \times 3 - 4 = 2$

3. **03 - Week 3**

   MULTI | 1.0 point | 0.10 penalty | Single | Shuffle

   X and Y are statistically independent random variables. If $Var(X) = 4$ and $Var(Y) = 9$ what is $Var(2X - Y)$?

   (a) $2 \times 4 - 9 = -1$ → *Variance cannot be negative!*
   (b) $2^2 \times 4 - 9 = 7$
   (c) $2^2 \times 4 + 9 = 25$ (100%) → *Excellent*

   $Var(2X - Y) = 2^2 Var(X) + Var(Y) = 4 \times 4 + 9 = 25$

4. **04 - Week 3**

   NUMERICAL | 1.0 point | 0.10 penalty

   According to an expert, the inflation rate and the cash rate (the interest rate set by the Reserve Bank of Australia) in Australia in the next quarter is governed by the following joint probability density function:

cash rate $\downarrow \setminus$ inflation rate $\longrightarrow$	1.00%	1.25%	1.50%
0.05%	0.06	0.10	0.00
0.10%	0.04	0.50	0.10
0.25%	0.00	0.10	0.10

According to this expert, what is the expected value of the cash rate next quarter? Enter your answer to three decimal points in percentage points but without the % sign.

- 0.122 ✓

The marginal probability mass function of cash rate is 0.05 with probability 0.16, 0.10 with probability 0.64 and 0.25 with probability 0.20. Therefore, $E(\textit{cash rate}) = 0.05 * 0.16 + 0.10 * 0.64 + 0.25 * 0.20 = 0.008 + 0.064 + 0.050 = 0.122$.

5. **05 - Week 3**

NUMERICAL | 1.0 point | 0.10 penalty

According to an expert, the inflation rate and the cash rate (the interest rate set by the Reserve Bank of Australia) in Australia in the next quarter is governed by the following joint probability density function:

cash rate $\downarrow \setminus$ inflation rate $\longrightarrow$	1.00%	1.25%	1.50%
0.05%	0.06	0.10	0.00
0.10%	0.04	0.50	0.10
0.25%	0.00	0.10	0.10

According to this expert, what is the expected value of the inflation rate conditional on cash rate staying at 0.10% next quarter? Do all calculations to the highest precision and then round your final answer to two decimal points and enter it in percentage points but without the % sign.

- 1.27 ✓

The conditional probability mass function of inflation rate conditional on cash rate being 0.10% *is 1.00 with probability* $0.04/0.64 = 0.0625$, *1.25 with probability* $0.50/0.64 = 0.78125$ *and 1.50 with probability* $0.10/0.64 = 0.15625$. *Therefore,* $E(\textit{inflation rate} \mid \textit{cash rate} = 0.10) = 1 * 0.0625 + 1.25 * 0.78125 + 1.50 * 0.15625 = 1.2734375$.

Total of marks: 5

Introductory Econometrics

Tutorial 3 (suggested answers)

PART B: The following questions will be covered in the tutorial. It is a good idea to attempt these questions on your own before the tutorial.

1. X and Y are random variables with mean μ_X and μ_Y respectively. The covariance between X and Y is defined as $Cov\,(X,Y) = E\,[(X-\mu_X)\,(Y-\mu_Y)]$. Show that:

$$Cov\,(X,Y) = E\,[(X-\mu_X)\,Y] = E\,[X\,(Y-\mu_Y)] = E\,(XY) - \mu_X\mu_Y.$$

Discuss why we cannot simplify $E\,(XY) - \mu_X\mu_Y$ further to $E\,(X)\,E\,(Y) - \mu_X\mu_Y = \mu_X\mu_Y - \mu_X\mu_Y = 0$.

$$\begin{aligned} Cov\,(X,Y) &= E\,[(X-\mu_X)\,(Y-\mu_Y)] = E\,[(X-\mu_X)\,Y - (X-\mu_X)\,\mu_Y] \\ &= E\,[(X-\mu_X)\,Y] - E[(X-\mu_X)\,\mu_Y] \\ &= E\,[(X-\mu_X)\,Y] - \mu_Y\underbrace{E\,(X-\mu_X)}_{0} = E\,[(X-\mu_X)\,Y] \\ &= E\,(XY-\mu_XY) = E\,(XY) - \mu_XE\,(Y) = E\,(XY) - \mu_X\mu_Y. \end{aligned}$$

The last expression $E\,(XY)-\mu_X\mu_Y$ cannot be simplified further to $E\,(X)\,E\,(Y)-\mu_X\mu_Y$ because $E\,(XY) \neq E\,(X)\,E\,(Y)$ in general. (Those are equal when X and Y are independent.)

2. Diversification in everyday life: [A pokie machine is called a slot machine in America and a jackpot machine in some other countries. It is a random device that operates for a price, and draws an outcome randomly, and depending on the outcome, you may win different amounts of money (including zero). The probabilities are controlled such that the expected value of the monetary prize is slightly smaller than the price]. Most pokie machines give you the option of multiplying your bet up. For example, if the machine accepts 20 cents per round for having a go at winnings given by the random variable X, you have the option of paying one dollar to scale up your winnings to $5X$. Suppose you have one dollar only. Compare the expected return and risk of using all of your money at once and betting $5X$, with using it for playing X five times (i.e. $X_1+X_2+X_3+X_4+X_5$, where X_i, $i=1,2,\ldots,5$ are independent and have distribution identical to X).
Let $E\,(X)=\mu$ and $Var\,(X)=\sigma^2$. Then when we bet all of one dollar at once, the expected winning is $E\,(5X)=5\mu$, and variance of this bet is $Var\,(5X)=5^2Var\,(X)=25\sigma^2$. If we use our dollar to bet X five different times, we have the expected winning of $E\,(X_1+X_2+X_3+X_4+X_5) = \mu+\mu+\mu+\mu+\mu = 5\mu$, and the variance of the outcome of this strategy is $Var\,(X_1+X_2+X_3+X_4+X_5) = Var\,(X_1)+Var\,(X_2)+Var\,(X_3)+Var\,(X_4)+Var\,(X_5) = \sigma^2+\sigma^2+\sigma^2+\sigma^2+\sigma^2 = 5\sigma^2$. So, the first strategy has the same expected return as the second one, but has a much larger risk.

3. Diversification in econometrics and statistics: Suppose we are interested in estimating the mean of a random variable. We can take a sample of one observation from the random variable and use that as the estimate of the mean, or we can take a sample of 5 observations and take the average of those 5 observations as the estimate of the mean. What is the expected value of each of these estimators and which one is safer (i.e. less risky)? Discuss the similarity of this to the previous question.
Let's denote the unknown mean and variance of the random variable by μ and σ^2. Our sample of one observation is X. Its mean is obviously $E\,(X)=\mu$ and its variance is $Var\,(X)=\sigma^2$. A random sample of 5 observations can be denoted by $\{X_1,X_2,X_3,X_4,X_5\}$ and its average is

$\overline{X} = \frac{1}{5}(X_1 + X_2 + X_3 + X_4 + X_5) = \frac{1}{5}\sum_{i=1}^{i=5} X_i$. *The expected value of* $\overline{X}$ *is*

$$\begin{aligned} E\left(\overline{X}\right) &= E\left[\frac{1}{5}(X_1 + X_2 + X_3 + X_4 + X_5)\right] = \frac{1}{5}E(X_1 + X_2 + X_3 + X_4 + X_5) \\ &= \frac{1}{5}[E(X_1) + E(X_2) + E(X_3) + E(X_4) + E(X_5)] \\ &= \frac{1}{5}[\mu + \mu + \mu + \mu + \mu] = \mu \end{aligned}$$

and its variance is

$$\begin{aligned} Var\left(\overline{X}\right) &= Var\left[\frac{1}{5}(X_1 + X_2 + X_3 + X_4 + X_5)\right] = \frac{1}{25}Var(X_1 + X_2 + X_3 + X_4 + X_5) \\ &= \frac{1}{25}[Var(X_1) + Var(X_2) + Var(X_3) + Var(X_4) + Var(X_5)], \text{ because the sample is randor} \\ &= \frac{1}{25}\left[\sigma^2 + \sigma^2 + \sigma^2 + \sigma^2 + \sigma^2\right] = \frac{1}{25} \times 5\sigma^2 = \frac{\sigma^2}{5} \end{aligned}$$

So both a sample of one observation and the average of the sample of 5 observations have the same expected value (which is the parameter that we want to estimate), but the average of 5 observations is less risky (i.e. it is less likely to be very far from μ*). This is very similar to the previous question. There we were comparing* $5X$ *and* $(X_1 + X_2 + X_3 + X_4 + X_5)$, *here we are comparing* X *and* $\frac{1}{5}(X_1 + X_2 + X_3 + X_4 + X_5)$. *Same principle: it is safer to diversify and not depend only on a single draw from the distribution.*