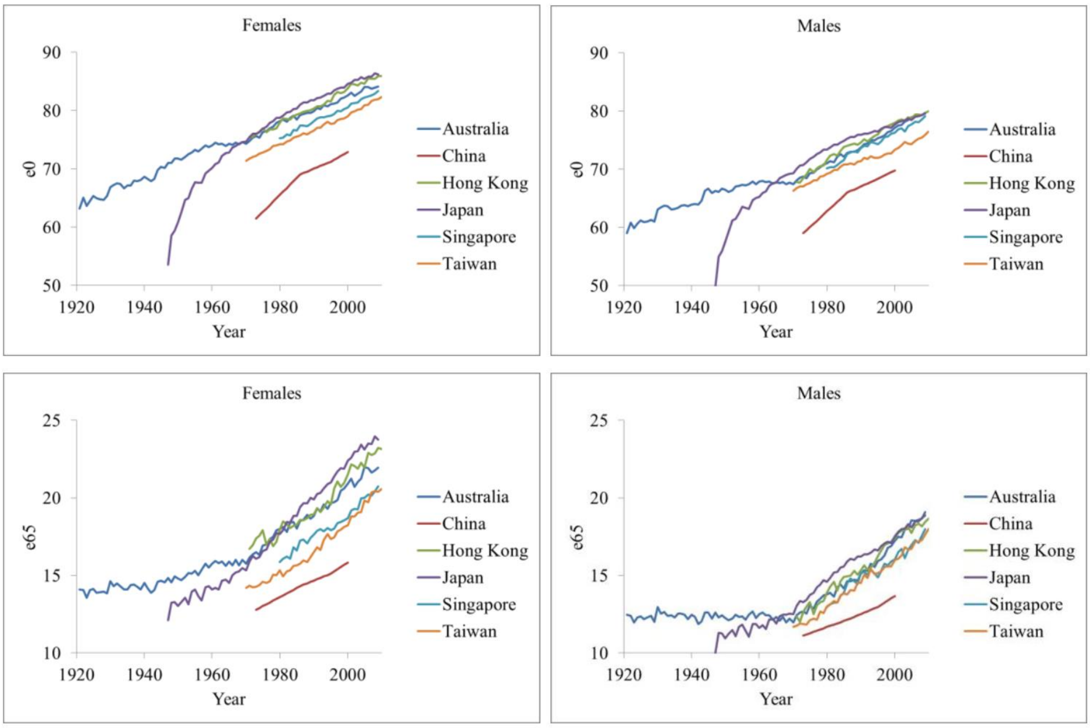
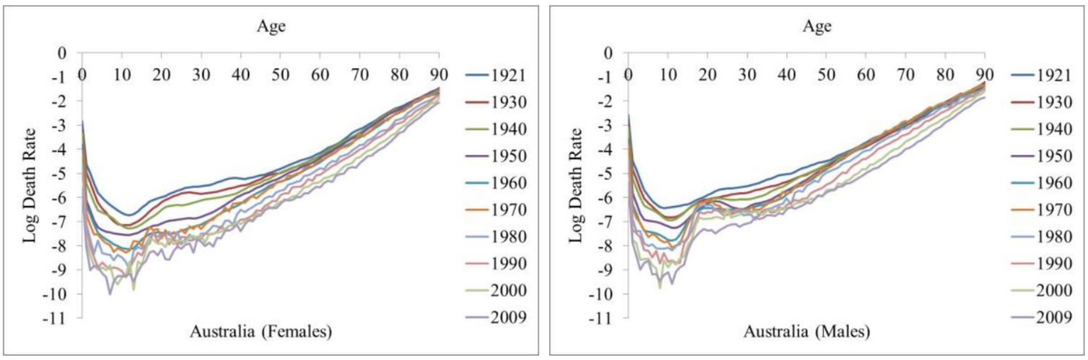
Mortality Improvements

– life expectancy $\uparrow$ since 20th century
– persistent upward trend
– improved nutrition, hygiene, medical care, living environment, education, lifestyle
– offsetting effects such as obesity, epidemics, catastrophes
– major causes of death shifted from infectious diseases to chronic diseases
– before (say) 1970, mortality decline at young ages more rapid
– in recent decades, mortality decline at old ages more important
– mortality projection

Mortality Improvements





Mortality Projection – Expectation

– using individuals' expectations and experts' opinions

– considering future structural changes and unexpected events

– pessimistic bias historically

– present knowledge on only current limitations but not future means of overcoming them

– limited to aggregate measures without more specific details

Mortality Projection – Explanation

- based on theories
- relationships between demographic quantities and economic, social, environmental factors
- fundamental solution
- to date mostly hypotheses
- lack of data
- high risk of model misspecification

Mortality Projection – Extrapolation

- most popular approach
- assuming continuation of past trends
- Lee-Carter, Cairns-Blake-Dowd etc
- perhaps too simple without experts' opinions or structural relationships
- still reasonable first step or benchmark for further analysis
- division between different approaches not that clear-cut
- suitable for short to medium term
- experts' opinions, informed judgement, comparison with other populations, scenario testing to supplement the analysis for long term

Lee-Carter (LC) Model, 1992

– $\ln m_{x,t} = a_x + b_x k_t + \varepsilon_{x,t}$

$m_{x,t}$: central death rate at age x in year t

a_x : overall mortality level at age x

b_x : age-specific sensitivity of log death rate to changes in k_t

k_t : mortality index in year t

$\varepsilon_{x,t}$: normal error term at age x in year t with mean 0 and variance σ^2

– $\Sigma_x b_x = 1$

– $\Sigma_t k_t = 0$

– $m_{x,t} \approx d_{x,t} / E^C_{x,t}$ (e.g. HMD data)

– simple structure

– applicable to all ages

– rather linear mortality index

Lee-Carter (LC) Model

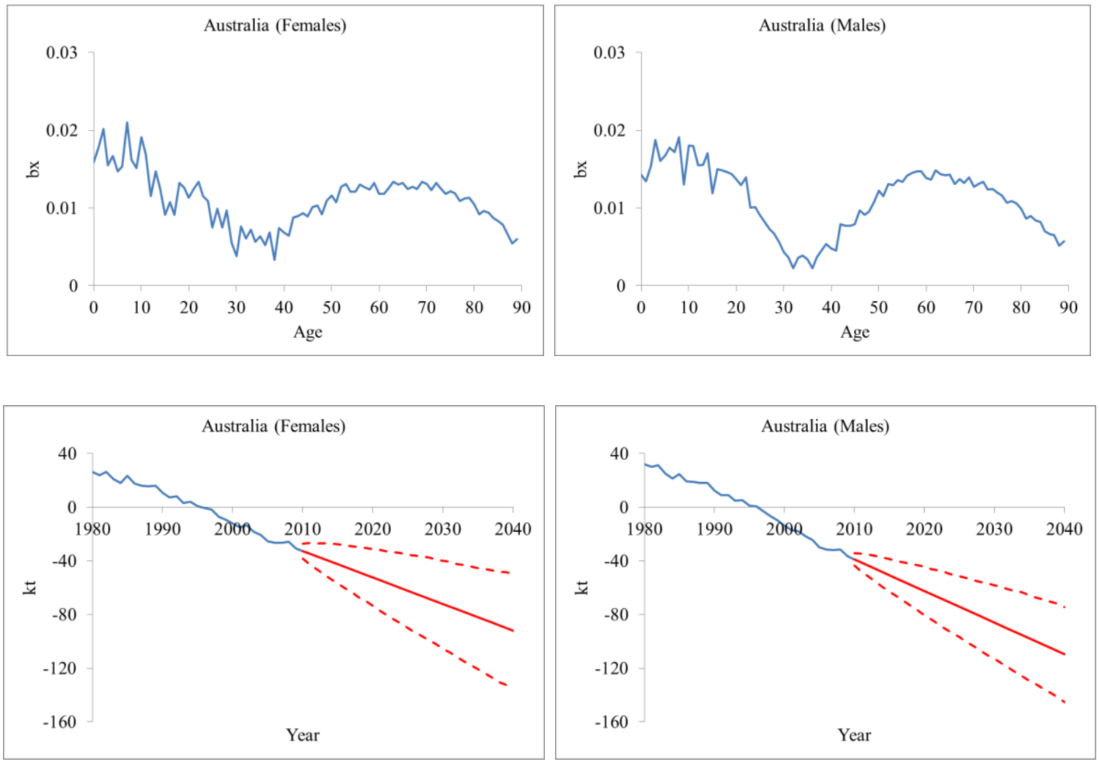
– a_x estimated as average ln $m_{x,t}$ over time
– b_x and k_t estimated by singular value decomposition (SVD)
– $b_x\, k_t$ = first principal component
– random walk with drift :

  $k_t = \mu + k_{t\text{-}1} + e_t$

  μ : drift term

  e_t : normal error term in year t with mean 0 and variance σ_k^2
– negative estimated μ
– projection and simulation of future death rates

Lee-Carter (LC) Model



Lee-Carter (LC) Model

```
a<-numeric()
for (x in 1:age) { a[x]=mean(log(m[x,1:year])) }

info<-array(NA,c(age,year))
for (x in 1:age) {
for (t in 1:year) {
info[x,t]=log(m[x,t])-a[x]
}}

pca<-svd(info,1,1)
b=pca$u/sum(pca$u)
k=sum(pca$u)*pca$d[1]*pca$v
```

Cairns-Blake-Dowd (CBD) Model, 2006

– logit $q_{x,t} = k_t^{(1)} + k_t^{(2)} (x - \bar{x}) + \varepsilon_{x,t}$

$q_{x,t}$: mortality rate at age x in year t

$k_t^{(1)}$: level of mortality curve in year t

$k_t^{(2)}$: slope of mortality curve in year t

$\bar{x}$: average age of age range

$\varepsilon_{x,t}$: normal error term at age x in year t

with mean 0 and variance σ^2

– $q_{x,t} \approx 1 - \exp(-m_{x,t})$

– simple structure

– applicable to older ages

– rather linear $k_t^{(1)}$

Cairns-Blake-Dowd (CBD) Model

- $k_t^{(1)}$ and $k_t^{(2)}$ estimated by least squares
- bivariate random walk with drift :

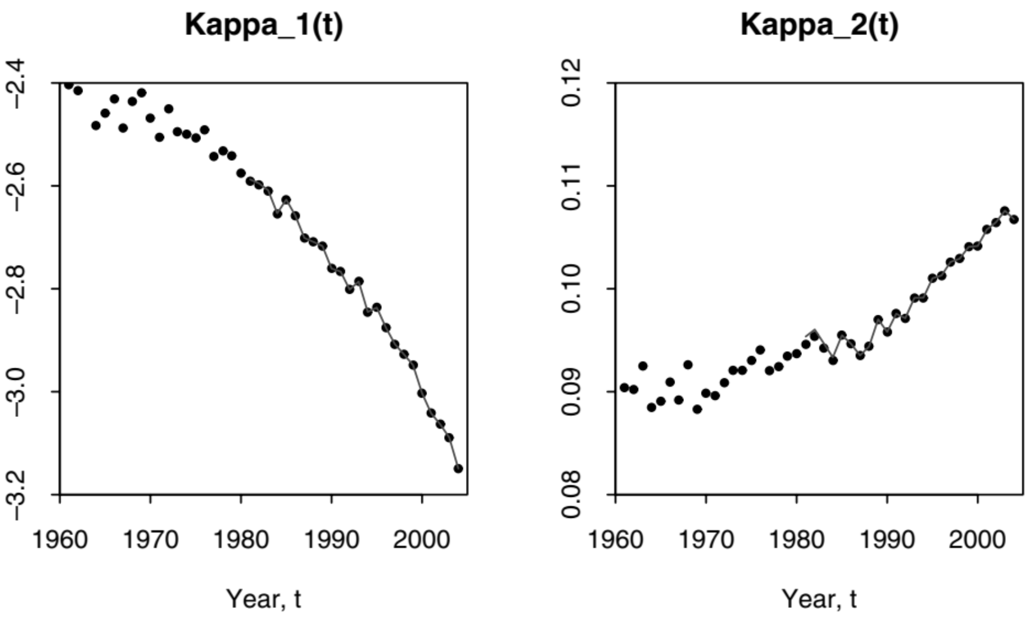
  $k_t^{(i)} = \mu^{(i)} + k_{t-1}^{(i)} + e_t^{(i)}$

  $\mu^{(i)}$: drift term

  $e_t^{(i)}$: bivariate normal error term in year t with mean $\mathbf{0}$ and covariance matrix $\mathbf{\Omega}$
- negative estimated $\mu^{(1)}$
- projection and simulation of future mortality rates

Cairns-Blake-Dowd (CBD) Model

England and Wales Data



Cairns-Blake-Dowd (CBD) Model

```
xbar=mean(c(60:89))
co=c(60:89)-xbar


k1<-numeric()
k2<-numeric()
for (t in 1:year) {
fit=lm(log(q[1:age,t]/(1-q[1:age,t]))~co)
k1[t]=fit$coef[1]
k2[t]=fit$coef[2]
}
```

Other Models

– Booth, Maindonald, Smith (2002) :

$\ln m_{x,t} = a_x + \Sigma_i\, b_x^{(i)}\, k_t^{(i)} + \varepsilon_{x,t}$

– Bray (2002), Currie (2006)

$\ln m_{x,t} = a_x + k_t + \gamma_{t-x} + \varepsilon_{x,t}$

– Renshaw, Haberman (2006) :

$\ln m_{x,t} = a_x + b_x^{(1)}\, k_t + b_x^{(2)}\, \gamma_{t-x} + \varepsilon_{x,t}$

– Cairns, Blake, Dowd et al (2009) :

$\text{logit } q_{x,t} = k_t^{(1)} + k_t^{(2)}\, (x - \bar{x}) + \gamma_{t-x} + \varepsilon_{x,t}$

$\text{logit } q_{x,t} = k_t^{(1)} + k_t^{(2)}\, (x - \bar{x})$

$+ k_t^{(3)}\, [(x - \bar{x})^2 - \sigma_x^2] + \gamma_{t-x} + \varepsilon_{x,t}$

– Li, Lee (2005) :

$\ln m_{x,t}^{(i)} = a_x^{(i)} + B_x\, K_t + b_x^{(i)}\, k_t^{(i)} + \varepsilon_{x,t}^{(i)}$

– Li (2012) :

$\ln m_{x,t}^{(i)} = a_x^{(i)} + B_x\, K_t + \Sigma_j\, b_x^{(i,j)}\, k_t^{(i,j)} + \varepsilon_{x,t}^{(i)}$

Other Estimation Methods

– Brouhns, Denuit, Vermunt (2002) :

$D_{x,t} \sim \text{Poisson}(E^C_{x,t}\, m_{x,t})$

Newton-Raphson method

$$\theta^* = \theta - \frac{\frac{\partial l}{\partial \theta}}{\frac{\partial^2 l}{\partial \theta^2}}$$

– Czado, Delwarde, Denuit (2005) :

Bayesian MCMC

$a_x \sim \text{Normal}(0, \sigma_a^2)$

$b_x \sim \text{Normal}(1/\text{no. of age groups}, \sigma_b^2)$

$\mu \sim \text{Normal}(\mu_0, \sigma_\mu^2)$

$e_t \sim \text{Normal}(0, \sigma_k^2)$

$\sigma_k^{-2} \sim \text{Gamma}(\alpha, \beta)$

Projection

```
mu=(k[year]-k[1])/(year-1)

for (t in 1:30) {
k[year+t]=k[year+t-1]+mu
for (x in 1:age) {
m[x,year+t]=exp(a[x]+b[x]*k[year+t])
}}
```

Simulation

```
sigma=sd(k[2:year]-k[1:(year-1)])

mf<-array(NA,c(age,year+30,1000))
kf<-array(NA,c(year+30,1000))
for (z in 1:1000) {
mf[1:age,1:year,z]=m[1:age,1:year]
kf[1:year,z]=k[1:year]
for (t in 1:30) {
kf[year+t,z]=kf[year+t-1,z]+mu+sigma*rnorm(1)
for (x in 1:age) {
mf[x,year+t,z]=exp(a[x]+b[x]*kf[year+t,z])
}}}
```

Prediction Intervals

```
upper<-numeric()
lower<-numeric()
for (t in 1:30) {
upper[t]=quantile(mf[65,year+t,1:1000],0.975)
lower[t]=quantile(mf[65,year+t,1:1000],0.025)
}
```