DHS Algebra 1

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Welcome to Algebra 1

Welcome to Algebra 1 at Frederick Douglass High School!

This book will guide you through the most important math skills you'll need to succeed in high school and beyond. Algebra is more than just solving equations — it's a powerful way to understand patterns, solve problems, and think logically.

Whether you're reviewing old ideas or learning something brand new, this book is here to help you every step of the way.

What You'll Find in This Book

Each unit includes:

- Clear goals to help you focus
- Examples and explanations
- Practice problems
- Activities to explore and talk through ideas

We'll start with the basics — like working with numbers and fractions — and build up to more complex ideas like equations, graphs, and even quadratics.

You don't have to be a "math person" to do well here. Just bring your curiosity, a little patience, and the willingness to try.

Let's get started!

Part I

Unit 1: Foundations

Introduction

Welcome to Unit 1! This is where you'll build your Algebra toolkit — skills like working with integers, simplifying fractions, and following the order of operations.

You'll use these tools again and again to solve expressions, equations, and real-world problems throughout the year.

What You'll Learn

By the end of this unit, you'll be able to:

- Work with positive and negative numbers on a number line
- Use factor trees to find prime factorizations
- Identify and use the greatest common factor (GCF)
- Convert between fractions, decimals, and percents
- Multiply, divide, and compare fractions
- Follow the correct order of operations to simplify expressions

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Topics in This Unit

Integers & Number Lines
Understand and use positive and negative numbers, and how to place them on a number line.

Factors, Multiples & Prime Factorization
Break numbers into their prime building blocks using factor trees.

GCF & Simplifying Fractions
Use prime factorization to find the GCF and simplify fractions to their simplest form.

Fractions, Decimals & Percents – Conversions
Convert between different number forms and compare them.

Multiply, Divide & Compare Fractions
Work with fractions in ways that actually show up in Algebra — simplify, multiply, divide, and compare.

Solving Problems with Fractions, Decimals & Percents
Solve real-world problems using these different number forms.

Using the Order of Operations
Follow the rules (PEMDAS) to simplify expressions with integers and fractions.

Let's build those Algebra muscles — you'll need them for everything that follows!

1.1 - Integers & Number Lines

Did you know that all of mathematics is actually built up from simple things like counting? Even advanced topics like algebra and calculus are just clever ways of organizing and extending basic ideas — like moving forward and backward on a number line.

In this lesson, we'll use the number line not just to count, but to add, subtract, and compare positive and negative numbers. That might sound basic, but it's the foundation of nearly everything else you'll do in Algebra.

Negative numbers can be tricky, especially when the rules don't always match what your gut tells you. But if you can master the way they work on the number line — including things like opposites, absolute value, and comparison — you'll be setting yourself up for success in the rest of the course.

- ☐ I know how to read and use a number line
- ☐ I can find and describe the opposite of a number
- ☐ I can compare positive and negative numbers using $>$, $<$, and $=$
- ☐ I can add and subtract integers on a number line

absolute value, greater than, integer, less than, number line, negative, opposite, positive

Warm-Up

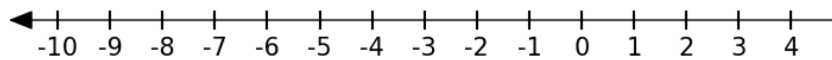
Answer as best you can – even if you aren't sure!

1. What is the opposite of 6?
2. Which is greater -4 or -9?
3. Which is farther from 0: -7 or 5?

Learn Together

1.1.1 - The Number Line Is More Than Just Counting

You already know how to count — 0, 1, 2, 3, and so on. The **number line** extends that idea in both directions.



Let's draw a number line from -10 to 10

Here, every tick mark is an integer — a whole number.

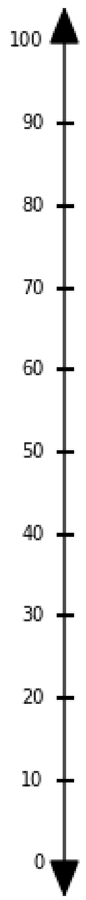
- Numbers to the **right** of zero are positive
- Numbers to the **left** of zero are negative

We can use this number line to *see* what happens when we add, subtract, or compare numbers.

Are there other ways to draw a number line?

Yes! Number lines can be drawn over different ranges and scales. For example, here is a number line that counts form -10 to 25 in steps of 5.

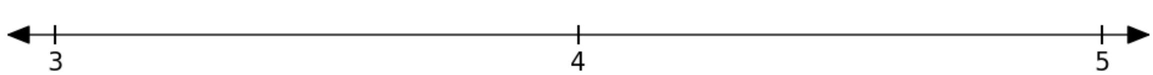
In fact, number lines don't even have to be horizontal. Here is a vertical number line that goes from 0 to 100 in steps of 10.

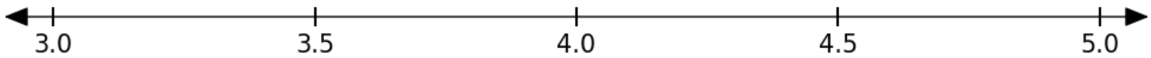


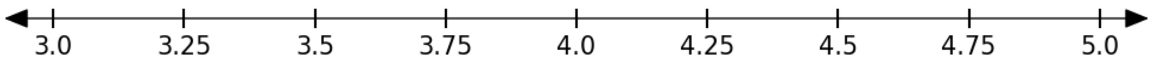
Here are a few examples:

- thermometer
- ruler
- timeline
- American football field
- volume slider on a phone

Though there are 2 integers between 3 and 5, the answer is not 2! There are infinitely many numbers between 3 and 5. Here are some number lines that might help convince you.



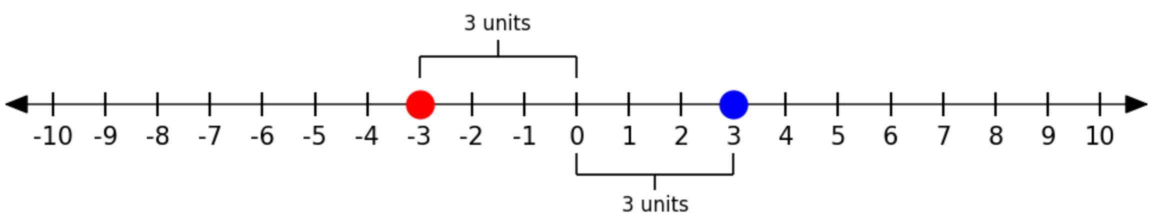




1.1.2 - Understanding Opposites

Let's look at a pair of numbers, 3 and -3.

These are called opposite numbers. They are the **same distance** from zero but on **opposite sides** of it.

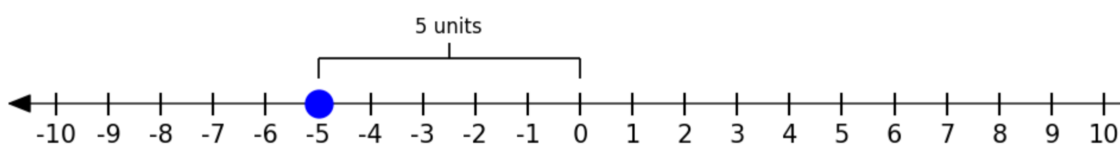


The opposite of zero is zero. Zero is the only number that is its own opposite!

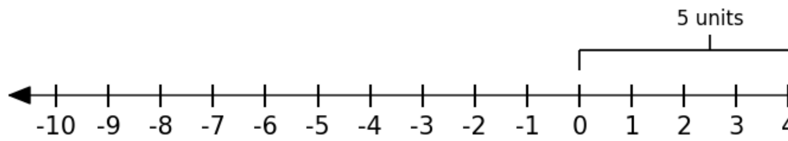
1.1.3 - What Is Absolute Value?

Absolute value ($|number|$) measures the **distance from zero**, no matter the direction.

Take a look at the number -5. The number line shows that it's absolute value is 5 because it is 5

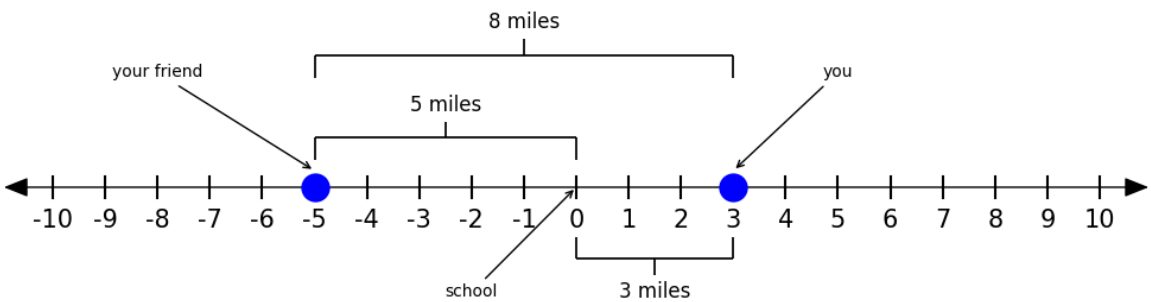


units away from zero.



You can see that |5| is also 5 for the same reason!

Absolute value is often used for describing the distance between two points. Suppose you live 3 miles to the east of the school and your best friend lives 5 miles to the west. How far apart are your houses? This is easy to see with a number line.

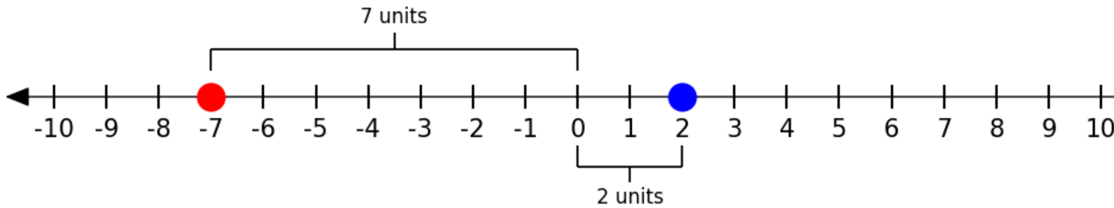


You can compute your distances by adding |-5| + |3|, by |-5 - 3|, or by |3 - (-5)|. All three of these give the same answer, 8 miles. What would change if we did not use absolute value?

Absolute value is **never** negative, because distance is never negative.

1.1.4 - Comparing Integers

We can also use the number line to compare values.



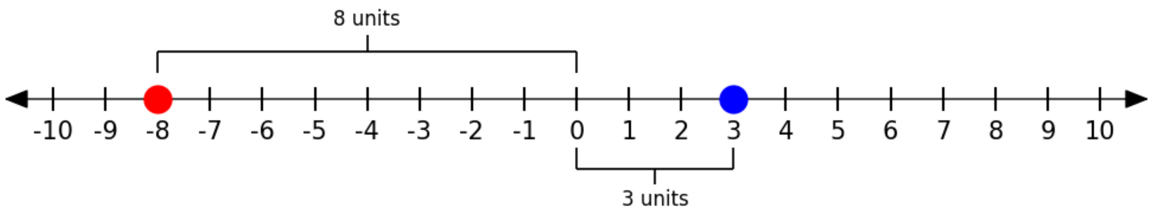
Let's compare 2 to -7.

You can see from the number line that 2 is greater than (>) -7 because 2 is to the right of -7.

You can also see that -7 is further from zero than 2 and so |-7| > |2|.

It is easy to get confused here. When we say which is "bigger" we are asking which number is further to the right on the number line, **not** which one is furthest from zero.

3 > -8 because it is farther to the right but |-8| > |3| because -8 is further from zero.

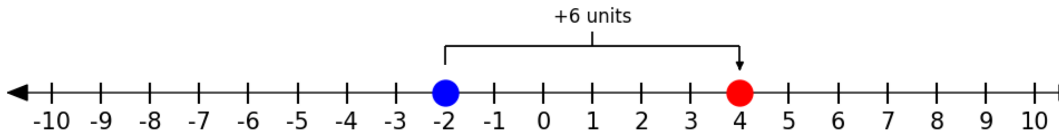


1.1.5 - Number Lines and Arithmetic

We can also use the number line to model **adding and subtracting** integers.

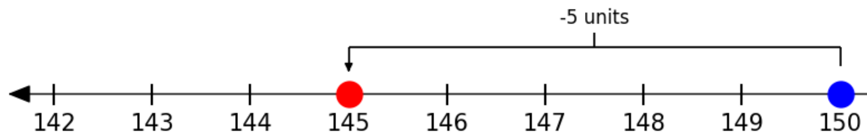
- To add a positive number, move **right**
- To add a negative number, move **left**

Examples:



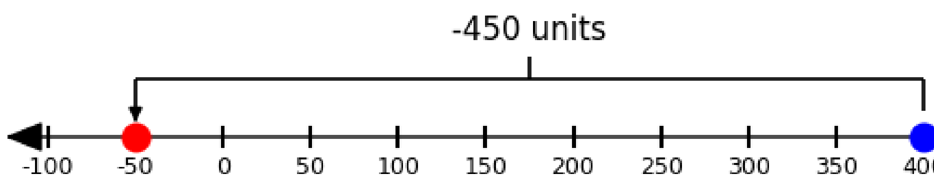
1. Addition: $-2+6=4$

Imagine that you are \$2 in debt. If someone pays you \$6 you can pay off the debt and have \$4 left over.



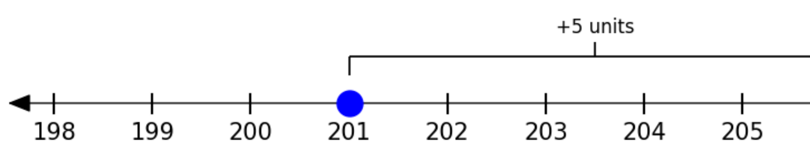
2. Adding a negative: $150+(-5)=145$

You have \$150 in the bank. The bank ads a fee for being under their \$200 minimum balance. You now have \$145.



3. Subtraction: $400-450=-50$

If you only have \$400 but spend \$450 on a credit card. You are now \$50 in debt.



4. Subtracting a negative: $201-(-5)=206$

   Example: The bank made a mistake, you had \$201 in your account so they took off the \$5 fee. Now you have \$206.

Practice On Your Own

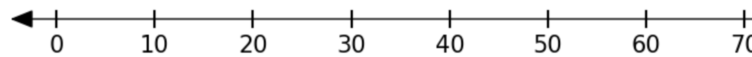
Working With Number Lines

1. Draw a number line that shows:

    a. -4, 0, and 3.

    b. Your age

    c. The number halfway between 5 and 9.



2. What question could match this number line?

Opposites

3. What is the opposite of 42?
4. What is the opposite of -3?
5. Draw a number line with two numbers that are opposites.
6. Does 3.5 have an opposite? If yes, what is it?

Comparing Numbers

7. Which number is **greater**, 5 or -10?
8. Which number has the greater absolute value, 5 or -10?
9. Is 28 bigger than -30?
10. Use (>) or (<) to compare:

    a. -11 _____ -13

    b. 7 _____ -2

    c. |-3| _____ |5|

11. Which is bigger?

a. -4 or -5

b. 3 or the opposite of 7

c. |-5| or |4|

12. Use a number line to compare:

    a. -7 to 2.

    b. The year you were born and the current year

Addition and Subtraction

13. Show these on a number line:

    a. $-3 + 5$

    b. $3 - 5$

    c. $-3 + (-3)$

    d. $3 - (-3)$

Word Problems

14. Solve using a number line

    a. The temperature was -12°F. It warms up by 20°. What is the new temperature?

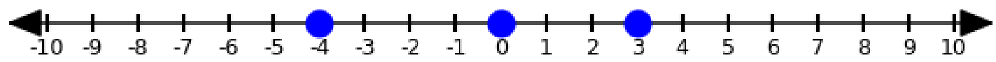
    b. A diver is 45 feet below sea level. She dives 30 feet deeper. How far down is she?

    c. Your bank account is at -$8. You deposit $5. What is your new balance?

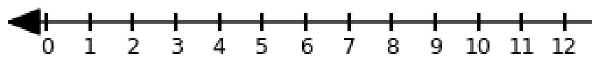
Warm-Up

1. -6
2. -4
3. -7

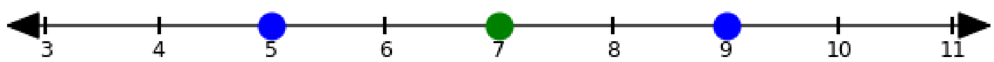
Working With Number Lines



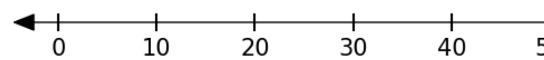
1. a.



   b. *Answers vary. Here is what a 15 year old would show*



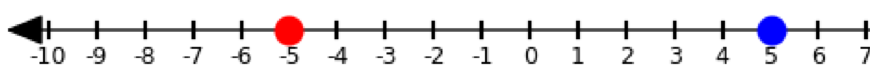
   c. *The answer is 7*



2. *Answers vary. We could say "Plot the temperature on July 4th"*

3. -42

4. 3



5. *Answers vary. Here is an example:*

6. Yes! The opposite is -3.5.

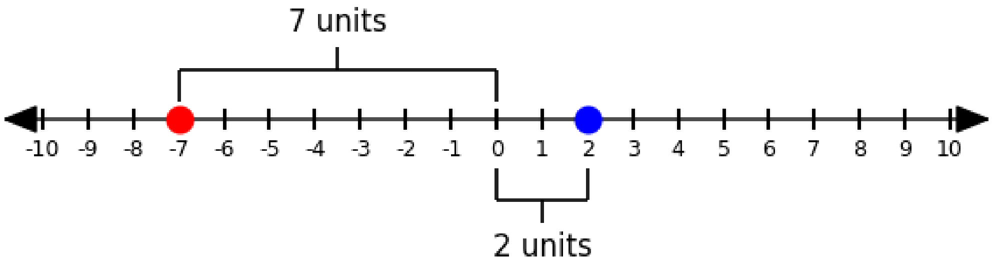
7. 5 is greater

8. -10 has a greater absolute value

9. Yes, because it is further from zero

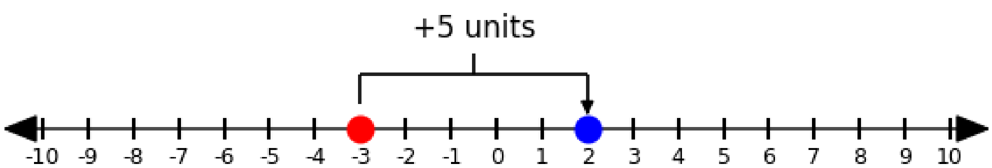
10. a. $-11 > -13$
    b. $7 > -2$
    c. $|-3| < |5|$

11. a. -4
    b. 3
    c. $|-5|$

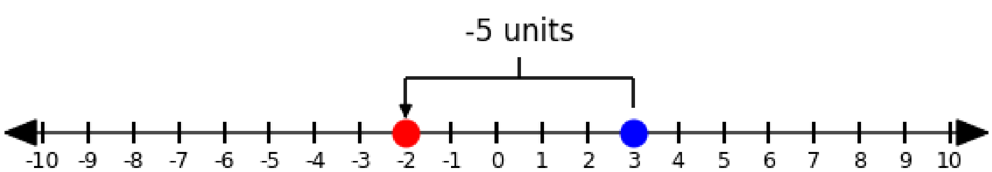


12. a.

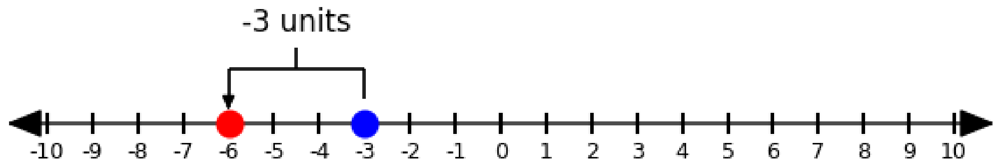
b. *Answers vary. I was born in 1982. The current year is 2025.*



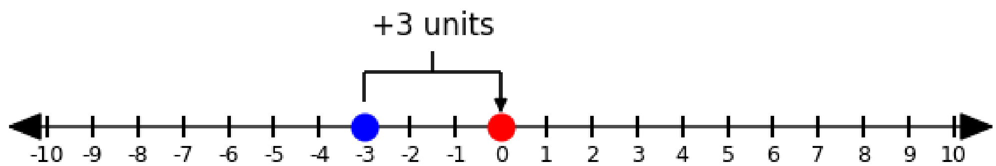
13. a.



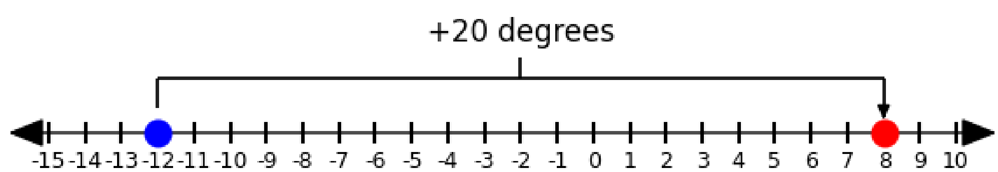
b.



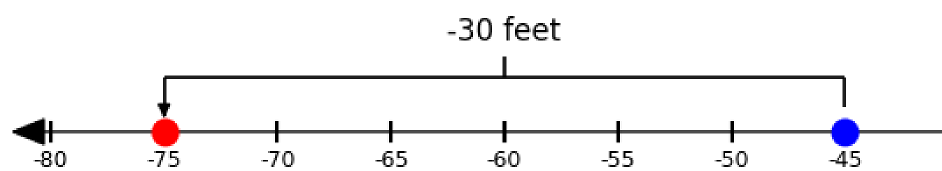
c.



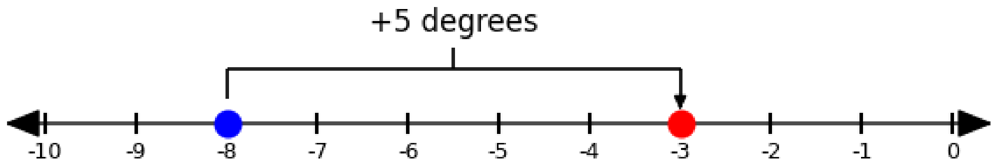
d.



14. a. It is now 8 degrees



b. She is now 75 feet down.



c. You now have -$3.

1.2 - Factors, Multiples & Prime Factorization

Have you ever had to split something up evenly — like slices of pizza or players on a team? That's really what factors are about: dividing numbers into equal parts.

In this lesson, you'll learn how to:

- Spot factors and multiples.
- Tell if a number is prime number or composite number.
- Break numbers into their basic building blocks using a factor tree.

You'll use these skills again and again — from simplifying fractions to solving equations.

☐ I can find the factors and multiples of integers
☐ I can tell if a number is prime or composite
☐ I can break numbers into prime factors using a factor tree

composite number, factor, factor tree, multiple, prime factorization, prime number

Warm-Up

1. List all the whole-number factors of 12.
2. Find a multiple of 7 that is less than 50.
3. Is 11 a prime number? How do you know?

Learn Together

1.2.1 - What Are Factors?

A factor of a number is a whole number that divides it evenly — with no remainder.

Example:

The factors of 12 are: 1, 2, 3, 4, 6 and 2

That's because:

$1 \times 12 = 12$

$2 \times 6 = 12$

$3 \times 4 = 12$

Only one number does: **1**. It only has itself as a factor and so it is neither prime nor composite!

1.2.2 - What Are Multiples?

A multiple is what you get when you multiply a number by 1, 2, 3, 4...

Example:

Here are the first few multiples of 5:

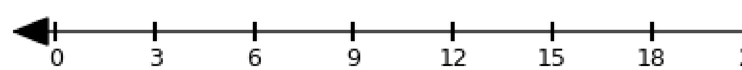
$5, 10, 15, 20, 25, 30, \ldots$

Multiples are useful when finding common denominators or common multiples later in algebra.

Where have we seen this before?

Multiples show up all over the place. When you skip count, you are using multiples. In the previous lesson, we used multiples to constuct number lines!

Example:



Here is a number line that shows multiples of three.

1.2.3 - Prime vs. Composite

A **prime number** has only 2 factors: 1 and itself.

Examples: 2, 3, 5, 7, 11, 13…

A **composite number** has more than 2 factors.

Examples: 4, 6, 8, 9, 10…

Prime numbers play a big role in **encryption**, which keeps your data safe when you shop or message online.

1.2.4 - Prime Factorization and Factor Trees

Every number can be broken into a **product of prime numbers** — sort of like breaking a LEGO® sculpture into individual bricks. These prime factors are the basic building blocks of all whole numbers.

We use **factor trees** to find these prime factors. This isn't just a fun trick — it builds your number sense: your ability to see patterns, understand how numbers are structured, and work confidently with them.

That number sense will come in handy later when you:

- Simplify fractions
- Solve equations
- Factor algebraic expressions
- Find common denominators

Let's build a factor tree for **360** to see how it works.

Figure 1

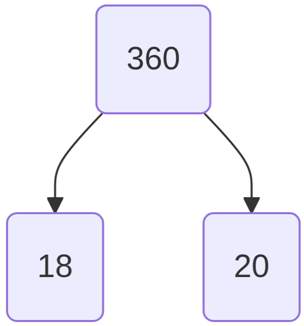


Figure 2

Steps to Make a Factor Tree

1. Start with a number:

2. Find any two numbers that multiply to give the number: $360 = 18 \times 20$

3. Break each of those numbers down further:

   - $18 = 3 \times 6$
   - $20 = 4 \times 5$

4. Keep going until all branches end in **prime numbers** (numbers that can't be factored anymore, like 2, 3, 5, 7…). We call the ends of the branches "leaves".

5. The prime factorization is the product of the leaves of the tree:

$$2 * 2 * 2 * 3 * 3 * 5 = 360$$

   This can be written more compactly by using the factor counts as exponents. There are three 2s and two 3s in this case and so we get…

$$2^3 * 3^2 * 5 = 360$$

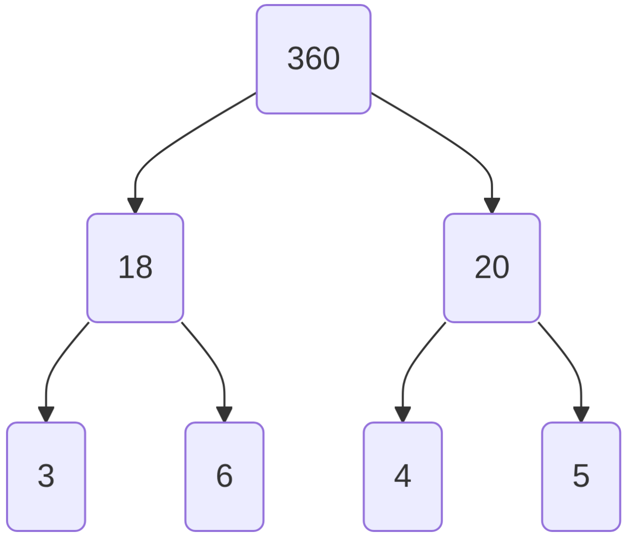


Figure 3

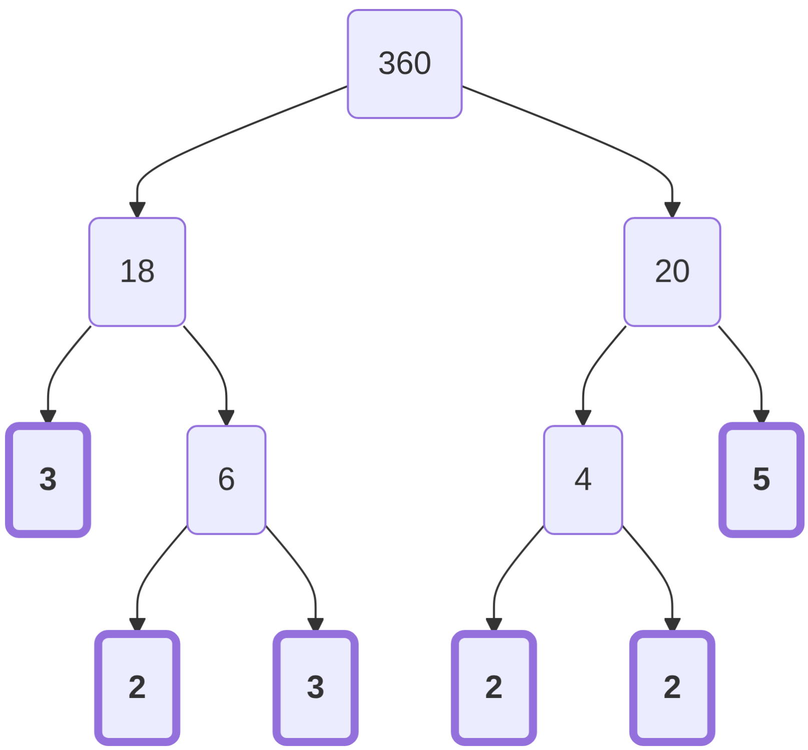


Figure 4

There are *many* factor trees for the number 360. For example, you could also have started with $360 = 3 * 120$.

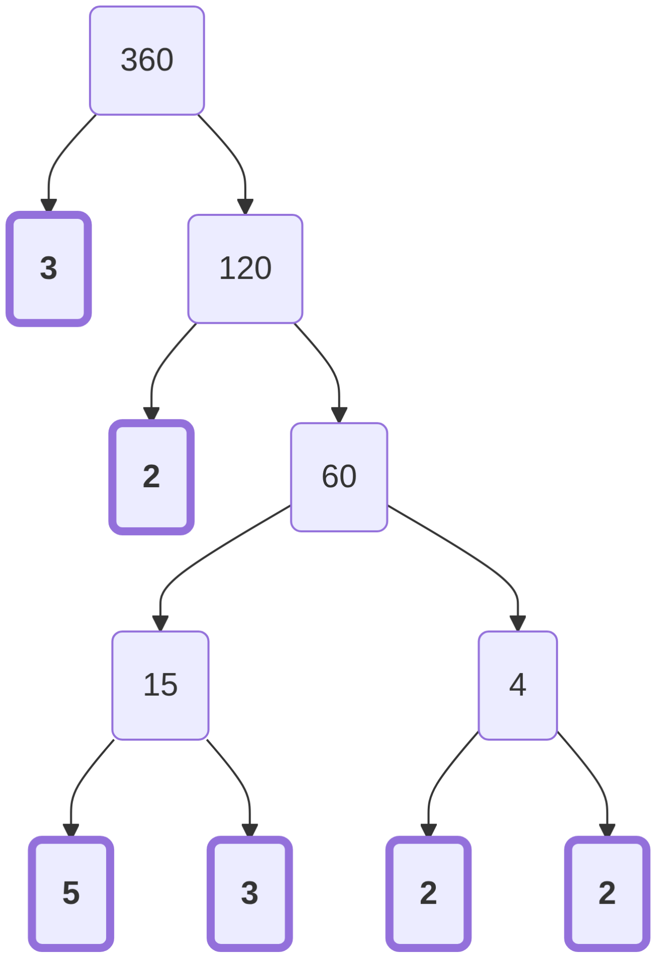


Figure 5

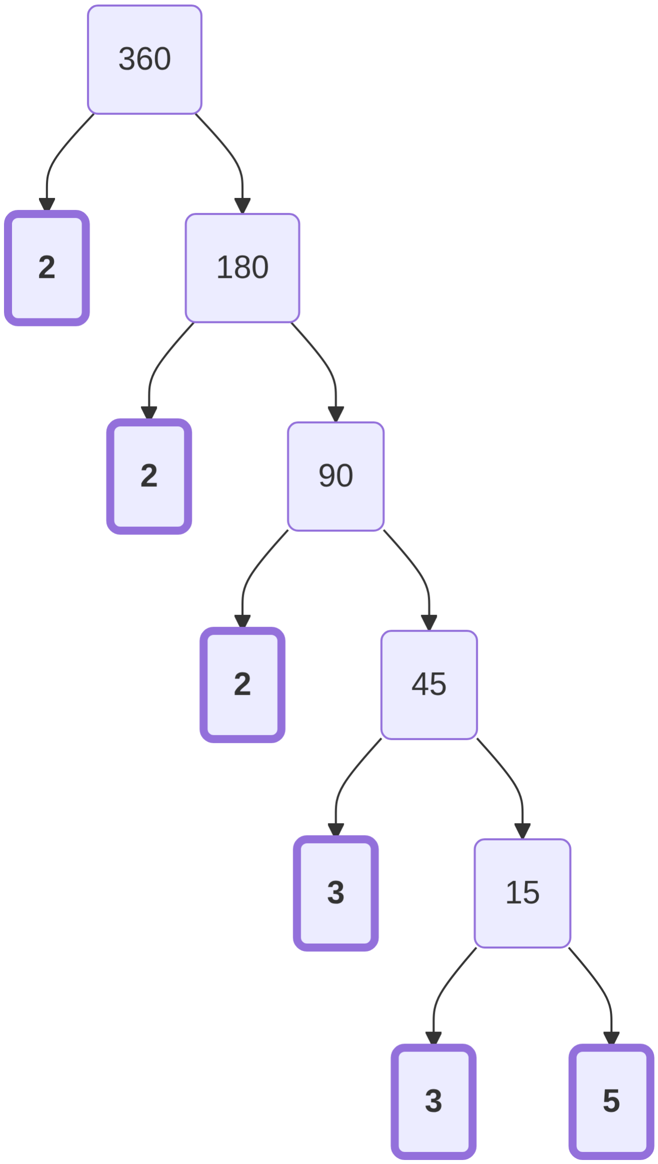
There are still three 2s, two 3s, and one 5, so the prime factorization does not change!

$$2^3 * 3^2 * 5 = 360$$

As long as you end up with the same prime numbers, the tree is correct!

Which factors should I start with?

There is no one right answer to this question. It depends on your goal. If your goal is finding easy numbers, you might start small. Notice that 360 is even that means it is divisible by 2. We could divide by 2 and keep going that way.



When you divide by the smallest (or biggest) factors, the tree tends to become deep. If you want smaller trees, you should start with factor pairs that are closer together like we did with the first factor tree for 360, splitting first with 18 and 20.

What About Negative Numbers?

If the number is negative, factor out a -1 first. Here is one possible factor tree for -24:

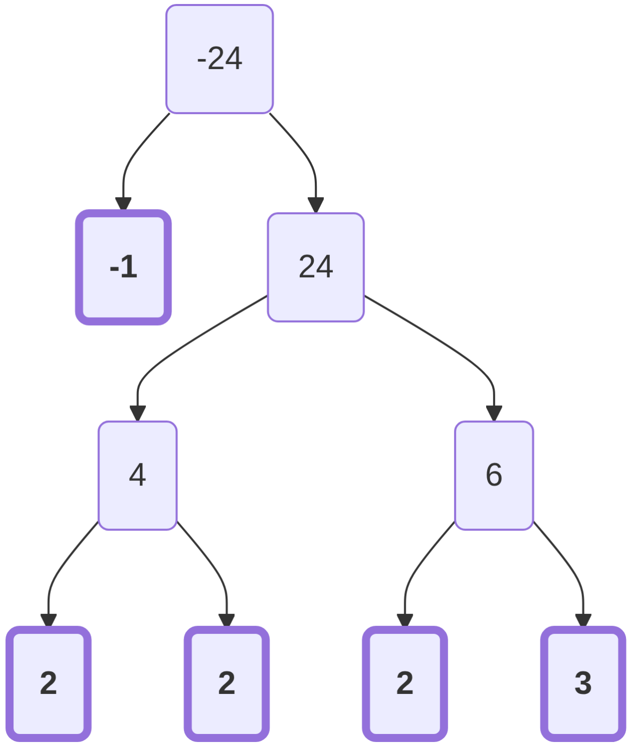


Figure 6

So, the prime factorization of -24 is…

$$-1 * 2^3 * 3 = -24$$

This will come in handy later when we factor algebraic expressions like $-x^2 + 4x$. It's often helpful to pull out a negative first!

Feeling overwhelmed?

If you struggle to come up with the factors for a number, you should check out the factor chart in the resources section of this book. It shows all of the factor pairs for many composite numbers!

I have only shown you 3 of the 60 unique factor trees for 360!

Practice On Your Own

Factors & Multiples

1. List all the factors of:

   a. 16
   b. 18
   c. 27

2. List the first 5 multiples of:

   a. 4
   b. 9
   c. 12

Prime or Composite?

3. Label each number as **prime**, **composite**, or **neither**:

   a. 7
   b. 15
   c. 1
   d. 19
   e. 21

Complete the Factor Tree

4. Fill in the missing numbers.

   a.

b.

Factor Trees & Prime Factorization

5. Use a factor tree to find the prime factorization of:

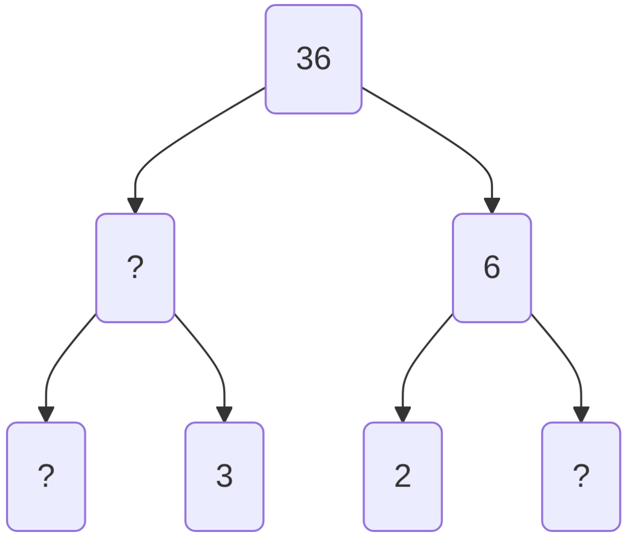


Figure 7

a. 24
b. 60
c. 100
d. 81
e. 72

Challenge

6. Can two different numbers have the same prime factorization? Why or why not?

Factors & Multiples

1. a. 1, 2, 4, 8, 16
   b. 1, 2, 3, 6, 9, 18
   c. 1, 3, 9, 27
2. a. 4, 8, 12, 16, 20
   b. 8, 18, 27, 36, 45
   c. 12, 24, 36, 48, 60
3. a. Prime

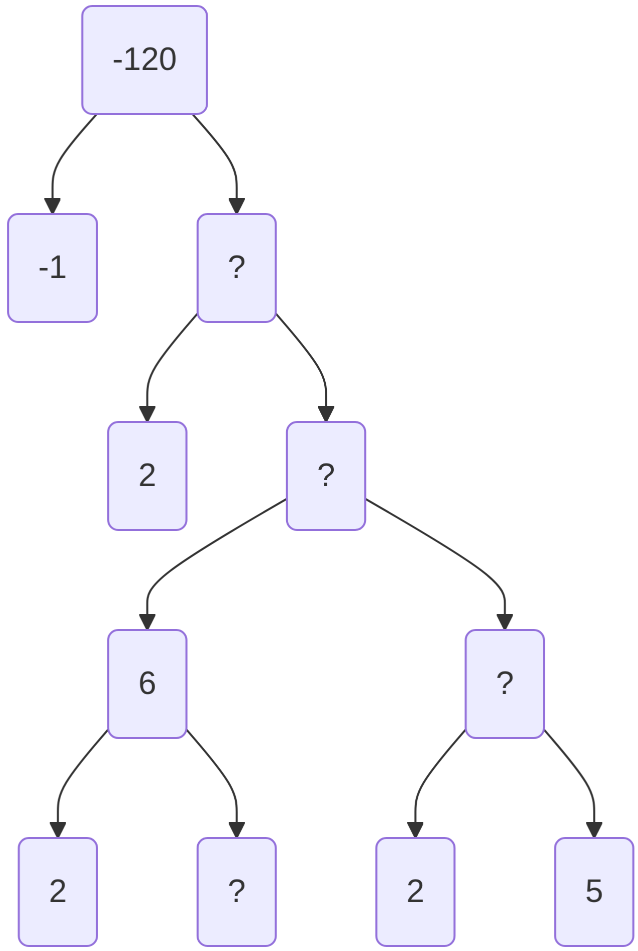
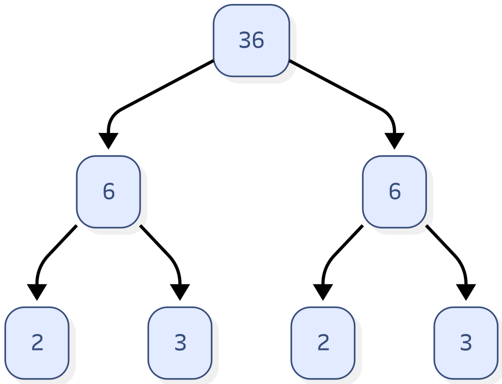


Figure 8

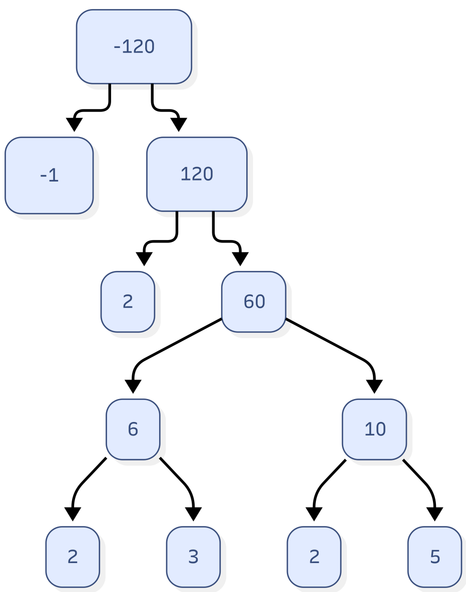
b. Composite
c. Neither
d. Prime
e. Composite

4.

a.



b.



5. a. $2 \times 2 \times 2 \times 3 = 2^3 \times 3$
b. $2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$
c. $2 \times 2 \times 5 \times 5 = 2^2 \times 5^2$
d. $3 \times 3 \times 3 \times 3 = 3$

e. $2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$

6. No. Each number has a **unique** prime factorization. This is called the **Fundamental Theorem of Arithmetic**.

1.3 - GCF & Simplifying Fractions

Have you ever needed to divide things up fairly — like sharing snacks or making equal teams? The greatest common factor (GCF) helps you figure out the largest group size that works for both numbers.

In this lesson, you'll learn how to use prime factorization to find the GCF, and how that can help us simplify fractions. Learning GCF helps us make fractions and math problems simpler so they are easier to understand and solve.

- ☐ I can find the GCF using factor trees
- ☐ I can simplify fractions using the GCF
- ☐ I can solve real-world problems using the GCF

equivalent, greatest common factor, prime factorization, relatively prime, simplify

---

Warm-Up

1. Which number do you *think* has the most prime factors? What makes you think so?

   a. 20

   b. 30

   c. 45

   d. 53

2. What's the largest number that you think might divide **both** 12 and 18 evenly?

3. Here are four fractions. Which one doesn't belong and why?

   a. $\frac{12}{18}$

   b. $\frac{4}{9}$

   c. $\frac{2}{3}$

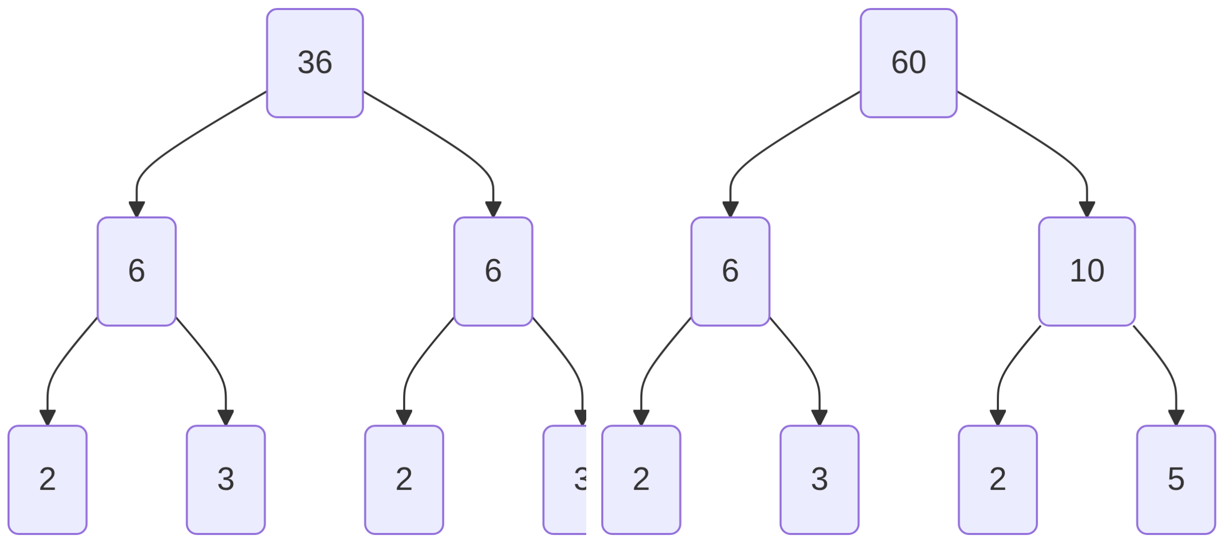
d. $\frac{24}{36}$

Learn Together

1.3.1 - Finding the GCF Using Factor Trees

To find the greatest common factor, we can break numbers into their **prime factors** using factor trees.

Let's try it with 36 and 60:



Now that we have the factor trees, we can use them to easily find the GCF by circling leaves that they both share.

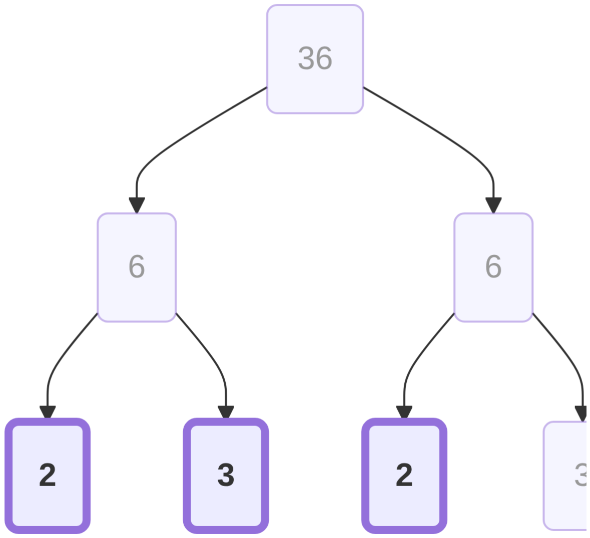
The numbers 36 and 60 share two 2s and one 3. We find the GCF by multiplying those shared factors.

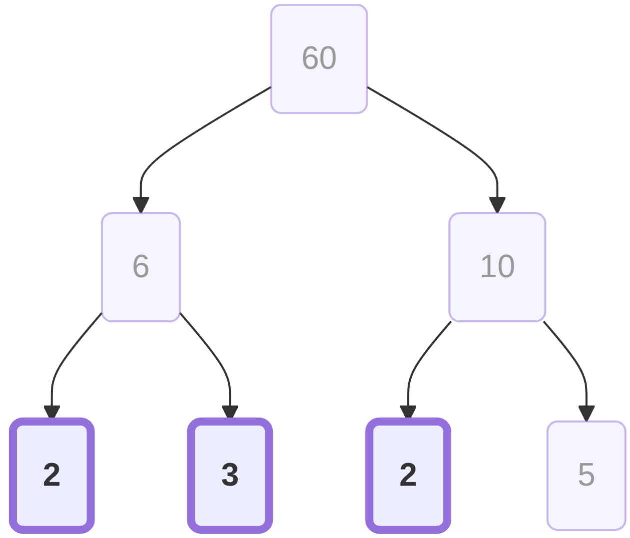
$$2 * 2 * 3 = 12$$

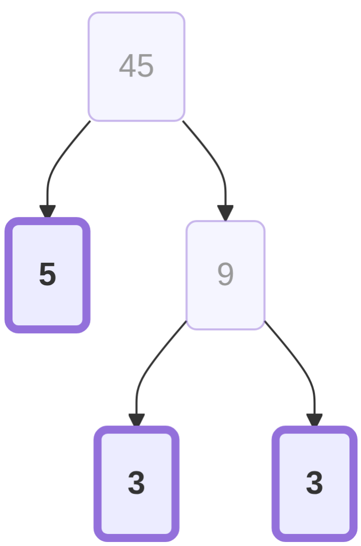
The GCF for 36 and 60 is 12!

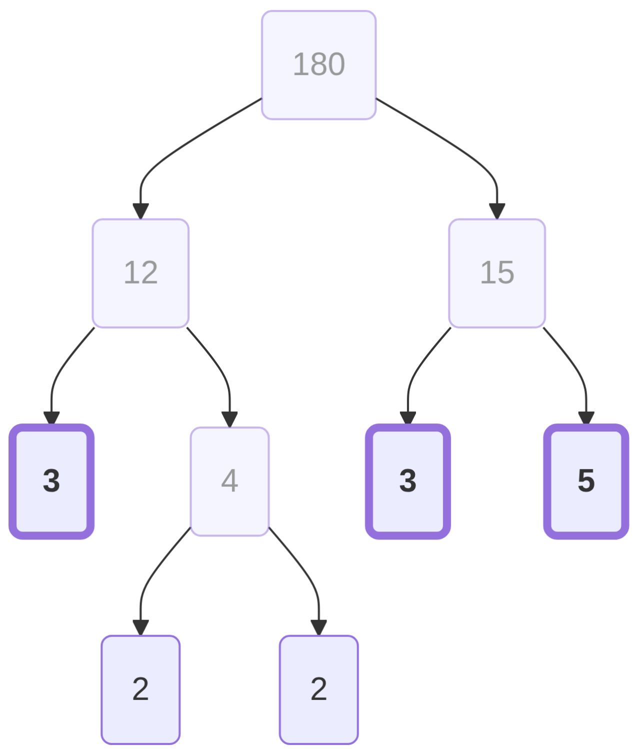
Both 45 and 180 share two 3s and one 5. So the GCF for 45 and 180 is:

$$3 * 3 * 5 = 45$$


36
6
6
2
3
2
3

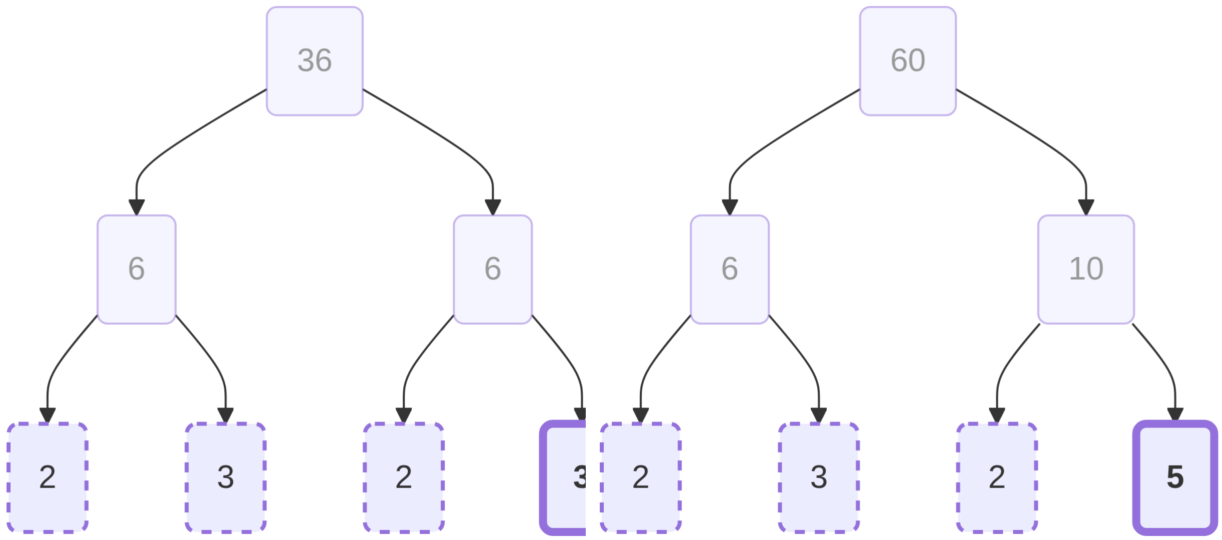

60
6
10
2
3
2
5


45
5
9
3
3


180
12
15
3
4
3
5
2
2

1.3.2 - Using Factor Trees to Divide

You might have noticed that we did not circle **all** of the prime factors for 36 and 60. What did we leave behind and what does that mean?

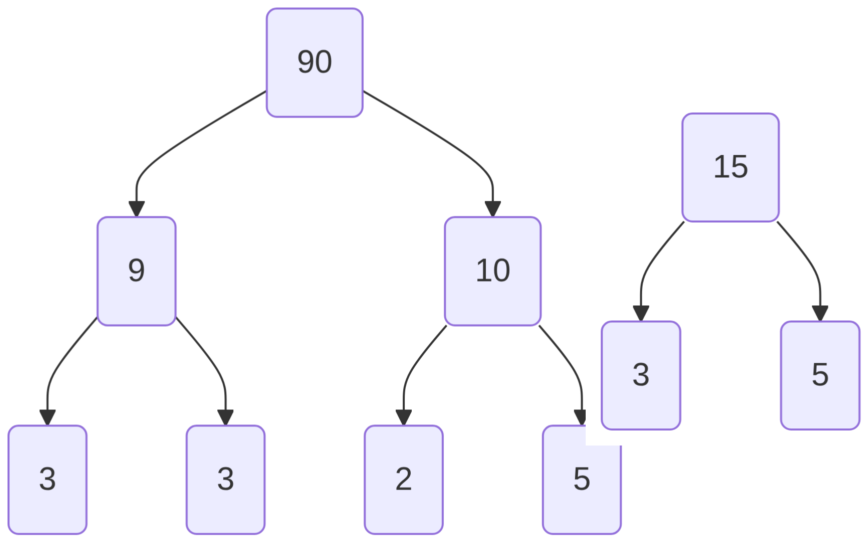


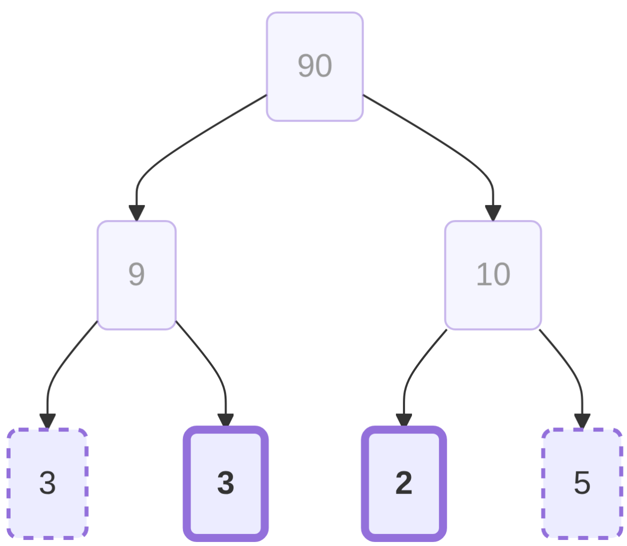
For 36 we left behind a 3 and for 60 we left behind a 5. What this means is that 36 ÷ 12 is 3 and 60 ÷ 12 is 5!"

Let's try another one

This time we will divide 90 by 15. Here are some factor trees to help us.

The number 15 has prime factors 3 and 5. We can divide 90 by 15 by crossing out those shared factors and multiplying what is left.



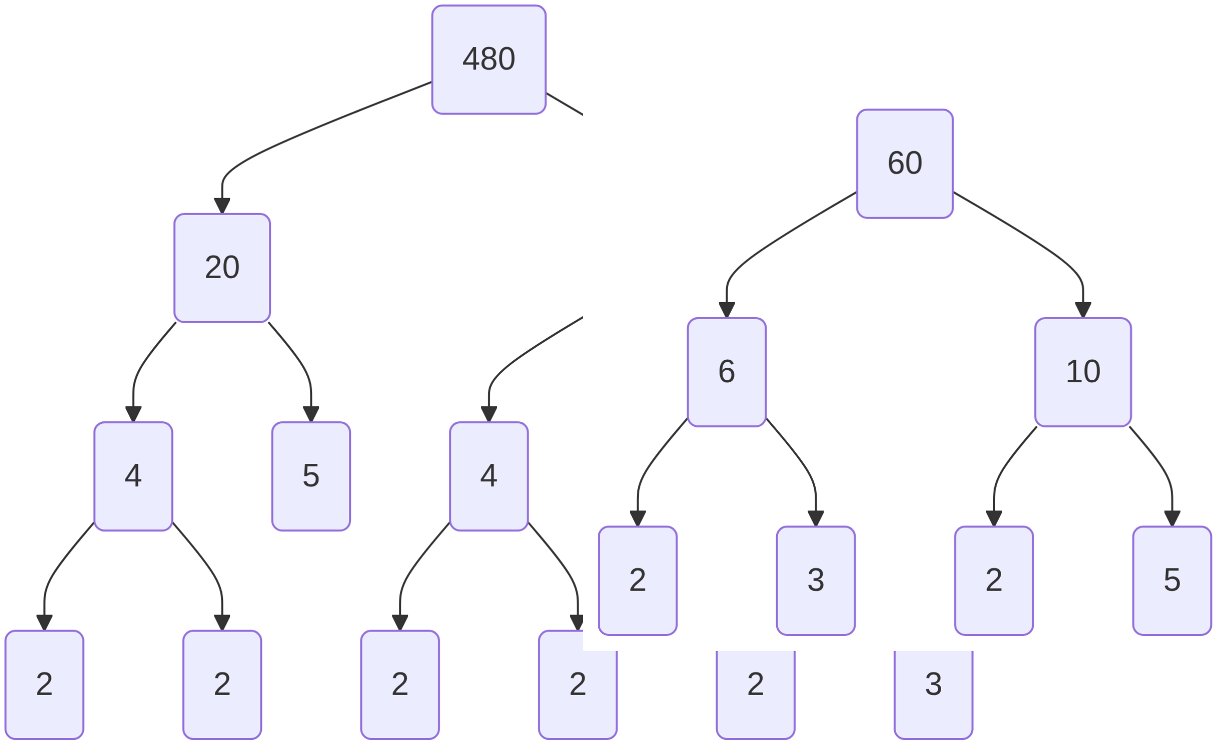


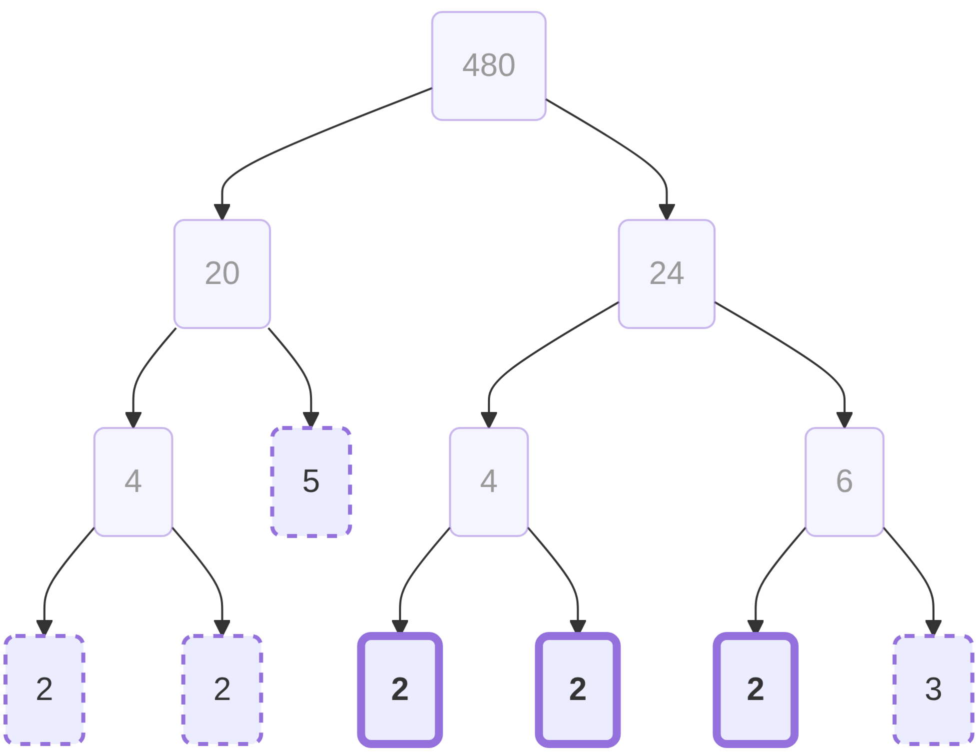
After crossing out 3 and 5 (the factors of 15) we are left with one 3 and one 2. This gives us...

$$90 \div 15 = 3 * 2 = 6$$

1. Find the factor trees for each number

2. Cross out the prime factors 480 shares with 60


480
60
20
6
10
4
5
4
2
3
2
5
2
2
2
2
2
3


480
20
24
4
5
4
6
2
2
2
2
2
3

3. Multiply the remaining prime factors to get the answer.

$$2 * 2 * 2 = 8$$

1.3.3 - Simplifying Fractions with the GCF

In the last section, dividing two numbers showed how to "cancel out" or eliminate shared factors. This is helpful when you want to simplify a fraction!

Simplifying a fraction means rewriting it in its simplest form. This makes a fraction as "small" or "basic" as possible without changing its value. To do that, we divide both the numerator and the denominator by their GCF. When a fraction is in its simplest form, the numerator and denominator don't share any factors other than 1

Let's go back to one we've already seen:

$$\frac{36}{60}$$

We found earlier that the GCF of 36 and 60 is 12. So to simplify, we divide both the top and bottom by 12:

$$\frac{36 \div 12}{60 \div 12} = \frac{3}{5}$$

This is just like the division you saw in the factor trees. We canceled out the factors they both had — two 2s and one 3 — and kept what was left.

Here's another:

$$\frac{90}{15}$$

We know that 15 is the GCF of 90 and 15. So:

$$\frac{90 \div 15}{15 \div 15} = \frac{6}{1} = 6$$

This tells us the fraction $\frac{90}{15}$ is just another way to write the number 6.

In other words:

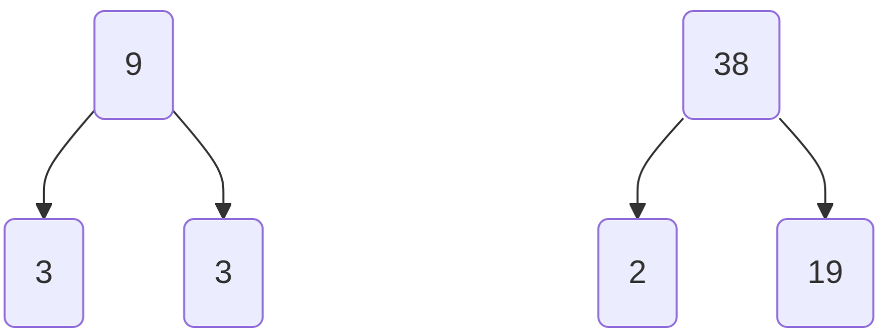
> Simplifying a fraction is just another way of dividing the numerator and denominator by their greatest common factor.

Factor trees help you *see* why this works by breaking the numbers into their building blocks.

If the numerator and denominator of a fraction share no common factors (besides 1) the two numbers are called relatively prime. In this case, the fraction cannot be simplified.

Example:

Simplify $\frac{9}{38}$.



Since 9 and 38 share no factors, $\frac{9}{38}$ is already in simplest form.

1.3.4 – Application: Simplifying with Recipes

Imagine you're following a recipe that makes a giant batch of cookies — way more than you need. You decide to cut the recipe down to a smaller size, but the measurements are a little awkward.

Here's what the recipe says:

- 36 cups of flour
- 60 cups of sugar

You don't want to bake that much — just a smaller, simpler version of the same cookie. But how do you shrink the recipe without changing how the cookies taste?

Let's treat the ingredients like a ratio:

$$\frac{\text{36 cups of flower}}{\text{60 cups sugar}}$$

This ratio tells us how much flour to use per amount of sugar. But the numbers are too big — and a little messy.

Just like with fractions, we can simplify this ratio by dividing both parts by their GCF. We already know the GCF of 36 and 60 is 12.

$$\frac{36 \div 12}{60 \div 12} = \frac{3}{5}$$

So for every 3 cups of flour, you need 5 cups of sugar.

Now you can make a smaller batch that keeps the same balance by finding multiples of the numerator and denominator. For example:

- 3 cups of flour
- 5 cups of sugar

Or double that:

- 6 cups of flour
- 10 cups of sugar

Simplifying the original recipe helped you find a cleaner ratio — one that's easier to scale up or down, depending on how many cookies you want.

Simplifying isn't just a math trick — it helps you work with numbers more easily in the real world. Whether you're adjusting recipes, mixing paint, or scaling blueprints, understanding fractions and simplifying them makes life easier.

Practice On Your Own

GCF Practice

1. Find the greatest common factor (GCF) of each pair:

   a. 20 and 30
   b. 36 and 45
   c. 18 and 48
   d. 30 and 42
   e. 50 and 65
   f. 72 and 90
   g. 81 and 108

2. Two numbers have a GCF of 6. One of the numbers is 18. What could the other number be? Give two possible answers.

3. Two numbers have a GCF of 1. What does that mean? Give an example.

Simplifying Fractions

4. Simplify each fraction:

   a. $\frac{18}{27}$
   b. $\frac{50}{100}$
   c. $\frac{14}{49}$
   d. $\frac{48}{60}$
   e. $\frac{84}{36}$
   f. $\frac{75}{90}$
   g. $\frac{99}{121}$
   h. $\frac{16}{40}$

5. Can a fraction be simplified if the GCF is 1? Explain your answer and give an example.

Word Problems

6. You have 72 juice boxes and 60 cookies. You want to make snack packs with the **same number** of each. You must use **all** the items.

   a. What's the greatest number of snack packs you can make?

b. How many juice boxes and cookies go in each pack?

7. A store is making bundles using 108 pairs of socks and 144 shirts. Each bundle must have the same number of socks and the same number of shirts. There should be no leftovers.

   a. What is the greatest number of bundles they can make?
   b. How many socks and shirts will go in each bundle?

8. A painter mixes 84 ounces of red paint and 36 ounces of blue paint. He wants to pour the paint into small jars that are all the same. Each jar must have the same mix of red and blue paint.

   a. What is the greatest number of jars he can make with no paint left over?
   b. How many ounces of red and blue paint will go in each jar?

Challenge Problems

9. Two numbers multiply to make 180. Their GCF is 6. What could the numbers be?

10. A teacher has 150 pencils and 100 pens. She wants to make gift bags with the same number of each. What is the most gift bags she can make with no leftovers?

---

1. a. 10
   b. 9
   c. 6
   d. 6
   e. 5
   f. 18
   g. 27

2. Example: 30 and 42

3. The numbers are relatively prime. For example: 8 and 15.

4. a. $\frac{2}{3}$
   b. $\frac{1}{2}$
   c. $\frac{2}{7}$
   d. $\frac{4}{5}$
   e. $\frac{7}{3}$
   f. $\frac{5}{6}$

g. $\frac{9}{11}$
h. $\frac{2}{5}$

5. No. The numerator and denominator are relatively prime and so nothing can cancel out.
6. a. 12 snack packs
   b. 6 juice boxes and 5 cookies
7. a. 12 bundles
   b. 9 pairs of socks and 4 shirts
8. a. 12 jars
   b. 7 ounces of red paint and 2 ounces of blue paint
9. 30 and 6 or -30 and -6
10. 30 gift bags with 3 pencils and 5 pens in each with 10 pencils left over

1.4 – Fractions, Decimals & Percents: Conversions

Fractions, decimals, and percents are just different ways of showing the same thing — a part of a whole. Whether you're splitting a pizza, measuring a distance, or shopping during a sale, these numbers are everywhere.

In this lesson, you'll learn how to move between these forms and understand how they relate to each other. This will be an important skill for solving many types of problems that involve parts of a whole.

- ☐ I can convert between fractions, decimals, and percents.
- ☐ I can recognize repeating and terminating decimals and write them as fractions.
- ☐ I can compare and order fractions, decimals, and percents in real-world situations.

fraction, decimal, percent, convert, equivalent, place value

Warm-Up

1. Which one doesn't belong?

   a. $\frac{1}{2}$
   b. 0.25
   c. 50%
   d. 0.5

   (Explain your reasoning. There's more than one right answer.)

2. Which two of these do you think are closest in value?

   a. $\frac{2}{3}$
   b. 70%
   c. 0.25
   d. $\frac{3}{4}$

Learn Together

1.4.1 – What Are Fractions, Decimals, and Percents?

Fractions, decimals, and percents are all ways of showing a **part of a whole**.

A **fraction** shows how many parts out of a total. We often use them when:

- Measuring ingredients: "Add $\frac{2}{3}$ of a cup of milk."
- Position in a ranking: "She ranked 2 out of 350 or $\frac{2}{350}$."
- Splitting or sharing something: "We each got $\frac{3}{8}$ of the pizza"

A **decimal** uses place value and powers of 10. Decimals could be used for:

- Dealing with money: "This costs $4.75."
- Measuring length or weight: "The board is 2.5 feet long."
- Reporting data or averages: "The average was 3.6 stars"

A **percent** means "per 100". Percents are used when:

- Talking about sales or discounts: "It's 20% off!"
- Describing test scores: "She got 95% on the quiz."
- Comparing populations or change: "Unemployment dropped by 2%."

We think about and visualize fractions, decimals, and percents differently too. Here is the same number shown in three different ways!

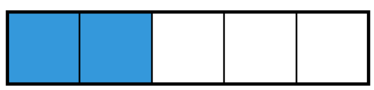


Figure 9: The fraction $\frac{2}{5}$

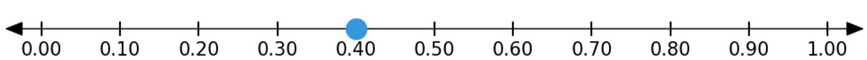


Figure 10: The decimal 0.4

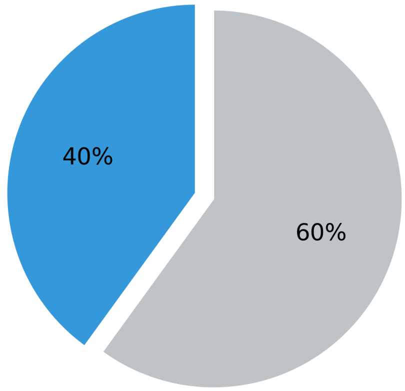


Figure 11: The percentage 40%

You can always use decimals, fractions, and percents interchangeably. Which one you pick often depends on the situation and so it is useful to be able to turn one type into another. The following sections show how to do just that!

1.4.2 – Converting Fractions to Decimals

Have you ever wondered why the division symbol ($\div$) looks the way it does? It has two dots separated by a bar because it represents a fraction! This is because fractions are just a different way to show division. To convert a fraction to a decimal, all we have to do is **divide** the numerator by the denominator.

Let's try:

$$\frac{3}{4} = 3 \div 4 = 0.75$$

When we convert to a decimal, we will see that some decimals **end**, and some **repeat forever**.

Fraction	Decimal
$\frac{1}{4}$	0.25
$\frac{1}{3}$	0.333...
$\frac{2}{3}$	0.666...
$\frac{1}{5}$	0.2

If the decimal repeats, we write a bar over the repeating part: $\frac{1}{3} = 0.\overline{3}$

When you use a calculator to turn a fraction into a decimal, the result might look like it stops, but that can be misleading!

Some fractions have repeating decimals that go on forever — but calculators round or cut them off.

For example:

$\frac{2}{3}$ really equals $0.\overline{6}$ (0.6666...) but your calculator might show 0.6666666667

This can cause small errors if you're not careful — especially when comparing values or converting back to fractions.

Tip: If the decimal looks like it's repeating (like 0.6666666667 or 0.14285714286...), it probably is!

> **What fractions have repeating decimals?**
>
> Fractions with denominators that only use multiples of 2 and 5 will end. Others repeat.
> **Example 1:**
> The fraction $\frac{3}{10}$ has a denominator with factors 2 and 5. It ends and gives 0.3.
> **Example 2**
> The fraction $\frac{2}{6}$ has a denominator with factors 2 and 3. It repeats and gives $0.\overline{3}$.

Using a calculator, we find that...

$$\frac{2}{5} = 2 \div 5 = 0.4$$

1.4.3 - Converting Fractions and Decimals to Percents

Turning a decimal into a percent is easy. We simply move the decimal point to the right twice (the same as multiplying by 100) and then add a % sign.

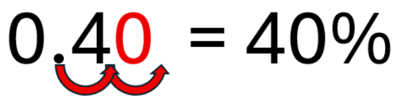
Example: Convert 0.234 to a percent

To do this, we move the decimal two places to the right and then add a % sign. This gives us 23.4%.

$$0.234 = 23.4\%$$

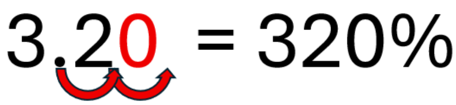
But what if there isn't a digit in the hundredths place? When we don't have enough digits to the right of the decimal, we just fill them in with zeros.

Example: Convert 0.4 to a percent.



But what if the number is bigger than 1? When the number is bigger than 1.0, we do exactly the same thing and end up with a percentage that is larger than 100%!

Example: Convert 3.2 to a percent.



We multiply by 100 (move the decimal to the right twice) and add a % sign. This gives...

$$1.342 \times 100 = 134.2\%$$

To convert a fraction to a percent, we first convert the fraction to a decimal.

Example: Convert $\frac{3}{5}$ to a percent.

$$\frac{3}{5} \Rightarrow 3 \div 5 = 0.6$$

Now we convert the decimal to a percent.

$$0.6 \Rightarrow 60\%$$

First convert to a decimal:

$$\frac{2}{3} \Rightarrow 2 \div 3 = 0.\overline{6}$$

Now convert to a percent:

$$0.\overline{6} \Rightarrow 66.\overline{6}\%$$

1.4.4 Converting Decimals to Fractions

What if we want to go the other direction and convert a decimal to a fraction? The key to doing this is understanding place value.

Suppose we have the number 123.456. The following table shows the place value for each digit.

Place	Value	Explanation
Hundreds	1	1 hundred = 100
Tens	2	2 tens = 20
Ones	3	3 ones = 3
Decimal Point	.	Separates whole from part
Tenths	4	4 tenths = $\frac{4}{10}$
Hundredths	5	5 hundredths = $\frac{5}{100}$
Thousandths	6	6 thousandths = $\frac{6}{1000}$

Place value helps us say the number differently. Instead of saying "123 **point** 456" we could say "123 **and** 456 **one thousandths**". This is the trick to converting decimals to fractions.

Example 1:

Say we want to convert 0.75 to a fraction. We could say this is "zero **point** seven five" but it is more useful to say this is "75 one hundredths". This gives us our fraction!

$$0.75 = 75 \text{ hundredths} \Rightarrow \frac{75}{100} = \frac{3}{4}$$

Example 2

Let's convert 0.1 into a fraction.

$$0.1 = 1 \text{ tenth} \Rightarrow \frac{1}{10}$$

What if we have a number bigger than one?

If the number is bigger than one (like we saw with 4.23) we do the same thing, but we end up with a mixed number.

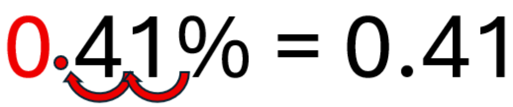
$$4.23 = 4 \text{ and } 23 \text{ hundredths} \Rightarrow 4\frac{23}{100}$$

$$0.51 = 51 \text{ hundredths} \Rightarrow \frac{51}{100}$$

1.4.5 – Converting Percents to Decimals and Fractions

To convert a percent to a decimal simply move the decimal two places left (the same as dividing by 100).

For example, here is what it looks like to convert 41% to a decimal. Notice that we sometimes have to add zeros to the left so the decimal has a place to go. Also notice that if there is no decimal, we put it just after the number before moving to the right.



Here are some more examples:

- $3.4\% \to 0.034$
- $75\% \to 0.75$
- $251\% \to 2.51$
- $0.2\% \to 0.002$

We move the decimal to the left twice and remove the % sign. This gives...

$$35\% = 0.35$$

To write a percent as a fraction, first convert to a decimal and then convert to a fraction, simplifying if possible:

- $3.4\% \to 0.034 \to \frac{34}{1000} \to \frac{17}{500}$
- $75\% \to 0.75 \to \frac{75}{100} \to \frac{3}{4}$
- $251\% \to 2.51 \to \frac{251}{100}$
- $0.2\% \to 0.002 \to \frac{2}{1000} \to \frac{1}{500}$

First, convert to a decimal and then to a fraction.

$$35\% = 0.35 \Rightarrow 35 \text{ hundredths} \Rightarrow \frac{35}{100} = \frac{7}{20}$$

1.4.5 – Bringing It All Together

Now that you've seen how to convert between fractions, decimals, and percents, here's a summary table to help you remember the steps:

From	To	How to Convert
Fraction	Decimal	Divide the numerator by the denominator
Fraction	Percent	First convert to a decimal then convert to a percent
Decimal	Fraction	Use place value (write the fraction as you would say it)
Decimal	Percent	Move the decimal two places right and add % sign
Percent	Decimal	Move the decimal two places left and remove % sign
Percent	Fraction	First convert to a decimal and then convert to a fraction

Some conversions are so common that it helps to **memorize** them. These are called **benchmark values**:

Fraction	Decimal	Percent
$\frac{1}{2}$	0.5	50%
$\frac{1}{4}$	0.25	25%
$\frac{3}{4}$	0.75	75%
$\frac{1}{3}$	0.333...	33.3%
$\frac{2}{3}$	0.666...	66.6%
$\frac{1}{5}$	0.2	20%
$\frac{1}{9}$	0.111...	11.1%
$\frac{1}{10}$	0.1	10%

Practice On Your Own

Conversions Practice

1. Convert each fraction to a decimal:

    a. $\frac{1}{2}$
    b. $\frac{3}{5}$
    c. $\frac{3}{4}$
    d. $\frac{1}{8}$
    e. $\frac{2}{3}$

f. $2\frac{2}{9}$

2. Convert each decimal to a percent:

   a. 0.42
   b. 0.1
   c. 0.125
   d. 0.01
   e. 0.005
   f. 5.03

3. Convert each percent to a decimal:

   a. 15%
   b. 30%
   c. 0.5%
   d. 66%
   e. 3.5%
   f. 132%

4. Convert each percent to a fraction (in simplest form):

   a. 50%
   b. 25%
   c. 80%
   d. 12.5%
   e. 0.5%
   f. 2.75%

Comparison Questions

5. Show the following numbers on the same number line

   - 0.25
   - $\frac{4}{5}$
   - 10%

6. Which is greatest? *(Explain how you know.)*

   a. 0.65
   b. 65%
   c. $\frac{2}{3}$
   d. $\frac{13}{20}$

7. A test has 100 points. Which student did the best?

- Kai scored $\frac{4}{5}$ of the points
- Aaliyah scored 78%
- Zoe scored 0.76

Challenge Problems

8. Which is closest to $\frac{3}{4}$ (show your reasoning)?

   a. 70%
   b. 0.72
   c. $\frac{4}{5}$
   d. 0.68

9. A science quiz has 20 questions. Three students earned the following scores:

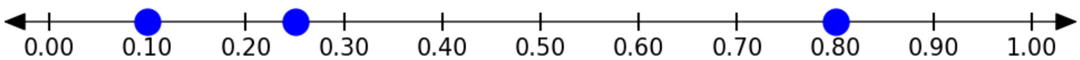
   - Emily got 15 correct
   - Omar got 75% correct
   - Jalen got $\frac{14}{20}$ correct

   Who had the highest score and who had the lowest?

---

1. a. 0.5
   b. 0.6
   c. 0.75
   d. 0.125
   e. $0.\overline{6}$
   f. $2.\overline{2}$
2. a. 42%
   b. 10%
   c. 12.5%
   d. 1%
   e. 0.5%
   f. 503%
3. a. 0.15
   b. 0.3
   c. 0.005
   d. 0.66
   e. 0.035
   f. 1.32
4. a. $\frac{1}{2}$
   b. $\frac{1}{4}$

c. $\frac{4}{5}$
d. $\frac{1}{8}$
e. $\frac{1}{200}$
f. $2\frac{3}{4}$

5. 

6. $\frac{2}{3}$ is greatest because it equals $0.\overline{6}$ all of the others are equal to 0.65.

7. Kai because he scored 80% and the others scored 78% and 76%.

8. B. $\frac{3}{4} = 0.75$. 0.72 is only 0.03 units away from 0.75. The rest are further away.

9. Both Emily and Omar tied for highest at 75%. Jalen got the lowest at 70%.

1.5 – Multiplying & Dividing Fractions

In previous lessons, you've learned how fractions, decimals, and percents relate to each other. But to solve real-world problems—like adjusting recipes, calculating discounts, or determining grades—you also need to multiply and divide these numbers.

In this lesson, you'll learn how to multiply and divide fractions and mixed numbers. We'll explore why multiplying fractions is actually simpler than it seems, and why dividing by a fraction is the same as multiplying by something called a reciprocal. These skills will help you tackle percent problems and practical applications in upcoming lessons.

- ☐ I can multiply fractions and mixed numbers.
- ☐ I can divide fractions by using reciprocals.
- ☐ I can explain what a reciprocal is and why it's useful.

improper fraction, mixed number, reciprocal

Warm-Up

1. What is half of a half?
2. How many fourths are in one whole?
3. Predict: What happens when you multiply two numbers that are smaller than 1?
4. If you divide 1 by $\frac{1}{2}$, will the anser be bigger or smaller?

Learn Together

1.5.1 – Multiplying Fractions

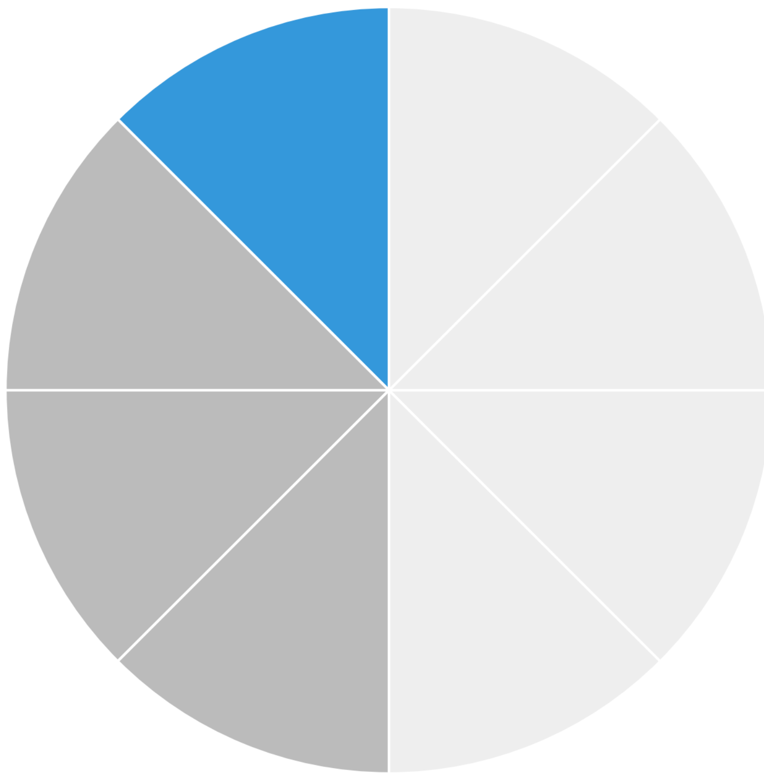
Multiplying by a fraction means finding a fraction **of** something else. In other words, you're finding a **part of a part**.

Example: $\frac{1}{4} \times \frac{1}{2}$

This means:

> **one-fourth of one-half**

Here is what it looks like to find how many fourths are in one half:



That blue slice shows one of the four parts of the darker half.
Since there are 8 total pieces, and only 1 is blue, the answer is $\frac{1}{8}$ of the whole.

Drawing pie charts every time we multiply would take forever—so here's the shortcut:

- Multiply numerators together.
- Multiply denominators together.

$$\frac{1}{4} \times \frac{1}{2} = \frac{1 \times 1}{4 \times 2} = \frac{1}{8}$$

$$\frac{3}{5} \times \frac{4}{7} = \frac{3 \times 4}{5 \times 7} = \frac{12}{35}$$

Multipying Whole Numbers by Fractions

No denominator? No problem! Just put a 1 underneath:

Example: $3 \times \frac{2}{5}$

$$3 \times \frac{2}{5} = \frac{3}{1} \times \frac{2}{5} = \frac{6}{5}$$

$$5 \times \frac{3}{4} = \frac{5}{1} \times \frac{3}{4} = \frac{5 \times 3}{1 \times 4} = \frac{15}{4}$$

Simplifying After Multiplying

The smaller the numbers are, the easier they are to work with. Always simplify fractions when you can.

Example: $\frac{2}{3} \times \frac{9}{4}$

$$\frac{2}{3} \times \frac{9}{4} = \frac{2 \times 9}{3 \times 4} = \frac{18}{12}$$

Cancel out any common factors:

$$\frac{18}{12} = \frac{\cancel{2} \times 3 \times \cancel{3}}{\cancel{2} \times \cancel{3} \times 2} = \frac{3}{2}$$

> Tip
>
> If you factor before you multiply, you get to skip a step.
>
> $$\frac{2}{3} \times \frac{9}{4} = \frac{2 \times 9}{3 \times 4} = \frac{\cancel{2} \times (3 \times \cancel{3})}{\cancel{3} \times (\cancel{2} \times 2)} = \frac{3}{2}$$

$$\frac{8}{9} \times \frac{3}{4} = \frac{8 \times 3}{9 \times 4} = \frac{(\cancel{2} \times \cancel{2} \times 2) \times \cancel{3}}{(\cancel{3} \times 3) \times (\cancel{2} \times \cancel{2})} = \frac{2}{3}$$

1.5.2 – Multiplying Mixed Numbers

A mixed number is a number that has a whole part and a fraction part, like $4\frac{2}{3}$. Before multiplying, we first need to convert to an improper fraction.

Example: Multiply $4\frac{2}{3} \times \frac{5}{6}$

We have 2 thirds, but how many thirds are in 4? To find out, multiply the whole number by the denominator:

$$4 \times 3 = 12$$

Then add the numerator:

$$12 + 2 = 14$$

Now we know that there are 14 thirds.

$$4\tfrac{2}{3} = \frac{14}{3}$$

Now we are ready to multiply:

$$\frac{14}{3} \times \frac{2}{3} = \frac{28}{9}$$

Sometimes we need to **convert back** to a mixed number. Here is how we do that:

First, **divide**:

$$28 \div 9 = 3 \text{ remainder } 1$$

Now we know we have 3 wholes and 1 remaining ninth. Together that is $3\frac{1}{9}$.

Another Example: $2\frac{1}{4} \times \frac{4}{5}$

Step 1: Convert the mixed number.

$$2 \times 4 = 8, \quad 8 + 1 = 9 \Rightarrow 2\tfrac{1}{4} = \frac{9}{4}$$

Step 2: Multiply:

$$\frac{9}{4} \times \frac{4}{5} = \frac{9 \times \cancel{4}}{\cancel{4} \times 5} = \frac{9}{5} = 1\frac{4}{5}$$

Mixed numbers show up often, especially in cooking. If a recipe for 12 needs $3\frac{1}{2}$ cups of flour, and we only want half (for 6 servings), we multiply:

$$3\frac{1}{2} \times \frac{1}{2} = \frac{7}{2} \times \frac{1}{2} = \frac{7}{4}$$

Instead of measuring 7 times with a $\frac{1}{4}$ cup, convert back to a mixed number:

$$\frac{7}{4} = 1\frac{3}{4} \text{ cups}$$

$$3\tfrac{2}{3} \times 4\tfrac{1}{8} = \frac{11}{3} \times \frac{9}{2} = \frac{11 \times (\cancel{3} \times 3)}{\cancel{3} \times 2} = \frac{33}{2} = 8\frac{1}{2}$$

1.5.3 – Division & Reciprocals

Dividing fractions? Easy—meet the reciprocal. It flips fractions upside down!

- Reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$
- Reciprocal of 5 is $\frac{1}{5}$

To divide fractions, multiply by the reciprocal of the divisor:

Example: $\frac{2}{3} \div \frac{1}{4}$

$$\frac{2}{3} \div \frac{1}{4} = \frac{2}{3} \times \frac{4}{1} = \frac{8}{3}$$

Dividing by asks, “How many fourths are there in $\frac{2}{3}$?”

Dividing fractions is as easy as 1-2-3:

1. **Keep** the first fraction.
2. **Change** the division sign to multiplication.
3. **Flip** the second fraction (use the reciprocal).

Practice On Your Own

1. Multiply:

   a. $\frac{2}{5} \times \frac{3}{4}$

   b. $3 \times \frac{4}{7}$

   c. $\frac{5}{6} \times \frac{1}{2}$

   d. $\frac{3}{8} \times \frac{4}{9}$

2. Multiply (mixed numbers):

   a. $1\frac{1}{2} \times \frac{2}{3}$

   b. $2\frac{2}{5} \times 3$

   c. $1\frac{3}{4} \times \frac{2}{3}$

   d. $5\frac{2}{9} \times 4\frac{3}{8}$

3. Divide:

   a. $\frac{3}{4} \div \frac{1}{2}$

   b. $2 \div \frac{2}{5}$

   c. $\frac{5}{6} \div \frac{1}{3}$

   d. $\frac{7}{8} \div \frac{7}{8}$

4. Word Problems:

   a. You have $\frac{3}{4}$ of a pie. If you give everyone $\frac{1}{8}$ of a pie, how many people can you serve?

   b. A recipe calls for $\frac{2}{3}$ cup of flour. If you're making only $\frac{1}{2}$ of the recipe, how much flour will you need?

   c. A board is 6 feet long. Each shelf needs $\frac{3}{4}$ feet. How many shelves can you make?

Challenge Problems

5. What two fractions multiply to give exactly $\frac{3}{8}$ if one of them is $\frac{3}{4}$?
6. A pancake recipe calls for $2\frac{1}{4}$ cups of sugar per batch. If you have exactly 9 cups of sugar, what is the greatest number of full batches you can make?

---

1. Multiply:
   a. $\frac{3}{10}$
   b. $\frac{12}{7}$
   c. $\frac{5}{12}$
   d. $\frac{1}{6}$
2. Multiply (mixed numbers):
   a. 1
   b. $\frac{36}{5} = 7\frac{1}{5}$
   c. $\frac{7}{6} = 1\frac{1}{6}$
   d. $\frac{1645}{72} = 22\frac{61}{72}$
3. Divide:
   a. $\frac{3}{2}$
   b. 5
   c. $\frac{5}{2}$
   d. 1
4. Word Problems:
   a. 6
   b. $\frac{1}{3}$
   c. 8
5. $\frac{3}{4}$ and $\frac{1}{2}$
6. 4

1.6 – Solving Problems With Fractions, Decimals & Percents

Now that you know how to multiply and divide fractions, you're ready to apply those skills to real-life problems.

Fractions, decimals, and percents all show parts of a whole — but they're used in different ways. We see them every day: in grades, prices, sales, surveys, and more.

In this lesson, you'll practice choosing the right form for the situation, estimating or calculating accurately, and solving common problems like "What percent of this is that?" or "What do I need to score to get an A?"

- ☐ I can use fractions, decimals, and percents to solve real-world problems.
- ☐ I can find a part, a whole, or a percent in situations like "30 is 10% of what number?"
- ☐ I can use percents to solve problems involving grades, sales, and survey results.

discount, equivalent, grade, markup, part, percent, proportion, rate, survey, whole

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Warm-Up

Use mental math if you can!

1. What is 10% of 60?
2. If a student scores 45 out of 50 points on a test, what percent did they earn?
3. A shirt costs $40. It's 25% off. What's the sale price?

Learn Together

1.6.1 – What's the Part, Whole, or Percent?

In many real-world problems, you're given some information and asked to find what's missing. Maybe you know the total price and the discount, and you want to know how much money you saved. Or you know how many questions you got right on a quiz, and you want to figure out your percent score.

These types of problems usually involve three pieces:

- part (how much you have),
- whole (the total or full amount),
- percent (the portion out of 100).

If you know two of them, you can figure out the third. This formula helps:

$$\text{part} = \text{percent} \times \text{whole}$$

You can also rearrange it to solve for the percent or the whole. We'll walk through all three types of problems step by step.

> **Tip**
>
> Converting the percent to a decimal will make calculations easier!

Example 1 – Finding the Part

What is 15% of 80?

You're looking for a small part of 80 — just 15 out of every 100. That's what 15% means.

We can turn this into a math sentence by translating the words:

- "is" becomes equals (=)
- "of" becomes times (×)

So:

what	is	15%	of	80?
?	$=$	0.15	$\times$	80

Now solve it:

$$0.15 \times 80 = 12$$

So **12** is 15% of 80.

Turn it into a math sentence:

what	is	10%	of	50?
?	$=$	0.10	$\times$	50

Then multiply:

$$0.10 \times 50 = 5$$

Answer: 5

Example 2 – Finding the Percent

Out of 50 students, 30 say they like soccer. **What percent is that?**"

The phrase **out of** tells us that 50 is the **whole** and 30 is the **part**. We can write that as a fraction, then convert it into a percent.

$$\frac{\text{part}}{\text{whole}} = \frac{30}{50} = 0.6 = 60\%$$

So **60%** of students like soccer.

> **Tip**
>
> To find a percent, divide the part by the whole, then multiply by 100.

$$\frac{3}{100} = 0.03 = 3\%$$

So only 3% of students get enough sleep!

Example 3 – Finding the Whole

25% of a number is 10. What is the number?

We're told that 10 is **25% of the total**. That means 10 is just a piece — one-fourth — of the whole. (Since $25\% = \frac{1}{4}$.)

If one-fourth of something is 10, then we can picture the whole as being made up of **4 equal parts** of 10:

$$10 + 10 + 10 + 10 = 40$$

So the whole is **40**.

But what if the fractions are harder to think about than $\frac{1}{4}$? Here's a trick:

25% as a decimal is 0.25. If we want to know,

> "25% of what number gives me 10?"

we can work backward by **dividing**:

$$10 \div 0.25 = 40$$

So the total is still **40** — it is just another way to get the same answer.

Start by thinking:

$20\% = \frac{1}{4}$ → So if one-fifth is 12, then the whole must be:

$$12 \times 5 = 60$$

Or use division:

$$12 \div 0.2 = 60$$

Answer: 60

1.6.2 – Real-Life Examples

Grades

Mr. Ross hands you back your Algebra quiz. You got 19 out of 20 points! **What is your grade?**

Grades are given as percentages so we want to figure out what $\frac{19}{20}$ is as a percent.

$$\frac{19}{20} = 0.95 = 95\%$$

You got **95%** on that assignment, that's an A!

$$\frac{20}{25} = 0.8 = 80\%$$

John got an 80% which is a B.

Sales & Discounts

A clothing store is having a back-to-school sale. A \$60 jacket is on sale for 25% off.

What's the new price?

First, find the discount by asking "What is 25% of \$60?":

what	is	25%	of	\$60?
?	=	0.25	×	60

$$0.25 \times 60 = 15$$

This tells us we will **save** \$15. To find the new price we subtract:

$$60 - 15 = 45$$

The jacket now costs **\$45**.

Find the discount:

what	is	65%	of	$300?
?	=	0.65	$\times$	300

$$0.65 \times 300 = 195$$

Now subtract to get the sale price:

$$300 - 195 = 105$$

Swanhilda pays $105 for the dress. What a steal!

Surveys & Data

A survey of teachers asked whether they prefer cookies or cake. The survey found that 75% prefer cookies. If 120 of the teachers prefered cookies, how many teachers were surveyed?

Here we want to know the **whole** when we have a percent. First, convert 75% to a decimal:

$$75\% = 0.75$$

Then divide:

$$120 \div 0.75 = 160$$

160 teachers were surveyed.

First, convert to a decimal:

$$55\% = 0.55$$

Then divide:

$$550 \div 0.55 = 1000$$

1000 voters were surveyed.

Practice On Your Own

Find the part whole or percent

1. What is 60% of 95?
2. Eight is what percent of 64?
3. One is what percent of 6?
4. What percent of 40 is 25?
5. Fifteen is what percent of 40?
6. What number is 85% of 40?
7. 85% of a number is 510. What is the number?
8. 20% of a number is 150. What is the number?

Real-Life Scenarios

9. A driver's test has 30 questions. To pass, you must score at least 24 points. What percent do you need to pass?
10. A student scores 27 out of 30 on one test and 42 out of 50 on another. What percent of the total points did they earn?
11. A backpack is 25% off. The original price was $80. What is the discount? What's the new price?
12. A restaurant offers 30% off drinks during happy hour. A soda usually costs $3.50. What is the discount? What is the happy hour price?
13. A survey shows 68% of people prefer cats to dogs. If 250 people were asked, how many chose dogs?
14. A company sells fidget spinners. In a quality control test, they found that 3% of the fidget spinners tested were defective. If 60 spinners were defective, how many fidget spinners did they test?

Challenge Problems

15. A class has 24 students. 1/3 are in choir, 25% are in band. The rest of the students are in art.
    What percent of students are in art class?
16. A shirt is marked down by 40%. It now costs $27. What was the original price?
17. You have 88% of 325 points in Algebra. The last exam has 50 questions and is worth 100 points. If it takes 90% to get an A, is it possible to get an A in the class? If so, how many points do you need to score on the exam?

---

1. 57

2. 12.5%
3. $16.\overline{6}$%

4. 62.5%
5. 37.5%

6. 34
7. 600
8. 750

9. 80%
10. 86.25%
11. Discount: $20, Price: $60
12. Discount: $1.05, Price: $2.45
13. 80 chose dogs
14. 2000 fidget spinners

15. $41.\overline{6}$%
16. $45
17. Yes. You need to get 49 out of 50 correct.

1.7 – Order of Operations

What does this equal?

$$6 + 2 \times 3$$

If you said 24, you're not alone—but that's not the correct answer. Math has specific rules for what to do first. These rules are called the order of operations, and they help make sure everyone simplifies expressions the same way.

In this lesson, you'll learn how to follow those rules correctly—even when negatives, fractions, and grouping symbols are involved.

- ☐ I follow the correct order of operations (PEMDAS).
- ☐ I simplify expressions with fractions and negatives.
- ☐ I avoid common mistakes when simplifying expressions.

expression, order of operations, parentheses, exponent, multiplication, division, addition, subtraction

---

Warm-Up

1. Simplify: $3 + 6 \times 2$
2. True or False: $(4 + 3) \times 2 = 4 + (3 \times 2)$
3. What does $\frac{1}{2} \times (4 + 2)$ equal?

Learn Together

1.7.1 – The Order Matters

In math, the order you do things **really matters** — doing steps out of order can completely change the answer.

Take this simple-looking problem:

$$10 + 3 \times 5$$

Let's try solving it from **left to right**:

$$10 + 3 \times 5 = 13 \times 5 = 65$$

Now let's try doing the **multiplication first**:

$$10 + 3 \times 5 = 10 + 15 = 25$$

So which one is correct — **65 or 25**?

The correct answer is **25**, because we follow a specific order of operations. Without these rules, people could get different answers to the same problem!

Let's look at the rules:

We often remember the order using **PEMDAS**:

- **P** – Parentheses (...)
- **E** – Exponents (like 3^2)
- **M/D** – Multiplication ($\times$) and Division ($\div$)
- **A/S** – Addition ($+$) and Subtraction ($-$)

Note:

- Multiplication and division are on the **same level**. Do them **left to right**.
- The same goes for addition and subtraction.

You can think of it like a ladder. You start at the top and climb your way down!

Because they are **two sides of the same operation**. Any division problem can be rewritten as multiplication by using the **reciprocal**.

Example: $12 \div \frac{3}{2}$ becomes $12 \times \frac{2}{3} = 8$

The same is true for addition and subtraction. Subtraction is really just adding the opposite.

Example: $5 - 2$ becomes $5 + (-2) = 3$

1.7.2 – Examples with Integers

Let's look at some expressions that follow the order of operations. These use only whole numbers — no fractions or negatives yet.

Example 1: $5 + 3 \times 2$

First, do the multiplication:

$$5 + 3 \times 2 = 5 + 6$$

Then do the addition:

$$5 + 6 = 11$$

Example 2: $(5 + 3) \times 2$

First, simplify the parentheses:

$$(5 + 3) \times 2 = 8 \times 2$$

Then multiply:

$$8 \times 2 = 16$$

Example 3: $8 - 12 \div 3$

First, divide:

$$8 - 12 \div 3 = 8 - 4$$

Then subtract:

$$8 - 4 = 4$$

Start with parentheses:

$$5(8-1) + 2 \times 3 = 5 \times 7 + 2 \times 3$$

Next, do the multiplication:

$$5 \times 7 + 2 \times 3 = 35 + 6$$

Finally, add:

$$35 + 6 = 41$$

1.7.3 – With Negatives and Fractions

Once you're comfortable with the basics, we can add in **negative numbers** and **fractions**. The order of operations still works the same way — you just have to be more careful.

Example 1: $-3 \times (4 - 7)$

First, simplify the parentheses:

$$-3 \times (4 - 7) = -3 \times -3$$

Then multiply. Remember: a **negative times a negative is positive**.

$$-3 \times -3 = 9$$

What does multiplying by a negative number do?
It **reverses the sign** of the number it's multiplied by.

Example:
$3 \times -1 = -3 \rightarrow$ the positive 3 becomes negative

Now try it with a negative number:

Example:
$-3 \times -1 = 3 \rightarrow$ the negative 3 becomes positive

So multiplying by -1 always gives the **opposite sign**.
That's why $-3 \times -1 = 3$ — the opposite of negative 3 is positive 3.

Example 2: $\frac{1}{2} \times (6+2)$

First, simplify the parentheses:

$$\frac{1}{2} \times \color{red}{(6+2)} \color{black}{=} \frac{1}{2} \times \color{red}{8}$$

Then multiply:

$$\frac{1}{2} \times 8 = \frac{1}{2} \times \frac{8}{1} = \frac{8}{2} = 4$$

Example 3: $\frac{3+5}{2}$

This fraction means to divide **after** you do the work on top.

Even though there are no parentheses, the **fraction bar acts like grouping symbols** — just like parentheses.

So:

$$\frac{\color{red}{3+5}}{2} = \frac{\color{red}{8}}{2} = 4$$

-3^2 means $-(3^2)$, which is:

$$-(3^2) = -9$$

But $(-3)^2$ means the negative is part of the base:

$$(-3)^2 = 9$$

First do the parentheses:

$$-3 \times (4 - 7) = -3 \times -3$$

Then multiply:

$$-3 \times -3 = 9$$

1.7.4 – More Complex Expressions

Let's try putting all the steps together. When expressions involve grouping, exponents, fractions, and multiple operations, PEMDAS really helps keep things organized.

Example: $4 + \frac{1}{2} \times (6 - 2)^2$

Step 1: Parentheses

$$4 + \frac{1}{2} \times (6 - 2)^2 = 4 + \frac{1}{2} \times 4^2$$

Step 2: Exponents

$$4 + \frac{1}{2} \times 4^2 = 4 + \frac{1}{2} \times 16$$

Step 3: Multiplication

$$4 + \frac{1}{2} \times 16 = 4 + 8$$

Step 4: Addition

$$4 + 8 = 12$$

Without parentheses, you might square the wrong number. Always follow PEMDAS and work **inside parentheses first** — especially when there are **multiple layers**.

Nested Parentheses

Sometimes, expressions use **more than one layer of grouping**. When that happens:

- Always work **from the inside out**
- Brackets [ ] or braces { } might be used to help keep things clear

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Example:
$[3 + (2^2 + 1)] \times 2$

Step 1: Inner parentheses

$$[3 + (2^2 + 1)] \times 2 = [3 + 5] \times 2$$

Step 2: Brackets

$$[3 + 5] \times 2 = 8 \times 2$$

Step 3: Multiply

$$8 \times 2 = 16$$

PEMDAS starts with **grouping**, and that means more than just parentheses!

These all group parts of an expression:

- (...) — parentheses
- [ ] or { } — brackets/braces (used for nesting)

- Fraction bars $\frac{a}{b}$
- Square roots $\sqrt{a+b}$

Always simplify **grouped expressions** before applying exponents or multiplying.

Step 1: Inner parentheses

$$[5 + (3^2 - 1)] \times 2 = [5 + 8] \times 2$$

Step 2: Brackets

$$[5 + 8] \times 2 = 13 \times 2$$

Step 3: Multiply

$$13 \times 2 = 26$$

Final Answer: **26**

1.7.5 – Why This Matters

This might feel like just number-crunching, but it lays the foundation for Algebra. You'll need these skills to simplify expressions, solve equations, and understand formulas.

Later in Algebra, you'll see variables, combining like terms, and the distributive property. If you can't simplify numbers correctly, the rest will fall apart.

You get a 25% off coupon and a $10 gift card. The item costs $40.

Which should be applied first?

- 25% off first → $40 × 0.75 = $30 → $30 - $10 = **$20**
- Gift card first → $40 - $10 = $30 → 25% off = $22.50

Same ingredients, different result. Order matters!

Practice On Your Own

Basic Order of Operations

1. Simplify:

    - a. $4 + 6 \times 2$
    - b. $(4 + 6) \times 2$
    - c. $12 \div 4 \times 3$
    - d. $12 \div (4 \times 3)$

With Negatives & Fractions

2. Simplify:

    - a. $-2 \times (3 - 5)$
    - b. $\frac{1}{2} \times (8 + 4)$
    - c. $(6 - 2)^2 \div 2$
    - d. $(3 + 5) \div 2$

Expression Breakdown

3. Simplify:

    - a. $5 + 2 \times (6 - 1)$
    - b. $(12 - 4)^2 \div 4$
    - c. $10 - 3 \times (2 + 1)$
    - d. $\frac{3}{4} \times (12 - 4)$

Challenge

4. Two students simplified this expression differently:

   $8 - 3 + 2$

   - Student A: $(8 - 3) + 2 = 7$
   - Student B: $8 - (3 + 2) = 3$

   Who is correct? What mistake did the other student make?

5. Simplify the expression:

   $6 + \frac{4 \times (2+1)}{3^2}$

   Be careful with grouping and exponents.

6. A student simplified this:

   $2 + 3^2 \times (4 - 1)$
   and got **27**.

   - What mistake did they make?
   - What is the correct answer?

7. Create your own expression using **at least three operations**, **one fraction**, and **a set of parentheses**.
   Swap with a partner — can they simplify it correctly?

1. a. **16**
   b. **20**
   c. **9**
   d. **1**

2. a. **4**
   b. **6**
   c. **8**
   d. **4**

3. a. **15**
   b. **16**
   c. **1**
   d. **6**

4. **7**

5. $\frac{22}{3}$ or $7\frac{1}{3}$

6. **29**

7. *Answers will vary*

Part II

Unit 2: Algebraic Expressions

Introduction

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Duis sagittis posuere ligula sit amet lacinia. Duis dignissim pellentesque magna, rhoncus congue sapien finibus mollis. Ut eu sem laoreet, vehicula ipsum in, convallis erat. Vestibulum magna sem, blandit pulvinar augue sit amet, auctor malesuada sapien. Nullam faucibus leo eget eros hendrerit, non laoreet ipsum lacinia. Curabitur cursus diam elit, non tempus ante volutpat a. Quisque hendrerit blandit purus non fringilla. Integer sit amet elit viverra ante dapibus semper. Vestibulum viverra rutrum enim, at luctus enim posuere eu. Orci varius natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus.

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2.1 Evaluating Expressions

You'll learn how to evaluate algebraic expressions by substituting values for variables.

Objectives

- ☐ Evaluate expressions with one or more variables
- ☐ Use correct substitution and order
- ☐ Check your work for accuracy

Vocabulary

expression, evaluate, substitute, variable

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

2.2 Inputs, Outputs & Function Machines (Intro)

This introductory lesson explains how functions work using simple input-output models. This is the foundation for understanding functions throughout the course.

Objectives

- ☐ Understand the concept of a function
- ☐ Match inputs with outputs
- ☐ Identify function rules from patterns

Vocabulary

input, output, function, function rule

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

Part III

Unit 3: Solving Equations

Introduction

This unit is where Algebra really begins to feel like solving puzzles. You'll learn how to isolate variables, understand balance, and make sense of problems that come up in everyday life.

What You'll Learn

By the end of this unit, you'll be able to:

- Solve one- and two-step equations using inverse operations
- Distribute and combine like terms in multi-step equations
- Move variables to one side of the equation
- Identify when equations have no or infinite solutions
- Write and solve equations from word problems and contexts

Topics in This Unit

3. Solving One- and Two-Step Equations

Use inverse operations to find solutions.

3. Multi-Step Equations with Distribution

Distribute, simplify, and solve more complex equations.

3. Equations with Variables on Both Sides

Move all variable terms to one side, then solve.

3. No Solution vs. Infinite Solutions

Learn to recognize when an equation has no solution or all numbers work.

3. Writing Equations from Contexts

Translate real-world problems into equations.

3. Solving with Tables, Graphs & Rules

Connect functions to equations and problem-solving.

How to Use This Unit

You'll find plenty of examples, visuals, and practice to help you develop confidence in solving equations from both numbers and words!

3.1 Solving One-Step & Two-Step Equations

In this lesson, students will learn how to solve one-step and two-step equations using inverse operations. This foundational skill sets the stage for solving more complex equations in future lessons.

Objectives

- ☐ Use inverse operations to isolate the variable
- ☐ Solve one-step and two-step equations involving addition, subtraction, multiplication, or division
- ☐ Check solutions by substitution

Vocabulary

equation, inverse operations, solution, variable

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

3.2 Multi-Step Equations with Distribution

This lesson extends equation solving to multi-step problems, including those that require the distributive property and combining like terms.

Objectives

- ☐ Apply the distributive property to simplify equations
- ☐ Combine like terms before solving
- ☐ Solve multi-step equations with multiple operations

Vocabulary

distributive property, like terms, combine, simplify

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

3.3 Equations with Variables on Both Sides

Students will learn how to solve equations where variables appear on both sides of the equals sign, reinforcing the concept of balancing and simplifying equations.

Objectives

- ☐ Move variable terms to one side of the equation
- ☐ Simplify both sides before solving
- ☐ Identify equations with no or infinite solutions

Vocabulary

combine like terms, variable, no solution, infinite solutions

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

3.4 No Solution vs. Infinite Solutions

This lesson focuses on identifying when equations have no solution or infinitely many solutions and how to justify those conclusions.

Objectives

- ☐ Recognize inconsistent equations with no solution
- ☐ Identify dependent equations with infinite solutions
- ☐ Justify solutions using substitution or reasoning

Vocabulary

identity, contradiction, solution set, consistent, inconsistent

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

3.5 Writing Equations from Real-Life Contexts

Students will translate real-world scenarios into algebraic equations, helping them understand the connection between math and everyday problem solving.

Objectives

- ☐ Identify quantities and relationships in word problems
- ☐ Write algebraic equations to represent situations
- ☐ Solve and interpret solutions in context

Vocabulary

context, representation, translate, real-world

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

3.6 Solving with Tables, Graphs & Rules (Function Tie-In)

This lesson introduces multiple representations of relationships — including tables, graphs, and rules — to show how equations can be connected to functions.

Objectives

- ☐ Solve equations by analyzing input-output tables
- ☐ Interpret relationships from graphs and equations
- ☐ Connect equations to real-world patterns

Vocabulary

input, output, table, function, rule, graph

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

Part IV

Unit 4: Graphs and Patterns

Introduction

In this unit, we'll use visual and numerical patterns to understand how algebraic relationships behave. This helps us prepare for graphing and working with functions in more depth.

What You'll Learn

- Recognize and extend arithmetic and geometric patterns
- Build and interpret tables
- Graph expressions and equations
- Compare linear models using graphs

Topics in This Unit

4. Graphing Expressions with Tables

Use input-output tables to generate points.

4. Interpreting Graphs in Context

Make sense of graphs in stories and real-life settings.

4. Arithmetic vs. Geometric Patterns

Identify whether change is constant or multiplicative.

4. Linear Modeling & Rate of Change

Build linear functions and interpret slope in context.

4. Estimating and Checking with Graphs

Use visuals to verify solutions.

How to Use This Unit

Graphing builds a strong link between abstract algebra and concrete understanding. Let's get visual!

4.1 Graphing Expressions with Tables

In this lesson, students will learn how to create tables of values for algebraic expressions and plot them on a coordinate plane. This builds foundational understanding of how algebraic rules connect to visual patterns.

Objectives

- ☐ Generate tables of values from algebraic expressions
- ☐ Graph ordered pairs on the coordinate plane
- ☐ Recognize linear patterns in tables and graphs

Vocabulary

expression, table, ordered pair, coordinate plane, input, output

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

4.2 Interpreting Graphs in Context

Students will examine graphs that represent real-world scenarios and learn how to describe the relationships shown. Emphasis is placed on labeling axes, identifying trends, and understanding what changes in slope mean.

Objectives

- ☐ Identify variables and units from graph labels
- ☐ Describe trends in linear graphs
- ☐ Interpret slope and intercepts in context

Vocabulary

x-axis, y-axis, slope, intercept, context, trend

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

4.3 Arithmetic vs. Geometric Patterns

Students will compare arithmetic and geometric patterns and recognize how they grow. This helps build pattern recognition and introduces exponential growth.

Objectives

- ☐ Identify arithmetic patterns using constant differences
- ☐ Identify geometric patterns using constant ratios
- ☐ Generate sequences and compare their growth

Vocabulary

arithmetic, geometric, sequence, common difference, common ratio, pattern

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

4.4 Linear Modeling & Rate of Change

This lesson focuses on creating linear models from real-life data. Students will identify constant rates of change and use equations to model situations.

Objectives

- ☐ Recognize and describe constant rate of change
- ☐ Write linear equations to represent situations
- ☐ Interpret slope and intercepts from data

Vocabulary

linear model, rate of change, slope, intercept, data

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

4.5 Estimating and Checking with Graphs

Students will use graphs to estimate values and verify solutions to equations. This lesson ties visual reasoning to algebraic work.

Objectives

- ☐ Estimate input or output values from a graph
- ☐ Use a graph to verify equation solutions
- ☐ Analyze how accurate a graph-based solution is

ℹ Vocabulary

estimate, graph, solution, verify, input, output

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

Part V

Unit 5: Inequalities

Introduction

Sometimes in life, it's not about finding the exact number — it's about knowing what's greater or less. In this unit, you'll explore how to express and graph inequalities.

What You'll Learn

- Solve and graph inequalities on number lines
- Write inequalities from real-world contexts
- Understand "greater than" and "less than" symbols
- Explore compound inequalities (optional)

Topics in This Unit

5. One- and Two-Step Inequalities

Use similar steps as equations to isolate variables.

5. Graphing on a Number Line

Use open and closed circles to represent solutions.

5. Writing Inequalities from Situations

Turn words into math using inequality symbols.

5. Interpreting Graphs with Constraints

Match real-world limits to graphs.

5. Compound Inequalities (Optional)

Handle ranges like "between 2 and 5".

How to Use This Unit

Use drawings and comparisons to make inequality concepts more concrete and real-world focused.

5.1 One- and Two-Step Inequalities

In this lesson, students will learn how to solve one-step and two-step inequalities and graph the solutions on a number line.

Objectives

- ☐ Solve one-step inequalities using addition, subtraction, multiplication, and division
- ☐ Solve two-step inequalities
- ☐ Graph the solution sets on a number line

Vocabulary

inequality, solution, greater than, less than, number line

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

5.2 Graphing on a Number Line

Students will practice representing solutions to inequalities by graphing them on a number line, including open and closed circles.

Objectives

- ☐ Understand the use of open and closed circles on a number line
- ☐ Graph simple inequalities
- ☐ Interpret solution sets visually

Vocabulary

number line, open circle, closed circle, graph, solution set

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

5.3 Writing Inequalities from Situations

This lesson teaches students to write inequalities based on verbal descriptions and real-world contexts.

Objectives

- ☐ Translate real-world problems into inequalities
- ☐ Identify keywords that signal inequality relationships
- ☐ Solve and interpret contextual inequalities

Vocabulary

verbal model, inequality, context, translate, interpret

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

5.4 Interpreting Graphs with Constraints

Students will explore how to read and make sense of graphs that include constraints or limited domains and ranges.

Objectives

- ☐ Analyze graphs that include limited domains or ranges
- ☐ Interpret constraints in real-world situations
- ☐ Relate inequalities to graphical representations

Vocabulary

constraint, domain, range, graph, inequality

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

5.5 Compound Inequalities (Optional)

Students will be introduced to compound inequalities, learning how to solve and graph problems with two connected inequalities.

Objectives

- ☐ Understand compound inequalities using ‘and’ and ‘or’
- ☐ Solve compound inequalities
- ☐ Graph compound inequalities on a number line

Vocabulary

compound inequality, and, or, solution set, number line

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

Part VI

Unit 6: Linear Relationships

Introduction

Linear equations are a powerful way to describe change. Whether it's cost, speed, or growth, this unit shows how lines help us understand the world.

What You'll Learn

- Graph lines using slope and intercepts
- Interpret slope as a rate of change
- Write equations from tables, graphs, or situations
- Compare different linear situations

Topics in This Unit

6. Coordinate Plane & Graphing

Plot ordered pairs and recognize axes.

6. Understanding Slope

Learn how steepness shows change.

6. Slope-Intercept Form

Graph and write lines using $y = mx + b$.

6. Writing Equations from Graphs or Words

Use information to build your own equations.

6. Comparing Models

See how different lines behave and what they represent.

6. Applications

Use linear models for real-world math.

How to Use This Unit

This unit brings it all together — tables, equations, and graphs help us tell a full story.

6.1 The Coordinate Plane and Graphing from Tables

This lesson introduces the coordinate plane and helps students practice plotting points and graphing from tables.

Objectives

- ☐ Identify and label the x- and y-axes
- ☐ Plot ordered pairs on the coordinate plane
- ☐ Graph data from tables

Vocabulary

coordinate plane, x-axis, y-axis, origin, ordered pair

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

6.2 Understanding Slope as Rate of Change

Students will explore slope as a measure of how one quantity changes in relation to another, using graphs and real-world contexts.

Objectives

- ☐ Define slope as a rate of change
- ☐ Interpret slope from a graph or context
- ☐ Calculate slope using tables or graphs

Vocabulary

slope, rate of change, rise, run, linear relationship

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

6.3 Slope-Intercept Form

This lesson introduces the slope-intercept form of a linear equation and how to use it to graph lines.

Objectives

- ☐ Understand the form $y = mx + b$
- ☐ Identify slope and y-intercept
- ☐ Graph a line using slope and intercept

Vocabulary

slope-intercept form, slope, y-intercept, linear equation

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

6.4 Writing Equations from Graphs or Words

Students learn to write linear equations from graphs, tables, or written descriptions of relationships.

Objectives

- ☐ Write linear equations from graphs or data
- ☐ Translate real-world relationships into equations
- ☐ Use slope and intercept in context

Vocabulary

linear equation, slope, y-intercept, context, model

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

6.5 Comparing Linear Models from Graphs or Data

Students compare multiple linear models by analyzing graphs and data sets.

Objectives

- ☐ Compare different linear relationships
- ☐ Analyze graphs and tables for patterns
- ☐ Interpret slope and intercept in context

Vocabulary

linear model, compare, rate of change, initial value

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

6.6 Applications: Cost, Speed, Growth

This lesson applies linear modeling to real-life contexts like cost, speed, and growth.

Objectives

- ☐ Apply linear equations to real-life situations
- ☐ Create and interpret graphs in context
- ☐ Understand the meaning of slope and intercept in real-life problems

Vocabulary

cost, speed, growth, context, linear relationship

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

Part VII

Unit 7: Exponents and Powers

Introduction

Exponents let us write repeated multiplication more easily. In this unit, you'll learn the rules for working with exponents to simplify expressions.

What You'll Learn

- Multiply and divide expressions with exponents
- Apply exponent rules (no scientific notation)
- Understand zero and negative exponents

Topics in This Unit

7. Multiplying with Exponents

Use the product rule.

7. Dividing with Exponents

Use the quotient rule.

7. Power of a Power

Apply powers to powers.

7. Zero & Negative Exponents

Learn their meaning and use them simply.

How to Use This Unit

Use guided examples and repetition to get comfortable with patterns in exponent rules.

7.1 Multiplying with Exponents

In this lesson, you'll learn how to multiply expressions that contain exponents. This is a key part of working with powers and simplifying expressions efficiently.

Objectives

- ☐ Multiply powers with the same base
- ☐ Understand and apply the product of powers rule
- ☐ Simplify expressions with exponents

Vocabulary

exponent, base, product of powers rule

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

7.2 Dividing with Exponents

This lesson focuses on how to divide expressions with the same base using exponents. You'll build on what you know about multiplication and simplify complex expressions.

Objectives

- ☐ Divide powers with the same base
- ☐ Apply the quotient of powers rule
- ☐ Simplify expressions involving division and exponents

Vocabulary

quotient, base, exponent

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

7.3 Power of a Power

You'll learn how to raise an exponent to another exponent. This is useful for simplifying more complex expressions and working with formulas.

Objectives

- ☐ Use the power of a power rule
- ☐ Simplify nested exponents
- ☐ Combine exponent rules to simplify expressions

Vocabulary

exponent, power of a power, simplify

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

7.4 Zero and Negative Exponents (Intro only)

This lesson introduces zero and negative exponents. You'll explore what these mean and how they behave in expressions.

Objectives

- ☐ Understand and apply the zero exponent rule
- ☐ Explore the meaning of negative exponents
- ☐ Simplify expressions with zero and negative exponents

Vocabulary

zero exponent, negative exponent, reciprocal

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

Part VIII

Unit 8: Quadratic Thinking

Introduction

Quadratic equations make parabolas, not lines! This unit introduces key forms and solution methods, especially factoring and the quadratic formula.

What You'll Learn

- Identify quadratic forms
- Factor simple trinomials
- Solve quadratics by factoring and formula
- Compare graphs of quadratics and lines

Topics in This Unit

8. Recognizing Quadratics

Understand what makes an equation quadratic.

8. Factoring

Break expressions into binomials.

8. Solving by Factoring

Set equal to zero and find solutions.

8. Quadratic Formula (Intro)

Use the formula to solve when factoring is hard.

8. Graphing Parabolas

See how the shape differs from linear graphs.

How to Use This Unit

This unit prepares students for what's tested and what's useful long-term.

8.1 Recognizing Quadratic Equations

In this lesson, students will learn to identify quadratic equations by their standard form and understand what makes them different from linear equations.

Objectives

- ☐ Recognize quadratic equations in standard form: $ax^2 + bx + c$
- ☐ Identify the key features that make an equation quadratic
- ☐ Distinguish between linear and quadratic relationships

Vocabulary

quadratic, parabola, standard form, coefficient

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

8.2 Factoring Simple Quadratics

This lesson introduces the process of factoring quadratic expressions where the leading coefficient is 1.

Objectives

- ☐ Factor simple quadratic expressions of the form $x^2 + bx + c$
- ☐ Use factoring to find the roots of a quadratic equation
- ☐ Check factored expressions by expanding

Vocabulary

factor, root, binomial, quadratic expression

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

8.3 Solving by Factoring

Students will learn how to solve quadratic equations by factoring and setting each factor equal to zero.

Objectives

- ☐ Solve quadratic equations using factoring
- ☐ Apply the zero product property
- ☐ Interpret solutions in context

Vocabulary

zero product property, solution, quadratic equation

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

8.4 The Quadratic Formula (Intro)

This lesson introduces the quadratic formula as a method for solving any quadratic equation, especially when factoring is not straightforward.

Objectives

- ☐ Identify the components of the quadratic formula
- ☐ Use the quadratic formula to solve quadratic equations
- ☐ Understand when the formula is useful compared to factoring

Vocabulary

quadratic formula, discriminant, solution, standard form

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

8.5 Graphing Parabolas by Table & Comparing with Linear

Students will use tables to graph quadratic functions and compare their shapes and behaviors with linear functions.

Objectives

- ☐ Graph quadratic functions using input-output tables
- ☐ Identify the vertex and axis of symmetry from a graph
- ☐ Compare quadratic and linear graphs

Vocabulary

vertex, axis of symmetry, parabola, table of values

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

Part IX

Unit 9: Systems of Equations

Introduction

Sometimes two equations work together. A system shows how two relationships interact. This unit is optional but powerful.

What You'll Learn

- Understand what a system is
- Solve systems by graphing or substitution
- Apply systems to real-life problems

Topics in This Unit

9. What Is a System?

Understand the idea of two equations and one solution.

9. Solving by Graphing

Find where lines intersect.

9. Substitution (Optional)

Plug one equation into another to find solutions.

9. Word Problems with Systems

Use systems to model stories or scenarios.

How to Use This Unit

Best taught after mastery of equations and graphing — use visuals and pair work!

9.1 What Is a System?

This lesson introduces the concept of a system of equations—two or more equations that share variables. Students learn how solutions to systems represent points that satisfy all equations involved.

Objectives

- ☐ Define what a system of equations is
- ☐ Identify solutions to systems from graphs and tables
- ☐ Understand consistent vs. inconsistent systems

Vocabulary

system of equations, solution, consistent, inconsistent

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

9.2 Solving by Graphing

Students learn to solve systems of equations by graphing each equation and identifying the intersection point. This visual approach builds on prior graphing skills and deepens conceptual understanding.

Objectives

- ☐ Graph linear equations
- ☐ Determine the solution to a system by finding where two lines intersect
- ☐ Interpret real-world meaning from the graph

Vocabulary

graphing, intersection, solution, coordinate

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

9.3 Substitution Method (Optional)

This lesson introduces substitution as an algebraic method to solve systems of equations. Students practice solving one equation for a variable and substituting into the other.

Objectives

- ☐ Solve one equation for one variable
- ☐ Substitute expressions to solve systems algebraically
- ☐ Check solutions for accuracy

Vocabulary

substitution, isolate, expression, solution

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

9.4 Word Problems with Systems

Students apply their knowledge of systems of equations to solve word problems. They learn to represent real-life situations with systems and interpret their solutions in context.

Objectives

- ☐ Translate real-world scenarios into systems of equations
- ☐ Solve using graphing or substitution
- ☐ Interpret solutions in context

Vocabulary

system, context, real-world, model

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

Part X

Unit 10: Cumulative Review and Projects

Introduction

The final unit ties everything together. Reflect, review, and show what you know through projects and EOC practice.

What You'll Learn

- Use vocabulary and concepts from the whole course
- Create graphs, tables, and equations for real-world data
- Review core topics for the final exam or state test

Topics in This Unit

10. Vocabulary Review

Define and use terms from the course.

10. Real-World Projects

Apply math to something meaningful.

10. Presentations

Explain your thinking visually and clearly.

10. Final Review or EOC Practice

Practice key problems to prepare for success.

How to Use This Unit

Encourage creativity and depth of understanding. Show off what you've learned!

10.1 Vocabulary Review

In this lesson, we'll review the key vocabulary from this course and reinforce understanding through matching, definitions, and real-world examples.

Objectives

- ☐ Review and define key algebra vocabulary terms
- ☐ Apply vocabulary in math contexts and explanations
- ☐ Recognize terms in problems and relate them to math operations

Vocabulary

term, coefficient, constant, expression, equation, solution, function

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

10.2 Real-World Projects (Graphs + Tables + Equations)

This lesson applies everything we've learned to real-world situations using data, graphs, tables, and equations to make connections and solve problems.

Objectives

- ☐ Interpret and analyze real-world data
- ☐ Represent situations with tables, graphs, and equations
- ☐ Explain connections between different representations

Vocabulary

data, table, graph, equation, relationship, pattern

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

10.3 Group Presentations or Visual Reports

Students will collaborate on a final presentation or report to demonstrate their learning, using mathematical vocabulary, visuals, and examples.

Objectives

- ☐ Create a visual or oral presentation using math content
- ☐ Work collaboratively to explain mathematical ideas
- ☐ Use accurate vocabulary and representations in communication

Vocabulary

presentation, visual, explanation, evidence, support

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

10.4 Final Assessment or EOC Practice

This lesson offers a chance to show mastery of key Algebra concepts through a final assessment or EOC-style practice problems.

Objectives

- ☐ Demonstrate understanding of major Algebra concepts
- ☐ Solve a variety of equations and interpret representations
- ☐ Apply skills learned to novel and test-like problems

Vocabulary

equation, function, graph, solution, expression

Warm-Up

Coming soon.

Learn Together

Coming soon.

Practice On Your Own

Coming soon.

Part XI

Supplemental

Supplemental Materials

Welcome to the **Supplemental Materials** section of this course! This is where you'll find all the fun, extra, and just plain interesting math content that doesn't quite fit into the main units — but still helps build understanding, spark curiosity, or offer a little challenge.

Use these resources to: - Practice your skills in new and creative ways - Explore math puzzles and logic games - Reinforce key concepts from class - Take a brain break with something still mathy (but fun!)

Math Games & Puzzles

Number grids, logic puzzles, equation word searches, and more.

Extra Practice Worksheets

Targeted drills and alternative problem sets.

Challenge Problems

For students who want to push their thinking further.

Math Activities

Open-ended or interactive things to try out

Happy exploring!

Math Games and Puzzles

Explore these fun and challenging math activities! Click on any worksheet to open the PDF.

Hidden Math Problems

Practice: Arithmetic operations, pattern recognition
How it works: Find groups of 3 numbers in the grid. Add, subtract, multiply, or divide the first two to get the third. Problems may be horizontal, vertical, or diagonal.

Download Worksheet

Want to suggest an activity or submit your own? Let me know!

Resources

Looking for extra support as you learn Algebra? Below you'll find helpful links to additional study materials and downloadable cheat sheets!

Factor Chart

Need help finding all the factor pairs of a number? This chart lists every composite number from 1 to 147 and lists all prime numbers up to 200. For each number, you'll see a list of its factor pairs.

Download Chart

Want to suggest a resource or submit your own? Let me know!

Glossary

Absolute value

The distance a number is from zero on a number line, always expressed as a positive number or zero.

Example:

The **absolute value** of `-7` is `7`.

Addition

An operation that combines two or more numbers into a total.

Example:

$5 + 7 = 12$

Algebra

Algebra is a branch of math that uses letters and symbols to represent numbers and relationships.

It lets us describe patterns, write rules, and solve problems that work in many different situations.

Calculus

Calculus is a branch of math that helps us understand change and motion.

It's used to study how fast things move, how things grow or shrink, and how to find exact areas or curves.

Composite number

A composite number has more than two factors.

That means it can be divided evenly by numbers other than 1 and itself.

> **Example:**
>
> 12 is composite because 2, 3, 4, and 6 all divide it evenly.

Convert

To convert means to **change a number from one form to another** — like from a fraction to a decimal, or a decimal to a percent.

> **Example:**
>
> $\frac{3}{4} = 0.75 = 75\%$ — all are equivalent, just written differently.

Decimal

A decimal is a number that uses a **dot (.)** to show values **less than 1**. It's based on powers of 10.

> **Example:**
>
> 0.5 means 5 tenths, and 3.14 means 3 and 14 hundredths.

Denominator

The denominator is the **bottom number** in a fraction. It tells **how many equal parts** the whole is divided into.

> **Example:**
>
> In the fraction 3/4, the **denominator** is 4.

Discount

A discount is an amount **taken off the original price**. It's usually shown as a percent.

> **Example:**
>
> A 25% discount on a $40 shirt means you save $10. You pay $30.

Division

An operation that splits a number into equal parts.

> **Example:**
>
> $12 \div 3 = 4$ (12 split into 3 equal parts)

Divisible

A number is **divisible** by another if it divides evenly — these rules help you check quickly.

- **Divisible by 2**: The number ends in **0, 2, 4, 6, or 8** (an even number).
- **Divisible by 3**: The **sum of the digits** is divisible by 3.
- **Divisible by 4**: The **last two digits** form a number divisible by 4.
- **Divisible by 5**: The number ends in **0 or 5**.
- **Divisible by 6**: The number is divisible by **both 2 and 3**.
- **Divisible by 9**: The **sum of the digits** is divisible by 9.
- **Divisible by 10**: The number ends in **0**.

Divisor

A divisor is the number you **divide by** in a division problem. It tells you how many parts to split something into.

Example:

In ($12 \div 3 = 4$), the number 3 is the divisor because it divides 12 into 3 equal parts.

Equation

An equation is a math sentence that says two things are equal.

It has an equals sign (=) and shows a relationship between numbers or expressions.

Examples:

$2 + 3 = 5$ and $x + 1 = 7$

Equivalent

Two numbers are equivalent if they have **the same value**, even if they look different.

Example:

$\frac{1}{2}$, 0.5, and 50% are all equivalent.

Even

An **even number** is any whole number that can be divided by 2 **with no remainder**. In other words, it ends in 0, 2, 4, 6, or 8.

Example: 6 is even because $6 $\div 2 = 3$ with no leftover. 14, 28, and 100 are also even.

Exponent

A number that tells you how many times to multiply a base number by itself.

Example:

$3^2 = 3 \times 3 = 9$

Expression

A math sentence made up of numbers, variables, operations, and sometimes parentheses — but **no equals sign**.

Example:

$4 + 2 \times (5 - 1)$ is an expression.

Another Example:

$4(x-3)^2$ is also an expression

Factor

A factor is a whole number that divides another number evenly — with no remainder.

If you can split something into equal groups with no leftovers, the group size is a factor.

Example:

3 is a factor of 12 because 3 × 4 = 12.

Factoring

Factoring means breaking something down into smaller parts that multiply together to make it.

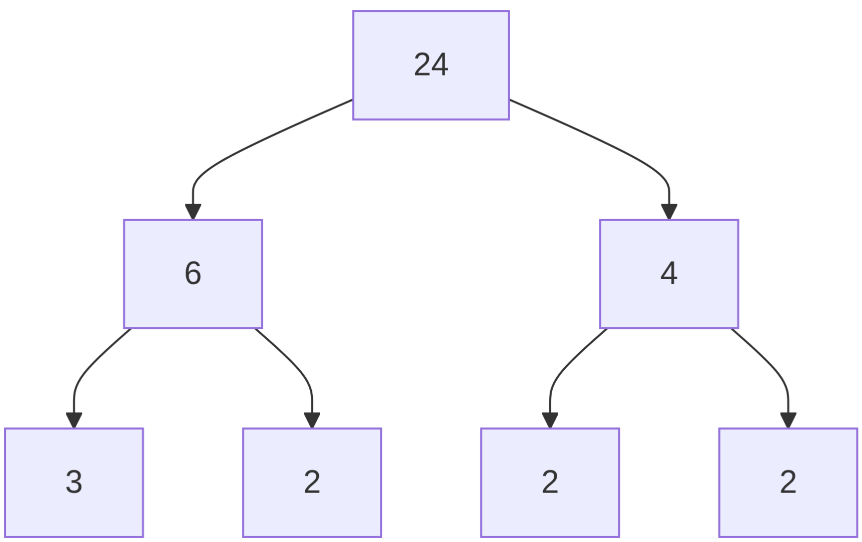
> **Examples:**
>
> - Factoring a number: 12 can be factored into 3×4 or 2×6.
> - Factoring an expression: $x^2 + 5x + 6$ can be factored into $(x + 2)(x + 3)$.

Factor tree

A factor tree is a way to break a number into its smallest building blocks — the prime numbers that multiply to make it.

You keep splitting the number into smaller factors until you can't go any further.

> Example: Here is a factor tree for 24:



> This tells us that the prime factorization of 24 is $2^3 \cdot 3$.

Fraction

A fraction shows a part of a whole. It has a numerator (top number) and a denominator (bottom number).

The denominator tells how many equal parts the whole is divided into, and the numerator tells how many of those parts you have.

Example:

In 3/4, the fraction means **3 out of 4** equal parts.

Grade

A grade often shows **how well someone did** using a percent or letter. It's usually based on **part out of whole**.

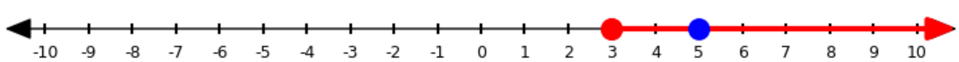
Example:

If you got 18 out of 20 questions right, your grade is $\frac{18}{20} = 90\%$.

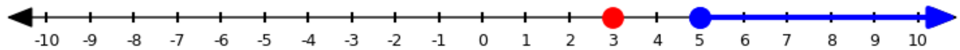
Greater than

A number is greater than ($>$) another number if it is further to the right on the number line.

Example:



$5 > 3$ is true



but $3 > 5$ is false

Greatest Common Factor

The **greatest common factor** (GCF) is the biggest number that divides evenly into two or more numbers. It is the largest factor they have in common.

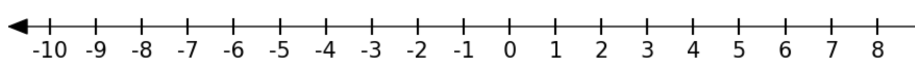
You can find the GCF by listing all the factors and finding the ones shared between the numbers.

> **Example:**
>
> The GCF of 18 and 24 is 6, because 6 is the largest number that goes into both 18 and 24 without a remainder.
>
> Factors of 18: 1, 2, 3, 6, 9, 18
>
> Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24
>
> Shared factors: 1, 2, 3, 6
>
> **The greatest is 6**

Horizontal

Side to side, like the horizon. Level ground is horizontal.

> **Example:**
>
> This is a horizontal number line.



Improper Fraction

An improper fraction is a fraction where the top number (numerator) is equal to or greater than the bottom number (denominator). That means the value is 1 or more.

> **Example:**
>
> $\frac{5}{4}$ is an improper fraction because 5 is greater than 4. It means "five fourths," or $1\frac{1}{4}$ as a mixed number.
>
> **Another example:**
>
> $\frac{8}{3}$ means "eight thirds," or $2\frac{2}{3}$.

Integer

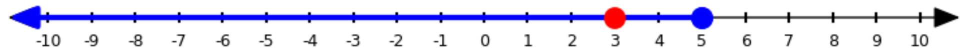
An integer is a whole number (not a fraction or decimal) that can be positive, negative, or zero.

> **Examples:**
>
> `-3`, `0`, `5`, `100`

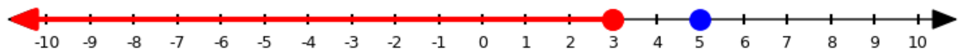
Less than

A number is less than ($<$) another number if it is further to the left on the number line.

> Example:



$3 < 5$ is true



but $5 < 3$ is false

Markup

A markup is the **amount added to a cost** to set a higher selling price, usually as a percent.

Example:

If a store buys a toy for $10 and adds a 50% markup, the price becomes $15.

Mixed Number

A mixed number has a **whole number and a fraction** combined into one value.

Example:

$2\frac{1}{4}$ means 2 wholes and 1 fourth. It's the same as $\frac{9}{4}$.

Multiple

A multiple is what you get when you multiply a number by 1, 2, 3, and so on.

Example:

5, 10, 15, and 20 are all multiples of 5.

Multiplication

A math operation that means repeated addition or combining equal groups.

Example:

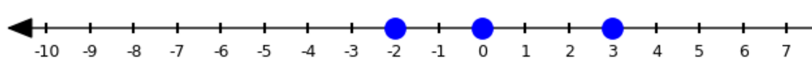
$4 \times 3 = 12$ (4 groups of 3)

Number line

A straight line used to show numbers in order. It usually has zero in the middle, with positive numbers to the right and negative numbers to the left.

Number lines help you visualize math operations and compare values.

Example:



-2, 0, and 3 are all on the number line.

Number sense

Number sense is a person's ability to understand, work with, and think about numbers.

In simple terms, it means having a good feel for how numbers work — like knowing:

- What numbers mean
- How they relate to each other
- How to break them apart or put them together
- What a reasonable answer might be

People with strong number sense can do mental math, estimate, recognize patterns, and spot when something "doesn't make sense."

It's kind of like having a good instinct for numbers — not just memorizing rules, but really *getting* how numbers behave.

Numerator

The numerator is the **top number** in a fraction. It tells **how many parts** you have.

> **Example:**
>
> In the fraction 3/4, the **numerator** is 3.

Negative

A number is negative if it is less than zero.

On a number line, negative numbers are to the left of zero.

> **Example:**
>
> `-4` is a negative number.

Odd

An **odd number** is a whole number that **cannot** be divided evenly by 2. It always has 1 left over. Odd numbers end in 1, 3, 5, 7, or 9.

> **Example:** 5 is odd because $5 ÷2 = 2$ with 1 left over. 11, 33, and 101 are also odd.

Operation

An operation is a **math action** you do with numbers — like adding, subtracting, multiplying, or dividing.

> **Example:**
>
> In (7 + 3), the operation is addition.
> In (12 ÷4), the operation is division.

Opposite

Two numbers that are the same distance from zero on a number line, but on opposite sides.

Their sum is always zero.

> **Example:**
>
> `-3` and `3` are opposites.

Order of Operations

The specific order you follow to simplify expressions: **P**arentheses $\rightarrow$ **E**xponents $\rightarrow$ **M**ultiply/**D**ivide $\rightarrow$ **A**dd/**S**ubtract, or **PEMDAS**.

Multiplication and division are done **left to right**, whichever comes first. Same with addition and subtraction — they are also done **left to right**.

Example:

In $3 + 4 \times 2$, you multiply first: $3 + 8 = 11$

Parentheses

A grouping symbol that tells you what to do first in an expression.

Example:

In $(2 + 3) \times 4$, you add first: $5 \times 4 = 20$

Place Value

Place value tells you **how much each digit is worth** based on where it is in the number.

Example:

In 23.4, the 2 is worth 20 (tens), the 3 is worth 3 (ones), and the 4 is worth 4 tenths.

Part

A part is one piece of the whole. It's often what you're looking for in a percent or fraction problem.

Example:

In 20% of 50, the part is 10. (Because 20% of 50 is 10.)

Percent

A percent is a way of expressing a number **out of 100**. The symbol **%** means "per hundred."

Example:

25% means 25 out of 100. It's the same as $\frac{1}{4}$ or 0.25.

Positive

A number is positive if it is greater than zero.

On a number line, positive numbers are to the right of zero.

Example:

5 is a positive number.

Prime Number

A number is prime if it has exactly two factors: 1 and itself.

Examples:

2, 3, 5, 7, 11

Prime factorization

Prime factorization means writing a number as a product of prime numbers.

Example:

$18 = 2 \times 3 \times 3$ or 2×3^2 These are the prime building blocks of 18.

Product

A product is the **result of multiplying** two or more numbers.

> **Example:**
>
> The product of 4 and 6 is `24`, because `4 × 6 = 24`.

Proportion

A proportion shows **two equal ratios**. It tells you that two fractions or comparisons are the same.

> **Example:**
>
> $\frac{2}{3} = \frac{4}{6}$ is a proportion.

Quotient

A quotient is the **result of dividing** one number by another.

> **Example:**
>
> The quotient of 20 divided by 5 is `4`, because `20 ÷ 5 = 4`.

Rate

A rate compares **two different units**, like miles per hour or cost per item.

> **Example:**
>
> If you drive 60 miles in 2 hours, your rate is 30 miles per hour.

Reciprocal

The reciprocal of a number is what you **multiply it by to get 1**. You can find it by **flipping the numerator and denominator** of a fraction.

Example:

The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$, because $\frac{2}{3} \times \frac{3}{2} = 1$.
The reciprocal of 5 is $\frac{1}{5}$.

Relatively Prime

Two numbers are relatively prime if they have **no common factors** other than 1.

This means their greatest common factor (GCF) is 1, even if neither number is prime.

Example:

8 and 15 are relatively prime because they share no factors except 1. 8 has factors: 1, 2, 4, 8 15 has factors: 1, 3, 5, 15

Remainder

A remainder is what's left over after dividing when the number doesn't go in evenly.

Example:

`10 ÷ 3 = 3` with a remainder of `1`, because `3 × 3 = 9` and there's `1` left.

Simplest Form

A fraction is in simplest form when the numerator and denominator have **no common factors** except 1.

In other words, the fraction can't be reduced any further.

Example:

$\frac{6}{8}$ is not in simplest form because both numbers can be divided by 2. $\frac{3}{4}$ is the simplest form of $\frac{6}{8}$.

Simplify

To **simplify** a number or expression means to **rewrite it in a cleaner or shorter way** — without changing its value. In math, we often simplify fractions, expressions, or equations to make them easier to work with.

Example:

The fraction $\frac{12}{20}$ can be simplified by dividing both the numerator and denominator by 4:

$$\frac{12}{20} = \frac{3}{5}$$

This simpler fraction means the same thing — it's just written with smaller numbers.

Subtraction

An operation that finds the difference between two numbers.

Example:

$9 - 4 = 5$

Sum

A sum is the **result of adding** two or more numbers.

Example:

The sum of 7 and 8 is `15`, because `7 + 8 = 15`.

Survey

A survey is a way to **collect information** from people by asking questions.

Example:

A survey might ask 100 students what their favorite lunch is.

Vertical

Up and down, like a flagpole.

Example:

This is a **vertical** number line.

Whole

The whole is the **total amount** — it's everything being considered before taking parts.

> **Example:**
>
> If 30 is 60% of a number, then the whole is 50.