

**MIDSEM TEST**

Missing Values

X	A	B	C
5.1	6	1.8	2.2

Question 1

$$N_t \sim \text{Poisson}(5.1t)$$

$$N_t - N_s \sim \text{Poisson}(5.1(t - s))$$

- a. The inter-event time follows a Poisson Distribution
- b.  $E(N_t - N_s) = 5.1(t - s)$
- c.  $E(N_6) = \text{Mean}(\text{Poisson}(5.1 * 6)) = 30.599999999999998 \approx 30.6$
- d.  $\Pr(N_3 > 1.8 + E(N_2)) =$   
 $E(X_2) = 10.2$   
 $\Pr[N_3 > 10.2 + 1.8] = 0.7564968855759098$
- e.  $\Pr(N_2 > 2.2) = 0.9976500924527895$

Question 2

- a. For the chain to be an absorbing Markov process, every state must be able to reach a state that is absorbing. So  $a, b$  both have to be 1 since we could start at either state 1 or state 2, and since both states are absorbing the chain will be considered an absorbing Markov process. Another option would be to direct to only one state, i.e.  $a \rightarrow b$  only or  $b \rightarrow a$  only. So  $(a = 0, b = 1)$  or  $(a = 1, b = 0)$  respectively.
- b. For a continuous-time Markov chain to be an absorbing Markov process the generator matrix,  $A = [\{(-a), a\}, \{b, (-b)\}]$ , the rows should equal to 0 and However the
- c. Given enough cycles, the distribution matrix for the Markov chain will eventually become a stationary distribution

Question 3

a.  $P =$

$[0, 0.5, 0.5,$   
 $0.5, 0, 0.5,$   
 $0.5, 0.5, 0]$

b.  $\Pr(\text{period} = 2) = 1/4$

$a \rightarrow b \rightarrow a$

$a \rightarrow c \rightarrow a$

$b \rightarrow a \rightarrow b$

$b \rightarrow c \rightarrow b$

$c \rightarrow a \rightarrow c$

$c \rightarrow b \rightarrow c$

c.  $D = [$

$0.5$

$0.5$

$0.5]$

Final distribution  $= D * P = [0.5, 0.5, 0.5]$

The final distribution does not change regardless of where the starting state is, since every final position is equally likely.

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#### Question 4

Missing Values

0.076404	0.195743	0.371898
X	Y	Z

$$6q_{58} = 58p * (1 - 6p_{58})$$

$$_tq_x = (1 - _tp_x) = 1 - E\left(-\int_0^5 (\mu_{55+s}) ds\right)$$

$$\mu_{60} \rightarrow 0.51434404563185$$

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Question 5