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A Multi-Array Antenna System with Optimal Lattice for Rectangular Pyramidal Scanning of Space Debris

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Abstract—**The European Space Agency (ESA) is developing surveillance radars for detecting and cataloguing space debris objects. The surveillance radar, based on a phased-array antenna system, is typically required to scan a specific region of interest, called Field of Regard (FoR). The number of elementary radiators (array elements) in the phased arrays, expected to be very large to provide an adequate link margin to detect objects orbiting at low Earth orbits, has a strong impact on the cost of the final antenna. Therefore, strategies aimed at reducing the array element density, while maintaining uniform lattices and avoiding grating lobes (GLs), are largely welcomed.**

This work shows the preliminary design of a multi-array antenna system, given a FoR defined by a rectangular section. The array orientations in terms of polar coordinate angles will be provided in a local coordinate system, having z axis directed along the zenith. Then, the geometrical lattice of each array, analytically optimized for the minimization of the number of elements, will be explicitly derived in terms of array lattice parameters (horizontal and vertical distance, and skew angle).

Index Terms— **array lattice optimization, field of regard, grating lobes, multi array system, phased array, pyramidal scanning, space debris, surveillance radar.**

I. INTRODUCTION

Space debris are out-of-control objects orbiting around the Earth at different altitudes, from Low Earth Orbit (LEO) to Geostationary Earth Orbit (GEO). They include large objects, such as non-operational spacecrafts and parts of launchers, as well as smaller fragments resulting from intentional and unintentional explosions and/or collisions. Space debris, travelling at a speed that may approach 10 km/s, pose serious problems to operational missions and spacecrafts, which usually implement a collision avoidance system to reduce the probability of a critical impact with space debris [1].

The key point for a collision avoidance system is the detailed knowledge of the orbit parameters of the space debris objects, smaller LEO fragments down to a few cm being the most difficult to detect, track, and catalogue, while still being able to represent a fatal hazard because of their higher orbiting speed. While a number of space-based radars have recently been investigated and proposed [2, 3], the large part of the work, aimed to detect, track, and catalogue LEO objects, still relies on ground-based radars [4]. In particular, the European Space Agency (ESA) is developing a ground-based phased-array radar system, in the framework of the Space Situational Awareness (SSA) program, with the aim of providing independent access to this type of critical information [5].

In this type of surveillance-radar applications, a certain "window" in the sky, called Field of Regard (FoR), is continuously monitored. A phased-array architecture can provide the vital advantage of rapid electronic scanning of a narrow beam, at the cost of large number of radiating elements to maintain the required radar link budget [6]. While the number of elements can be reduced using sparse or thinned arrays [7, 8], a regular lattice can provide uniform performance in terms of UV coverage and a flexible upgradability from initial small arrays to final large operational arrays, spreading the cost and effort of such huge systems over several years.

This paper presents the design process of a phased-array antenna system for rectangular pyramidal scan (Fig. 1), i.e. the FoR is delimited by a rectangular section, whose scan limits are determined by two angles, called deflection angles, measured in a coordinate system x'y'z' where the FoR is symmetrical about the z' axis (Fig. 2). In this work, we present the uniform sectorization of the rectangular pyramidal FoR in the azimuthal direction and the assignation of each sector to a separate array antenna.

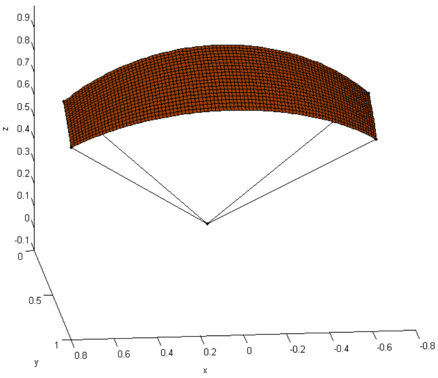

Fig. 1. Rectangular pyramidal Field of Regard (FoR).

In transmission, this system allows to perform parallel scanning of all sectors of the FoR, so reducing the revisit time, i.e. the time needed to perform a complete scanning of the region of interest. A multi-array system is designed and the orientation of each array provided in terms of azimuth and elevation angles of the array axis in a local coordinate system, where zenith coincides with the z axis, and x and y axes lie in the plane of the horizon. After calculating the orientation of each array in the multi-array system, an array-lattice optimization procedure is applied. In particular, the array lattice is optimized in order to minimize the number of radiating elements in the array, without allowing grating lobes to show up in the array factor, while maintaining a regular lattice. Eventually, analytical formulas for the main array-lattice parameters will be provided.

II. A MULTI-ARRAY SYSTEM

This section shows the calculation of the orientation of each array in a multi-array system. The orientation is provided in terms of a couple of polar angles defining the direction of the array axis, where such angles are measured in a local coordinate system (xyz) having z axis coinciding with the local zenith. Let the FoR, as in Fig. 2, be defined between –B and B in the y'z' plane, and between –A and A in the x'z' plane, in the FoR-referenced coordinate system. Now, let the FoR be divided into N_S identical sectors.

Each array in a multi-array system will point orthogonally to its sector of the FoR. An example of a two-array system is depicted in Fig. 3. At the outset, without losing generality, let us assume that the FoR is oriented such that the z axis points orthogonally to the FoR (case 1). In this case, the determination of each array orientation is simpler. In fact, it is possible to calculate the angle α_n formed by the axis $\mathbf{v}_n$ of the n-th array with respect to the z axis on the xz plane (Fig. 4). This angle will be equal to:

$$\alpha_n = -A + \frac{A}{N_s} + 2(n-1)\frac{A}{N_s} \qquad (1)$$

Generally, in the local coordinate system, the FoR is located at an elevation of (90°-δ), measured between the FoR's axis (vector pointing at the FoR's center) and the plane of the horizon (Fig. 5). Also assume that y axis of this coordinate system divides into two halves the projection of the FoR onto the xy plane. This FoR is nothing but the FoR as in case 1, rotated by an angle δ from the z axis in the yz plane. Hence, to calculate the new orientation of the n-th array's axis, it is sufficient to apply a rotation of $\mathbf{v}_n$ by an angle α_n about the x axis. Hence, overall to calculate the precise orientation of vector $\mathbf{v}_n$, it is sufficient to start from a vector equally oriented to z axis, say $\mathbf{v}_0 = (0\ 0\ 1)^T$, and rotate it twice, firstly by an angle α_n about y axis and secondly by an angle δ about x axis. The corresponding rotation matrix is:

$$\mathbf{R} = \mathbf{R}_x \times \mathbf{R}_y$$

where

$$\mathbf{R}_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\delta & \sin\delta \\ 0 & -\sin\delta & \cos\delta \end{bmatrix}$$

$$\mathbf{R}_y = \begin{bmatrix} \cos\alpha_n & 0 & \sin\alpha_n \\ 0 & 1 & 0 \\ -\sin\alpha_n & 0 & \cos\alpha_n \end{bmatrix}$$

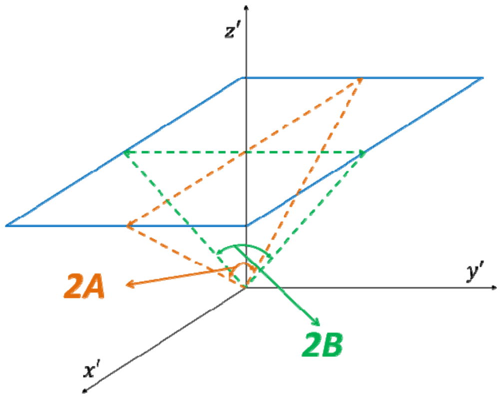

Fig. 2. Definition of a rectangular pyramidal scan.

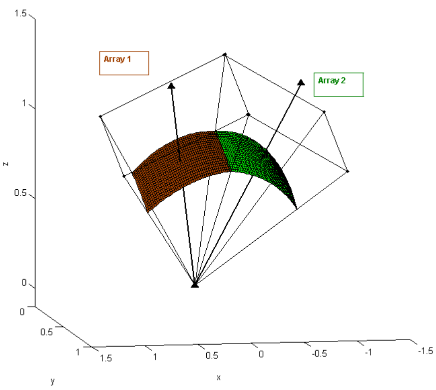

Fig. 3. FoR split into two sectors, in the case of a two-array

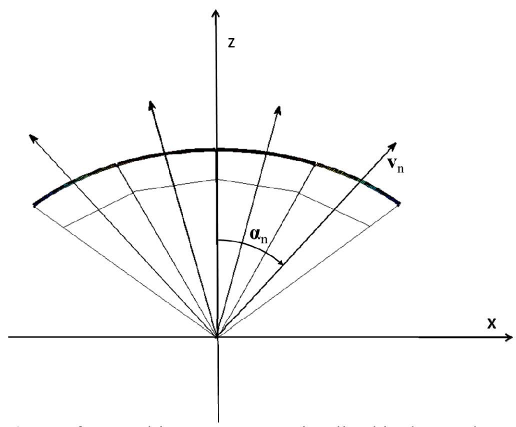

Fig. 4. FoR for a multi-array system, visualized in the xz plane (case

The vector $\mathbf{v}_n$ will be equal to:

$$\mathbf{v}_n = \mathbf{R} \times \mathbf{v}_0 = \begin{bmatrix} \sin\alpha_n \\ \cos\alpha_n \sin\delta \\ \cos\alpha_n \cos\delta \end{bmatrix}$$

Eventually, as shown in Fig. 6, the direction of vector $\mathbf{v}_n$ is described by the two polar coordinate angles (ϑ_n, φ_n):

$$\vartheta_n = \cos^{-1}(\cos\alpha_n \cos\delta) \tag{2}$$

$$\varphi_n = \begin{cases} \tan^{-1}\left(\dfrac{\cos\alpha_n \sin\delta}{\sin\alpha_n}\right) + \pi & \text{if } -\dfrac{\pi}{2} \le \alpha_n < 0 \\ \dfrac{\pi}{2} & \text{if } \alpha_n = 0 \\ \tan^{-1}\left(\dfrac{\cos\alpha_n \sin\delta}{\sin\alpha_n}\right) & \text{if } 0 < \alpha_n \le \dfrac{\pi}{2} \end{cases} \tag{3}$$

III. ARRAY-LATTICE OPTIMIZATION

The array lattice can be optimized for a rectangular pyramidal scan (or FoR) to minimize the required number of elements, while maintaining a regular lattice, shown in Fig. 7, described by three parameters a, b and γ [9]. To ease the analysis of the array and better understand the aspects related to grating lobes, it is more convenient to pass from the angle space to the so-called UV space, through the following mathematical relations:

$$u = \sin\theta \cdot \cos\phi$$
$$v = \sin\theta \cdot \sin\phi$$

It is observed that the rectangular pyramidal sector, as in Fig. 2, is mapped into the interception of two mutually orthogonal ellipses (Fig. 8), one having the semi-major axis vertically oriented (vertical ellipse), and the other horizontally oriented (horizontal ellipse), reported for sake of completeness in (4), where $\psi = \tan^{-1}(T_y/T_x)$, with $T_y = \tan B$ and $T_x = \tan A$.

$$(u, v)_{p.s.} = \begin{cases} \left(\dfrac{u}{\sin A}\right)^2 + v^2 = 1 & \text{if } 0° \le \phi \le \psi \\ u^2 + \left(\dfrac{v}{\sin B}\right)^2 = 1 & \text{if } \psi \le \phi \le 90° \end{cases} \tag{4}$$

The analysis of grating lobes was performed in the UV space, by considering the tangential condition for the closest grating lobe contours. This condition imposes that grating lobe contours must not enter but at most be tangent to the unit circle. It can be found [10] that the centres of GL contours are given by:

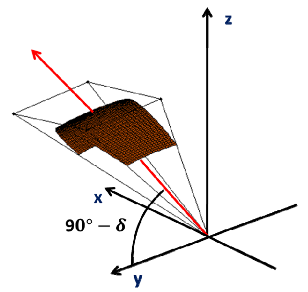

Fig. 5 Location of the FoR in the local coordinate system

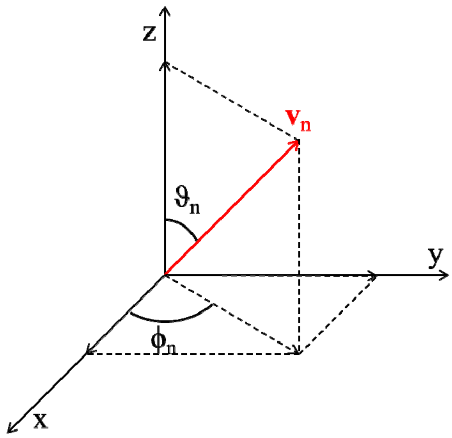

Fig. 6 Location of the FoR in the local coordinate system

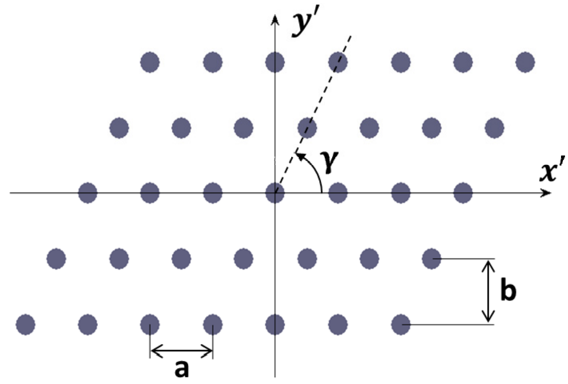

Fig. 7. General triangular array lattice.

$$\begin{aligned} (u_{GL})_p &= \frac{p\lambda}{a} \quad \text{with } p = \pm1, \pm2, \ldots \\ (v_{GL})_{p,q} &= \frac{q\lambda}{b} - \frac{(u_{GL})_P}{\tan\gamma} \quad \text{with } q = \pm1, \pm2, \ldots. \end{aligned} \tag{5}$$

and forcing grating lobe contours (Fig. 9) to be tangent to the unit circle, the following optimal array parameters are calculated [9]:

$$\gamma = \tan^{-1}\left(\frac{2b}{a}\right) \qquad (6)$$

$$b = \frac{\lambda}{1+\sin B} \qquad (7)$$

The optimum value for a is found by imposing that either critical points CP2 or CP3 are touching the unit circle. This calculation, not derived in [9], is presented here. The coordinates of CP2 are given by:

$$u_{CP2} = u_{P1} + u_{GL2}$$
$$v_{CP2} = v_{P1} + u_{GL2}$$

where point P1 is the bottom-left corner point of the scan region (Fig. 9) and $u_{GL2} = \lambda/a$, $v_{GL2} = \lambda/(2b)$ are the UV coordinates of grating lobe GL2 from Eq. (5). Coordinates of point P1 can be calculated by solving the following system of two equations:

$$(u,v)_{P1}: \begin{cases} v = T_{\phi} \cdot u \\ \left(\frac{u}{\sin A}\right)^2 + v^2 = 1 \end{cases} \qquad (8)$$

where T_{ϕ} = (tan B)/(tan A) is the angular coefficient of the straight line in Fig. 8. P1 is given by the intersection of this line with the vertical ellipse. Coordinates of P1 are obtained by keeping the negative solutions to system (8):

$$\begin{cases} u_{P1} = -\sqrt{\frac{\sin^2 A}{1+(\sin A \cdot T_{\phi})^2}} \\ v_{P1} = -T_{\phi} \cdot u_{P1} \end{cases} \qquad (9)$$

Coordinates of CP2 can be obtained by translation of point P1 by an amount equal to GL2:

$$\begin{aligned} u_{CP2} &= u_{P1} + u_{GL2} \\ v_{CP2} &= v_{P1} + v_{GL2} \end{aligned} \qquad (10)$$

Now, the condition for CP2 to touch the unit circle is that its distance from the origin equals 1:

$$(u_{CP2})^2 + (v_{CP2})^2 = 1 \qquad (11)$$

After plugging (9) and (10) into (11) and substituting the coordinates of GL2, the optimum value for a is retrieved as:

$$a = \frac{\lambda}{\sqrt{1-\left(v_{P1}+\frac{\lambda}{2b}\right)^2} - u_{P1}} \qquad (12)$$

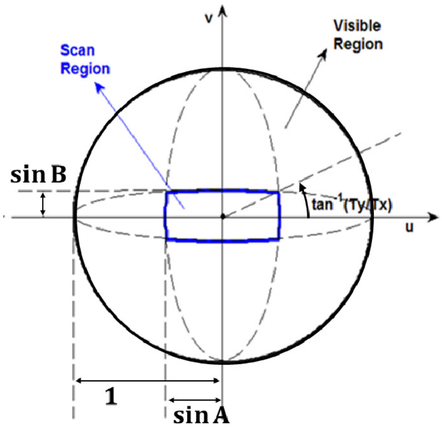

Fig. 8. UV mapping of a rectangular pyramidal FoR.

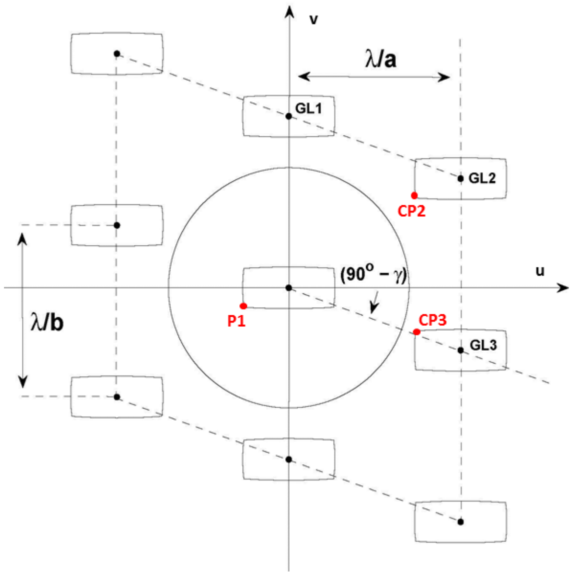

Fig. 9. UV mapping of grating lobes for rectangular pyramidal scan.

The three lattice parameters are now determined. To summarize, the procedure to calculate the optimum array-grid geometrical parameters can be structured in three steps:

1. Compute u_{P1} and v_{P1} according to (9);
2. Use (7) and (12) to determine the optimum vertical and horizontal element spacings b and a;
3. Calculate the skew angle γ by using Eq. (6).

IV. Conclusions

A multi-array-antenna system for parallel scanning of a certain field of regard, aimed at space debris detection, has been designed in the framework of the European Space Agency Space Situational Awareness program, providing the orientation of each array in the local coordinate system. A complete procedure to calculate the optimal array-lattice geometrical parameters, given some rectangular pyramidal scan specifications, has been developed. The optimization has been performed through the analysis of grating lobes versus scan specifications in the UV space. Analytical expressions for the three array parameters have been developed, in contrast to purely graphical techniques relying on iterative optimization procedures.

REFERENCES

[1] "Technical Report on Space Debris," *United Nations*, New York 1999.

[2] J. Lv, J. Wu, and B. Sun, "Space debris detection by space borne radar in the low earth orbit," *Remote Sens. Technol. Appl.*, Vol. 21, No. 2, pp. 103–108, 2006.

[3] R. Battaglia, M. Ferri, and V. Dainelli, "W-band advanced radar for debris early notification from ISS," *Proc. IEEE Aerospace Conf.*, Big Sky, Montana, March 8–15, 2003.

[4] Dieter Mehrholz, "Radar Observations in Low Earth Orbit," *Advances in Space Research*, Vol. 19, No. 2, pp. 203–212, 1997.

[5] "Declaration on the Space Situational Awareness (SSA) Preparatory Programme," *European Space Agency Council*, ESA/C(2008)192, Att.: ESA/C/SSA-PP/VII/Dec. 1 (Final), Paris, 8 December 2008.

[6] J. Garcia-Gasco Trujillo, S. Halte, M. S. Perez, and P. Besso, "On the design of a planar phased array radar antenna architecture for space debris situational awareness," *7th European Conference on Antennas and Propagation (EuCAP)*, Gothenburg, Sweden, April 8-12, 2013.

[7] P. Angeletti and G. Toso, "Synthesis of circular and elliptical sparse arrays," *Electronics Letters*, Vol. 47, No. 5, pp. 304–305, March 2011.

[8] G. Toso, C. Mangenot, and A. G. Roederer, "Sparse and Thinned Arrays for Multiple Beam Satellite Applications," *2nd European Conference on Antennas and Propagation (EuCAP)*, Nice, France, November 11–16, 2007.

[9] S. Zinka, Il-Bong Jeong, Jong-Hoon Chun and Jeong-Phill Kim, "A novel geometrical technique for determining optimal array antenna lattice configuration", *IEEE Trans. Antennas Propagat.*, vol. 58, no. 2, pp. 404-412, 2010.

[10] A. Bhattacharyya, *Phased Array Antennas, Floquet Analysis, Synthesis, BFNs, and Active Array Systems*. Hoboken, NJ: Wiley-Interscience, 2006.