

Statistical Inference Project: Central Limit Theorem in an Exponential Distribution

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Overview

This report aims to demonstrate how the Central Limit Theorem applies to the exponential distribution. In the simulation of 1000 runs, the mean of 40 observations in the distribution was computed. These sample means converge to the theoretical mean of 0.5.

Simulations

The exponential distribution can be simulated in R with `rexp(n, lambda)` where `lambda` is the rate parameter. The mean of exponential distribution is $1/\lambda$ and the standard deviation is also $1/\lambda$. For all simulations, $\lambda = 0.2$ was used.

The following is the exponential distribution for 1000 observations:

```
n=1000
lambda = 0.2
mean = 1/lambda
sd = 1/lambda
var = (sd^2)

expDist = rexp(n, lambda)
head(expDist)
```

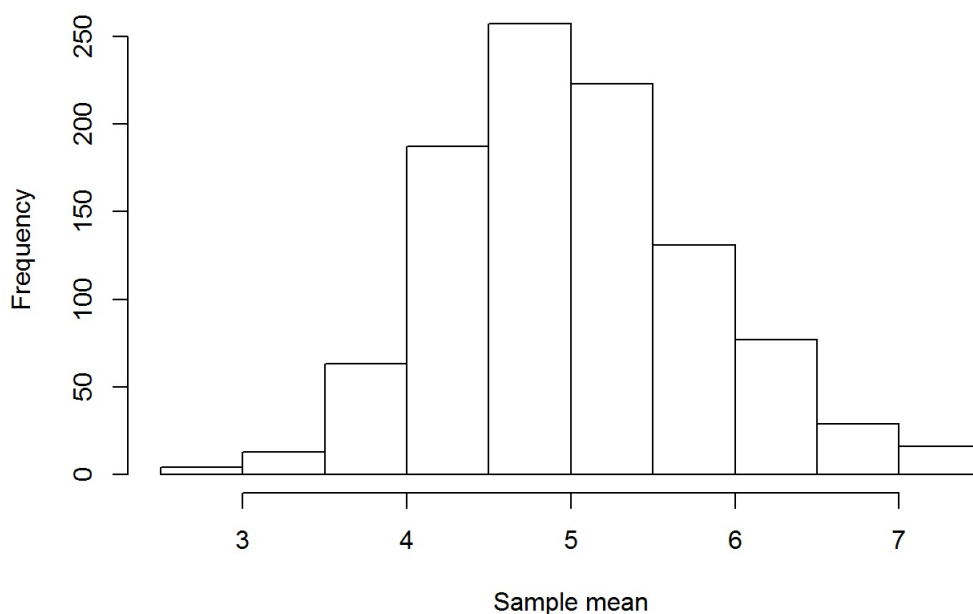
```
## [1] 3.4423946 16.4007094 0.4159244 0.4749522 5.5146261 1.1685493
```

The theoretical mean is 5 and the theoretical variance is 25.

Then we get the sample means from 1000 exponential distribution simulations with 40 observations each, which results in a distribution that looks like a normal distribution also centered at 5.

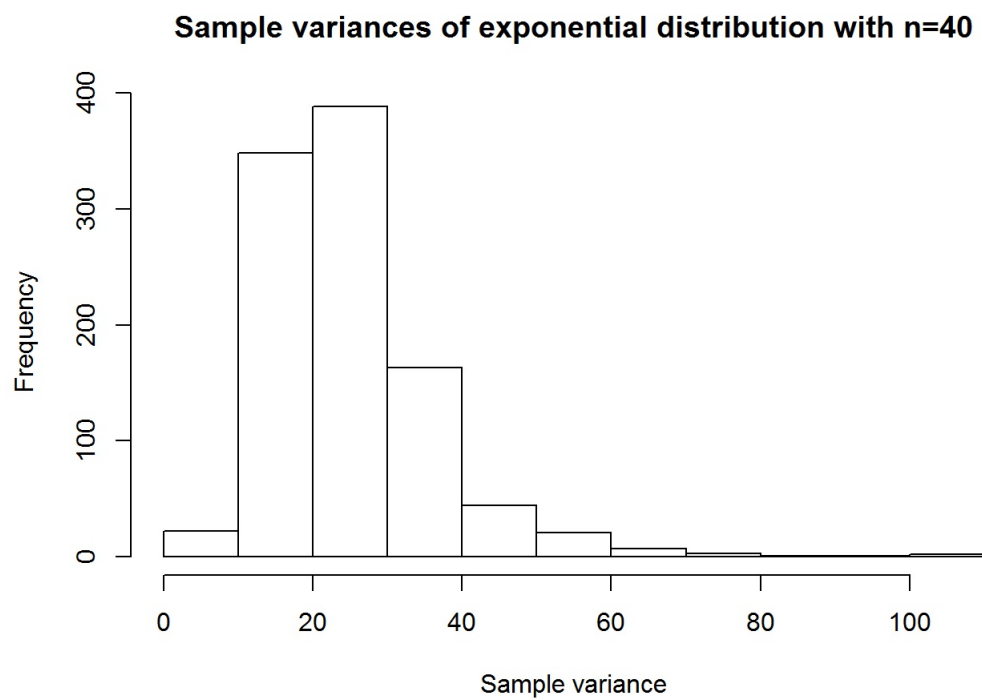
```
mns = NULL
for (i in 1 : 1000) mns = c(mns, mean(rexp(40, lambda)))
hist(mns, xlab = "Sample mean", main = "Sample means of exponential distribution with n=40")
```

Sample means of exponential distribution with n=40



The variance for this also looks like a normal distribution:

```
vars = NULL
for (i in 1 : 1000) vars = c(vars, var(rexp(40, lambda)))
hist(vars, xlab = "Sample variance", main = "Sample variances of exponential distribution with n=40")
```



For these two distributions of sample mean and sample variance:

- The sample mean is centered on the theoretical mean 5.
- The sample variance is also centered on the theoretical variance 25.