Week 1 Tutorial Questions

2020

1. For a stochastic process X_n with time set J and state space S, define the terms:
   (a) stationary
   (b) weakly stationary
   (c) increment
   (d) Markov property.

2. A moving average (stochastic) process, X_n, has a discrete time set and a continuous state space and is defined as:

$$X_n = Z_n + \alpha_1 Z_{n-1} + \alpha_2 Z_{n-2} + \alpha_3 Z_{n-3}$$

   where $\{Z_n, n \in Z\}$ are independent and identically distributed $N(0, \sigma^2)$ random variables and α_1 , α_2 , α_3 are constants.
   (a) Prove that X_n is weakly stationary.
   (b) Explain whether the Markov property holds.
   (c) Deduce whether the process has independent increments.

3. Explain whether a random walk has the Markov property.

4. (a) i. Define a Poisson process with rate λ.
   ii. Define a compound Poisson process.
   (b) Identify the circumstances in which a compound Poisson process is also a Poisson process.
   (c) The cumulative amount of claims reaching an insurance company is modelled using a compound Poisson process.
   i. Explain why the compound Poisson process has the Markov property.
   ii. Comment on whether this seems reasonable for the given insurance model.
   iii. State whether the compound Poisson process is weakly stationary.

iv. Explain whether you expect the cumulative insurance claims to follow a weakly stationary process.

5. Define a simple symmetric random walk and identify its time set and state space.

6. The price of an ordinary share is modelled as a stochastic process $X_n; n = 0, 1, 2, 3, \ldots$ with initial condition $x_0 = x_0 > O$, where:

$$X_n = x_0 \prod_{j=1}^{n} U_j \quad n \geq 1$$

and U_n is a white noise process.

(a) Show that the process lnXn, n 2': 0 has independent increments.

(b) Explain why Xn is a Markov process.

7. Calculate the covariance between the values $X(t)$ and $X(t + s)$ taken by a Poisson process $X(t)$ with constant rate λ at the two times t and $t + s$, where $s > 0$.

8. (a) X_n is a stochastic process with a discrete state space and a discrete time set. Show that if non-overlapping increments of this process are independent, then the process satisfies the Markov property.

(b) Show that a white noise process in discrete time with a discrete state space does not have independent increments, but is a Markov process.

9. An insurer has initial capital of u and receives premium income continuously at the rate of c per annum. Let $S(t)$ denote the total claim amount up to time t .

(a) Describe a model that would allow the insurer to estimate its probability of ruin (*ie* the probability that its claims outgo is more than its available funds). State any assumptions that you make.

(b) Write down an expression for the probability of ruin in terms of u, c and $S(t)$.

10. (a) In the context of a stochastic process denoted by $\{X_t : t \in J\}$, define the terms:

i. state space

ii. time set

iii. sample path.

(b) Stochastic process models can be placed in one of four categories according to whether the state space is continuous or discrete, and whether the time set is continuous or discrete. For each of the four categories:

i. state a stochastic process model of that type
ii. give an example of a problem an actuary may wish to study using a model from that category.