# Introductory Econometrics

Tutorial 12

**PART A:** This homework is a review of time series and asymptotic properties of the OLS estimator. After reviewing Week 10 and 11 lecture slides and studying chapter 5 and 11 of the textbook, attempt Week 12 Part A quiz on Moodle. You need to submit the quiz before your tutorial and attend the tutorial to obtain 1 point for Week 12 participation.

**Part B: This part will be covered in the tutorial. It is still a good idea to attempt these questions before the tutorial.**

1. **This is based on a question from S1, 2018 final exam:** A researcher wants to test the Efficient Market Hypothesis (EMH) using weekly percentage returns, denoted by $r_t$, on the New York Stock Exchange composite index. In its strict form the EMH states that information observable to the market prior to week $t$ should not help to predict the return during week $t$. If we use only past information on $r$, the EMH is stated as

$$E\left(r_t | r_{t-1}, r_{t-2}, \ldots\right) = E\left(r_t\right). \tag{1}$$

One simple way to test that (1) holds is to specify the following alternative AR(1) model to describe $r_t$:

$$r_t = \beta_0 + \beta_1 r_{t-1} + u_t, \tag{2}$$

where $E\left(u_t | r_{t-1}, r_{t-2}, \ldots\right) = 0$ and $Var\left(u_t | r_{t-1}, r_{t-2}, \ldots\right) = \sigma^2$. Using data from the first week of January 2004 to the third week of April 2018, estimation of (2) gives:

$$\begin{aligned} \hat{r}_t &= \underset{(0.096)}{0.086} - \underset{(0.038)}{0.059} r_{t-1}, \\ n &= 689,\ R^2 = 0.0035,\ \bar{R}^2 = 0.0020. \end{aligned} \tag{3}$$

(standard errors are reported in parentheses underneath the parameter estimates).

(a) i. How would you formulate the null hypothesis that the EMH holds based on (2)? Briefly explain your intuition behind your choice of $H_0$.

ii. Given the OLS regression results in (3) do you reject or not reject $H_0$ from (i)? Briefly explain.

(4 marks)

(b) The alternative AR(1) model in (2) does not preclude that potentially there could be dependence between returns that are more than one week apart.

i. If (2) were the correct specification for describing returns, what type of process would you expect $u_t$ to follow? Provide the properties of this process.

ii. If the researcher suspects that returns 3 and 4 weeks apart individually add power to the prediction of $r_t$, what problem do you think that he would be worried about with regard to the behaviour of $u_t$? Briefly explain.

(5 marks)

(c) The researcher is also interested in the behaviour of the squared residuals from regression (3) because he is concerned that the variance given past information might not be constant. For this purpose he runs a regression of $\hat{u}_t^2$ on $r_{t-1}$ and obtains the following results:

$$\begin{aligned} \hat{u}_t^2 &= \underset{(0.43)}{4.66} - \underset{(0.201)}{1.104} r_{t-1} + \hat{v}_t, \\ n &= 689,\ R^2 = 0.042. \end{aligned} \tag{4}$$

i. Which problem is the researcher worried about in this case? Define the problem and set up a formal test that makes use of the goodness-of-fit of (4). Clearly state the steps involved in the implementation of this test, the null and alternative hypotheses of the test, the statistic(s) of interest and corresponding distribution(s).

ii. What advice would you give this researcher based on your analysis of c.(i)? Briefly explain your answer.

(6 marks)

2. **This is based on a question from S2, 2016 final exam:**

(a) A researcher who wished to study the behavior of a stationary time series $\{y_t\}$ estimated both an AR(2) model,

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + u_t,$$

and an ADL(2,1) model,

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \beta_1 x_{t-1} + u_t.$$

The researcher obtained the results reported in Table 1 below (standard errors are reported below the estimated coefficients). Based on the information in Table 1, which model do you prefer? Briefly explain.

Table 1

| | AR(2) | ADL(2,1) |
|---|---|---|
| $\widehat{c}$ | 1.28 (0.53) | 1.30 (0.44) |
| $\widehat{\phi}_1$ | $-0.31$ (0.09) | $-0.42$ (0.08) |
| $\widehat{\phi}_2$ | $-0.39$ (0.08) | $-0.37$ (0.08) |
| $\widehat{\beta}_1$ | – | $-2.64$ (0.46) |
| $\overline{R}^2$ | 0.55 | 0.71 |
| $SSR$ | 475 | 462 |
| $AIC$ | 1.08 | 1.04 |
| $BIC$ | 1.09 | 1.11 |
| $n$ | 200 | 200 |

(4 marks)

(b) When the researcher estimated the model

$$y_t = c + \beta_0 D_t + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \beta_1 x_{t-1} + \beta_2 (D_t x_{t-1}) + u_t \qquad (2)$$

by OLS, where

$$D_t = \begin{cases} 1 \text{ for } t = 1, 2, ...100 \\ 0 \text{ for } t = 101, 102, ...200 \end{cases},$$

he obtained the results reported in (3) below (standard errors are reported in parentheses):

$$\begin{aligned} \widehat{y}_t &= \underset{(0.44)}{1.32} + \underset{(0.04)}{0.30} D_t - \underset{(0.07)}{0.39} y_{t-1} - \underset{(0.06)}{0.31} y_{t-2} + \underset{(0.41)}{2.15} x_{t-1} + \underset{(0.07)}{0.15} (D_t x_{t-1}), \qquad (5) \\ SSR &= 450, \overline{R}^2 = 0.69, n = 200. \end{aligned}$$

i. What is

$$\widehat{E}(y_t | y_{t-1}, y_{t-2}, x_{t-1})$$

for different values of $t$? Briefly explain.

ii. What are the estimated immediate and long run effect of a one unit increase in $x$ on $y$ before and after time $t = 100$? Briefly explain.

iii. Test the null **hypothesis** that there is no structural break in either the intercept or the coefficient attached to $x_{t-1}$ in the ADL(2,1) model. State the null and the alternative hypothesis, the form and asymptotic distribution of the test statistic under the null, the sample value and critical value of the test statistic and you conclusion.

(7 marks)

(c) Carefully describe the steps involved in performing a **Breush-Godfrey test** for autocorrelation up to order two in the error term in the ADL(2,1) model

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \beta_1 x_{t-1} + u_t. \tag{4}$$

Make sure you specify the form and distribution of the test statistic.

(5 marks)