Tutorial 6

6.1 (a) $S_0(50)=(1-\frac{50}{110})^{\frac{1}{3}}=0.8171$

(b) $S_0(60)-S_0(70)=(1-\frac{60}{110})^{\frac{1}{3}}-(1-\frac{70}{110})^{\frac{1}{3}}=0.0551$

(c) $S_{40}(10)=\frac{S_0(50)}{S_0(40)}=\frac{(1-\frac{50}{110})^{\frac{1}{3}}}{(1-\frac{40}{110})^{\frac{1}{3}}}=0.9499$

(d) $S_{50}(10)-S_{50}(30)=\frac{S_0(60)-S_0(80)}{S_0(50)}=\frac{(1-\frac{60}{110})^{\frac{1}{3}}-(1-\frac{80}{110})^{\frac{1}{3}}}{(1-\frac{50}{110})^{\frac{1}{3}}}=0.1473$

(e) $S_{60}(t)=\frac{S_0(60+t)}{S_0(60)}=\frac{(1-\frac{60+t}{110})^{\frac{1}{3}}}{(1-\frac{60}{110})^{\frac{1}{3}}}=0.6$ $\quad t=39.2 \quad$ $\text{age}=99.2$

(f) $\frac{d}{dx}\,{}_xp_0=-{}_xp_0\ \mu_x$ $\quad \mu_x=-\frac{-\frac{1}{3}\frac{1}{110}(1-\frac{x}{110})^{-\frac{2}{3}}}{(1-\frac{x}{110})^{\frac{1}{3}}}=\frac{1}{330}(1-\frac{x}{110})^{-1}$ $\quad \mu_{30}=0.00417$

6.2 $S_x(t)=\frac{S_0(x+t)}{S_0(x)}=\frac{\exp(-\lambda(x+t))}{\exp(-\lambda x)}=\exp(-\lambda t)=S_0(t)$

6.3 (i) $p_{55}\ p_{56}\ p_{57}=0.98013$

(ii) $1-{}_4p_{56}=0.03107$

(iii) $p_{55}\ p_{56}(1-p_{57})+p_{55}\ p_{56}\ p_{57}(1-p_{58})=0.01536$

6.4 $F_x(t)=1-{}_tp_x=1-\exp\left(-\int_0^t\frac{3}{1+x+s}ds\right)=1-\exp\left(-3\ln\frac{1+x+t}{1+x}\right)=1-\left(\frac{1+x}{1+x+t}\right)^3$

$f_x(t)={}_tp_x\mu_{x+t}=\left(\frac{1+x}{1+x+t}\right)^3\frac{3}{1+x+t}=\frac{3(1+x)^3}{(1+x+t)^4}$

Based on Pareto(3,1+*x*),

$\mathrm{E}(T_x)=\frac{1+x}{2}$ $\quad \mathrm{Var}(T_x)=\frac{3(1+x)^2}{4}$

6.5 ${}_{15}p_{65}(1-{}_3p_{80})=\exp(-0.01\times5-0.015\times10)(1-\exp(-0.025\times3))=0.05916$

6.6 $\mu_x^* = (1+0.01k)\mu_x$ $\quad {}_{10}p_{40}^* = \exp(-\int_0^{10} \mu_{40+s}^* ds) = \exp(-\int_0^{10} (1+0.01k)\mu_{40+s} ds) = ({}_{10}p_{40})^{1+0.01k}$

$0.97247 = 0.973^{1+0.01k}$ $\quad k \approx 2$

6.7 $\mu_{50+s}^* = \mu_{50+s} + 0.002 - 0.0002s$

$p_{50}^* = \exp(-\int_0^1 \mu_{50+s}^* ds) = \exp(-\int_0^1 (\mu_{50+s} + 0.002 - 0.0002s) ds) = p_{50} \exp(-0.002 + 0.0001) = 0.9911$

6.8 (i) $\mathring{e}_x = \int_0^\infty {}_t p_x dt = \int_0^\infty \frac{{}_{x+t}p_0}{{}_x p_0} dt = \int_0^{100-x} \frac{1-\frac{x+t}{100}}{1-\frac{x}{100}} dt = 50 - \frac{x}{2}$

(ii) $\mathring{e}_x = \int_0^\infty {}_t p_x dt = \int_0^\infty \exp(-0.01t) dt = 100$

6.9 $\Pr(K_{105} = 0) = q_{105} = \frac{33}{82}$

$\Pr(K_{105} = 1) = p_{105}\ q_{106} = \frac{49}{82}\frac{23}{49} = \frac{23}{82}$

$\Pr(K_{105} = 2) = {}_2p_{105}\ q_{107} = \frac{49}{82}\frac{26}{49}\frac{14}{26} = \frac{7}{41}$

$\Pr(K_{105} = 3) = {}_3p_{105}\ q_{108} = \frac{49}{82}\frac{26}{49}\frac{12}{26}\frac{8}{12} = \frac{4}{41}$

$\Pr(K_{105} = 4) = {}_4p_{105}\ q_{109} = \frac{49}{82}\frac{26}{49}\frac{12}{26}\frac{4}{12} = \frac{2}{41}$

$\mathrm{E}(K_{105}) = \frac{23\times 1}{82} + \frac{7\times 2}{41} + \frac{4\times 3}{41} + \frac{2\times 4}{41} = \frac{91}{82}$

$\mathrm{E}(K_{105}^2) = \frac{23\times 1}{82} + \frac{7\times 2^2}{41} + \frac{4\times 3^2}{41} + \frac{2\times 4^2}{41} = \frac{215}{82}$

$\mathrm{Var}(K_{105}) = \frac{215}{82} - \frac{91^2}{82^2} = \frac{9349}{6724}$

6.10 (i) ${}_{0.5}p_{40} = 1 - 0.5q_{40} = 0.99885$ $\quad {}_{0.25}p_{40.5} = 1 - \frac{0.25q_{40}}{1-0.5q_{40}} = 0.99942434$

(ii) ${}_{0.5}p_{40} = \frac{1-q_{40}}{1-0.5q_{40}} = 0.99884868$ $\quad {}_{0.25}p_{40.5} = \frac{{}_{0.75}p_{40}}{{}_{0.5}p_{40}} = \frac{\frac{1-q_{40}}{1-0.25q_{40}}}{\frac{1-q_{40}}{1-0.5q_{40}}} = 0.99942467$

6.11 $\mu_x^* = 2\mu_x$ $\quad {}_n p_x^* = \exp(-\int_0^n \mu_{x+s}^* ds) = \exp(-\int_0^n 2\mu_{x+s} ds) = ({}_n p_x)^2 = g^{2c^x(c^n-1)}$

${}_n p_{x+a} = g^{c^{x+a}(c^n-1)}$ $\quad c^a = 2$ $\quad a = \frac{\ln 2}{\ln c}$

6.12 $\mu_{50} = Bc^{50}$ $\quad \mu_{70} = Bc^{70}$ $\quad c^{20} = 0.0380 / 0.0045$ $\quad c = 1.11257$ $\quad B = 0.0000217162$

$${}_{20}p_{30} = \exp(-\frac{Bc^{30}(c^{20}-1)}{\ln c}) = 0.9635$$

6.13 $\mu_{20} = A + Bc^{20}$ $\quad \mu_{40} = A + Bc^{40}$ $\quad \mu_{60} = A + Bc^{60}$

$\mu_{60} - \mu_{40} = B(c^{60} - c^{40})$ $\quad \mu_{40} - \mu_{20} = B(c^{40} - c^{20})$

$c^{20} = 3.52570694$ $\quad c = 1.06503115$ $\quad B = 0.00069894$ $\quad A = 0.00120274$

6.14 $${}_{10}p_{30} = \exp(-10A - \frac{Bc^{30}(c^{10}-1)}{\ln c}) \quad {}_{10}p_{40} = \exp(-10A - \frac{Bc^{40}(c^{10}-1)}{\ln c}) \quad {}_{10}p_{50} = \exp(-10A - \frac{Bc^{50}(c^{10}-1)}{\ln c})$$

$$\ln {}_{10}p_{50} - \ln {}_{10}p_{40} = -\frac{B(c^{10}-1)(c^{50}-c^{40})}{\ln c} \qquad \ln {}_{10}p_{40} - \ln {}_{10}p_{30} = -\frac{B(c^{10}-1)(c^{40}-c^{30})}{\ln c}$$

$c^{10} = 1.79085$ $\quad c = 1.06$ $\quad B = 0.0006$ $\quad A = 0.001$ $\quad \mu_{70} = 0.0364456$