DHS Algebra 1

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Welcome to Algebra 1

Welcome to Algebra 1 at Frederick Douglass High School!

This book will guide you through the most important math skills you'll need to succeed in high school and beyond. Algebra is more than just solving equations — it's a powerful way to understand patterns, solve problems, and think logically.

Whether you're reviewing old ideas or learning something brand new, this book is here to help you every step of the way.

What You'll Find in This Book

Each unit includes:

- Clear goals to help you focus
- Examples and explanations
- Practice problems
- Activities to explore and talk through ideas

We'll start with the basics — like working with numbers and fractions — and build up to more complex ideas like equations, graphs, and even quadratics.

You don't have to be a "math person" to do well here. Just bring your curiosity, a little patience, and the willingness to try.

Let's get started!

Part I

Unit 1: Foundations

Introduction

Welcome to Unit 1! In this unit, we'll build the foundation you need to succeed in Algebra. Think of this as preparing your math toolkit.

You'll explore integers, factors, fractions, and the rules of simplification. These skills are the building blocks that will help you solve more complex problems with confidence — especially when working with expressions and equations in later units.

What You'll Learn

By the end of this unit, you'll be able to:

- Work with positive and negative numbers on a number line
- Use factor trees to find prime factorizations
- Identify and use the greatest common factor (GCF)
- Convert between fractions, decimals, and percents
- Multiply, divide, and compare fractions
- Follow the correct order of operations to simplify expressions

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Topics in This Unit

Integers & Number Lines Understand and use positive and negative numbers, and how to place them on a number line.

Factors, Multiples & Prime Factorization Break numbers into their prime building blocks using factor trees.

GCF & Simplifying Fractions Use prime factorization to find the GCF and simplify fractions to their simplest form.

Fractions, Decimals & Percents Convert between different number forms and apply them in real-world problems.

Multiply, Divide & Compare Fractions Work with fractions in ways that actually show up in Algebra — simplify, multiply, divide, and compare.

Order of Operations Follow the rules (PEMDAS) to simplify numeric expressions with integers and fractions.

Let's build those Algebra muscles — you'll need them for everything that follows!

1.1 - Integers & Number Lines

Did you know that all of mathematics is actually built up from simple things like counting? Even advanced topics like algebra and calculus are just clever ways of organizing and extending basic ideas — like moving forward and backward on a number line.

In this lesson, we'll use the number line not just to count, but to add, subtract, and compare positive and negative numbers. That might sound basic, but it's the foundation of nearly everything else you'll do in Algebra.

Negative numbers can be tricky, especially when the rules don't always match what your gut tells you. But if you can master the way they work on the number line — including things like opposites, absolute value, and comparison — you'll be setting yourself up for success in the rest of the course.

Objectives

- ☐ Know what a number line represents
- ☐ Understand and identify opposite numbers
- ☐ Compare integers using greater than and less than
- ☐ Use a number line to perform addition and subtraction

Vocabulary

absolute value, greater than, integer, less than, number line, negative, opposite, positive

Warm-Up

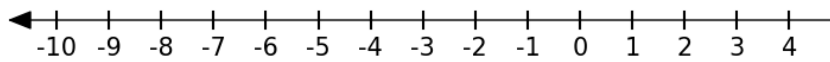
Answer as best you can – even if you aren't sure!

1. What is the opposite of 6?
2. Which is greater -4 or -9?
3. Which is farther from 0: -7 or 5?

Learn Together

1.1.1 - The Number Line Is More Than Just Counting

You already know how to count — 0, 1, 2, 3, and so on. The **number line** extends that idea in both directions.



Let's draw a number line from -10 to 10

Here, every tick mark is an integer — a whole number.

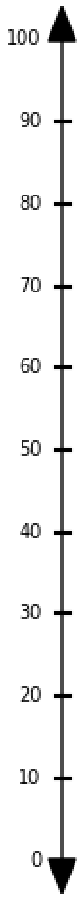
- Numbers to the **right** of zero are positive
- Numbers to the **left** of zero are negative

We can use this number line to *see* what happens when we add, subtract, or compare numbers.

Are there other ways to draw a number line?

Yes! Number lines can be drawn over different ranges and scales. For example, here is a number line that counts form -10 to 25 in steps of 5.

In fact, number lines don't even have to be horizontal. Here is a vertical number line that goes from 0 to 100 in steps of 10.



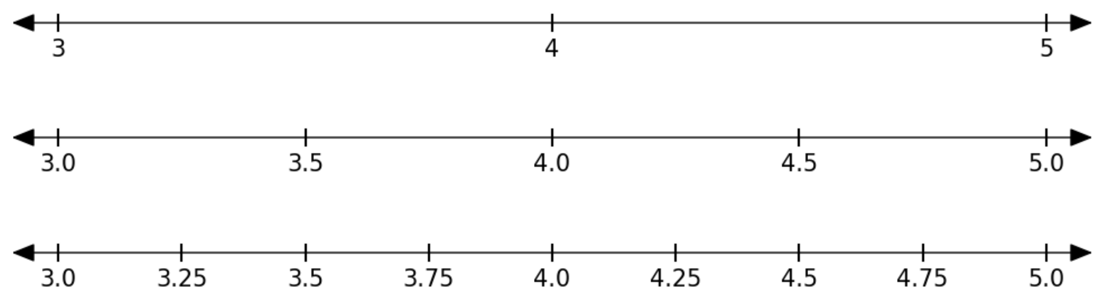
Can you think of any real world examples of number lines?

Here are a few examples:

- thermometer
- ruler
- timeline
- American football field
- volume slider on a phone

How many numbers are between 3 and 5?

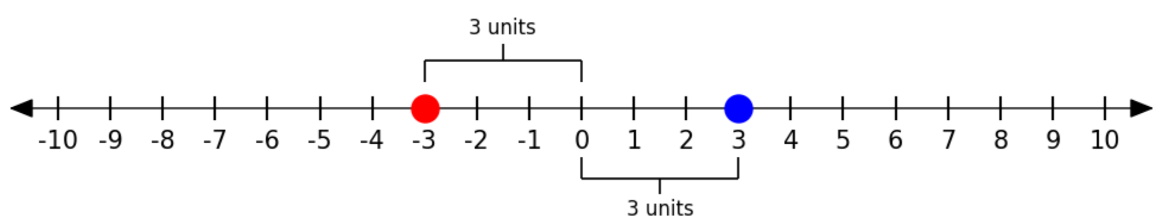
Though there are 2 integers between 3 and 5, the answer is not 2! There are infinitely many numbers between 3 and 5. Here are some number lines that might help convince you.



1.1.2 - Understanding Opposites

Let's look at a pair of numbers, 3 and -3.

These are called opposite numbers. They are the **same distance** from zero but on **opposite sides** of it.



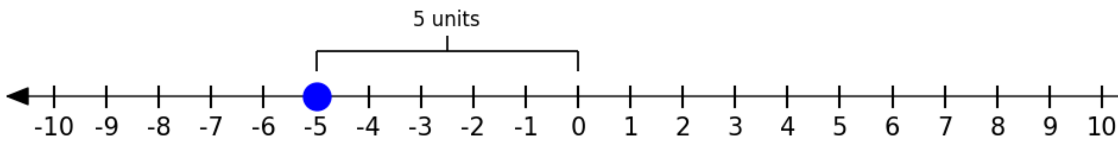
What is the opposite of zero?

The opposite of zero is zero. Zero is the only number that is its own opposite!

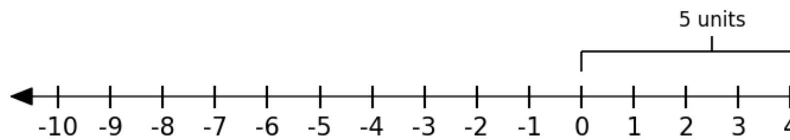
1.1.3 - What Is Absolute Value?

Absolute value ($|number|$) measures the **distance from zero**, no matter the direction.

Take a look at the number -5. The number line shows that it's absolute value is 5 because it is 5



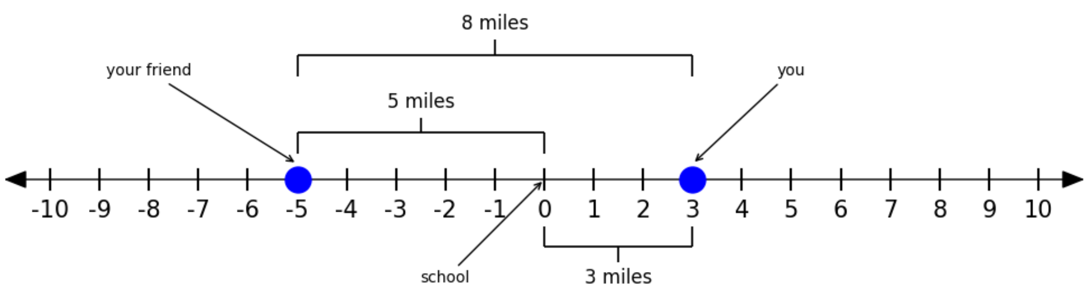
units away from zero.



You can see that |5| is also 5 for the same reason!

In the Real World

Absolute value is often used for describing the distance between two points. Suppose you live 3 miles to the east of the school and your best friend lives 5 miles to the west. How far apart are your houses? This is easy to see with a number line.



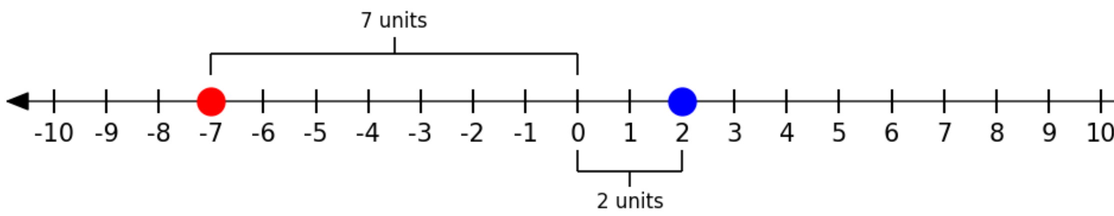
You can compute your distances by adding |-5| + |3|, by |-5 - 3|, or by |3 - (-5)|. All three of these give the same answer, 8 miles. What would change if we did not use absolute value?

Can the absolute value ever be negative?

Absolute value is **never** negative, because distance is never negative.

1.1.4 - Comparing Integers

We can also use the number line to compare values.



Let's compare 2 to -7.

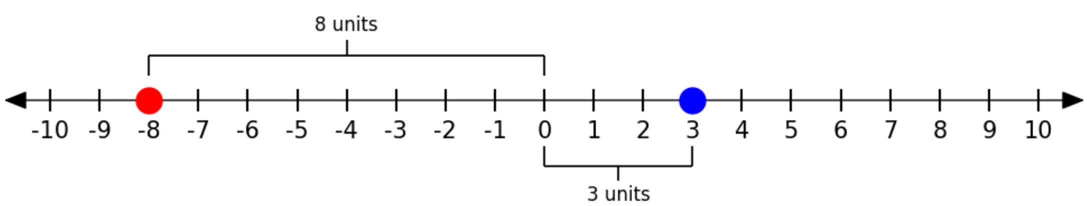
You can see from the number line that 2 is greater than (>) -7 because 2 is to the right of -7.

You can also see that -7 is further from zero than 2 and so |-7| > |2|.

> Which is **bigger**?
>
> It is easy to get confused here. When we say which is "bigger" we are asking which number is further to the right on the number line, **not** which one is furthest from zero.

Try comparing 3 to -8 using a number line.

3 > -8 because it is farther to the right but |-8| > |3| because -8 is further from zero.

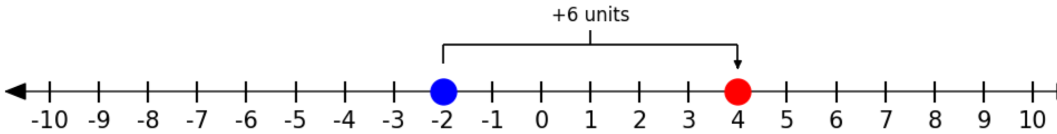


1.1.5 - Number Lines and Arithmetic

We can also use the number line to model **adding and subtracting** integers.

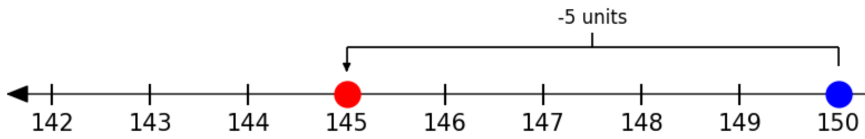
- To add a positive number, move **right**
- To add a negative number, move **left**

Examples:



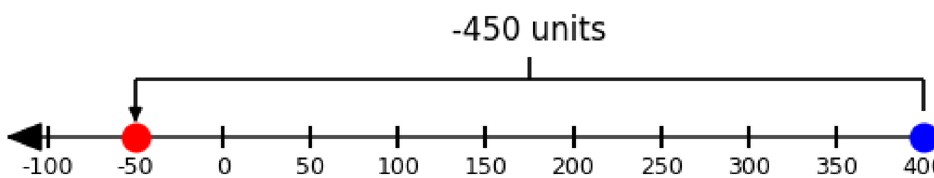
1. Addition: $-2+6=4$

> In the Real World
>
> Imagine that you are \$2 in debt. If someone pays you \$6 you can pay off the debt and have \$4 left over.



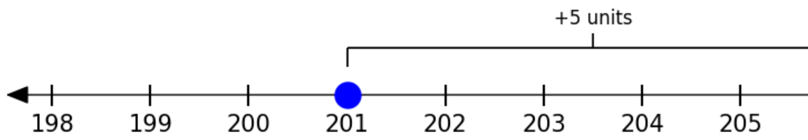
2. Adding a negative: $150+(-5)=145$

> In the Real World
>
> You have \$150 in the bank. The bank ads a fee for being under their \$200 minimum balance. You now have \$145.



3. Subtraction: $400-450=-50$

> In the Real World
>
> If you only have \$400 but spend \$450 on a credit card. You are now \$50 in debt.



4. Subtracting a negative: $201-(-5) = 206$

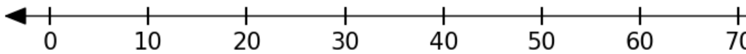
> **Can You think of a real-world example for the previous example?**
>
> Example: The bank made a mistake, you had \$201 in your account so they took off the \$5 fee. Now you have \$206.

Practice On Your Own

Working With Number Lines

1. Draw a number line that shows:
   a. -4, 0, and 3.
   b. Your age
   c. The number halfway between 5 and 9.

2. What question could match this number line?



Opposites

3. What is the opposite of 42?
4. What is the opposite of -3?

5. Draw a number line with two numbers that are opposites.
6. Does 3.5 have an opposite? If yes, what is it?

Comparing Numbers

7. Which number is **greater**, 5 or -10?
8. Which number has the greater absolute value, 5 or -10?
9. Is 28 bigger than -30?
10. Use (>) or (<) to compare:
    a. -11 _____ -13
    b. 7 _____ -2
    c. |-3| _____ |5|
11. Which is bigger?
    a. -4 or -5
    b. 3 or the opposite of 7
    c. |-5| or |4|
12. Use a number line to compare:
    a. -7 to 2.
    b. The year you were born and the current year

Addition and Subtraction

13. Show these on a number line:
    a. $-3 + 5$
    b. $3 - 5$
    c. $-3 + (-3)$
    d. $3 - (-3)$

Word Problems

14. Solve using a number line

    a. The temperature was -12°F. It warms up by 20°. What is the new temperature?

    b. A diver is 45 feet below sea level. She dives 30 feet deeper. How far down is she?

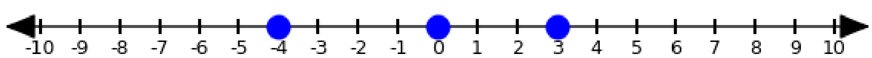
    c. Your bank account is at -\$8. You deposit \$5. What is your new balance?

Answer key

Working With Number Lines

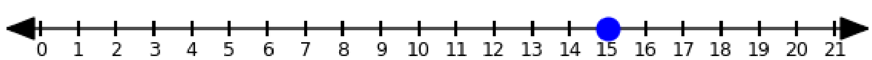
1. Draw a number line that shows:

   a. -4, 0, and 3.

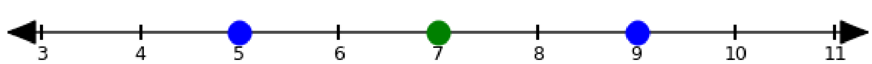


   b. Your age

   Answers vary. Here is what a 15 year old would show



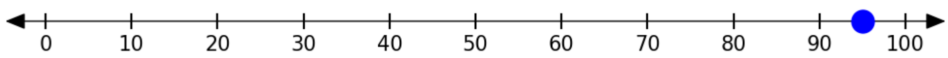
   c. The number halfway between 5 and 9.



   The answer is 7

2. What question could match this number line?

Answers vary. We could say "Plot the temperature on July 4th"



Opposites

3. What is the opposite of 42?

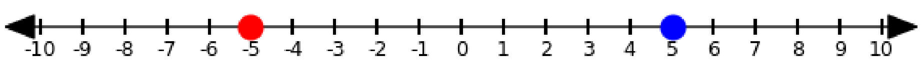
   -42

4. What is the opposite of -3?

   3

5. Draw a number line with two numbers that are opposites.

   Answers vary. Here is an example:



6. Does 3.5 have an opposite? If yes, what is it?

   Yes! The opposite is -3.5.

Comparing Numbers

7. Which number is **greater**, 5 or -10?

   5 is greater

8. Which number has the greater absolute value, 5 or -10?

   -10 has a greater absolute value

9. Is 28 bigger than -30?

   Yes, because it is further from zero

10. Use (>) or (<) to compare:

 a. -11 _____ -13

 -11 > -13

 b. 7 _____ -2

 7 > -2

 c. |-3| _____ |5|

 |-3| < |5|

11. Which is bigger?

 a. -4 or -5

 -4 is bigger because it is further to the right

 b. 3 or the opposite of 7

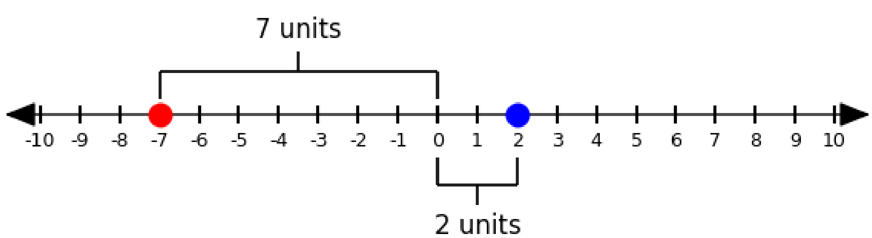
 3 is bigger. The opposite of 7 is -7 and 3 is further to the right.

 c. |-5| or |4|

 |-5| is bigger. |-5| is 5 which is further to the right than |4| which is 4.

12. Use a number line to compare:

 a. -7 to 2.



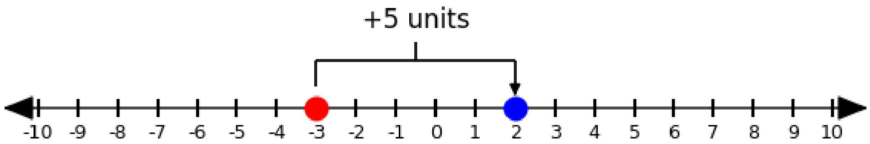
 b. The year you were born and the current year

Answers vary. I was born in 1982. The current year is 2025.

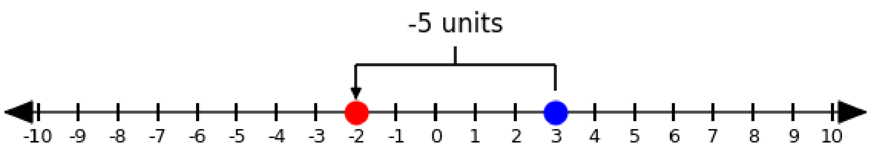
Addition and Subtraction

13. Show these on a number line:

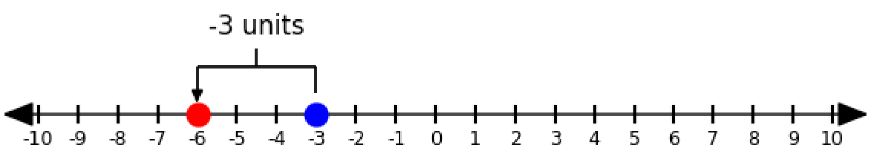
    a. $-3+5$



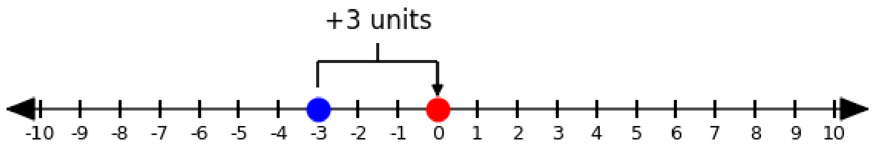
    b. $3-5$



    c. $-3+(-3)$



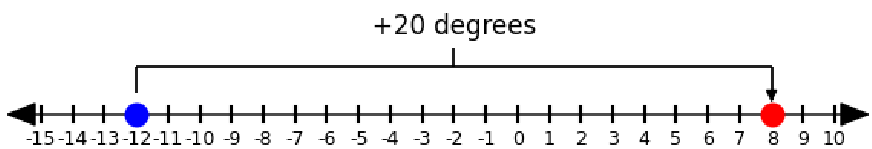
d. $3 - (-3)$



Word Problems

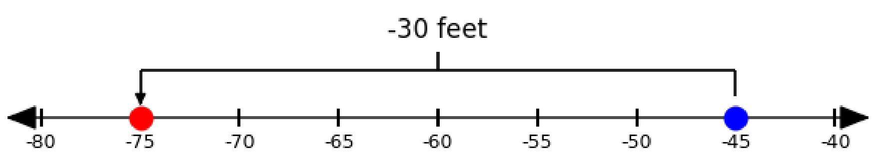
14. Solve using a number line

a. The temperature was -12°F. It warms up by 20°. What is the new temperature?



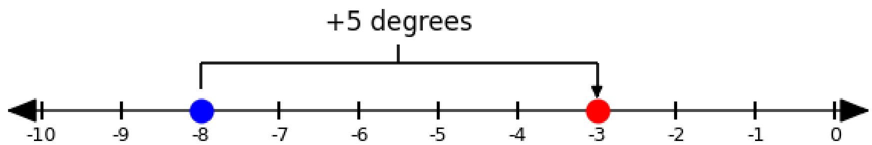
It is now 8 degrees

b. A diver is 45 feet below sea level. She dives 30 feet deeper. How far down is she?



She is now 75 feet down.

c. Your bank account is at -\$8. You deposit \$5. What is your new balance?



You now have -$3.

1.2 - Factors, Multiples & Prime Factorization

Have you ever had to split something up evenly — like slices of pizza or players on a team? That's really what factors are about: dividing numbers into equal parts.

In this lesson, you'll learn how to:

- Spot factors and multiples.
- Tell if a number is prime number or composite number.
- Break numbers into their basic building blocks using a factor tree.

You'll use these skills again and again — from simplifying fractions to solving equations.

Objectives

- ☐ Identify factors and multiples of integers
- ☐ Determine whether a number is prime or composite
- ☐ Use factor trees to find the prime factorization of a number

Vocabulary

composite number, factor, factor tree, multiple, prime factorization, prime number

Warm-Up

1. List all the whole-number factors of 12.
2. Find a multiple of 7 that is less than 50.
3. Is 11 a prime number? How do you know?

Learn Together

1.2.1 - What Are Factors?

A factor of a number is a whole number that divides it evenly — with no remainder.

Example:

The factors of 12 are: 1, 2, 3, 4, 6 and 2

That's because:

$1 \times 12 = 12$

$2 \times 6 = 12$

$3 \times 4 = 12$

Can a number have just one factor?

Only one number does: **1**. It only has itself as a factor and so it is neither prime nor composite!

1.2.2 - What Are Multiples?

A multiple is what you get when you multiply a number by 1, 2, 3, 4...

Example:

Here are the first few multiples of 5:

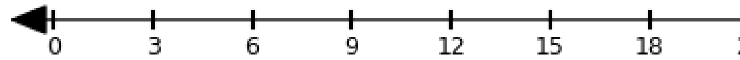
$5, 10, 15, 20, 25, 30, \ldots$

Multiples are useful when finding common denominators or common multiples later in algebra.

Where have we seen this before?

Multiples show up all over the place. When you skip count, you are using multiples. In the previous lesson, we used multiples to constuct number lines!

Example:



Here is a number line that shows multiples of three.

1.2.3 - Prime vs. Composite

A **prime number** has only 2 factors: 1 and itself.

Examples: 2, 3, 5, 7, 11, 13…

A **composite number** has more than 2 factors.

Examples: 4, 6, 8, 9, 10…

> In the Real World
>
> Prime numbers play a big role in **encryption**, which keeps your data safe when you shop or message online.

1.2.4 - Prime Factorization and Factor Trees

Every number can be broken into a **product of prime numbers** — sort of like breaking a LEGO® sculpture into individual bricks. These prime factors are the basic building blocks of all whole numbers.

We use **factor trees** to find these prime factors. This isn't just a fun trick — it builds your number sense: your ability to see patterns, understand how numbers are structured, and work confidently with them.

That number sense will come in handy later when you:

- Simplify fractions
- Solve equations
- Factor algebraic expressions
- Find common denominators

Let's build a factor tree for **360** to see how it works.

Steps to Make a Factor Tree

1. Start with a number:

Figure 1

2. Find any two numbers that multiply to give the number: $360 = 18 \times 20$

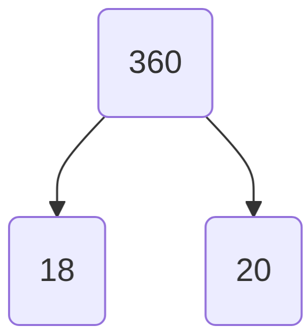


Figure 2

3. Break each of those numbers down further:

   - $18 = 3 \times 6$
   - $20 = 4 \times 5$

4. Keep going until all branches end in **prime numbers** (numbers that can't be factored anymore, like 2, 3, 5, 7…). We call the ends of the branches "leaves".

5. The prime factorization is the product of the leaves of the tree:

$$2 * 2 * 2 * 3 * 3 * 5 = 360$$

   This can be written more compactly by using the factor counts as exponents. There are three 2s and two 3s in this case and so we get…

$$2^3 * 3^2 * 5 = 360$$

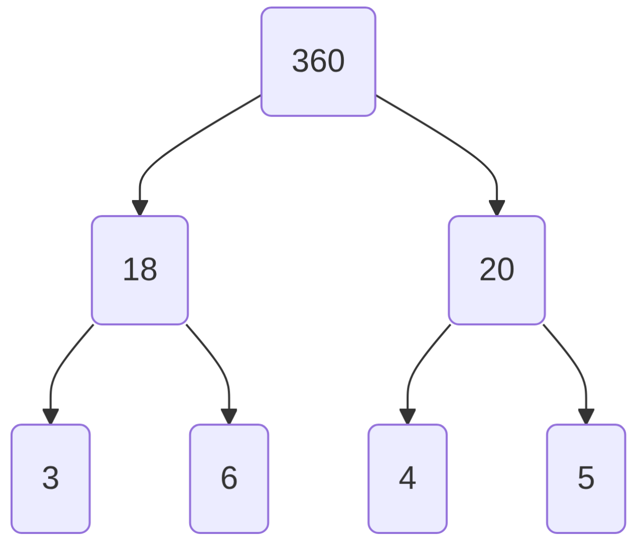


Figure 3

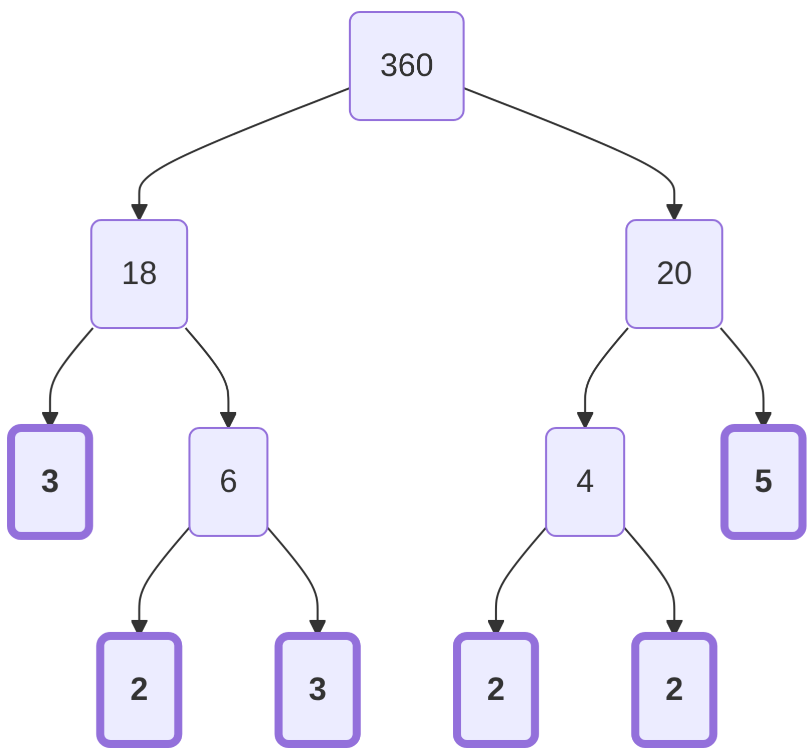


Figure 4

There are *many* factor trees for the number 360. For example, you could also have started with $360 = 3 * 120$.

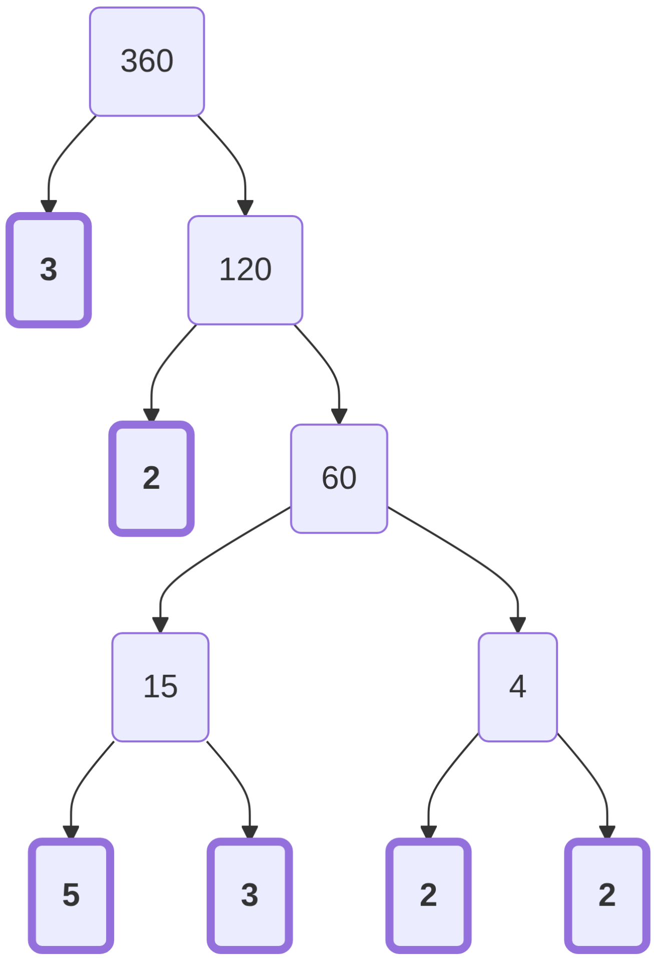


Figure 5

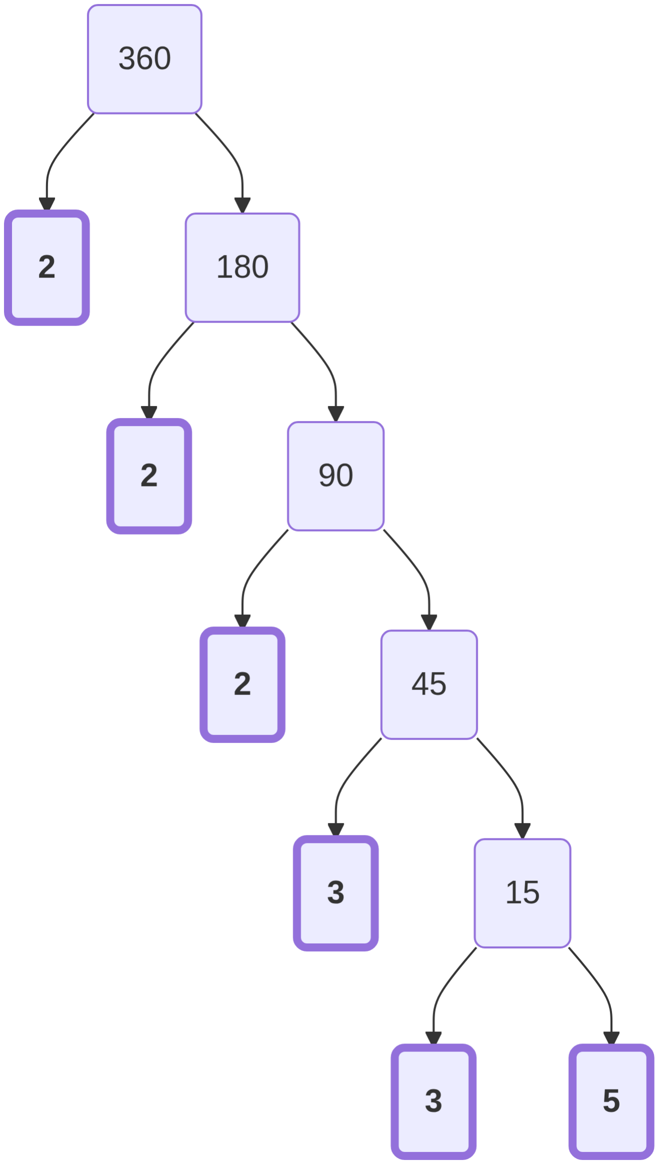
There are still three 2s, two 3s, and one 5, so the prime factorization does not change!

$$2^3 * 3^2 * 5 = 360$$

As long as you end up with the same prime numbers, the tree is correct!

Which factors should I start with?

There is no one right answer to this question. It depends on your goal. If your goal is finding easy numbers, you might start small. Notice that 360 is even that means it is divisible by 2. We could divide by 2 and keep going that way.



When you divide by the smallest (or biggest) factors, the tree tends to become deep. If you want smaller trees, you should start with factor pairs that are closer together like we did with the first factor tree for 360, splitting first with 18 and 20.

What About Negative Numbers?

If the number is negative, factor out a -1 first. Here is one possible factor tree for -24:

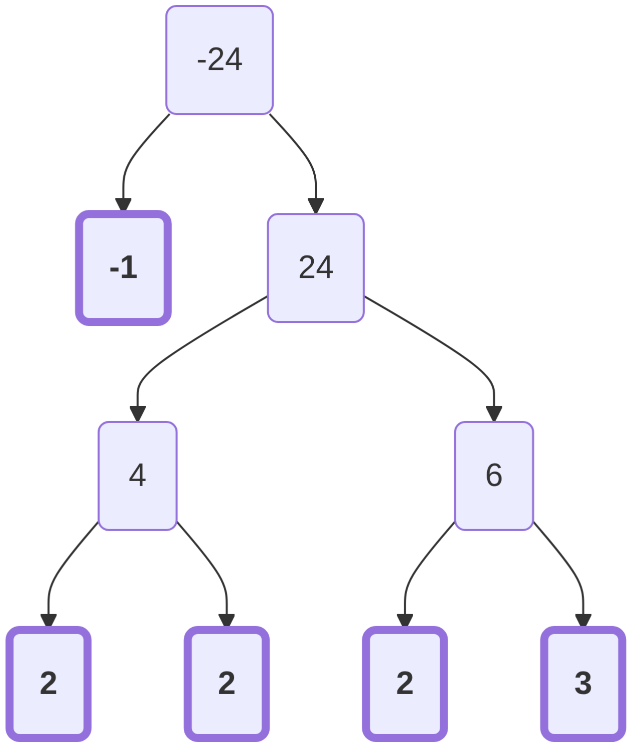


Figure 6

So, the prime factorization of -24 is…

$$-1 * 2^3 * 3 = -24$$

This will come in handy later when we factor algebraic expressions like $-x^2 + 4x$. It's often helpful to pull out a negative first!

Feeling overwhelmed?

If you struggle to come up with the factors for a number, you should check out the factor chart in the resources section of this book. It shows all of the factor pairs for many composite numbers!

> **Can you find at least one more factor tree for 360?**
>
> I have only shown you 3 of the 60 unique factor trees for 360!

Practice On Your Own

Factors & Multiples

1. List all the factors of:

   a. 16
   b. 18
   c. 27

2. List the first 5 multiples of:

   a. 4
   b. 9
   c. 12

Prime or Composite?

3. Label each number as **prime**, **composite**, or **neither**:

   a. 7
   b. 15
   c. 1
   d. 19
   e. 21

Complete the Factor Tree

4. Fill in the missing numbers.

   a.

b.

Factor Trees & Prime Factorization

5. Use a factor tree to find the prime factorization of:

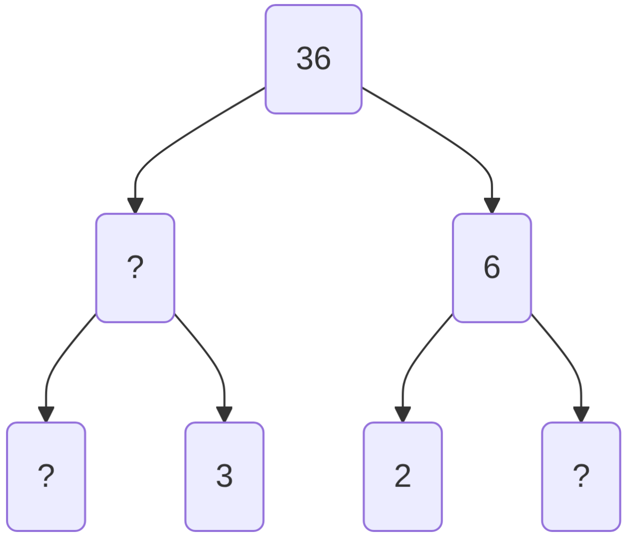


Figure 7

a. 24
b. 60
c. 100
d. 81
e. 72

Challenge

6. Can two different numbers have the same prime factorization? Why or why not?

Answer Key

Factors & Multiples

1. List all the factors of:

   a. 16

      1, 2, 4, 8, 16

   b. 18

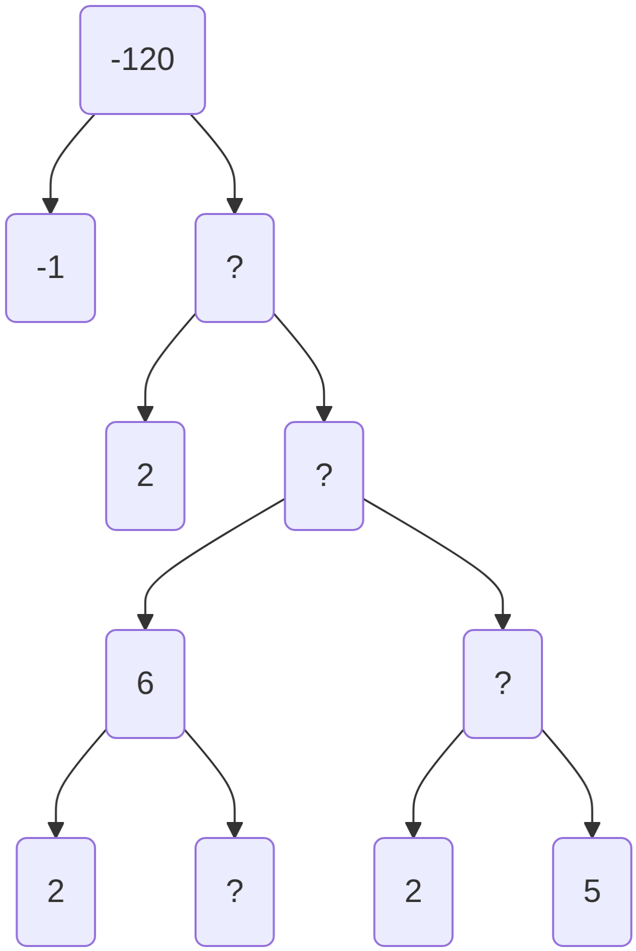


Figure 8

1, 2, 3, 6, 9, 18

c. 27

1, 3, 9, 27

2. List the first 5 multiples of:

a. 4

4, 8, 12, 16, 20

b. 9

8, 18, 27, 36, 45

c. 12

12, 24, 36, 48, 60

Prime or Composite?

3. Label each number as **prime**, **composite**, or **neither**:

a. 7

Prime

b. 15

Composite

c. 1

Neither

d. 19

Prime

e. 21

Composite

Complete the Factor Tree

4. Fill in the missing numbers.

a.

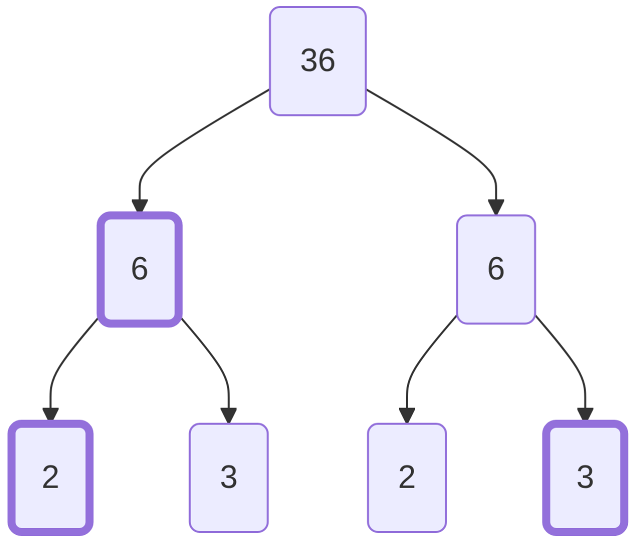


Figure 9

b.

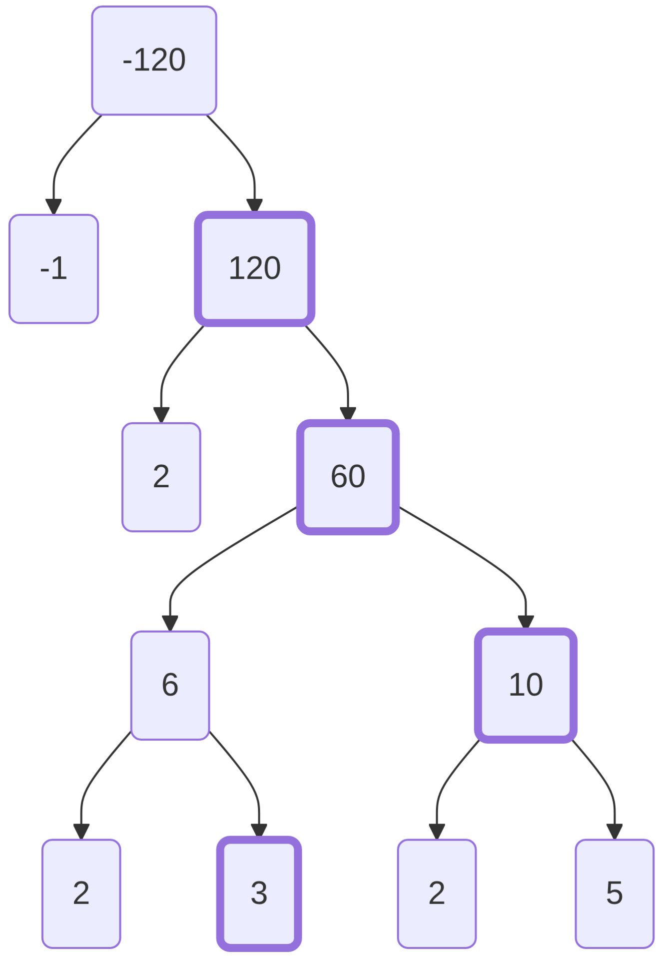


Figure 10

Factor Trees & Prime Factorization

5. Use a factor tree to find the prime factorization of:

   a. 24

      $2 \times 2 \times 2 \times 3 = 2^3 \times 3$

   b. 60

      $2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$

   c. 100

$2 \times 2 \times 5 \times 5 = 2^2 \times 5^2$

d. 81

$3 \times 3 \times 3 \times 3 = 3$

e. 72

$2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$

Challenge

6. Can two different numbers have the same prime factorization? Why or why not?

   No. Each number has a **unique** prime factorization. This is called the **Fundamental Theorem of Arithmetic**.

1.3 - GCF & Simplifying Fractions

Have you ever needed to divide things up fairly — like sharing snacks or making equal teams? The greatest common factor (GCF) helps you figure out the largest group size that works for both numbers.

In this lesson, you'll learn how to use prime factorization to find the GCF, and how that can help us simplify fractions. Learning GCF helps us make fractions and math problems simpler so they are easier to understand and solve.

Objectives

- ☐ Find the GCF using prime factorization
- ☐ Apply GCF to simplify fractions
- ☐ Solve problems involving shared quantities

Vocabulary

equivalent, greatest common factor, prime factorization, relatively prime, simplify

---

Warm-Up

1. Which number do you *think* has the most prime factors? What makes you think so?

   a. 20

   b. 30

   c. 45

   d. 53

2. What's the largest number that you think might divide **both** 12 and 18 evenly?
3. Here are four fractions. Which one doesn't belong and why?

a. $\frac{12}{18}$

b. $\frac{4}{9}$

c. $\frac{2}{3}$

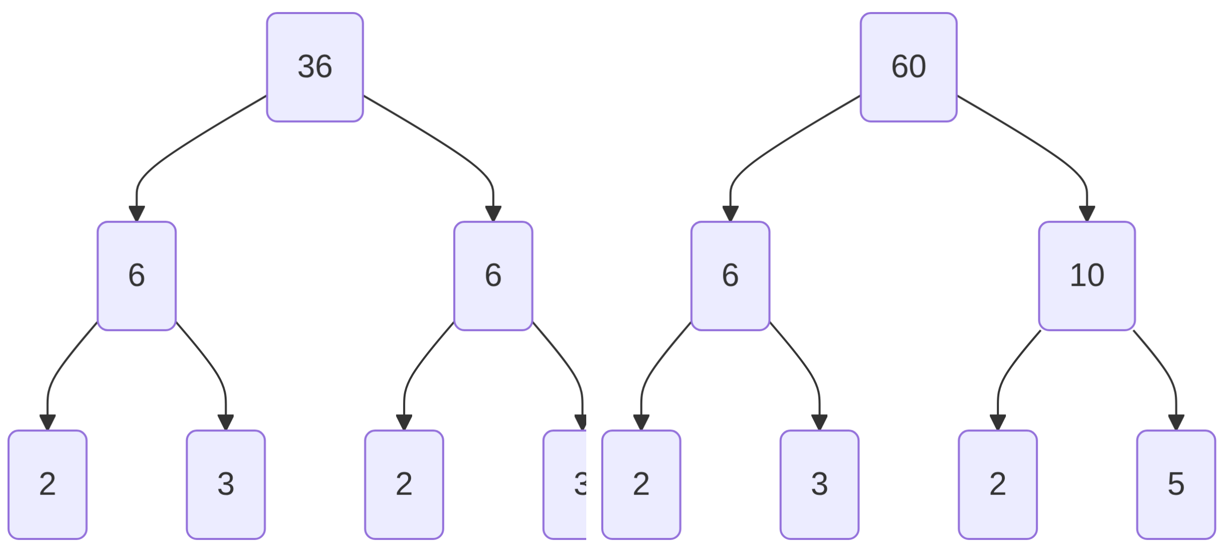
d. $\frac{24}{36}$

Learn Together

1.3.1 - Finding the GCF Using Factor Trees

To find the greatest common factor, we can break numbers into their **prime factors** using factor trees.

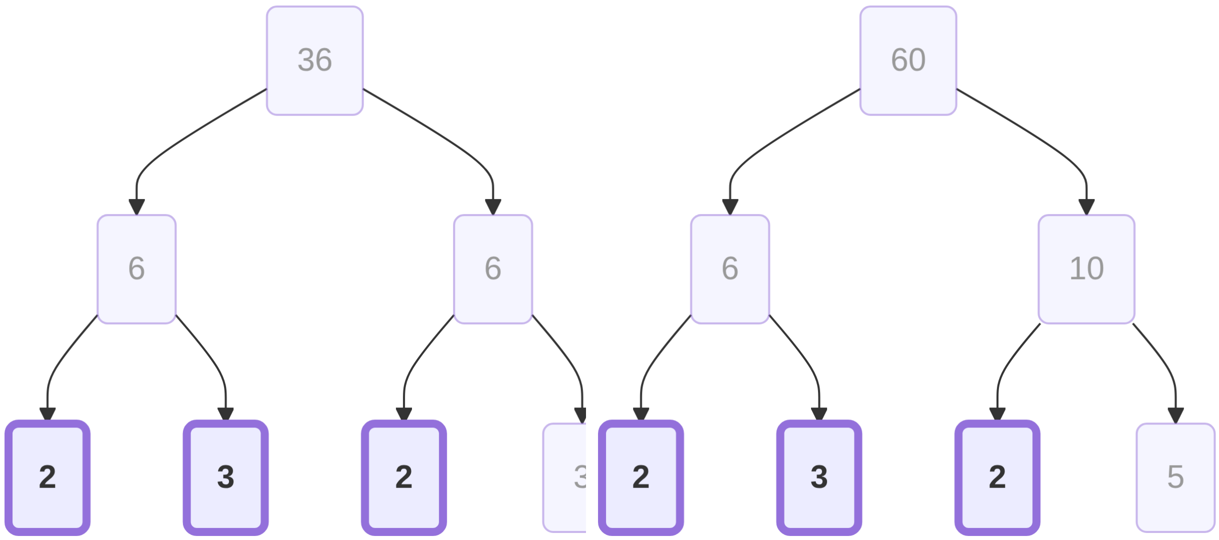
Let's try it with 36 and 60:



Now that we have the factor trees, we can use them to easily find the GCF by circling leaves that they both share.

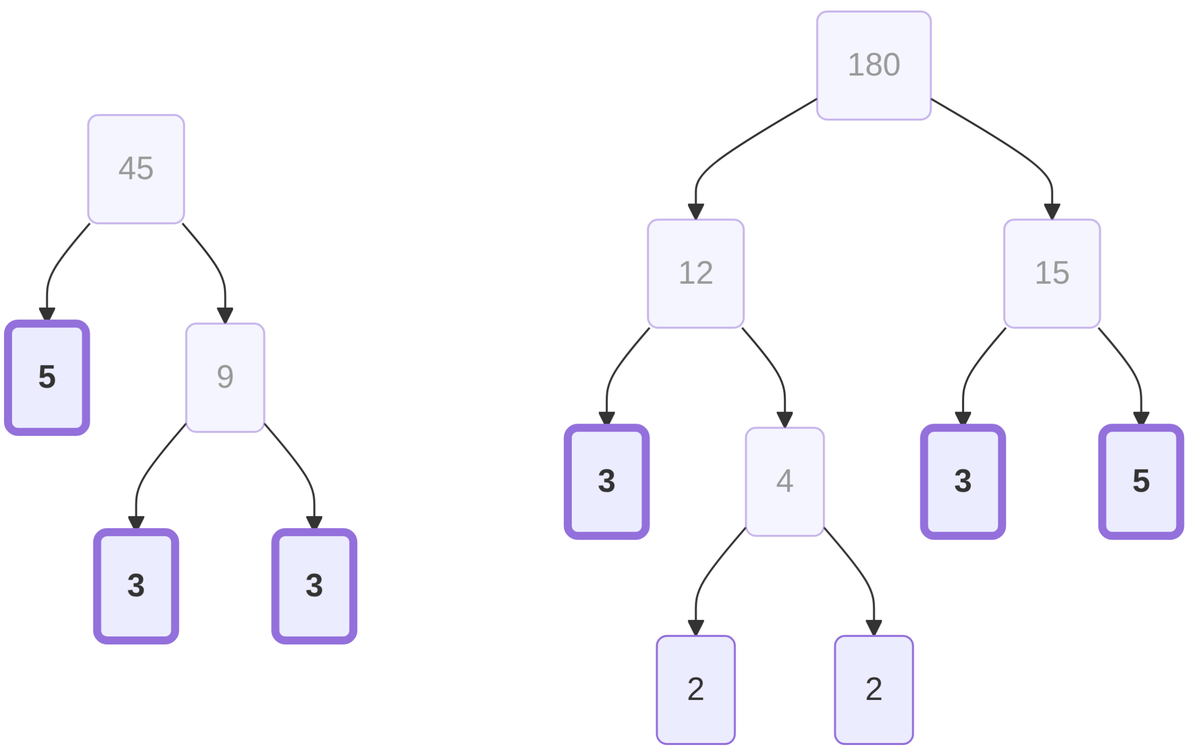
The numbers 36 and 60 share two 2s and one 3. We find the GCF by multiplying those shared factors.

$$2 * 2 * 3 = 12$$



The GCF for 36 and 60 is 12!

Can you find the greatest common factor between 45 and 180?

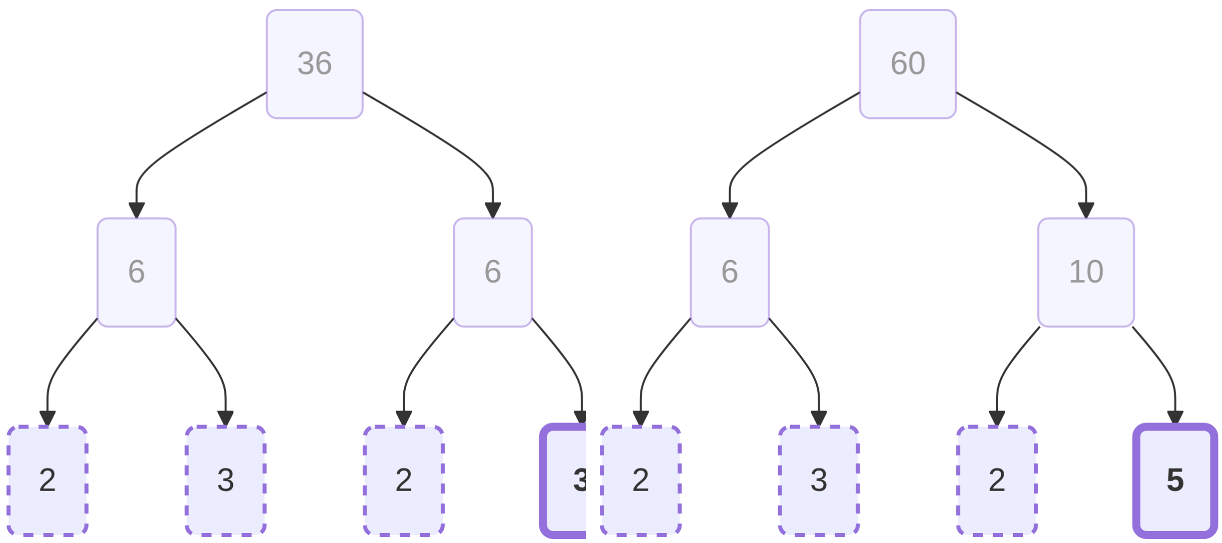


Both 45 and 180 share two 3s and one 5. So the GCF for 45 and 180 is:

$$3 * 3 * 5 = 45$$

1.3.2 - Using Factor Trees to Divide

You might have noticed that we did not circle **all** of the prime factors for 36 and 60. What did we leave behind and what does that mean?

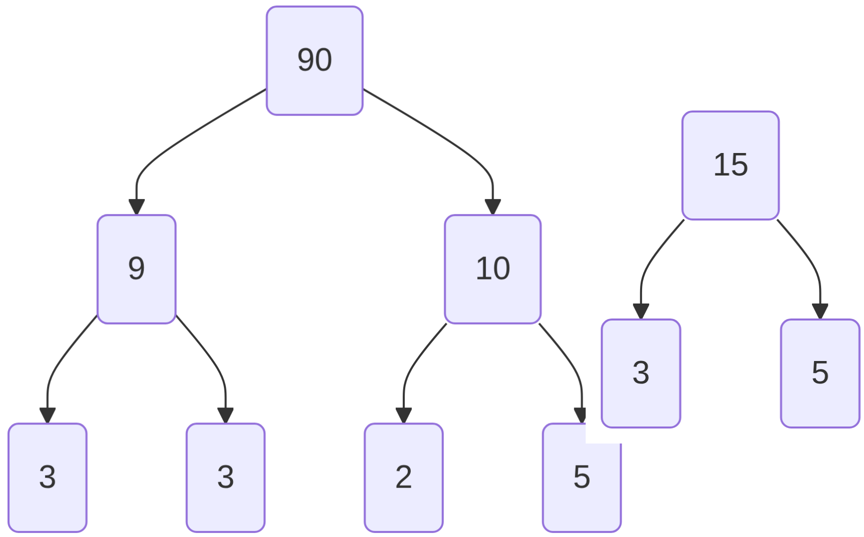


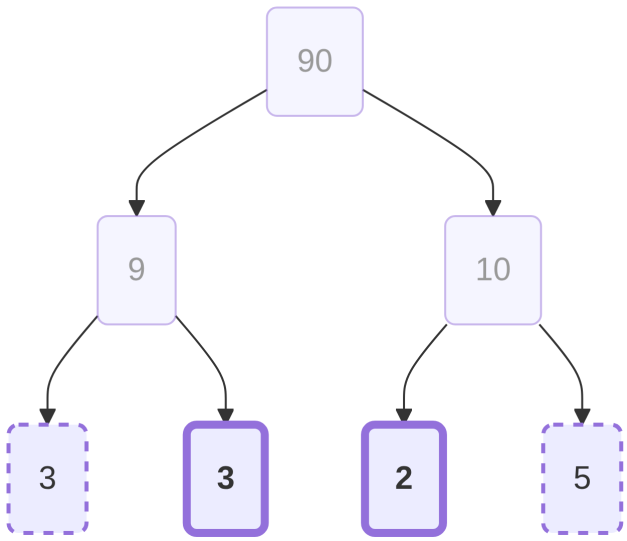
For 36 we left behind a 3 and for 60 we left behind a 5. What this means is that $36 \div 12$ is 3 and $60 \div 12$ is 5!"

Let's try another one

This time we will divide 90 by 15. Here are some factor trees to help us.

The number 15 has prime factors 3 and 5. We can divide 90 by 15 by crossing out those shared factors and multiplying what is left.



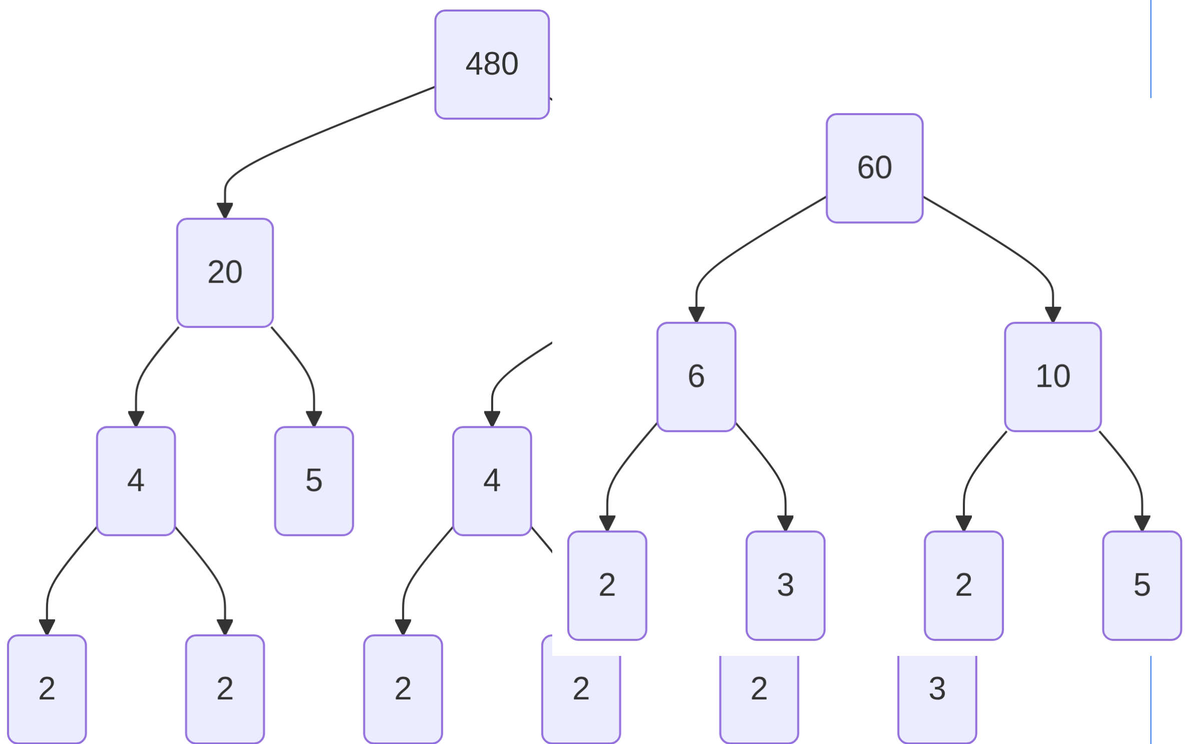


After crossing out 3 and 5 (the factors of 15) we are left with one 3 and one 2. This gives us...

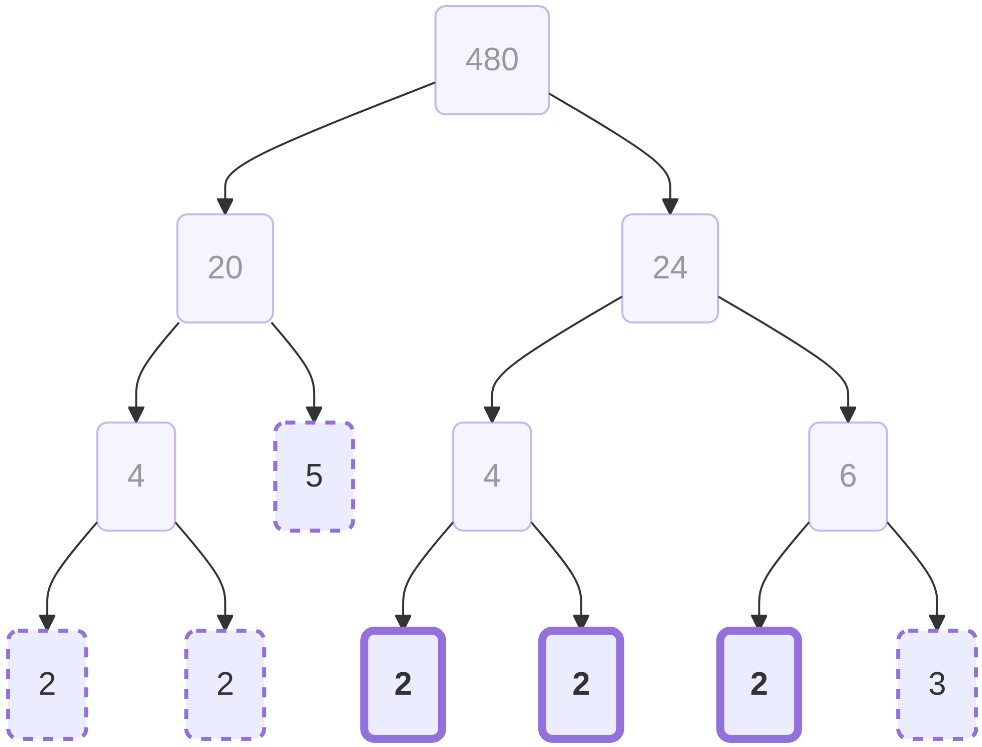
$$90 \div 15 = 3 * 2 = 6$$

Give it a try! Use factor trees to divide 480 by 60.

1. Find the factor trees for each number



2. Cross out the prime factors 480 shares with 60



3. Multiply the remaining prime factors to get the answer.

$$2 * 2 * 2 = 8$$

1.3.3 - Simplifying Fractions with the GCF

In the last section, dividing two numbers showed how to "cancel out" or eliminate shared factors. This is helpful when you want to simplify a fraction!

Simplifying a fraction means rewriting it in its simplest form. This makes a fraction as "small" or "basic" as possible without changing its value. Divide both the numerator (top number) and the denominator (bottom number) by their greatest common factor (GCF) – like finding the biggest number to divide both whole numbers in the previous examples. When a fraction is in its simplest form, the numerator and denominator don't share any factors other than 1

Let's go back to one we've already seen:

$$\frac{36}{60}$$

We found earlier that the GCF of 36 and 60 is 12. So to simplify, we divide both the top and bottom by 12:

$$\frac{36 \div 12}{60 \div 12} = \frac{3}{5}$$

This is just like the division you saw in the factor trees. We canceled out the factors they both had — two 2s and one 3 — and kept what was left.

Here's another:

$$\frac{90}{15}$$

We know that 15 is the GCF of 90 and 15. So:

$$\frac{90 \div 15}{15 \div 15} = \frac{6}{1} = 6$$

This tells us the fraction $\frac{90}{15}$ is just another way to write the number 6.

In other words:

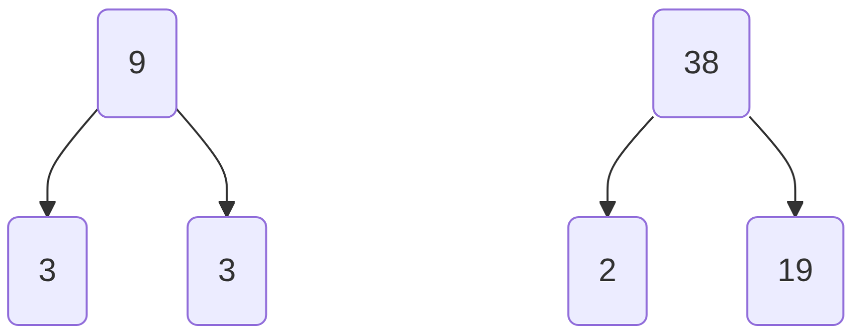
> Simplifying a fraction is just another way of dividing the numerator and denominator by their greatest common factor.

Factor trees help you *see* why this works by breaking the numbers into their building blocks.

What if the numerator and denominator don't share any factors?

If the numerator and denominator of a fraction share no common factors (besides 1) the two numbers are called relatively prime. In this case, the fraction cannot be simplified.
Example:

Simplify $\frac{9}{38}$.



Since 9 and 38 share no factors, $\frac{9}{38}$ is already in simplest form.

1.3.4 – Application: Simplifying with Recipes

Imagine you're following a recipe that makes a giant batch of cookies — way more than you need. You decide to cut the recipe down to a smaller size, but the measurements are a little awkward.

Here's what the recipe says:

- 36 cups of flour
- 60 cups of sugar

You don't want to bake that much — just a smaller, simpler version of the same cookie. But how do you shrink the recipe without changing how the cookies taste?

Let's treat the ingredients like a ratio:

$$\frac{\text{36 cups of flower}}{\text{60 cups sugar}}$$

This ratio tells us how much flour to use per amount of sugar. But the numbers are too big — and a little messy.

Just like with fractions, we can simplify this ratio by dividing both parts by their GCF. We already know the GCF of 36 and 60 is 12.

$$\frac{36 \div 12}{60 \div 12} = \frac{3}{5}$$

So for every 3 cups of flour, you need 5 cups of sugar.

Now you can make a smaller batch that keeps the same balance by finding multiples of the numerator and denominator. For example:

- 3 cups of flour
- 5 cups of sugar

Or double that:

- 6 cups of flour
- 10 cups of sugar

Simplifying the original recipe helped you find a cleaner ratio — one that's easier to scale up or down, depending on how many cookies you want.

> In the Real World
>
> Simplifying isn't just a math trick — it helps you work with numbers more easily in the real world. Whether you're adjusting recipes, mixing paint, or scaling blueprints, understanding fractions and simplifying them makes life easier.

Practice On Your Own

GCF Practice

1. Find the greatest common factor (GCF) of each pair:

   a. 20 and 30
   b. 36 and 45
   c. 18 and 48
   d. 30 and 42
   e. 50 and 65
   f. 72 and 90
   g. 81 and 108

2. Two numbers have a GCF of 6. One of the numbers is 18. What could the other number be? Give two possible answers.
3. Two numbers have a GCF of 1. What does that mean? Give an example.

Simplifying Fractions

4. Simplify each fraction:

    a. $\frac{18}{27}$
    b. $\frac{50}{100}$
    c. $\frac{14}{49}$
    d. $\frac{48}{60}$
    e. $\frac{84}{36}$
    f. $\frac{75}{90}$
    g. $\frac{99}{121}$
    h. $\frac{16}{40}$

5. Can a fraction be simplified if the GCF is 1? Explain your answer and give an example.

Word Problems

6. You have 72 juice boxes and 60 cookies. You want to make snack packs with the **same number** of each. You must use **all** the items.

    a. What's the greatest number of snack packs you can make?
    b. How many juice boxes and cookies go in each pack?

7. A store is making bundles using 108 pairs of socks and 144 shirts. Each bundle must have the same number of socks and the same number of shirts. There should be no leftovers.

    a. What is the greatest number of bundles they can make?
    b. How many socks and shirts will go in each bundle?

8. A painter mixes 84 ounces of red paint and 36 ounces of blue paint. He wants to pour the paint into small jars that are all the same. Each jar must have the same mix of red and blue paint.

    a. What is the greatest number of jars he can make with no paint left over?

b. How many ounces of red and blue paint will go in each jar?

Challenge Problems

9. Two numbers multiply to make 180. Their GCF is 6. What could the numbers be?
10. A teacher has 150 pencils and 100 pens. She wants to make gift bags with the same number of each. What is the most gift bags she can make with no leftovers?

Answer Key

GCF Practice

1. Find the greatest common factor (GCF) of each pair:
   a. 20 and 30

      10

   b. 36 and 45

      9

   c. 18 and 48

      6

   d. 30 and 42

      6

   e. 50 and 65

      5

   f. 72 and 90

      18

   g. 81 and 108

      27

2. Two numbers have a GCF of 6. One of the numbers is 18. What could the other number be? Give two possible answers.

   Example: 30 and 42

3. Two numbers have a GCF of 1. What does that mean? Give an example.

> **The numbers are relatively prime. For example: 8 and 15.**

Simplifying Fractions

4. Simplify each fraction:

   a. $\frac{18}{27}$

      $\frac{2}{3}$

   b. $\frac{50}{100}$

      $\frac{1}{2}$

   c. $\frac{14}{49}$

      $\frac{2}{7}$

   d. $\frac{48}{60}$

      $\frac{4}{5}$

   e. $\frac{84}{36}$

      $\frac{7}{3}$

   f. $\frac{75}{90}$

      $\frac{5}{6}$

   g. $\frac{99}{121}$

      $\frac{9}{11}$

   h. $\frac{16}{40}$

      $\frac{2}{5}$

5. Can a fraction be simplified if the GCF is 1? Explain your answer and give an example.

   No. The numerator and denominator are relatively prime and so nothing can cancel out.

Word Problems

6. You have 72 juice boxes and 60 cookies. You want to make snack packs with the **same number** of each. You must use **all** the items.

a. What's the greatest number of snack packs you can make?

12 snack packs

b. How many juice boxes and cookies go in each pack?

6 juice boxes and 5 cookies

7. A store is making bundles using 108 pairs of socks and 144 shirts. Each bundle must have the same number of socks and the same number of shirts. There should be no leftovers.

   a. What is the greatest number of bundles they can make?

   12 bundles

   b. How many pairs of socks and how many shirts will go in each bundle?

   9 pairs of socks and 4 shirts

8. A painter mixes 84 ounces of red paint and 36 ounces of blue paint. He wants to pour the paint into small jars that are all the same. Each jar must have the same mix of red and blue paint.

   a. What is the greatest number of jars he can make with no paint left over?

   12 jars

   b. How many ounces of red and blue paint will go in each jar?

   7 ounces of red paint and 2 ounces of blue paint

Challenge Problems

9. Two numbers multiply to make 180. Their GCF is 6. What could the numbers be?

   30 and 6 or -30 and -6

10. A teacher has 150 pencils and 100 pens. She wants to make as many identical gift bags as possible, but she also wants to save at least 10 pencils for future use. What is the greatest number of gift bags she can make now?

    30 gift bags with 3 pencils and 5 pens in each with 10 pencils left over

1.4 – Fractions, Decimals & Percents

Fractions, decimals, and percents are just different ways of showing the same thing — a part of a whole. Whether you're splitting a pizza, measuring a distance, or shopping during a sale, these numbers are everywhere.

In this lesson, you'll learn how to move between these forms and understand how they relate to each other. We'll build on what you already know about simplifying fractions to make conversions easier.

Objectives

- ☐ Convert between fractions, decimals, and percents
- ☐ Use place value and division to convert fractions into decimals
- ☐ Understand and use common benchmark values like ½ = 0.5 = 50%

Vocabulary

fraction, decimal, percent, convert, equivalent, place value

Warm-Up

1. Which one doesn't belong?

   a. $\frac{1}{2}$
   b. 0.25
   c. 50%
   d. 0.5

   (Explain your reasoning. There's more than one right answer.)

2. You drink half a bottle of water. Your friend drinks 60% of a bottle. Who drank more?

3. Which two of these do you think are closest in value?

a. $\frac{2}{3}$
b. 70%
c. 0.25
d. $\frac{3}{4}$

Learn Together

1.4.1 – What Are Fractions, Decimals, and Percents?

Fractions, decimals, and percents are all ways of showing a **part of a whole**.

- A **fraction** shows how many parts out of a total (e.g. $\frac{2}{5} = 2$ out of 5 equal parts).
- A **decimal** uses place value and powers of 10 (e.g. $0.4 = 4$ tenths).
- A **percent** means "per 100" — it's like a fraction with a denominator of 100 (e.g. 25% $= \frac{25}{100}$).

Let's look at one example in all three forms:

Name	Example	Meaning
Fraction	$\frac{1}{2}$	1 out of 2 parts
Decimal	0.5	5 tenths
Percent	50%	50 out of 100 parts

These are all equivalent – they mean the same thing, just written in different ways.

Here's another example:

All of the following represent the same value: $\frac{2}{5} = 0.4 = 40\%$.

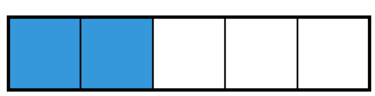


Figure 11: Fraction grid showing the fraction $\frac{2}{5}$

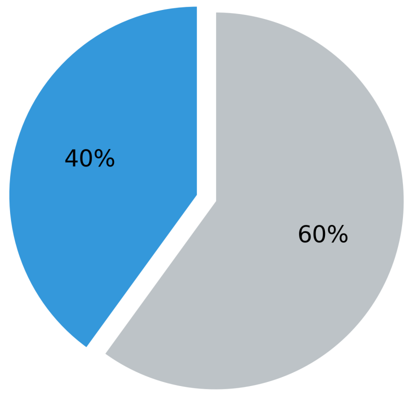


Figure 12: Pie chart highlighting 40%

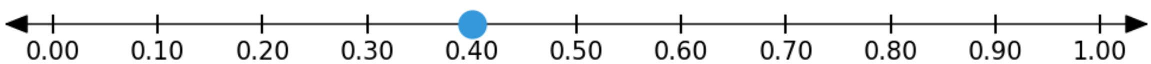


Figure 13: Number line with 0.4 marked

Each image shows a **part of a whole** in a different way — but they all represent the same quantity.

1.4.2 – Converting Fractions to Decimals

To convert a fraction to a decimal, divide the numerator by the denominator.

Let's try:

$$\frac{3}{4} = 3 \div 4 = 0.75$$

Use a calculator if needed!

Try these:

- $\frac{1}{2} = 1 \div 2 =$ _____
- $\frac{2}{5} = 2 \div 5 =$ _____
- $\frac{1}{8} = 1 \div 8 =$ _____

Some fractions turn into decimals that **stop** (terminate), and some **repeat forever**.

Fraction	Decimal
$\frac{1}{4}$	0.25
$\frac{1}{3}$	0.333...
$\frac{2}{3}$	0.666...
$\frac{1}{5}$	0.2

If the decimal repeats, we write a bar over the repeating part: $\frac{1}{3} = 0.\overline{3}$

1.4.3 – Converting Decimals to Percents

To turn a decimal into a percent, multiply by 100 (move the decimal two places right) and add a percent sign:

- $0.5 \to 50$
- $0.75 \to 75$
- $0.08 \to 8$

Try this: What percent is 0.6?

$$0.6 \times 100 = 60$$

1.4.4 – Converting Percents to Decimals and Fractions

To convert a percent to a decimal, divide by 100 (move the decimal two places left):

- 75
- 20
- 5

To write it as a fraction, think of the percent as "out of 100," then simplify if possible:

- 75
- 20
- 5

Tip

If the percent ends in a 0 or 5, it often simplifies nicely as a fraction.

1.4.5 – Using a Conversion Triangle

Here's a quick triangle you can draw to help you remember how to switch between all three forms.

```
     Fraction
     /      \
    /        \
Decimal <--> Percent
```

- **Fraction → Decimal**: divide top ÷ bottom
- **Decimal → Percent**: × 100
- **Percent → Decimal**: ÷ 100
- **Fraction → Percent**: Convert to decimal first, then multiply by 100

1.4.6 – Benchmark Values

Some conversions are so common that it helps to **memorize** them. These are called **benchmark values**:

Fraction	Decimal	Percent
$\frac{1}{2}$	0.5	50%
$\frac{1}{4}$	0.25	25%
$\frac{3}{4}$	0.75	75%
$\frac{1}{3}$	0.333...	33.3%
$\frac{2}{3}$	0.666...	66.6%
$\frac{1}{5}$	0.2	20%
$\frac{1}{10}$	0.1	10%

Practice On Your Own

Conversions Practice

1. Convert each fraction to a decimal:

   a. $\frac{1}{2}$
   b. $\frac{2}{5}$
   c. $\frac{3}{4}$
   d. $\frac{1}{8}$
   e. $\frac{2}{3}$

2. Convert each decimal to a percent:

   a. 0.4
   b. 0.75
   c. 0.125
   d. 0.01
   e. 0.6

3. Convert each percent to a fraction (in simplest form):

   a. 50%
   b. 25%
   c. 80%
   d. 12.5%
   e. 5%

4. Convert each percent to a decimal:

   a. 15%
   b. 30%
   c. 0.5%
   d. 66%
   e. 3.5%

Real-World Problems

5. A store is offering 25% off all items. If a shirt costs $20, how much is the discount? What's the sale price?

6. A survey shows that 60% of students like pizza. If 120 students were asked, how many said they like pizza?

7. A recipe calls for 0.75 cups of sugar. Write this as a fraction and as a percent.
8. A basketball player makes 18 out of 24 free throws. What fraction of shots did they make? Write it as a percent.

Challenge Problems

9. Write a fraction that equals 0.625 and simplify it.
10. A student scored 36 out of 45 on a quiz. What percent did they get?

Answer Key

Conversions Practice

1. Fractions to Decimals:
   a. 0.5
   b. 0.4
   c. 0.75
   d. 0.125
   e. 0.666...
2. Decimals to Percents:
   a. 40%
   b. 75%
   c. 12.5%
   d. 1%
   e. 60%
3. Percents to Fractions:
   a. $\frac{1}{2}$
   b. $\frac{1}{4}$
   c. $\frac{4}{5}$
   d. $\frac{1}{8}$
   e. $\frac{1}{20}$
4. Percents to Decimals:
   a. 0.15

b. 0.3
c. 0.005
d. 0.66
e. 0.035

Real-World Problems

5. 25% of \$20 is \$5, so the sale price is \$15.
6. 60% of 120 is 72 students.
7. $0.75 = \frac{3}{4} = 75\%$
8. $\frac{18}{24} = \frac{3}{4} = 75$

Challenge Problems

9. $0.625 = \frac{625}{1000} = \frac{5}{8}$
10. $\frac{36}{45} = \frac{4}{5} = 80$

1.5 – Multiply, Divide & Compare Fractions

In Algebra, you'll work with fractions all the time — but not usually adding and subtracting them. What matters most is knowing how to **multiply**, **divide**, and **simplify** fractions to make expressions easier to work with.

In this lesson, you'll learn how to multiply and divide fractions with confidence, and how to compare them logically. These are skills you'll use throughout Algebra — especially when solving equations and simplifying rational expressions.

Objectives

- ☐ Multiply and divide fractions and mixed numbers
- ☐ Simplify results using the GCF
- ☐ Compare fractions using multiplication or benchmarks

Vocabulary

multiply, divide, fraction, reciprocal, simplify, compare

Warm-Up

1. What's   of 12?
2. What's the reciprocal of 4/5?
3. Is ¾ greater than ? How do you know?

Learn Together

1.5.1 – Multiplying Fractions

Multiply straight across:

Numerators × Numerators Denominators × Denominators

Example:

$$\frac{2}{3} \times \frac{5}{6} = \frac{2 \times 5}{3 \times 6} = \frac{10}{18}$$

Simplify: 10/18 → **5/9**

If possible, **simplify before multiplying**:

$$\frac{4}{5} \times \frac{10}{12} = \frac{2}{5} \times \frac{5}{6} = \frac{2 \times 5}{5 \times 6} = \frac{10}{30} = \frac{1}{3}$$

1.5.2 – Multiplying Mixed Numbers

Step 1: Convert to improper fractions **Step 2:** Multiply **Step 3:** Simplify if needed

Example: 1½ × 2/3 → 1½ = 3/2 → 3/2 × 2/3 = 6/6 = **1**

1.5.3 – Dividing Fractions

To divide fractions, **multiply by the reciprocal** (flip the second fraction):

Example:

$$\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \times \frac{5}{2} = \frac{15}{8}$$

Mixed number: **1**

> Don't flip the first fraction!
>
> Only flip the second fraction — the one you're dividing by.

1.5.4 – Comparing Fractions

Method 1: Use Benchmarks

Estimate which is closer to ½, , or 1.

Method 2: Use Common Denominators

Compare and ¾ → = 8/12 → ¾ = 9/12 So ¾ is greater.

Method 3: Cross Multiply

$$\frac{3}{5} \quad \text{vs.} \quad \frac{2}{3} 3 \times 3 = 9 5 \times 2 = 10 \text{Since } 10 > 9,\ \frac{2}{3} > \frac{3}{5}$$

1.5.5 – Why This Matters in Algebra

You'll use fraction multiplication and division when:

- Solving equations with fractional coefficients
- Simplifying rational expressions
- Finding slope
- Working with proportions

In the Real World

You're painting a wall. The can says it covers of a room. You buy 3 cans. How much will it cover?
3 × = **2** → Enough for **2 rooms**

Practice On Your Own

Multiplying Fractions

1. Multiply:

   a. $2/3 \times 3/5$
   b. $4/7 \times 14/15$
   c. $1½ \times$
   d. $3/10 \times 5/6$

Dividing Fractions

2. Divide:

   a. $¾ \div ½$
   b. $2/5 \div ¾$
   c. $1 \div$
   d. $9/10 \div 3/5$

Compare the Fractions

3. Use any method to compare. Write $>$, $<$, or $=$:

   a. ___ ¾
   b. 3/5 ___ 2/3
   c. 5/6 ___ 10/12
   d. 7/10 ___ 2/3

Challenge

4. A recipe uses ¾ cup of flour for one batch. You want to make 2½ batches. How much flour will you need?

5. A board is 5/6 meter long. You cut it into 3 equal pieces. How long is each piece?

Answer Key

1. Multiplication

a. 6/15 = 2/5
b. 4/7 × 14/15 → cancel 7 & 14 → 4 × 2/1 × 15 = 8/15
c. 3/2 × 4/5 = 12/10 = 6/5 or 1
d. 15/60 = ¼

---

2. Division

a. ¾ ÷ ½ = ¾ × 2/1 = 6/4 = 1½
b. 2/5 × 4/3 = 8/15
c. 4/3 ÷ 2/3 = 4/3 × 3/2 = 12/6 = 2
d. 9/10 ÷ 3/5 = 9/10 × 5/3 = 45/30 = 3/2 = 1½

---

3. Comparisons

a. < ¾
b. 3/5 < 2/3
c. 5/6 = 10/12
d. 7/10 > 2/3

---

4. Recipe
¾ × 2½ = ¾ × 5/2 = 15/8 = 1 cups
5. Cutting the board
5/6 ÷ 3 = 5/6 × 1/3 = 5/18 meter per piece

1.6 – Order of Operations

What does this equal?

$$6 + 2 \times 3$$

If you said 24 — you're not alone. But that's **not** the correct answer. Math has rules for what to do first. These rules are called the **order of operations**, and they're essential for simplifying expressions the right way.

In this lesson, you'll learn to follow those rules consistently — even when negatives, fractions, and grouping are involved.

Objectives

- ☐ Apply the correct order of operations (PEMDAS)
- ☐ Evaluate expressions involving integers and fractions
- ☐ Recognize common mistakes and avoid them

Vocabulary

expression, order of operations, parentheses, exponent, multiply, divide, add, subtract

Warm-Up

1. Simplify: $3 + 6 \times 2$
2. True or False: $(4 + 3) \times 2 = 4 + (3 \times 2)$
3. What does $\frac{1}{2} \times (4 + 2)$ equal?

Learn Together

1.6.1 – The Order Matters

The correct order is:

P – Parentheses **E** – Exponents **MD** – Multiply or Divide (left to right) **AS** – Add or Subtract (left to right)

This is often remembered as **PEMDAS**.

> Multiply doesn't always come before divide. Same with add and subtract. You go **left to right** when they appear together.

1.6.2 – Basic Examples

1. $$5 + 3 \times 2 = 5 + 6 = 11$$

2. $$(5 + 3) \times 2 = 8 \times 2 = 16$$

3. $$8 - 12 \div 3 = 8 - 4 = 4$$

1.6.3 – Include Negatives & Fractions

Watch how signs and simplification interact:

Example 1:

$$-3 \times (4 - 7) = -3 \times (-3) = 9$$

Example 2:

$$\frac{1}{2} \times (6 + 2) = \frac{1}{2} \times 8 = 4$$

Example 3:

$$\frac{3 + 5}{2} = \frac{8}{2} = 4$$

> 🔥 Be careful with negatives!
>
> -3^2 means $-(3^2) = -9$ but $(-3)^2 = 9$

1.6.4 – Complex Expressions

Put it all together:

Example:

$$4 + \frac{1}{2} \times (6 - 2)^2$$

Step 1: Parentheses → (6 - 2) = 4 Step 2: Exponents → 4^2 = 16 Step 3: Multiply → ½ × 16 = 8 Step 4: Add → 4 + 8 = **12**

1.6.5 – Why This Matters in Algebra

Later, expressions will include:

- **Variables**
- **Distributive property**
- **Combining like terms**

If students can't simplify numbers correctly, the algebra won't work either.

In the Real World

You get a 25% off coupon and a \$10 gift card. The item costs \$40. Which should be applied first?
25% of 40 = \$10 → pay \$30, then gift card = \$20 But **40 - 10 = 30, then 25% off = \$22.50**
The order changes the result!

Practice On Your Own

Basic Order of Operations

1. Simplify:

   a. $4 + 6 \times 2$
   b. $(4 + 6) \times 2$
   c. $12 \div 4 \times 3$
   d. $12 \div (4 \times 3)$

Include Negatives & Fractions

2. Simplify:

   a. $-2 \times (3 - 5)$
   b. $\frac{1}{2} \times (8 + 4)$
   c. $(6 - 2)^2 \div 2$
   d. $(3 + 5) \div 2$

Expression Breakdown

3. Simplify:

   a. $5 + 2 \times (6 - 1)$
   b. $(12 - 4)^2 \div 4$
   c. $10 - 3 \times (2 + 1)$
   d. $\frac{3}{4} \times (12 - 4)$

Challenge

4. Show two different ways to simplify:

$$8 - 3 + 2$$

Which is correct, and why?

5. Simplify:

$$\frac{4 + 6 \div 3}{2}$$

Answer Key

1. Basic

a. 16
b. 20
c. 9
d. 1

---

2. Negatives/Fractions

a. $-2 \times (-2) = 4$
b. $\frac{1}{2} \times 12 = 6$

c. $(4)^2 \div 2 = 16 \div 2 = 8$
d. $8 \div 2 = 4$

---

3. Expressions

a. $5 + 2 \times 5 = 5 + 10 = 15$
b. $8^2 \div 4 = 64 \div 4 = 16$
c. $10 - 3 \times 3 = 10 - 9 = 1$
d. $\frac{3}{4} \times 8 = 6$

---

4. Challenge
$8 - 3 + 2 = (8 - 3) + 2 = 5 + 2 =$ **7** → This is correct because subtraction and addition go **left to right**

---

5. Final Problem
$6 \div 3 = 2 \rightarrow 4 + 2 = 6 \rightarrow 6 \div 2 =$ **3**

Part II

Unit 2: Algebraic Expressions

Introduction

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2.1 Evaluating Expressions

You'll learn how to evaluate algebraic expressions by substituting values for variables.

Objectives

- ☐ Evaluate expressions with one or more variables
- ☐ Use correct substitution and order
- ☐ Check your work for accuracy

Vocabulary

expression, evaluate, substitute, variable

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

2.2 Inputs, Outputs & Function Machines (Intro)

This introductory lesson explains how functions work using simple input-output models. This is the foundation for understanding functions throughout the course.

Objectives

- ☐ Understand the concept of a function
- ☐ Match inputs with outputs
- ☐ Identify function rules from patterns

Vocabulary

input, output, function, function rule

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

Part III

Unit 3: Solving Equations

Introduction

This unit is where Algebra really begins to feel like solving puzzles. You'll learn how to isolate variables, understand balance, and make sense of problems that come up in everyday life.

What You'll Learn

By the end of this unit, you'll be able to:

- Solve one- and two-step equations using inverse operations
- Distribute and combine like terms in multi-step equations
- Move variables to one side of the equation
- Identify when equations have no or infinite solutions
- Write and solve equations from word problems and contexts

Topics in This Unit

3. Solving One- and Two-Step Equations

Use inverse operations to find solutions.

3. Multi-Step Equations with Distribution

Distribute, simplify, and solve more complex equations.

3. Equations with Variables on Both Sides

Move all variable terms to one side, then solve.

3. No Solution vs. Infinite Solutions

Learn to recognize when an equation has no solution or all numbers work.

3. Writing Equations from Contexts

Translate real-world problems into equations.

3. Solving with Tables, Graphs & Rules

Connect functions to equations and problem-solving.

How to Use This Unit

You'll find plenty of examples, visuals, and practice to help you develop confidence in solving equations from both numbers and words!

3.1 Solving One-Step & Two-Step Equations

In this lesson, students will learn how to solve one-step and two-step equations using inverse operations. This foundational skill sets the stage for solving more complex equations in future lessons.

Objectives

- ☐ Use inverse operations to isolate the variable
- ☐ Solve one-step and two-step equations involving addition, subtraction, multiplication, or division
- ☐ Check solutions by substitution

Vocabulary

equation, inverse operations, solution, variable

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

3.2 Multi-Step Equations with Distribution

This lesson extends equation solving to multi-step problems, including those that require the distributive property and combining like terms.

Objectives

- ☐ Apply the distributive property to simplify equations
- ☐ Combine like terms before solving
- ☐ Solve multi-step equations with multiple operations

Vocabulary

distributive property, like terms, combine, simplify

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

3.3 Equations with Variables on Both Sides

Students will learn how to solve equations where variables appear on both sides of the equals sign, reinforcing the concept of balancing and simplifying equations.

Objectives

- ☐ Move variable terms to one side of the equation
- ☐ Simplify both sides before solving
- ☐ Identify equations with no or infinite solutions

Vocabulary

combine like terms, variable, no solution, infinite solutions

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

3.4 No Solution vs. Infinite Solutions

This lesson focuses on identifying when equations have no solution or infinitely many solutions and how to justify those conclusions.

Objectives

- ☐ Recognize inconsistent equations with no solution
- ☐ Identify dependent equations with infinite solutions
- ☐ Justify solutions using substitution or reasoning

Vocabulary

identity, contradiction, solution set, consistent, inconsistent

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

3.5 Writing Equations from Real-Life Contexts

Students will translate real-world scenarios into algebraic equations, helping them understand the connection between math and everyday problem solving.

Objectives

- ☐ Identify quantities and relationships in word problems
- ☐ Write algebraic equations to represent situations
- ☐ Solve and interpret solutions in context

Vocabulary

context, representation, translate, real-world

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

3.6 Solving with Tables, Graphs & Rules (Function Tie-In)

This lesson introduces multiple representations of relationships — including tables, graphs, and rules — to show how equations can be connected to functions.

Objectives

- ☐ Solve equations by analyzing input-output tables
- ☐ Interpret relationships from graphs and equations
- ☐ Connect equations to real-world patterns

Vocabulary

input, output, table, function, rule, graph

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

Part IV

Unit 4: Graphs and Patterns

Introduction

In this unit, we'll use visual and numerical patterns to understand how algebraic relationships behave. This helps us prepare for graphing and working with functions in more depth.

What You'll Learn

- Recognize and extend arithmetic and geometric patterns
- Build and interpret tables
- Graph expressions and equations
- Compare linear models using graphs

Topics in This Unit

4. Graphing Expressions with Tables

Use input-output tables to generate points.

4. Interpreting Graphs in Context

Make sense of graphs in stories and real-life settings.

4. Arithmetic vs. Geometric Patterns

Identify whether change is constant or multiplicative.

4. Linear Modeling & Rate of Change

Build linear functions and interpret slope in context.

4. Estimating and Checking with Graphs

Use visuals to verify solutions.

How to Use This Unit

Graphing builds a strong link between abstract algebra and concrete understanding. Let's get visual!

4.1 Graphing Expressions with Tables

In this lesson, students will learn how to create tables of values for algebraic expressions and plot them on a coordinate plane. This builds foundational understanding of how algebraic rules connect to visual patterns.

Objectives

- ☐ Generate tables of values from algebraic expressions
- ☐ Graph ordered pairs on the coordinate plane
- ☐ Recognize linear patterns in tables and graphs

Vocabulary

expression, table, ordered pair, coordinate plane, input, output

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

4.2 Interpreting Graphs in Context

Students will examine graphs that represent real-world scenarios and learn how to describe the relationships shown. Emphasis is placed on labeling axes, identifying trends, and understanding what changes in slope mean.

Objectives

- ☐ Identify variables and units from graph labels
- ☐ Describe trends in linear graphs
- ☐ Interpret slope and intercepts in context

Vocabulary

x-axis, y-axis, slope, intercept, context, trend

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

4.3 Arithmetic vs. Geometric Patterns

Students will compare arithmetic and geometric patterns and recognize how they grow. This helps build pattern recognition and introduces exponential growth.

Objectives

- ☐ Identify arithmetic patterns using constant differences
- ☐ Identify geometric patterns using constant ratios
- ☐ Generate sequences and compare their growth

Vocabulary

arithmetic, geometric, sequence, common difference, common ratio, pattern

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

4.4 Linear Modeling & Rate of Change

This lesson focuses on creating linear models from real-life data. Students will identify constant rates of change and use equations to model situations.

Objectives

- ☐ Recognize and describe constant rate of change
- ☐ Write linear equations to represent situations
- ☐ Interpret slope and intercepts from data

Vocabulary

linear model, rate of change, slope, intercept, data

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

4.5 Estimating and Checking with Graphs

Students will use graphs to estimate values and verify solutions to equations. This lesson ties visual reasoning to algebraic work.

Objectives

- ☐ Estimate input or output values from a graph
- ☐ Use a graph to verify equation solutions
- ☐ Analyze how accurate a graph-based solution is

Vocabulary

estimate, graph, solution, verify, input, output

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

Part V

Unit 5: Inequalities

Introduction

Sometimes in life, it's not about finding the exact number — it's about knowing what's greater or less. In this unit, you'll explore how to express and graph inequalities.

What You'll Learn

- Solve and graph inequalities on number lines
- Write inequalities from real-world contexts
- Understand "greater than" and "less than" symbols
- Explore compound inequalities (optional)

Topics in This Unit

5. One- and Two-Step Inequalities

Use similar steps as equations to isolate variables.

5. Graphing on a Number Line

Use open and closed circles to represent solutions.

5. Writing Inequalities from Situations

Turn words into math using inequality symbols.

5. Interpreting Graphs with Constraints

Match real-world limits to graphs.

5. Compound Inequalities (Optional)

Handle ranges like “between 2 and 5”.

How to Use This Unit

Use drawings and comparisons to make inequality concepts more concrete and real-world focused.

5.1 One- and Two-Step Inequalities

In this lesson, students will learn how to solve one-step and two-step inequalities and graph the solutions on a number line.

Objectives

- ☐ Solve one-step inequalities using addition, subtraction, multiplication, and division
- ☐ Solve two-step inequalities
- ☐ Graph the solution sets on a number line

Vocabulary

inequality, solution, greater than, less than, number line

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

5.2 Graphing on a Number Line

Students will practice representing solutions to inequalities by graphing them on a number line, including open and closed circles.

Objectives

- ☐ Understand the use of open and closed circles on a number line
- ☐ Graph simple inequalities
- ☐ Interpret solution sets visually

Vocabulary

number line, open circle, closed circle, graph, solution set

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

5.3 Writing Inequalities from Situations

This lesson teaches students to write inequalities based on verbal descriptions and real-world contexts.

Objectives

- ☐ Translate real-world problems into inequalities
- ☐ Identify keywords that signal inequality relationships
- ☐ Solve and interpret contextual inequalities

Vocabulary

verbal model, inequality, context, translate, interpret

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

5.4 Interpreting Graphs with Constraints

Students will explore how to read and make sense of graphs that include constraints or limited domains and ranges.

Objectives

- ☐ Analyze graphs that include limited domains or ranges
- ☐ Interpret constraints in real-world situations
- ☐ Relate inequalities to graphical representations

Vocabulary

constraint, domain, range, graph, inequality

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

5.5 Compound Inequalities (Optional)

Students will be introduced to compound inequalities, learning how to solve and graph problems with two connected inequalities.

Objectives

- ☐ Understand compound inequalities using 'and' and 'or'
- ☐ Solve compound inequalities
- ☐ Graph compound inequalities on a number line

Vocabulary

compound inequality, and, or, solution set, number line

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

Part VI

Unit 6: Linear Relationships

Introduction

Linear equations are a powerful way to describe change. Whether it's cost, speed, or growth, this unit shows how lines help us understand the world.

What You'll Learn

- Graph lines using slope and intercepts
- Interpret slope as a rate of change
- Write equations from tables, graphs, or situations
- Compare different linear situations

Topics in This Unit

6. Coordinate Plane & Graphing

Plot ordered pairs and recognize axes.

6. Understanding Slope

Learn how steepness shows change.

6. Slope-Intercept Form

Graph and write lines using $y = mx + b$.

6. Writing Equations from Graphs or Words

Use information to build your own equations.

6. Comparing Models

See how different lines behave and what they represent.

6. Applications

Use linear models for real-world math.

How to Use This Unit

This unit brings it all together — tables, equations, and graphs help us tell a full story.

6.1 The Coordinate Plane and Graphing from Tables

This lesson introduces the coordinate plane and helps students practice plotting points and graphing from tables.

Objectives

- ☐ Identify and label the x- and y-axes
- ☐ Plot ordered pairs on the coordinate plane
- ☐ Graph data from tables

Vocabulary

coordinate plane, x-axis, y-axis, origin, ordered pair

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

6.2 Understanding Slope as Rate of Change

Students will explore slope as a measure of how one quantity changes in relation to another, using graphs and real-world contexts.

Objectives

- ☐ Define slope as a rate of change
- ☐ Interpret slope from a graph or context
- ☐ Calculate slope using tables or graphs

Vocabulary

slope, rate of change, rise, run, linear relationship

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

6.3 Slope-Intercept Form

This lesson introduces the slope-intercept form of a linear equation and how to use it to graph lines.

Objectives

- ☐ Understand the form $y = mx + b$
- ☐ Identify slope and y-intercept
- ☐ Graph a line using slope and intercept

Vocabulary

slope-intercept form, slope, y-intercept, linear equation

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

6.4 Writing Equations from Graphs or Words

Students learn to write linear equations from graphs, tables, or written descriptions of relationships.

Objectives

- ☐ Write linear equations from graphs or data
- ☐ Translate real-world relationships into equations
- ☐ Use slope and intercept in context

Vocabulary

linear equation, slope, y-intercept, context, model

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

6.5 Comparing Linear Models from Graphs or Data

Students compare multiple linear models by analyzing graphs and data sets.

Objectives

- ☐ Compare different linear relationships
- ☐ Analyze graphs and tables for patterns
- ☐ Interpret slope and intercept in context

Vocabulary

linear model, compare, rate of change, initial value

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

6.6 Applications: Cost, Speed, Growth

This lesson applies linear modeling to real-life contexts like cost, speed, and growth.

Objectives

- ☐ Apply linear equations to real-life situations
- ☐ Create and interpret graphs in context
- ☐ Understand the meaning of slope and intercept in real-life problems

Vocabulary

cost, speed, growth, context, linear relationship

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

Part VII

Unit 7: Exponents and Powers

Introduction

Exponents let us write repeated multiplication more easily. In this unit, you'll learn the rules for working with exponents to simplify expressions.

What You'll Learn

- Multiply and divide expressions with exponents
- Apply exponent rules (no scientific notation)
- Understand zero and negative exponents

Topics in This Unit

7. Multiplying with Exponents

Use the product rule.

7. Dividing with Exponents

Use the quotient rule.

7. Power of a Power

Apply powers to powers.

7. Zero & Negative Exponents

Learn their meaning and use them simply.

How to Use This Unit

Use guided examples and repetition to get comfortable with patterns in exponent rules.

7.1 Multiplying with Exponents

In this lesson, you'll learn how to multiply expressions that contain exponents. This is a key part of working with powers and simplifying expressions efficiently.

Objectives

- ☐ Multiply powers with the same base
- ☐ Understand and apply the product of powers rule
- ☐ Simplify expressions with exponents

Vocabulary

exponent, base, product of powers rule

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

7.2 Dividing with Exponents

This lesson focuses on how to divide expressions with the same base using exponents. You'll build on what you know about multiplication and simplify complex expressions.

Objectives

- ☐ Divide powers with the same base
- ☐ Apply the quotient of powers rule
- ☐ Simplify expressions involving division and exponents

Vocabulary

quotient, base, exponent

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

7.3 Power of a Power

You'll learn how to raise an exponent to another exponent. This is useful for simplifying more complex expressions and working with formulas.

Objectives

- ☐ Use the power of a power rule
- ☐ Simplify nested exponents
- ☐ Combine exponent rules to simplify expressions

Vocabulary

exponent, power of a power, simplify

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

7.4 Zero and Negative Exponents (Intro only)

This lesson introduces zero and negative exponents. You'll explore what these mean and how they behave in expressions.

Objectives

- ☐ Understand and apply the zero exponent rule
- ☐ Explore the meaning of negative exponents
- ☐ Simplify expressions with zero and negative exponents

Vocabulary

zero exponent, negative exponent, reciprocal

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

Part VIII

Unit 8: Quadratic Thinking

Introduction

Quadratic equations make parabolas, not lines! This unit introduces key forms and solution methods, especially factoring and the quadratic formula.

What You'll Learn

- Identify quadratic forms
- Factor simple trinomials
- Solve quadratics by factoring and formula
- Compare graphs of quadratics and lines

Topics in This Unit

8. Recognizing Quadratics

Understand what makes an equation quadratic.

8. Factoring

Break expressions into binomials.

8. Solving by Factoring

Set equal to zero and find solutions.

8. Quadratic Formula (Intro)

Use the formula to solve when factoring is hard.

8. Graphing Parabolas

See how the shape differs from linear graphs.

How to Use This Unit

This unit prepares students for what's tested and what's useful long-term.

8.1 Recognizing Quadratic Equations

In this lesson, students will learn to identify quadratic equations by their standard form and understand what makes them different from linear equations.

Objectives

- ☐ Recognize quadratic equations in standard form: $ax^2 + bx + c$
- ☐ Identify the key features that make an equation quadratic
- ☐ Distinguish between linear and quadratic relationships

Vocabulary

quadratic, parabola, standard form, coefficient

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

8.2 Factoring Simple Quadratics

This lesson introduces the process of factoring quadratic expressions where the leading coefficient is 1.

Objectives

- ☐ Factor simple quadratic expressions of the form $x^2 + bx + c$
- ☐ Use factoring to find the roots of a quadratic equation
- ☐ Check factored expressions by expanding

Vocabulary

factor, root, binomial, quadratic expression

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

8.3 Solving by Factoring

Students will learn how to solve quadratic equations by factoring and setting each factor equal to zero.

Objectives

- ☐ Solve quadratic equations using factoring
- ☐ Apply the zero product property
- ☐ Interpret solutions in context

Vocabulary

zero product property, solution, quadratic equation

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

8.4 The Quadratic Formula (Intro)

This lesson introduces the quadratic formula as a method for solving any quadratic equation, especially when factoring is not straightforward.

Objectives

- ☐ Identify the components of the quadratic formula
- ☐ Use the quadratic formula to solve quadratic equations
- ☐ Understand when the formula is useful compared to factoring

Vocabulary

quadratic formula, discriminant, solution, standard form

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

8.5 Graphing Parabolas by Table & Comparing with Linear

Students will use tables to graph quadratic functions and compare their shapes and behaviors with linear functions.

Objectives

- ☐ Graph quadratic functions using input-output tables
- ☐ Identify the vertex and axis of symmetry from a graph
- ☐ Compare quadratic and linear graphs

Vocabulary

vertex, axis of symmetry, parabola, table of values

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

Part IX

Unit 9: Systems of Equations

Introduction

Sometimes two equations work together. A system shows how two relationships interact. This unit is optional but powerful.

What You'll Learn

- Understand what a system is
- Solve systems by graphing or substitution
- Apply systems to real-life problems

Topics in This Unit

9. What Is a System?

Understand the idea of two equations and one solution.

9. Solving by Graphing

Find where lines intersect.

9. Substitution (Optional)

Plug one equation into another to find solutions.

9. Word Problems with Systems

Use systems to model stories or scenarios.

How to Use This Unit

Best taught after mastery of equations and graphing — use visuals and pair work!

9.1 What Is a System?

This lesson introduces the concept of a system of equations—two or more equations that share variables. Students learn how solutions to systems represent points that satisfy all equations involved.

Objectives

- ☐ Define what a system of equations is
- ☐ Identify solutions to systems from graphs and tables
- ☐ Understand consistent vs. inconsistent systems

Vocabulary

system of equations, solution, consistent, inconsistent

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

9.2 Solving by Graphing

Students learn to solve systems of equations by graphing each equation and identifying the intersection point. This visual approach builds on prior graphing skills and deepens conceptual understanding.

Objectives

- ☐ Graph linear equations
- ☐ Determine the solution to a system by finding where two lines intersect
- ☐ Interpret real-world meaning from the graph

Vocabulary

graphing, intersection, solution, coordinate

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

9.3 Substitution Method (Optional)

This lesson introduces substitution as an algebraic method to solve systems of equations. Students practice solving one equation for a variable and substituting into the other.

Objectives

- ☐ Solve one equation for one variable
- ☐ Substitute expressions to solve systems algebraically
- ☐ Check solutions for accuracy

Vocabulary

substitution, isolate, expression, solution

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

9.4 Word Problems with Systems

Students apply their knowledge of systems of equations to solve word problems. They learn to represent real-life situations with systems and interpret their solutions in context.

Objectives

- ☐ Translate real-world scenarios into systems of equations
- ☐ Solve using graphing or substitution
- ☐ Interpret solutions in context

Vocabulary

system, context, real-world, model

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

Part X

Unit 10: Cumulative Review and Projects

Introduction

The final unit ties everything together. Reflect, review, and show what you know through projects and EOC practice.

What You'll Learn

- Use vocabulary and concepts from the whole course
- Create graphs, tables, and equations for real-world data
- Review core topics for the final exam or state test

Topics in This Unit

10. Vocabulary Review

Define and use terms from the course.

10. Real-World Projects

Apply math to something meaningful.

10. Presentations

Explain your thinking visually and clearly.

10. Final Review or EOC Practice

Practice key problems to prepare for success.

How to Use This Unit

Encourage creativity and depth of understanding. Show off what you've learned!

10.1 Vocabulary Review

In this lesson, we'll review the key vocabulary from this course and reinforce understanding through matching, definitions, and real-world examples.

Objectives

- ☐ Review and define key algebra vocabulary terms
- ☐ Apply vocabulary in math contexts and explanations
- ☐ Recognize terms in problems and relate them to math operations

Vocabulary

term, coefficient, constant, expression, equation, solution, function

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

10.2 Real-World Projects (Graphs + Tables + Equations)

This lesson applies everything we've learned to real-world situations using data, graphs, tables, and equations to make connections and solve problems.

Objectives

- ☐ Interpret and analyze real-world data
- ☐ Represent situations with tables, graphs, and equations
- ☐ Explain connections between different representations

Vocabulary

data, table, graph, equation, relationship, pattern

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

10.3 Group Presentations or Visual Reports

Students will collaborate on a final presentation or report to demonstrate their learning, using mathematical vocabulary, visuals, and examples.

Objectives

- ☐ Create a visual or oral presentation using math content
- ☐ Work collaboratively to explain mathematical ideas
- ☐ Use accurate vocabulary and representations in communication

Vocabulary

presentation, visual, explanation, evidence, support

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

10.4 Final Assessment or EOC Practice

This lesson offers a chance to show mastery of key Algebra concepts through a final assessment or EOC-style practice problems.

Objectives

- ☐ Demonstrate understanding of major Algebra concepts
- ☐ Solve a variety of equations and interpret representations
- ☐ Apply skills learned to novel and test-like problems

Vocabulary

equation, function, graph, solution, expression

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

Part XI

Supplemental

Supplemental Materials

Welcome to the **Supplemental Materials** section of this course! This is where you'll find all the fun, extra, and just plain interesting math content that doesn't quite fit into the main units — but still helps build understanding, spark curiosity, or offer a little challenge.

Use these resources to: - Practice your skills in new and creative ways - Explore math puzzles and logic games - Reinforce key concepts from class - Take a brain break with something still mathy (but fun!)

Math Games & Puzzles

Number grids, logic puzzles, equation word searches, and more.

Extra Practice Worksheets

Targeted drills and alternative problem sets.

Challenge Problems

For students who want to push their thinking further.

Math Activities

Open-ended or interactive things to try out

Happy exploring!

Math Games and Puzzles

Explore these fun and challenging math activities! Click on any worksheet to open the PDF.

Hidden Math Problems

Practice: Arithmetic operations, pattern recognition
How it works: Find groups of 3 numbers in the grid. Add, subtract, multiply, or divide the first two to get the third. Problems may be horizontal, vertical, or diagonal.

Download Worksheet

Want to suggest an activity or submit your own? Let me know!

Resources

Looking for extra support as you learn Algebra? Below you'll find helpful links to additional study materials and downloadable cheat sheets!

Factor Chart

Need help finding all the factor pairs of a number? This chart lists every composite number from 1 to 147 and lists all prime numbers up to 200. For each number, you'll see a list of its factor pairs.

Download Chart

Want to suggest a resource or submit your own? Let me know!

Glossary

Absolute value

The distance a number is from zero on a number line, always expressed as a positive number or zero.

> **Example:**
>
> The **absolute value** of `-7` is `7`.

Algebra

Algebra is a branch of math that uses letters and symbols to represent numbers and relationships.

It lets us describe patterns, write rules, and solve problems that work in many different situations.

Calculus

Calculus is a branch of math that helps us understand change and motion.

It's used to study how fast things move, how things grow or shrink, and how to find exact areas or curves.

Composite number

A composite number has more than two factors.

That means it can be divided evenly by numbers other than 1 and itself.

Example:

12 is composite because 2, 3, 4, and 6 all divide it evenly.

Convert

To convert means to change a number from one form to another — like turning a fraction into a decimal, or a percent into a fraction. The value stays the same, but the way it's written changes.

Decimal

A decimal is a way to show part of a whole using the base-10 system. It uses a decimal point to separate whole numbers from fractional parts.

Each digit after the decimal point has a place value based on powers of 10:

- The first place is tenths (0.1)
- Then hundredths (0.01)
- Then thousandths (0.001), and so on

Decimals are closely related to fractions and can often be converted back and forth.

Example:

- 0.3 means 3 tenths, or $\frac{3}{10}$
- 0.25 means 2 tenths and 5 hundredths, or $\frac{25}{100}$

Denominator

The denominator is the **bottom number** in a fraction. It tells **how many equal parts** the whole is divided into.

Example:

In the fraction 3/4, the **denominator** is 4.

Divisible

A number is **divisible** by another if it divides evenly — these rules help you check quickly.

- **Divisible by 2**: The number ends in **0, 2, 4, 6, or 8** (an even number).
- **Divisible by 3**: The **sum of the digits** is divisible by 3.
- **Divisible by 4**: The **last two digits** form a number divisible by 4.
- **Divisible by 5**: The number ends in **0 or 5**.
- **Divisible by 6**: The number is divisible by **both 2 and 3**.
- **Divisible by 9**: The **sum of the digits** is divisible by 9.
- **Divisible by 10**: The number ends in **0**.

Equation

An equation is a math sentence that says two things are equal.

It has an equals sign (=) and shows a relationship between numbers or expressions.

Examples:

`2 + 3 = 5` and `x + 1 = 7`

Equivalent

Two numbers, fractions, or expressions are **equivalent** if they have the **same value**, even if the look different.

Example:

The fractions $\frac{2}{4}$ and $\frac{6}{12}$ are equivalent because both simplify to $\frac{1}{2}$.

The decimal for $\frac{2}{4}$ can be found using a calculator by entering $1 \div 2$, which gives 0.5

Likewise, $\frac{6}{12}$, can be found by entering $6 \div 12$, which also gives 0.5.

Even

A number is **even** if it can be divided by 2 with no remainder. Numbers like 2, 4, 6, 8, … are even. Numbers that are **not** even are called **odd** numbers.

Examples:

$24 \div 2$ is 12 with no remainder, so 24 is even.

$35 \div 2$ is 17 with a remainder of 1 so 35 is **odd**.

Expression

An expression is a math phrase made of numbers, variables, or both — but it doesn't have an equals sign.

Examples:

- `3x + 2`
- `7 - y`
- `5(a - 1)`

Factor

A factor is a whole number that divides another number evenly — with no remainder.

If you can split something into equal groups with no leftovers, the group size is a factor.

Example:

3 is a factor of 12 because $3 \times 4 = 12$.

Factoring

Factoring means breaking something down into smaller parts that multiply together to make it.

Examples:

- Factoring a number: 12 can be factored into `3 × 4` or `2 × 6`.
- Factoring an expression: $x^2 + 5x + 6$ can be factored into $(x + 2)(x + 3)$.

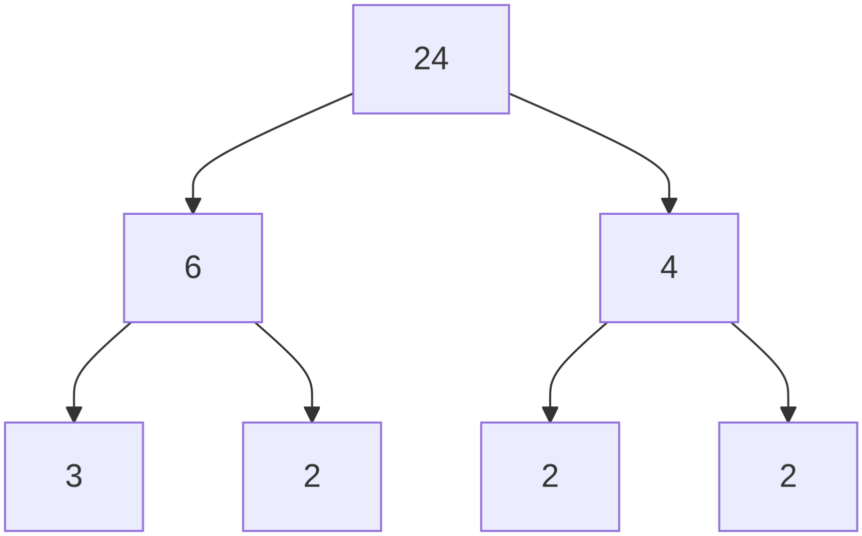
Factor tree

A factor tree is a way to break a number into its smallest building blocks — the prime numbers that multiply to make it.

You keep splitting the number into smaller factors until you can't go any further.

Example:

Here is a factor tree for 24:



This tells us that the prime factorization of 24 is $2^3 \cdot 3$.

Fraction

A fraction shows a part of a whole. It has a numerator (top number) and a denominator (bottom number).

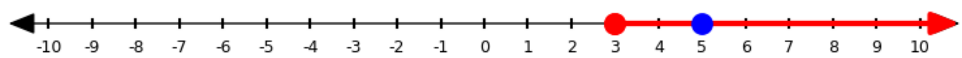
The denominator tells how many equal parts the whole is divided into, and the numerator tells how many of those parts you have.

> **Example:**
>
> In 3/4, the fraction means **3 out of 4** equal parts.

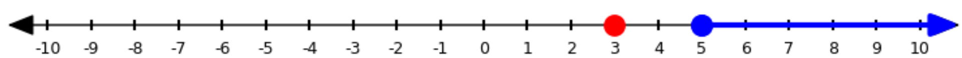
Greater than

A number is greater than (>) another number if it is further to the right on the number line.

Example:



$5 > 3$ is true



but $3 > 5$ is false

Greatest Common Factor

The **greatest common factor** (GCF) is the biggest number that divides evenly into two or more numbers. It is the largest factor they have in common.

You can find the GCF by listing all the factors and finding the ones shared between the numbers.

Example:

The GCF of 18 and 24 is 6, because 6 is the largest number that goes into both 18 and 24 without a remainder.

Factors of 18: 1, 2, 3, 6, 9, 18

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

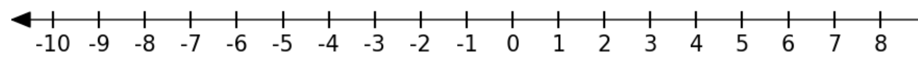
Shared factors: 1, 2, 3, 6

The greatest is 6

Horizontal

Side to side, like the horizon. Level ground is horizontal.

Example:

This is a horizontal number line. 

Integer

An integer is a whole number (not a fraction or decimal) that can be positive, negative, or zero.

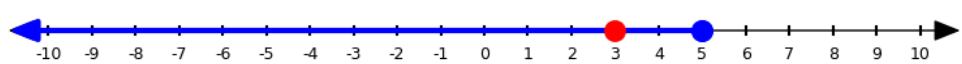
Example:

`-3`, `0`, `5`, and `100` are all integers.

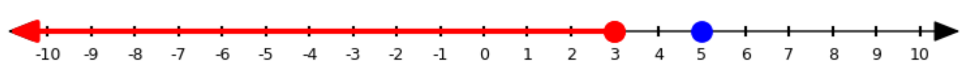
Less than

A number is less than ($<$) another number if it is further to the left on the number line.

Example:



`3 < 5` is true



but `5 < 3` is false

Multiple

A multiple is what you get when you multiply a number by 1, 2, 3, and so on.

Example:

5, 10, 15, and 20 are all multiples of 5.

Negative

A number is negative if it is less than zero.

On a number line, negative numbers are to the left of zero.

Example:

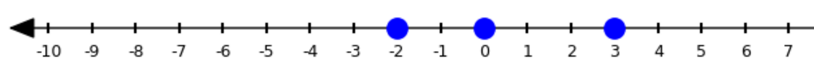
-4 is a negative number.

Number line

A straight line used to show numbers in order. It usually has zero in the middle, with positive numbers to the right and negative numbers to the left.

Number lines help you visualize math operations and compare values.

Example:



-2, 0, and 3 are all on the number line.

Number sense

Number sense is a person's ability to understand, work with, and think about numbers.

In simple terms, it means having a good feel for how numbers work — like knowing:

- What numbers mean
- How they relate to each other
- How to break them apart or put them together
- What a reasonable answer might be

People with strong number sense can do mental math, estimate, recognize patterns, and spot when something "doesn't make sense."

It's kind of like having a good instinct for numbers — not just memorizing rules, but really *getting* how numbers behave.

Numerator

The numerator is the **top number** in a fraction. It tells **how many parts** you have.

> **Example:**
>
> In the fraction 3/4, the **numerator** is 3.

Odd

A number is **odd** if dividing it by 2 leaves a remainder. Numbers like 3, 5, 7, 9, … are **odd** numbers. Numbers that are **not** odd are called **even** numbers.

> **Examples:**
>
> $35 \div 2$ is 17 with a remainder of 1 so 35 **is** odd.
>
> $24 \div 2$ is 12 with no remainder, so 24 is **even**.

Opposite

Two numbers that are the same distance from zero on a number line, but on opposite sides.

Their sum is always zero.

Example:

`-3` and `3` are opposites.

Percent

A percent is a number out of 100. It uses the percent symbol (%) and is another way to show part of a whole.

Example:

25% means 25 out of 100 parts.

Place Value

Place value tells you what each digit in a number means, based on its position. In a decimal like 0.4, the 4 is in the tenths place — it means 4 tenths. Each place to the right is worth ten times less than the one before it.

Positive

A number is positive if it is greater than zero.

On a number line, positive numbers are to the right of zero.

Example:

`5` is a positive number.

Prime Number

A number is prime if it has exactly two factors: 1 and itself.

Example:

`2`, `3`, `5`, `7`, and `11` are all prime numbers.

Prime factorization

Prime factorization means writing a number as a product of prime numbers.

Example:

$18 = 2 \times 3 \times 3$ or 2×3^2

These are the prime building blocks of 18.

Product

A product is the **result of multiplying** two or more numbers.

Example:

The product of 4 and 6 is `24`, because `4 × 6 = 24`.

Quotient

A quotient is the **result of dividing** one number by another.

Example:

The quotient of 20 divided by 5 is `4`, because `20 ÷ 5 = 4`.

Ratio

A ratio compares two amounts or quantities, showing how much there is of one thing compared to another. It tells us about the relationship between the numbers.

There are three common ways to write a ratio:

- Using a colon (like 2:3)
- As a fraction (like $\frac{2}{3}$)
- Using the word "to" (like 2 to 3)

  Example:

  If you have 4 red balloons and 6 blue balloons: The ratio of red balloons to blue balloons is 4:6 or 2:3 (when simplified). This means for every 2 red balloons, there are 3 blue balloons.

Relatively prime

Two numbers are **relatively prime** if their greatest common factor is **1**. That means they don't share any factors other than 1 - even if neither number is prime.

> **Example:**
>
> 8 and 15 are relatively prime because the only number that evenly divides both is 1.

Remainder

A remainder is what's left over after dividing when the number doesn't go in evenly.

> **Example:**
>
> `10 ÷ 3 = 3` with a remainder of `1`, because `3 × 3 = 9` and there's `1` left.

Simplest form

A fraction is in simplest form (or lowest terms) when you cannot divide the top number (numerator) and the bottom number (denominator) by an common factor other than 1.

We write fractions in simplest form to make the numbers as "small" and easy to understand as possible without changing the value.

> **Example:**
>
> $\frac{4}{8}$ is **not** in simplest form because both 4 and 8 can be divided by 4.
>
> $\frac{1}{2}$ is in simplest form because the only common factor for 1 and 2 is 1.

Simplify

To **simplify** a number or expression means to **rewrite it in a cleaner or shorter way** — without changing its value. In math, we often simplify fractions, expressions, or equations to make them easier to work with.

> **Example:**
>
> The fraction $\frac{12}{20}$ can be simplified by dividing both the numerator and denominator by 4:
>
> $$\frac{12}{20} = \frac{3}{5}$$
>
> This simpler fraction means the same thing — it's just written with smaller numbers.

Sum

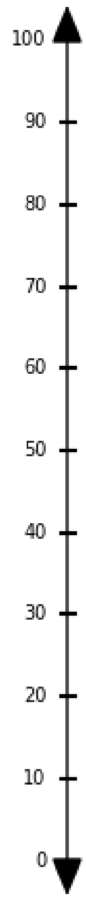
A sum is the **result of adding** two or more numbers.

> **Example:**
>
> The sum of 7 and 8 is `15`, because `7 + 8 = 15`.

Vertical

Up and down, like a flagpole.

Example:



This is a vertical number line.