# 1 Algebra

- Absolute function cannot be part of polynomial
- Linear equations: $a_1x_1 + \ldots + a_nx_n + b = 0$. (e.g. $0 = 8$)
- Homogeneous system: All constant terms are 0.
- $AA^{-1} = I$
- determinant = product of eigenvalues
- Matrix multiplication: flip second matrix by diagonal
- A subspace of $\mathbb{R}^n$ is a subset $V$ of $\mathbb{R}^n$ such that:
  - the zero vector $\mathbf{0}$ is in $V$
  - if $\mathbf{u}$, $\mathbf{v} \in V$, then $\mathbf{u} + \mathbf{v} \in V$
  - if $\mathbf{u} \in V$ and $c \in \mathbb{R}$, then $c\mathbf{u} \in V$
- Basis: Linearly independent set of vectors that span $V$
- $\det(\lambda I - A)$=0
- $\det(A) = \frac{1}{\det(A^{-1})}$
- trivial solution: the zero solution
- linear independence: $c_1\underline{v_1} + c_2\underline{v_2} + c_3\underline{v_3} = 0$
- $AI = A$
- $\det(ABC) = \det(A)\det(B)\det(C)$
- Must use pivot to find basic solution
- Don't use slack variable in vertex.
- Theorems:

  1.1 Any sequence of elementary operations applied to a linear system produces an equivalent system.

  3.1 Every matrix is row equivalent to a unique matrix in reduced row echelon form.

  5.1 The linear system with augmented matrix $[A|\mathbf{b}]$ is consistent if and only if $\mathbf{b}$ is a linear combination of the columns of $A$.

  5.2 Let $A$ be an $m \times n$ matrix.
  - (a) The linear system with augmented matrix $[A|\mathbf{b}]$ is consistent for every $\mathbf{b} \in \mathbb{R}^m$.
  - (b) The columns of $A$ span $\mathbb{R}^m$.
  - (c) The reduced row echelon form of $A$ does not have a row of zeros, that is, there is a pivot in every row of the reduced row echelon form of $A$.

  5.3 $\mathbb{R}^m$ cannot be spanned by fewer than $m$ vectors.

  7.2 A set of more than $m$ vectors in $\mathbb{R}^m$ is linearly independent.

  7.3 : Let $\mathbf{v}_1, \ldots, \mathbf{v}_n$ be vectors in $\mathbb{R}^m$. Let $A$ be the $m \times n$ matrix with columns $\mathbf{v}_n, \ldots, \mathbf{v}_n$.
  - (a) The following are equivalent
    - i. The vectors $\mathbf{v}_1, \ldots, \mathbf{v}_n$ span $\mathbb{R}^m$
    - ii. The reduced row echelon form of $A$ has a pivot in every row.
    - iii. The linear system $[A|\mathbf{b}]$ has at least one solution for every $\mathbf{b} \in \mathbb{R}^m$
  - (b) The following are equivalent
    - i. The vectors $\mathbf{v}_1, \ldots, \mathbf{v}_n$ are linearly independent.
    - ii. The reduced row echelon form of $A$ has a pivot in every column.
    - iii. The linear system $[A|\mathbf{b}]$ has at most one solution for every $\mathbf{b} \in \mathbb{R}^m$.
    - iv. The homogenous linear system $[A|\mathbf{0}]$ has only the trivial solution.
  - (c) A set of $m$ vectors in $\mathbb{R}^m$ is linearly independent if and only if it spans $\mathbb{R}^m$.

  13.1 A square matrix is invertible if and only if it is row equivalent to the identity matrix.

  13.3 Let $A$ be an $n \times n$ matrix. The following are equivalent:
  - (a) $A$ is invertible.
  - (b) The reduced row echelon form of $A$ is the identity matrix.
  - (c) The linear system $A\mathbf{x} = \mathbf{b}$ has a unique solution for every $\mathbf{b} \in \mathbb{R}^n$.
  - (d) The homogenous system $A\mathbf{x} = 0$ has only the trivial solution.
  - (e) The columns of $A$ span $\mathbb{R}^n$.
  - (f) The columns of $A$ are linearly independent.
  - (g) The columns of $A$ form a basis for $\mathbb{R}^n$.
  - (h) $A$ is a product of elementary matrices.
  - (i) $\det A \neq 0$

  15.4 Let $\Omega$ be a nonempty bounded feasible region in $\mathbb{R}^n$.
  - (a) The number of vertices of $\Omega$ is finite.
  - (b) Let $f(x_1, \ldots, x_n) = c_1x_1 + \cdots + c_nx_n$ be an objective function. For every point $\mathbf{x}$ in $\Omega$, there is a vertex $\mathbf{v}$ of $\Omega$ such that $f(\mathbf{v}) \geq f(\mathbf{x})$.
  - (c) The objective function has a largest value on $\Omega$ and it is taken at a vertex. Also ditto but smallest

  16.1 The vertices of the feasible region $\Omega$ correspond to the basic feasible solutions.

  18.3
  - (a) A square matrix $A$ is invertible if and only if $\det A \neq 0$.
  - (b) If $A$ and $B$ are square matrices of the same size, then $\det AB = \det A \det B$
  - (c) If $A$ is an invertible matrix, then $\det A^{-1} = (\det A)^{-1}$

## 2 Calculus

- $\cot\theta = \frac{1}{\tan\theta}$
- $\sec\theta = \frac{1}{\cos\theta}$
- $\csc\theta = \frac{1}{\sin\theta}$
- $\sin(2\theta) = 2\sin\theta\cos\theta$
- $\cos(2\theta) = \cos^2\theta - \sin^2\theta$
- $\sinh x = \frac{e^x - e^{-x}}{2}$
- $\cosh x = \frac{e^x + e^{-x}}{2}$
- $\tanh x = \frac{e^{2x}-1}{e^{2x}+1}$
- $H(x) = \begin{cases} 1 & \text{if } x \geq 0, \\ 0 & \text{if } x < 0, \end{cases}$
- $D(x) = \begin{cases} 1 & \text{if } x \text{ is a rational number,} \\ 0 & \text{otherwise} \end{cases}$
- $\log_b(xy) = \log_b(x) + \log_b(y)$
- $\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$
- $\log_b(\sqrt[y]{x}) = \frac{\log_b(x)}{y}$
- $x^{\log_b(y)} = y^{\log_b(x)}$
- $(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$
- Try something like $\frac{\mathrm{d}}{\mathrm{d}x}(\cosh x) = \frac{\mathrm{d}}{\mathrm{d}x}(\frac{1}{2}(e^x + e^{-x}))$ $= \frac{1}{2}(e^x - e^{-x}) = \sinh x$.
- Remember to look for previous given identities.
- Let $u$ and $v$ be differentiable function
- $\frac{\mathrm{d}}{\mathrm{d}x}\sin x = \cos x$
- $\frac{\mathrm{d}}{\mathrm{d}x}\cos x = -\sin x$
- $\frac{\mathrm{d}}{\mathrm{d}x}\tan x = \sec^2 x$
- $\frac{\mathrm{d}}{\mathrm{d}x}e^x = e^x$
- $\frac{\mathrm{d}}{\mathrm{d}x}\ln x = \frac{1}{x}$
- $\frac{\mathrm{d}}{\mathrm{d}x}(c_1 u + c_2 v) = c_1\frac{\mathrm{d}u}{\mathrm{d}x} + c_2\frac{\mathrm{d}v}{\mathrm{d}x}$
- Product rule: $\frac{\mathrm{d}}{\mathrm{d}x}(uv) = v\frac{\mathrm{d}u}{\mathrm{d}x} + u\frac{\mathrm{d}v}{\mathrm{d}x}$
- $\frac{\mathrm{d}}{\mathrm{d}x}x^r = rx^{r-1} \quad (r \neq 0)$
- Quotient rule: $\frac{\mathrm{d}}{\mathrm{d}x}\frac{u}{v} = \dfrac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$
- Chain rule: $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u}\cdot\frac{\mathrm{d}u}{\mathrm{d}x}$ or $\frac{\mathrm{d}}{\mathrm{d}x}f(g(x)) = f'(g(x))g'(x)$
- $\frac{\mathrm{d}}{\mathrm{d}x}x = 1$
- Exponential function rule: $\frac{\mathrm{d}}{\mathrm{d}x}a^u = a^u\ln a\frac{\mathrm{d}u}{\mathrm{d}x}$
- Derivative of constant is 0.
- Critical point of continuous function: a point at which the derivative is 0 or undefined.
- $\displaystyle\int x^r dx = \frac{x^{r+1}}{r+1} + c, r \neq -1$
- $\displaystyle\int \frac{1}{x}dx = \ln|x| + c$
- $\displaystyle\int e^x dx = e^x + c$
- $\displaystyle\int \sin x\ dx = -\cos x + c$
- $\displaystyle\int \cos x\ dx = \sin x + c$
- $\displaystyle\int \sec^2 x\ dx = \tan x + c$
- $\displaystyle\int \sinh x\ dx = \cosh x + c$
- $\displaystyle\int \cosh x\ dx = \sinh x + c$
- $\displaystyle\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}x + c, \quad |x| < 1$
- $\displaystyle\int \frac{dx}{1+x^2} = \tan^{-1}x + c$
- $\displaystyle\int a^u du = \frac{a^u}{\ln a}$
- The Intermediate Value Theorem: Let $f(x)$ be a continuous function defined on an interval $[a,b]$. Then $f(x)$ takes every value between $f(a)$ and $f(b)$ at least once.
- The Extreme Value Theorem: Let $f$ be a continuous function defined on a closed, bounded interval $[a,b]$. Then, $f$ has a global maximum and minimum on $[a,b]$.
- Find global maxima and minima by finding $f'(x) = 0$ or undefined and end points of the interval.
- The First Fundamental Theorem of Calculus: Let $f(t)$ be a continuous function on $[a,b]$, and let $a < x < b$. Then

$$G(x) = \int_a^x f(t)\ dt$$

is an antiderivative for $f$ on $(a,b)$ - i.e.

$$\frac{\mathrm{d}G}{\mathrm{d}x} = f(x)$$

- The Second Fundamental Theorem of Calculus: Let $f$ be a continuous function on $[a,b]$ and $G$ any antiderivative of $f$ on $[a,b]$. Then

$$\int_a^b f(t)dt = G(b) - G(a) = [G(x)]_a^b$$

- Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$
- Integration by substitution: $\displaystyle\int f(g(x))g'(x)dx \implies \int f(u)du$
- Integration by parts: $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}dx = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}dx$
- Integration by partial fractions: idk how to explain it im extremely tired but like compare numerators ok