Incoherent Scatter Radar Techniques-

an overview

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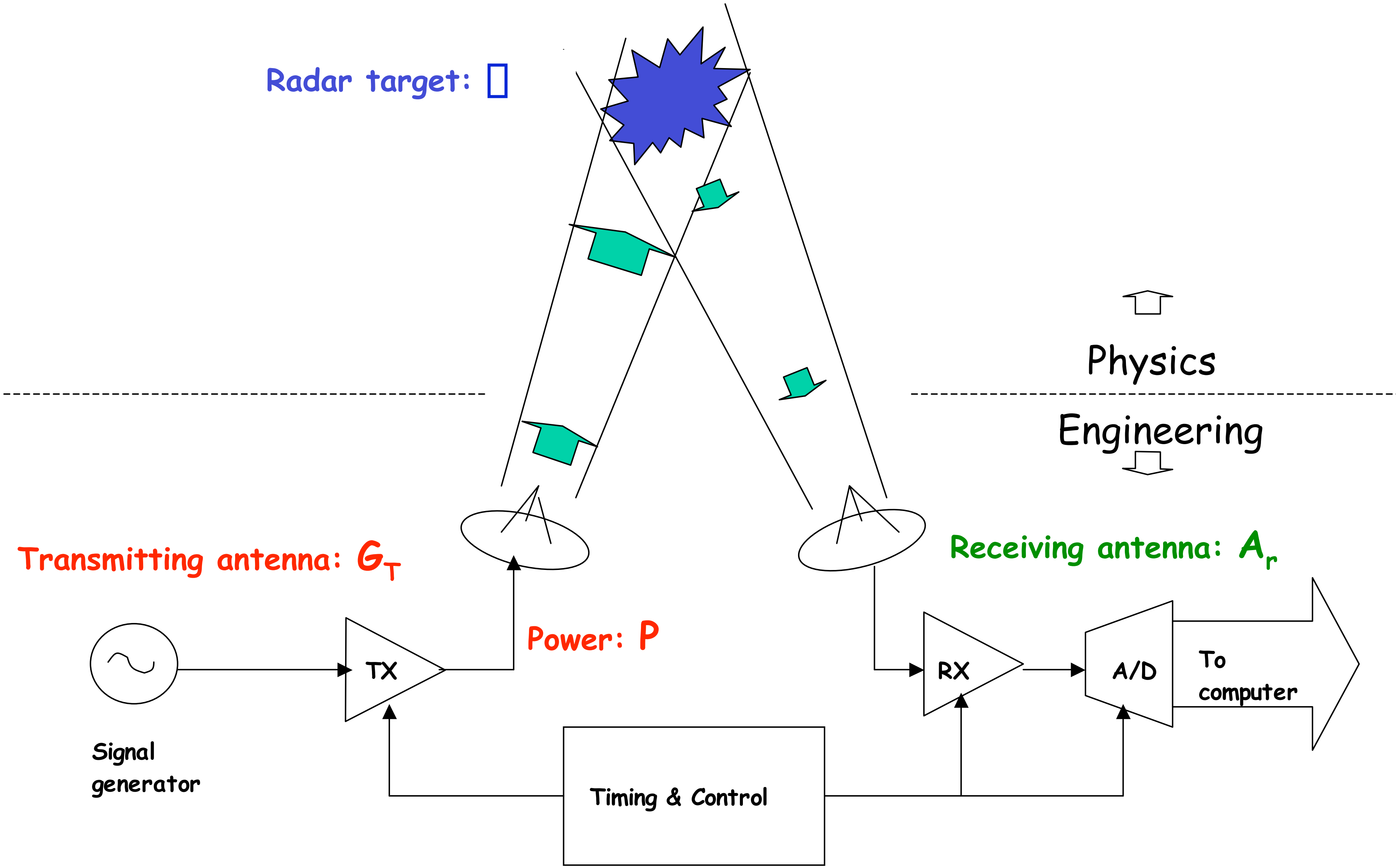
A generic radar system



ISR spectral estimates, physical parameters

The essential job of an ISR experiment is to estimate the *power spectral density* (PSD), or *autocorrelation* (ACF) of IS radar returns as functions of space and time, with sufficient spatial and temporal resolution to resolve the medium properly, such that we can model and understand the physics.

The *plasma dispersion relation,* which governs the shape of the *plasma power frequency spectrum,* is a function of

• Electron density	N_e
• Ion temperature	T_i
• Electron/ion temperature ratio	T_e/T_i
• Mean ion mass	m_i
• Ion-neutral collision frequency	ν_{in}

To extract physical parameter values, the measured ACFs are input to an inverse problem, which is then solved iteratively for $[N_e\ T_i\ T_e\ m_i\ \nu_{in}]$ by the GUISDAP system (more about that later...)

Planning an ISR experiment:

- What *part/region of the ionosphere* will we be looking at ?
  - D, E, F or the topside ? Vastly different densities, scale sizes...
- Which *radar system(s)* will be used ?
  - Different operating frequencies, antenna gains, power levels....
- What *time resolution* are we aiming for ?
  - what we can achieve in practice is essentially a function of S/N
- Which *physical parameters* do we plan to extract ?
  - Only densities and temperatures, or the full five parameter set ?
- What *error levels* can we accept ?
  - related to the power spectral resolution that can be achieved

Typical conditions in the ionosphere

When planning an experiment, you should start from some best-guess values for N_e, T_i and T_e. The Table below summarises what conditions in the ionosphere above Tromsø were thought to be like in pre-EISCAT times (*Bratteng and Haug, 1971*). While that model is now obsolete, the numbers are not too badly off the mark – so we use them for illustrative purposes:

Height [km]	Quiet night N	Min N	Model N	Model T_i	Model T_e	Max N	Max T_i	Max T_e
120	1.0 e10	1.0 e11	1.0 e11	330	330	1.0 e12	350	350
150	3.0 e10	2.7 e11	2.7 e11	480	520	2.3 e12	520	560
200	5.2 e10	4.0 e11	5.0 e11	760	1080	3.6 e12	1000	1580
250	6.6 e10	3.0 e11	6.0 e11	960	1680	4.7 e12	1400	2870
300	6.9 e10	1.8 e11	6.0 e11	1020	2400	4.6 e12	1720	4100
500	2.4 e10	2.4 e10	2.7 e11	1160	3000	1.6 e12	2500	4800

NOTE: The degree of ionisation is for the most part <u>very small</u>:

– At 300-350 km (the peak of the F layer) some 3 % ,

– at 100 km (the E layer) normally less than 10^{-4} ,

– at 70 km (D layer), in exceptional circumstances, it can be 10^{-5}

ISR systems parameters

When planning an experiment, you also need values for all the following parameters for the ISR system you will be using:

• Operating frequency / wavelength	f_{radar} / □$_{radar}$
• TX peak power	P
• TX duty cycle	□
• Max. and min. TX pulse length	t_{min}, t_{max}
• Max. pulse-repetition frequency	PRF_{max}
• Receiver noise temperature	T_{noise}
• Receiver bandwidth	B
• Antenna gain / collecting area	G, A

Figure-of-merit of ISR systems

f_{radar} , P, □, T_{noise} and G all influence the SNR that a radar will produce from a given plasma density at a given range, and thus indirectly the *time resolution*. For some examples see the Table below:

ISR system	f_{radar} [MHz]	$□_{radar}$ [m]	P [MW]	□ [%]	T_{noise} [K]	G [dBi]	FOM*
EISCAT UHF	928	0.33	1.3	12.5	100	48.1	2.8
EISCAT VHF	224	1.34	3.0	12.5	250	46	47.3
ESR 32-m	500	0.60	1.0	25.0	65	42.5	12.4
Sondrestrom	1290	0.23	3.0	3.0	90	50.8 ?	3.6

* FOM stands for "Figure Of Merit". It is a relative measure of the signal-to noise ratio that the radar will produce from some standard ionosphere in unit time and has the dimension [MW m^2 GHz^{-1} K^{-1}]. Debye effects are not considered.

t_{min}, t_{max} and B control the radar range and spectral resolution (more later)

Which parameters can we actually control ?

The notion of *designing* experiments implies that some aspects of how the ISR system works are actually under your control.

In most ISRs there are essentially only two subsystems that you can really manipulate freely –

- *Antenna*
  - pointing / scanning
  - dwelltimes
  - beam selection (when multiple beams available)

- *Transmitter modulation*
  - frequency
  - phase
  - coding

Scale heights, tidal wave modes and spatial resolution

The *scale height,* H, of an atmospheric constituent is the height over which its density varies by 1/e. For the plasma part, this is

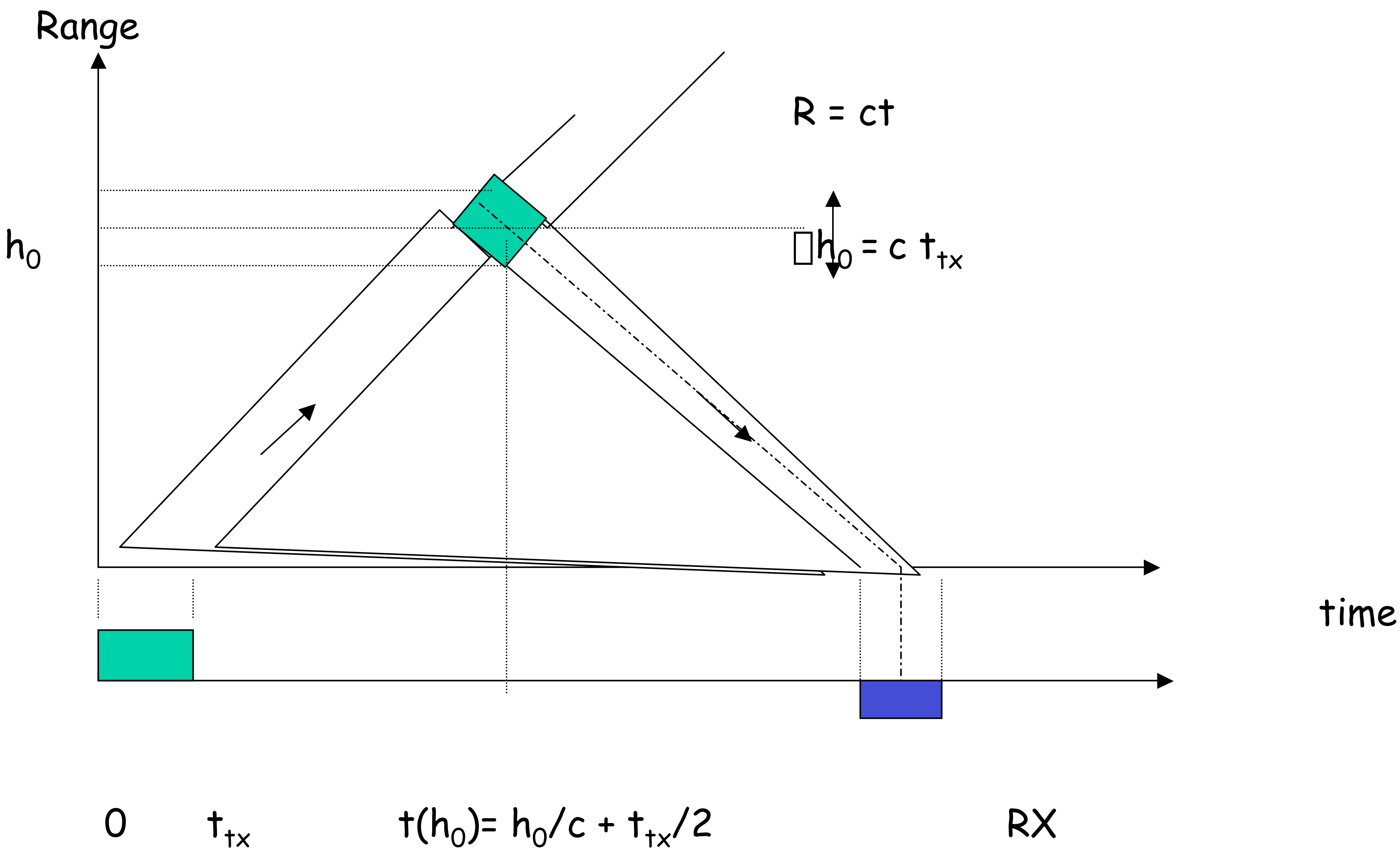
$$H_p = k\,(T_e + T_i)/\,(m_i\, g)$$

The atmosphere also supports different vertical *tidal wave modes.* Ballpark lower limits for scale heights and tidal wavelengths in the auroral zone atmosphere are tabulated below:

Height [km]	$H_{neutral}$ [km]	$H_{ionised}$ [km]	? $\square_{1,1}$ [km]	? $\square_{2,2}$ [km]	? $\square_{2,4}$ [km]
120	12	20	15	80	60
150	20	40	20	80	75
200	25	100	20	80	80
250	30	100	20	85	80
300	35	125	20	85	85
500	45	125	25	95	95

Well-designed experiments should provide height resolutions substantially better (~ 3-4x) than these values at each altitude – and here's how to...

Range resolution: the range-time diagram



A sample taken at $t = 2\, h_0/c + t_{tx}$ contains contributions from the range

Altitude/range resolution and pulse length :

Following the sampling theorem, *select the basic pulse length,* t_B, *such that (assuming a vertically directed antenna beam)*

$c\ t_B < \frac{1}{2}$ inf {

- the neutral scale height,
- the ionised scale height,
- the shortest tidal mode wavelength }

NOTE: The resulting upper bound on t_B is set *by the medium* and must be met *independently of* f_{radar} / λ_{radar} !!!

NOTE: In the E region, the scale height of the ionised component drops rapidly; below 100 km one often encounters very narrow E_s layers with 1/e widths of just a few hundreds of meters. In the *mesosphere*, height resolution down to < 100 m is useful $t_B <$ 0.6 μs !

Altitude resolution and modulation BW

A height resolution of dR meters requires a square pulse of length t:

$$t = 2\ dR/c$$

In the frequency domain, this pulse has a $S_t = c\ (\sin x/x)^2$ *PSD,* where the full width of the main lobe is

$$B_t \approx c\ /\ dR$$

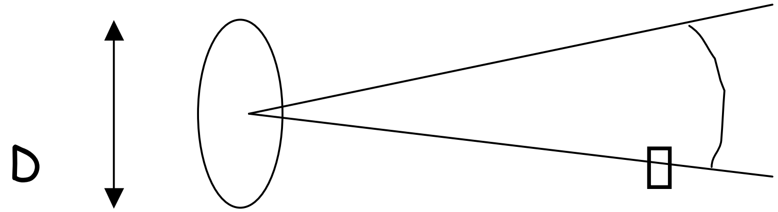
Examples:	dR	t	B_t
	150 m	1 μs	2 MHz
	1.5 km	10 μs	200 kHz
	15 km	100 μs	20 kHz
	150 km	1 ms	2 kHz

Beam (transverse) resolution

The transverse (cross-beam) resolution of an ISR is defined by the antenna beam pattern. The half-power width θ of the main beam of an aperture antenna of size D, operating at wavelength λ, is approximately (we will see later why that is so).

Example: For D = 32 m and λ = 0.33 m (EISCAT UHF), $\theta = 0.59^{o}$

$$\theta = \lambda / D$$



At a distance of R, this corresponds to a transverse resolution of

$$r_t = R\,\theta$$

Example: For R = 100 km $\quad r_t = 10^5 \,.\, 0.33/32 = 1.03$ km

The beam resolution eventually becomes the limiting factor when observing in the E region with a tilted beam ($\theta < 45^{o}$); then the beam covers about 2 km or

Radar equation, cross sections, noise and S/N

- To estimate the *statistical accuracy* of our experiment, we first need to establish what *signal-to-noise ratio* (S/N) to expect,

- From the *radar equation for a monostatic ISR and a beam-filling target* we derive an expression for the returned signal power,

- The *signal bandwidth* and *system noise temperature* determine the noise power according to Friis' formula. But to get a value for the signal bandwidth we must first estimate the *ion-acoustic frequency* at each altitude,

- Numerical values are inserted and actual S/N ratios computed.

- We will show later that *instantaneous S/N values in the order of the inverse of the number of element pulses in the modulation pattern are optimum* from the statistics point of view; larger S/N is uneconomical !

Ionospheric plasma as a radar target

- Ionospheric plasma is a *beam-filling radar target*:
  - The transmitter beam is always filled with scattering electrons at some average density N_e, regardless of beamwidth and distance to the measuring volume
  - So the total scattering cross section, $\square_{tot}$, is inversely proportional to transmitting antenna gain and proportional to distance squared, polarisation and average density of scatterers:

$$\square_{tot} \;\mu\; G_t^{-1} R^2 \;\square\; \sin^2 \chi \, N_e$$

Radar equation for beam-filling targets

Power scattered from a beam-filling target of thickness dR at R =

$$P_{sc} = P_t \, G_t \, \Box_{Tot} \, / \, (4\Box R^2) = \; P_t \, N_e \, dR \, \Box \, \sin^2 \chi$$

Power density of echo signal at radar:

$$dP_{sc}/dA = P_t \, N_e \, dR \, \Box \, \sin^2 \chi / \, (4\Box \, R^2)$$

Power captured by receiving antenna:

$$P_r = A_r \, dP_{sc}/dA = A_r \, P_t \, N_e \, dR \, \Box \, \sin^2 \chi / (4\Box \, R^2)$$

P_r now varies only as R^{-2}, not as R^{-4} as in the point target case!

Plasma radar cross section

The scattering cross section per plasma electron is σ :

$$\sigma = \sigma_e \{1 - (1 + \alpha^2)^{-1} + [(1 + \alpha^2)\,(1 + \alpha^2 + T_e/T_i)]^{-1}\},$$

$$\alpha = 4\pi\, D/\lambda$$ where D is the plasma Debye length.

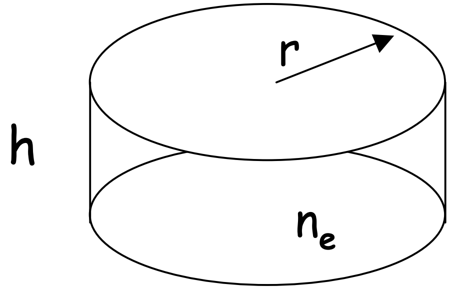
For $\lambda >> D$ and "normal" Te/Ti ratios this reduces to:

$$\sigma \sim \sigma_e\,(1 + T_e/T_i)^{-1} \approx (0.2\ .....\ 0.5)\,\sigma_e$$

(We will return to the case of $\lambda \sim D$ in a little while...)

Total radar cross section of a slab of F layer plasma

Plasma is a beam-filling, diffuse target:



h = slab height, *defined by sampling*

r = slab width, *defined by antenna beam*

n_e = average electron density in slab

Total cross section σ_{total} :

$$\sigma_{total} \approx 3/2 \, \pi \, r^2 \, h \, n_e \, \sigma_T$$

Typical parameter values at 300 km altitude:

h = 15 km

r = 2 km

$n_e = 3 \; 10^{11} \; m^{-3}$

$\sigma_{total} = 5.7 \; 10^{-6} \; m^2$!!!

Target, modulation and signal bandwidths

The *spectrum of the scattered signal*, S_s, is the convolution of the *modulation spectrum* S_m and the *target spectrum* S:

$$S_s = \text{conv}\,[S, S_m]$$

where $$S_m \ \square\ S_t$$

A practical approximation of the bandwidth of S_s is then:

$$B_s = B + B_t$$

The shape of the *target spectrum,* S *,* is defined by the *plasma dispersion relation*, which we will soon take a closer look at.

How SNR varies with range

To get a qualitative idea of how S/N varies with range, we combine all the ionospheric factors into a "figure of difficulty" $F_D(R)$, proportional to S/N when dR and the radar parameters are fixed:

$$F_D(R) = N_e(R) \,/\, \{ R^2 \,(1 + T_e(R)/T_i(R)) \;\; f_{IA}(R))\}$$

Figure 1 shows how $F_D(R)$ varies with altitude as a function of the ionospheric conditions, again assuming the *Bratteng and Haug* ionosphere and low-bandwidth modulation. We assume a radar frequency around 930 MHz and a vertical radar beam such that R=H.

OBSERVATION: In an average ionosphere, $F_D(R)$ varies by over two orders of magnitude over the (100-600) km range !!

Cosmic Radio Noise

An antenna pointing skywards sees a noise background of cosmic origin. At VHF and UHF, most of this is due to synchrotron radiation from free electrons orbiting in stellar and interstellar magnetic fields.

Over narrow frequency ranges below ~ 1 THz, the background radiation can be fairly accurately described as *blackbody radiati*

unit bandwidth), dP_N / dB, is:

$$dP_N = kT_{sky}\, dB$$

where T_{sky} is the *equivalent blackbody temperature* or the *equivalent noise temperature*. Since the background radiation is beam-filling, the antenna gain does not enter into the formula.

At 928 MHz (the EISCAT UHF frequency), T_{sky} is approximately 10 K =>

$dP_N/d\square$ = 1.38 10^{-22} [watts/Hz]

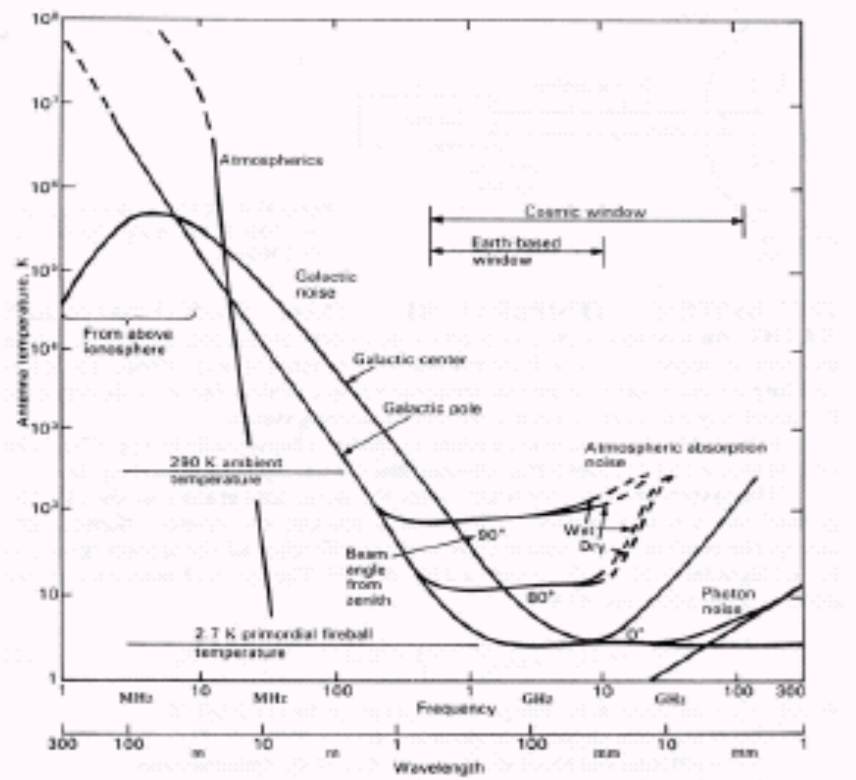
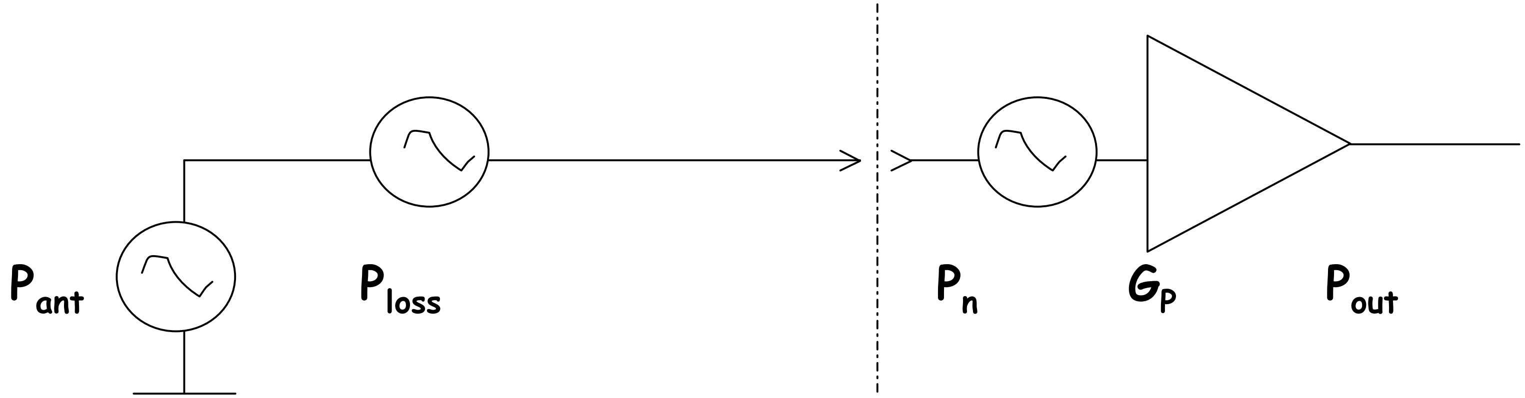


Figure 17-4 Antenna (noise) temperature from the sky as a function of frequency. See text for explanation. (From J. D. Kraus, Radio Astronomy, 2nd ed., Cygnus-Quasar, 1986.)

Antenna / Receiver Noise

The random thermal motion of charge carriers in lossy conductors generates noise power P_{loss} , which adds to the noise applied from the outside. The active devices in the receiver also add noise power, P_n , which is largely non-thermal:



The first generator is the antenna. The total noise power delivered by it (including cosmic noise) is P_{ant} . The second generator is the noise generated by transmission line losses between antenna and receiver. The third generator represents the receiver noise mapped back to the input. The amplifier is assumed to be noiseless with gain G_P

Over the bandwidth of an IS ion line, we can treat all noise generators as blackbodies and assign equivalent noise temperatures $\mathbf{T_{ant}}$, $\mathbf{T_{loss}}$ and $\mathbf{T_n}$ to them.

Signal-to-Noise Ratio

The total (signal + noise) power per Hertz at the receiver output is:

$$dP_{out}/\,dB = dP_r/dB + dP_{Noise}/dB$$

$$= G_P\,(dP_r/dB)$$

$$+\; G_P\; k\,(T_{ant} + T_{loss} + T_n)$$

$$= G_P\; A_r\, P_t\; N_e\; dR\; \square\; \sin^2\ \chi\, /\, (B_s\, 4\square\, R^2) +$$

$$+\; G_P\; k\,(T_{ant} + T_{loss} + T_n)$$

So the signal-to-noise ratio (S/N) becomes

$$S/N = A_r\, P_t\; N_e\; dR\; \square\; \sin^2\ \chi\, /\, (B_s\, 4\square\, R^2)\; k\,(T_{ant}+T_{loss}+T_n)$$

Dispersion relation for ion-acoustic waves

The bulk of the scattered power is contained in the *ion line*, which is produced by scattering off low-frequency *ion-acoustic waves* propagating along the scattering k-vector direction, towards or away from the radar.

Ion-acoustic waves are non-dispersive, i.e. their phase velocity does not depend on their wavelength. In one dimension, the *ion-acoustic branch of the plasma dispersion relation* looks approximately like this:

$$(\omega/\kappa)^2 = m_i^{-1}\, k\, (T_e + T_i) = c_s^2$$

where κ designates the ion-acoustic wave vector and k is Boltzmann's constant.

Solving for the wave frequency:

$$f_{IA} = (2\pi)^{-1}\, \kappa\, [m_i^{-1}\, k\, (T_e + T_i)]^{\frac{1}{2}} = \Lambda^{-1}\, [m_i^{-1}\, k\, (T_e + T_i)]^{\frac{1}{2}}$$

which shows that if we can measure the ion-acoustic spectrum, we should be able to deduce something about the temperatures.

Ion line spectral width

For radar back-scatter, where $\Lambda = \lambda_{radar}/2$, the frequency of undamped ion-acoustic waves matching the mono-static Bragg criterion $\Lambda = \lambda_{radar}/2$ is

$$f_{IA} = [m_i^{-1}\, k\, (T_e + T_i)]^{\frac{1}{2}} \cdot (2\, f_{radar} / c)$$

We can use this to compute a zeroth order estimate of the spectral width of the ion line return, B:

$N_e = 5.0\ 10^{11}\ m^{-3}$

$T_i = 1020$ K

$T_e = 2400$ K

$m_i = 16$ amu (100 % O^+)

$$f_{IA} \approx 8.89\ 10^{-6} * f_{radar}$$

For the EISCAT UHF (930 MHz) $f_{IA} = 8.27$ kHz =>

$B \approx 2 \cdot f_{IA} = 16.6$ kHz

Real (Landau-damped) ion line spectra are wider by ~ 60 % :

LEGEND:

Red – F region (300 km)

n_e = 3 .10^{11} T_e = 2000 K
O^+ T_i = 1000 K

Green - F region (300 km)

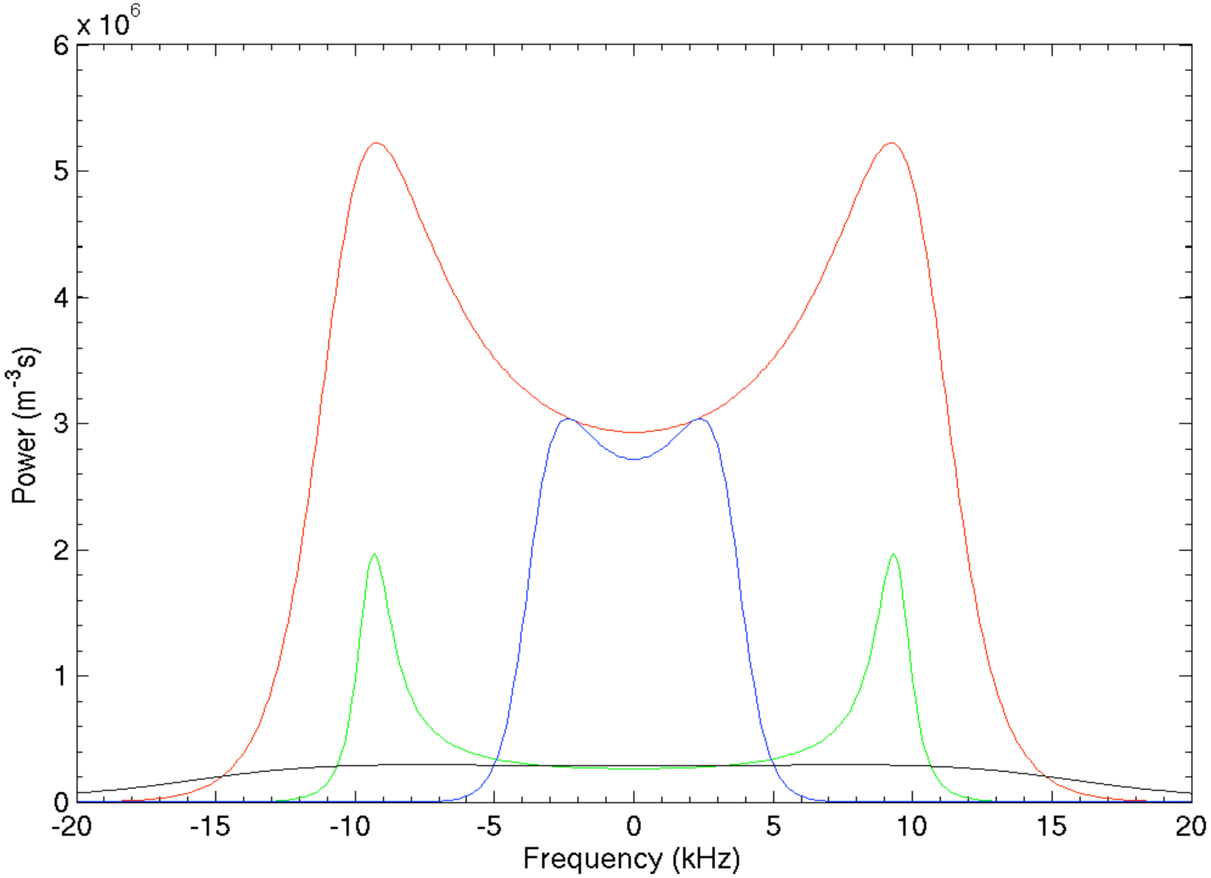
n_e = 1 .10^{11} T_e = 3000 K
O^+ T_i = 500 K

Blue – E region (120 km)

n_e = 5 .10^{10} T_e = 300 K
NO^+ / O_2^+ T_i = 300 K

Black – topside (1000 km)

n_e = 5 .10^{10} T_e = 4000 K
90%O^+ 10% H^+ T_i = 3000 K



Spectra computed for the EISCAT UHF radar wavelength of 0.33 m (930 MHz).

Power spectral density (y-axis) plotted to linear scale

How dP_r/dB varies with range

Monostatic radar equation for beam-filling targets:

$$P_r = A_r \; P_t \; N_e \, dR \; \square \; (T_{e,i}) \; / \; (4\square \, R^2)$$

The *target range*, R, affects the received power directly:

$$P_r \, \mu \; R^{-2}$$

The *electron number density* N_e is a function of R:

$$P_r \, \mu \; N_e(R)$$

The *effective scattering cross section* $\square$ is also a function of R:

$$P_r \, \mu \; \square(T_{e,i} \, (R)) = \square_e \, (1 + (T_e \, (R) \, / T_i \; (R))^{-1}$$

We assume that the scattered power is spread fairly uniformly across the *ion line bandwidth,* B, which in turn is a function of $T_{e,i}$:

$$dP_r/d\square \, \mu \; B^{-1}(T_{e,i}\;) = B^{-1}(T_{e,i}\,(R)) \; \square \; (2 \; f_{TA} \; (R))^{-1}$$

An added complication: the Debye cutoff

$F_D(R)$ in Figure 1 was computed assuming that

$$\alpha = 4\pi D/\lambda \ll 1$$

But at altitudes above 500 km, α becomes significant and we must use the full expression for σ_{ion} to estimate the S/N ratio:

$$\sigma_{ion}(\alpha) = \sigma_e \left[(1 + \alpha^2)(1 + \alpha^2 + T_e/T_i)\right]^{-1}$$

We note that (assuming $T_e/T_i = 1$),

$$\sigma_{ion}(\alpha = 1) = 0.33\ \sigma_{ion}(\alpha = 0)$$

This "Debye cutoff" can become a problem for UHF ISR systems. Refer to Figure 2, where the 33 % - cross section heights are indicated on two typical ionosphere profiles. At 930 MHz, measurements at all heights > 500 km will suffer badly

Now we can finally estimate S/N (h):

- $dR \rightarrow t \rightarrow B_t$ $\rightarrow$ B_s, dP_r/dB
- $h, N_e, T_e, T_i \rightarrow B, P_r$
- $T_{sky}, T_{loss}, T_n \rightarrow P_N \rightarrow$ $\boxed{S/N\ (h)}$

S/N: some numerical examples

We use the parameters of the EISCAT UHF system, the *Bratteng and Haug* model ionosphere, a slab height dR appropriate to the scale height and $\sin^2 \chi = 1$ (mono-static operation) to work a couple of examples:

$f = 9.28\ 10^8$ Hz

$P = 1.3\ 10^6$ W

$A = 560\ m^2$

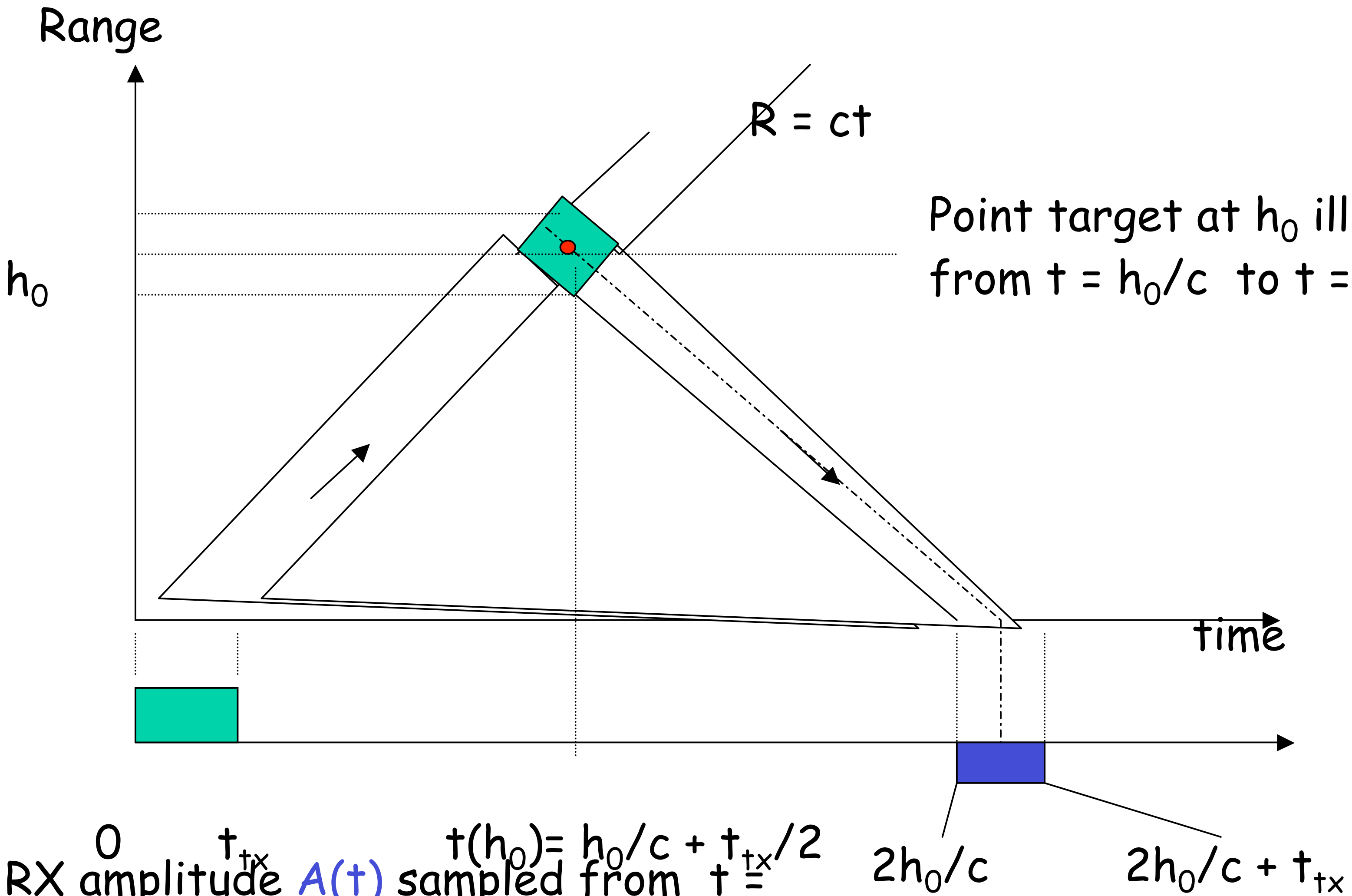
$T_{sky} = 10$ K

$T_n = 35$ K

$T_{loss} = 55$ K

Height	120	150	300	1000
N_e [m^{-3}]	5.0 e10	2.7 e11	3.0 e11	5.0 e10
T_i [K]	300	480	1000	3000
T_e/T_i	1	1.1	2	1.3
B [kHz]	10	12	30	50
dR [km]	1.5	1.5	15	150
B_m [kHz]	200	200	20	2
B_s [kHz]	210	212	50	52
S/N	0.05	0.17	2.3	0.27

How Spectral Information is Derived



Point target at h_0 illuminated from $t = h_0/c$ to $t = h_0/c + t_{tx}$

RX amplitude $A(t)$ sampled from $t = 2h_0/c$ to $t = 2h_0/c + t_{tx}$; **FFT** or **autocorrelation** computed

How fast, and for how long, must we sample?

Obviously, the receiver bandwidth BW must be > B_s to pass all spectral information on to the sampler.

For low-bandwidth modulations (i.e. long pulse), B_s □ B and we can argue as follows:

How fast: The *sampling theorem* tells us that

$$f_{samp} > 2 * f_{max} \quad \Longleftrightarrow \quad □_{samp} < (2 * f_{max})^{-1}$$

where $\mathbf{f_{max}}$ is the highest "nonzero-power" frequency in the signal spectrum (e.g. ~ 50 kHz in the 300-km model spectrum)

For how long: Depends on which, and how many, parameters we want to get out from the subsequent analysis !

This is best illustrated in the time domain...

The plasma autocorrelation function, $r_{xx}(\tau)$

is the Fourier transform of the ion line power spectral density. Using the plasma dispersion relation, we can compute model autocorrelation functions for different combinations of N_e, T_e and T_i (Figure 3).

An estimate of the target r_{xx} at *lag time* $n\tau_0$ can be computed from the time series of complex amplitude samples, $s(t)$, output from the receiver:

$$r_{xx}(n\tau_0) = s\,(t)\; s^*(t + n\tau_0)$$

Intuitively, it may appear natural to continue sampling at a given range for so long that the model ACF has decayed almost to zero. To see if that helps at all, let us first look at how the different plasma parameters influence the model ACF at different lag times (Figure 4):

Partial derivatives of the plasma dispersion function:

$\partial r_{xx}(\tau) / \partial Ne$

$\partial r_{xx}(\tau) / \partial T_i$

$\partial r_{xx}(\tau) / \partial (T_e/T_i)$

$\partial r_{xx}(\tau) / \partial m_i$

$\partial r_{xx}(\tau) / \partial \nu_{in}$

are shown in terms of τ/τ_0 , where τ_0 , the *plasma correlation time,* is the time to the first zero crossing of the ACF of a undamped ion-acoustic wave with wavelength = $\Lambda = \frac{1}{2} \lambda_{radar}$

NOTE: $\partial r_{xx}(\tau) / \partial T_i$ and $\partial r_{xx}(\tau) / \partial m_i$ are almost linearly dependent...

ACF estimate extent and errors

Figure 5 (from *Vallinkoski 1989*) shows how the errors of the different plasma parameters behave as functions of lag extent when measurement data are fitted to a five-parameter plasma model.

Comparing this to Figure 4, we see that *as the lag extent is increased to the point where the partial derivative of a given parameter goes through a complete cycle*, the error in that parameter suddenly drops dramatically.

If one is satisfied with slightly less than ultimate accuracy, extending the measurement to $\square/\square_0$ = 2.5 should be sufficient. By about $\square/\square_0$ = 3.5, all errors have settled down to their asymptotic value –

But to allow for surprises, design for $\square/\square_0$ $\square$ 4 !

ACF estimate, practical consequences

- $\tau/\tau_0 \approx 4$ is actually rather long, even at 930 MHz:
  - 280 μs at 150 km
  - 650 μs at 100 km
  - Several milliseconds at 95 km
- The required duration of illumination increases as the inverse of the radar frequency (~ 2700 μs @ 150 km/224 MHz)

- **Experiments should be designed to illuminate the plasma with RF for at least this long (*longer* the lower you go) !**
- **At the same time, the illumination must provide altitude resolution better than the smallest of (scale height, shortest tidal mode) at each altitude (and those get *shorter* the lower you go, see Table) !**

At some point we run into a conflict...

Summing up the various timing restrictions in one diagram (Figure 6):

Required ACF extents at different radar frequencies progress from upper left to lower right,

Pulse length limitations go from lower left to upper right.

Four different regions can be distinguished:

- Region I: Height resolution provided by long uncoded pulses OK, intra-pulse ACF measurement OK,
- Region II: ***Coding required*** *to get the desired height resolution*, intra-pulse ACF measurement OK,
- Region III: ***Pulse-to-pulse ACF measurement required***. Coding not mandatory, but advantageous,
- Region IV: Pulse lengths in this region do not meet the minimum ACF length requirement - BEWARE!

Pulse-to-pulse measurements

Whenever

$$2R/c < 4\tau_b(R)$$

it is impossible to do a full spectrum estimate of the target at R within the timespan of a single radar pulse (Region III in Figure 6).

Instead, we can illuminate the target repeatedly with a series of pulses transmitted at some repetition frequency PRF_{ptp} and form estimates of the target ACF by taking cross-products between samples from the same height taken in different interpulse periods:

$$r_{xx}(r, kt_{ptp}) = s(r,n_p=i) \;\; s^*(r,n_p=i+k)$$

where

$$t_{ptp} = 1/ PRF_{ptp}$$

NOTE: For this to work, the radar must be *pulse-to-pulse phase coherent* !

Statistical accuracy and averaging

The normalised variance of an individual ACF lag estimate has the general form:

$$[\mathrm{var}\,(r_{xx}(\square)) \,/\, r_{xx}^{2}(\square)] \;=\; (k_1\, n_{ind})^{-1}\;\; (n_p + (S/N)^{-1})^{2}$$

where k_1 is determined by the code and computation scheme used, n_p is the number of elements in the code and n_{ind} is the number of statistically independent estimates averaged:

When $S/N \;\square\; n_p^{-1}$, *both terms contribute equally to* var $(r_{xx}(\square))$

when $S/N > n_p^{-1}$, *its contribution to* var $(r_{xx}(\square))$ *tapers off,*

when $S/N \gg n_p^{-1}$, *no further improvement in* var $(r_{xx}(\square))$!

The high SNR case

When S/N >> 1

$$[\mathrm{var}\,(r_{xx}(\tau)) / r_{xx}^2(\tau)] \approx (k_1\, n_{ind})^{-1}\ n_p^2$$

and the only way to reduce the variance further is to increase n_{ind}

However:

Measurements repeated closer in time than ~ 4τ_0 are partially correlated; thus we can obtain at most

$$\mathbf{n_{ind} = 1/\ 4\tau_0}$$

totally independent estimates per unit time !

There are two ways to work around this restriction:

1) Use as high a radar frequency as possible (as this lowers τ_0),

PRF, max. range and rate of statistics

Since

$$[\mathrm{var}\ (r_{xx}(\Box)) \ /\ r_{xx}^{2}(\Box)]\ \ \mu\ \ n_{ind}^{-1}$$

it is smart to increase n_{ind} by using the highest PRF the radar can deliver.

However, since max range R_{max} and max useable PRF are related through

$$PRF_{max} = c/\ (2\ R_{max})$$

we cannot increase the PRF arbitrarily; there is a ceiling on the rate at which we can reduce variance by averaging:

R_{max}	150	300	1500	km	
PRF_{max}	1000	500	100	Hz	
Error in 1 σ	3.2	4.5	10	%	(asymptotic)

Range ambiguities and frequency hopping

When a modulation pattern is being repeated on a given frequency with a PRF > 50 Hz or so, *two or more RF packets will be in the dense part of the ionosphere simultaneously*, separated in range by

$$R_{amb} = c/(2\ PRF)$$

The more distant pulses will be at the so-called *ambiguous ranges* (1^{st}, 2^{nd}). *Returns from all illuminated ranges will be received simultaneously;* there is no way to prevent the ambiguous-range pulses from at least producing clutter –

But when the radar hardware allows it (as e.g.in EISCAT/ESR), four or more frequencies can be used in round-robin fashion, pushing the first ambiguous range out to well past 3000 km. Ambiguous returns from such ranges are normally so weak as to be negligible.

CAUTION: This obviously does not work in pulse-to-pulse experiments...

Frequency ambiguities in pulse-to-pulse

Since the basic sampling rate in a pulse-to-pulse experiment is PRF_{ptp}, the highest frequency which can be uniquely resolved in the received signal is:

$$f_{max} = \tfrac{1}{2}\, PRF_{ptp}$$

At first sight, this appears to violate the sampling theorem, as the short pulse we will be using is associated with a modulation bandwidth

$$B_m \gg f_{max} \qquad \Rightarrow \qquad B_s = \mathrm{conv}\,[B, B_m] \gg f_{max}$$

But the technique really works, since as long as $B \ll f_{max}$, sampling B_m at f_{max} folds all the scattered power back into the first Nyqvist zone (*constructive undersampling*).

However, if the nonzero part of the target power spectral distribution, B, extends beyond f_{max}, all power at frequencies > f_{max} will be folded in below f_{max}. The reconstructed spectrum then contains unresolvable *frequency ambiguities* and it

Blind ranges and staggered PRF

All pulsed radars have a *blind range* problem:

You cannot receive while you transmit,

The distant range being illuminated by the previous pulse during the transmission time is not observable,

The fraction of the theoretically observable range that is being blanked in this way is directly proportional to the duty cycle.

Difficult problem at the ESR ($\square_{max}$ = 25%), and in P-T-P work

Workaround: - change PRF at intervals, or use multiple, interlaced PRFs

Eliminates the total blindness, but the blind ranges will be sampled at a lower rate than the rest....

Always wastes some fraction of the available □.