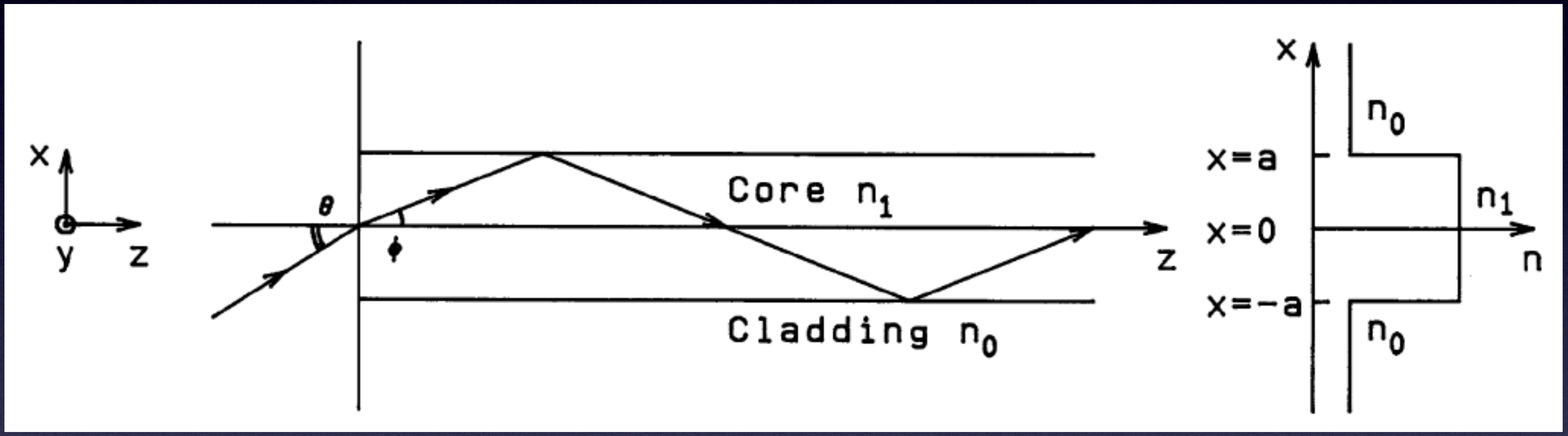
Planar Optical Waveguides

- Waveguide modes

Waveguide structure -Numerical Aperture (NA)



Total internal reflection $\theta \leqslant \sin^{-1}\sqrt{n_1^2 - n_0^2} \equiv \theta_{max}$ $\theta_{max} \cong \sqrt{n_1^2 - n_0^2}.$

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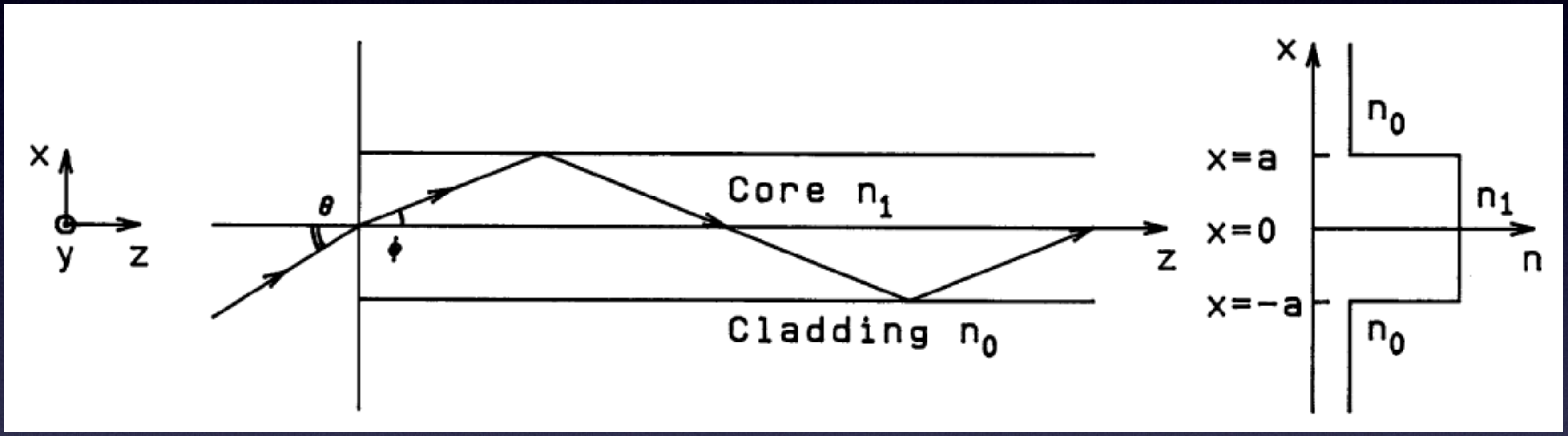

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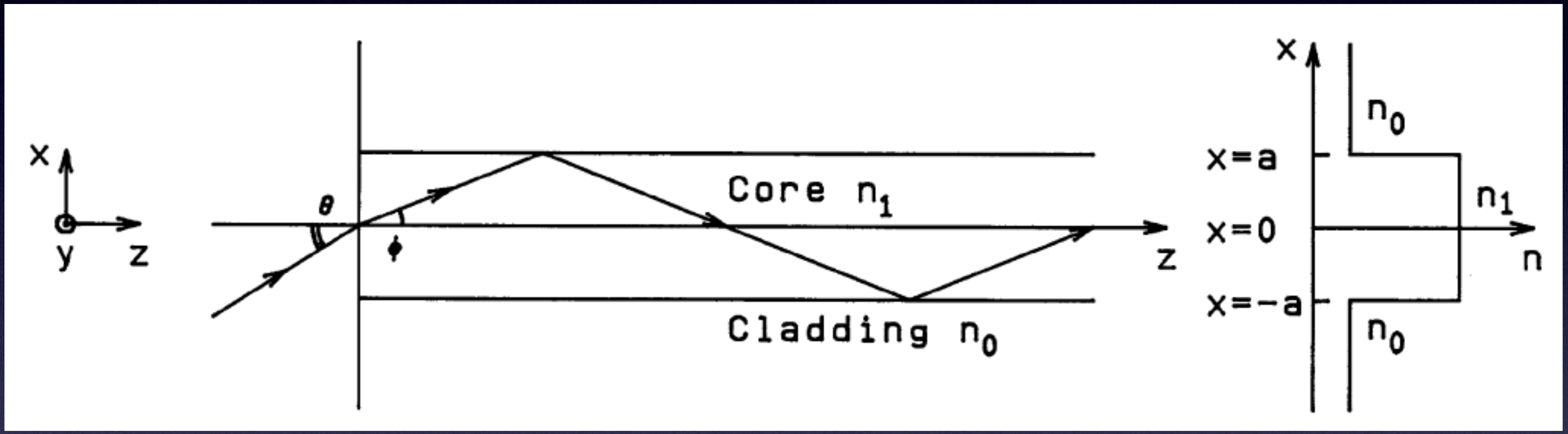

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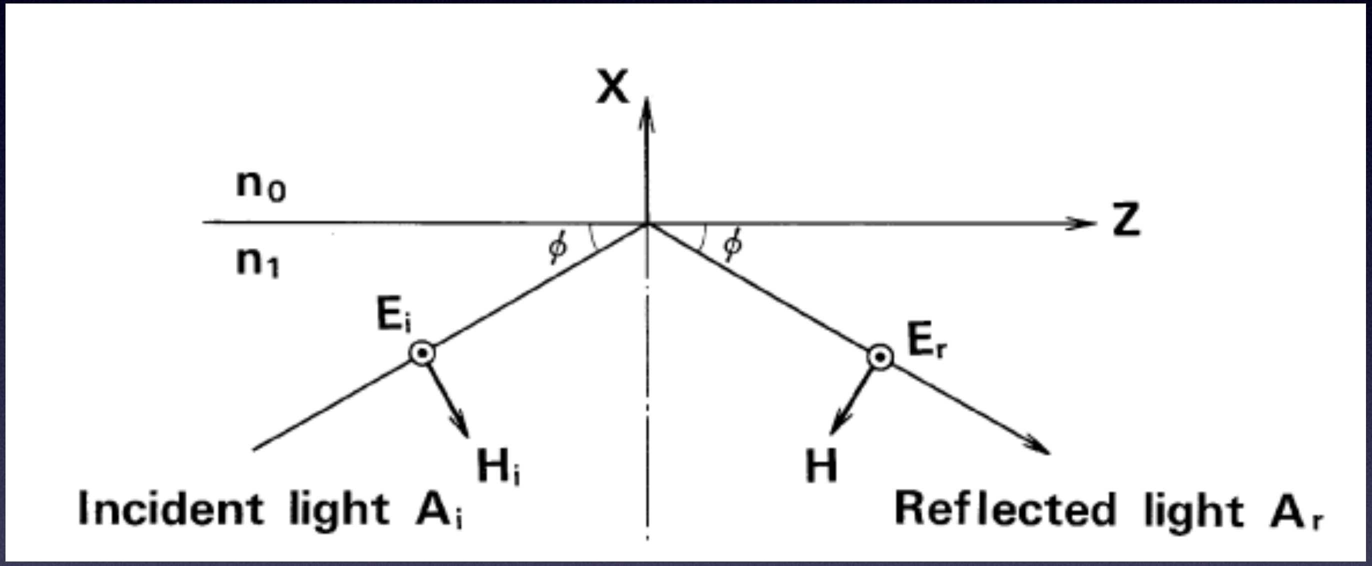
Numerical Aperture $\mathrm{NA} = \theta_{max} \cong n_1\sqrt{2\Delta}.$

Total internal reflection- Goos-Hanchen shift



Consider:
Field normal to the plane of incidence
(perpendicular polarization)
Transverse Electric(TE) waves

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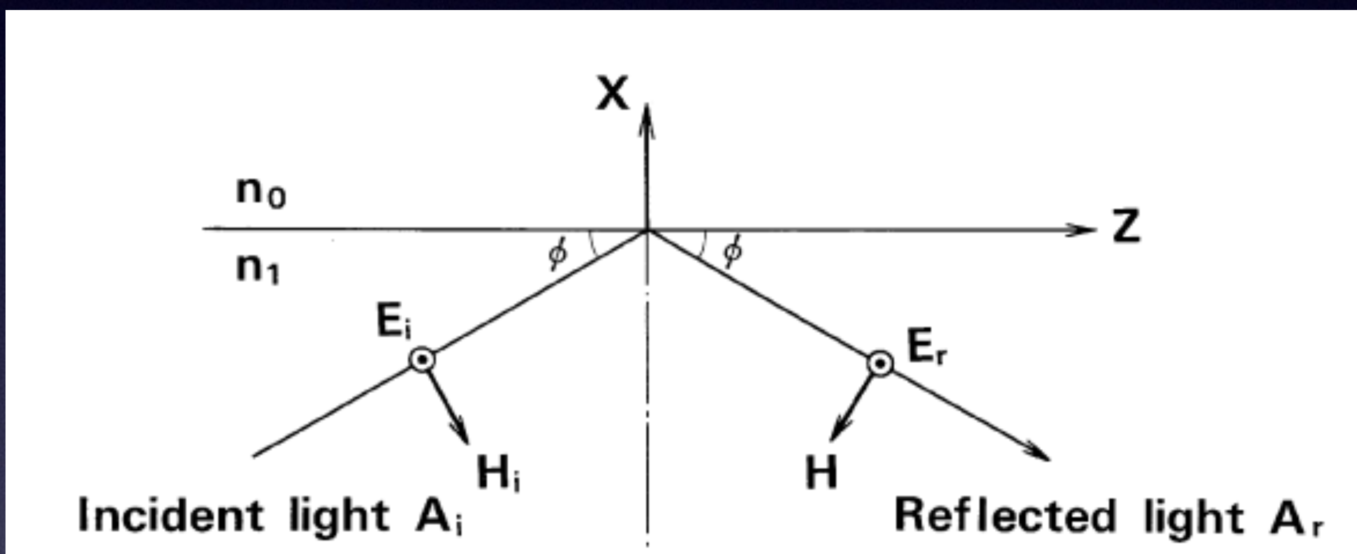

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Reflection coefficient of the TIR

$$r = \frac{A_r}{A_i} = \frac{n_1 \sin\phi + j\sqrt{n_1^2 \cos^2\phi - n_0^2}}{n_1 \sin\phi - j\sqrt{n_1^2 \cos^2\phi - n_0^2}}.$$

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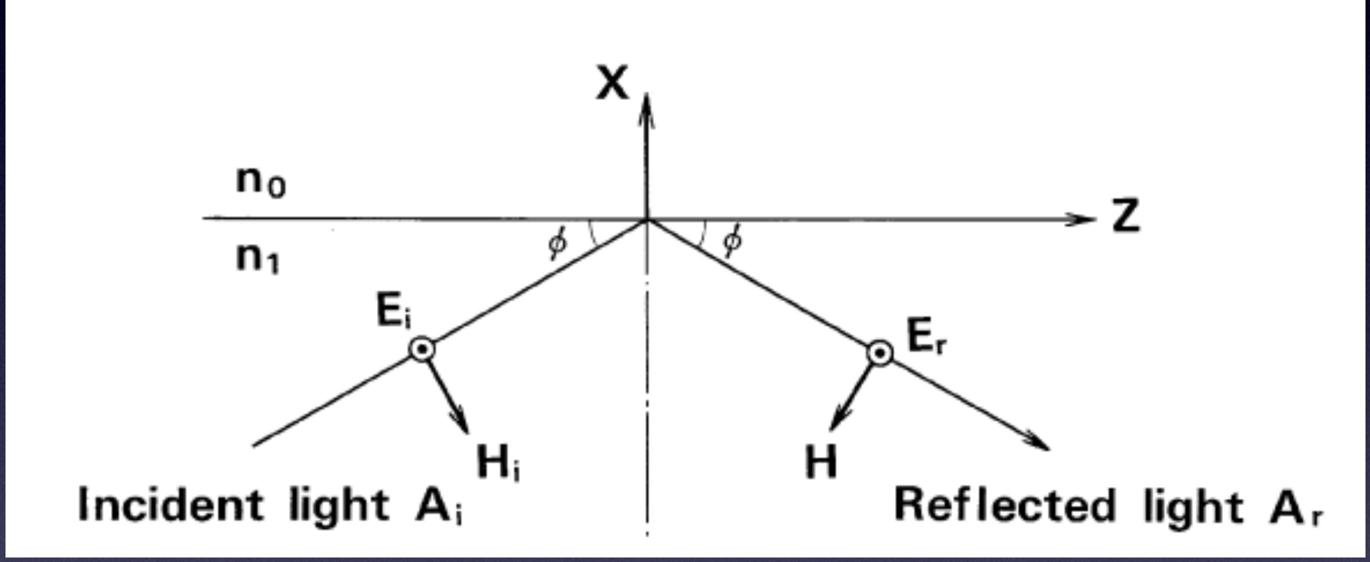

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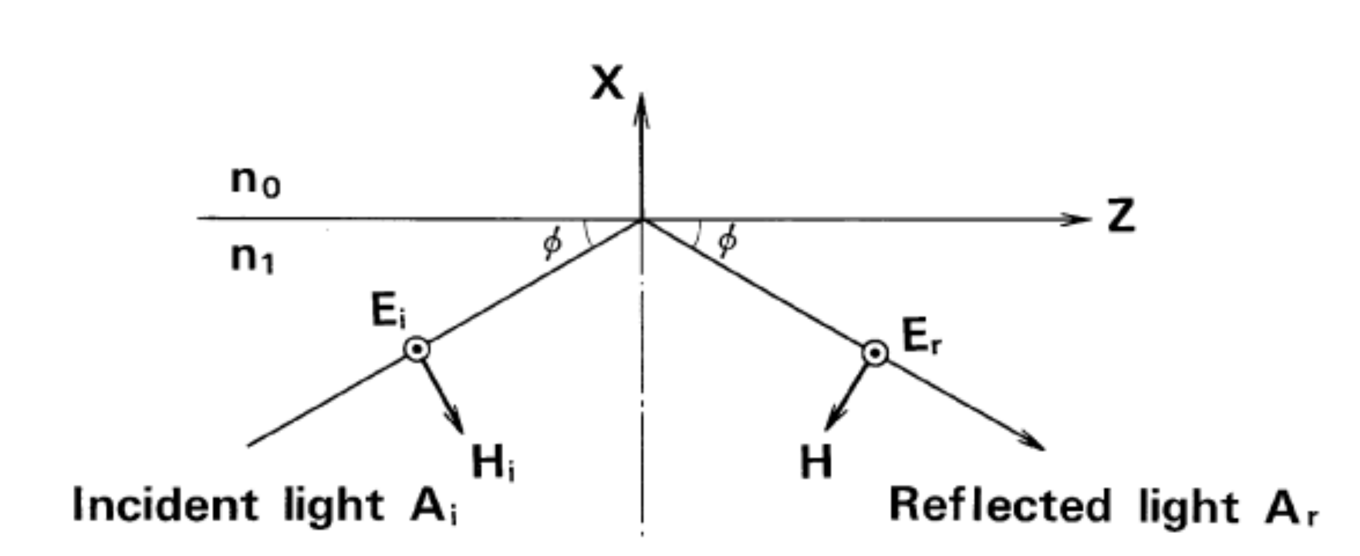

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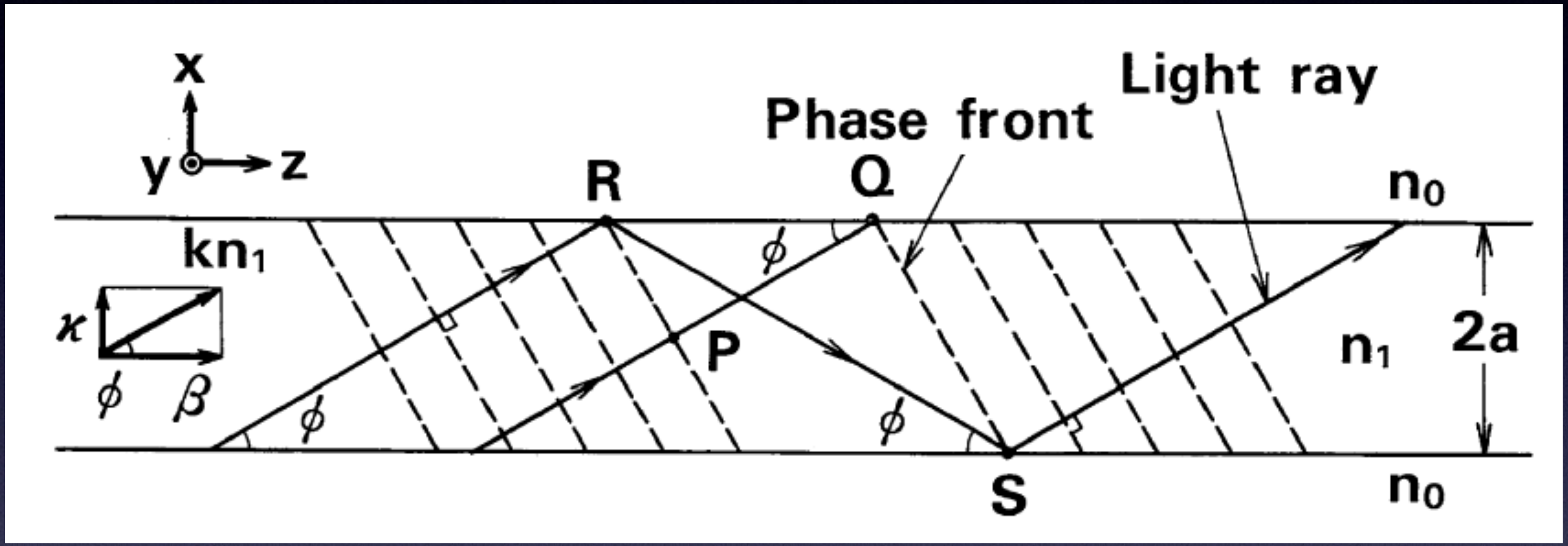
Goos-Hanchen shift

$$\Phi = -2\tan^{-1}\frac{\sqrt{n_1^2\cos^2\phi - n_0^2}}{n_1 \sin\phi} = -2\tan^{-1}\sqrt{\frac{2\Delta}{\sin^2\phi} - 1}.$$

Guided Modes-eigenvalues

$$\beta = kn_1 \cos\phi,$$
$$\kappa = kn_1 \sin\phi.$$



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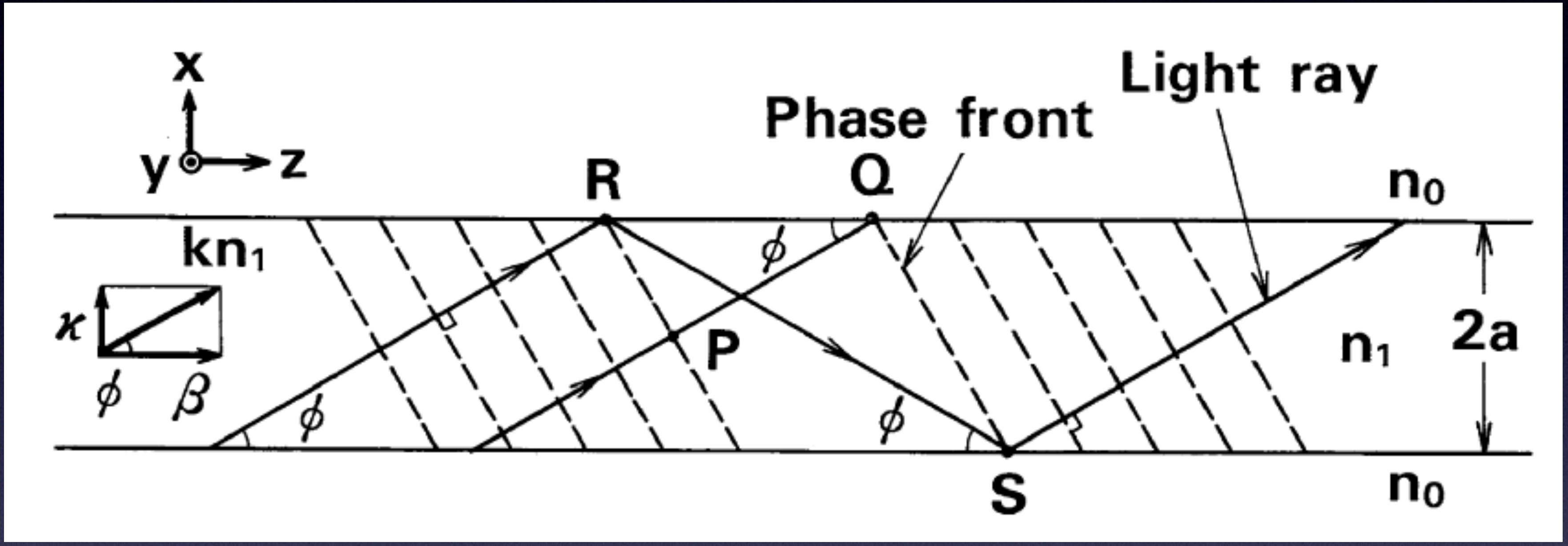

(P,R) and (Q,S) are on the same phase front

$$\tan\left(kn_1 a \sin\phi - \frac{m\pi}{2}\right) = \sqrt{\frac{2\Delta}{\sin^2\phi} - 1}.$$

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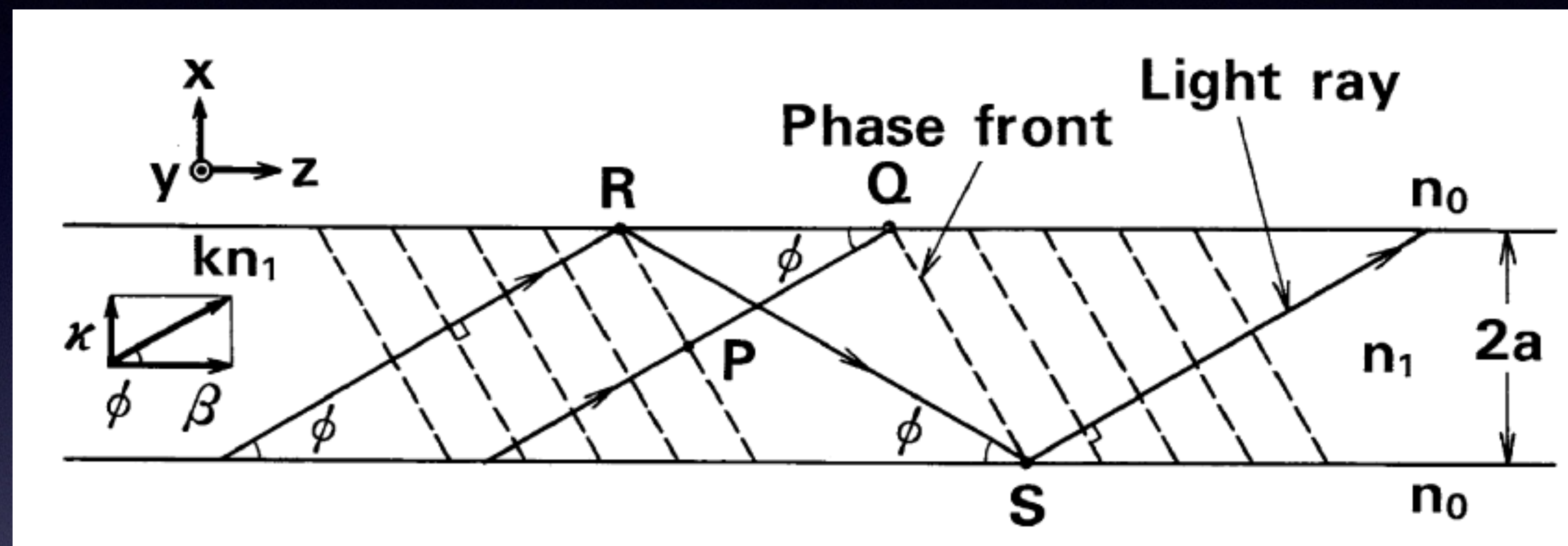

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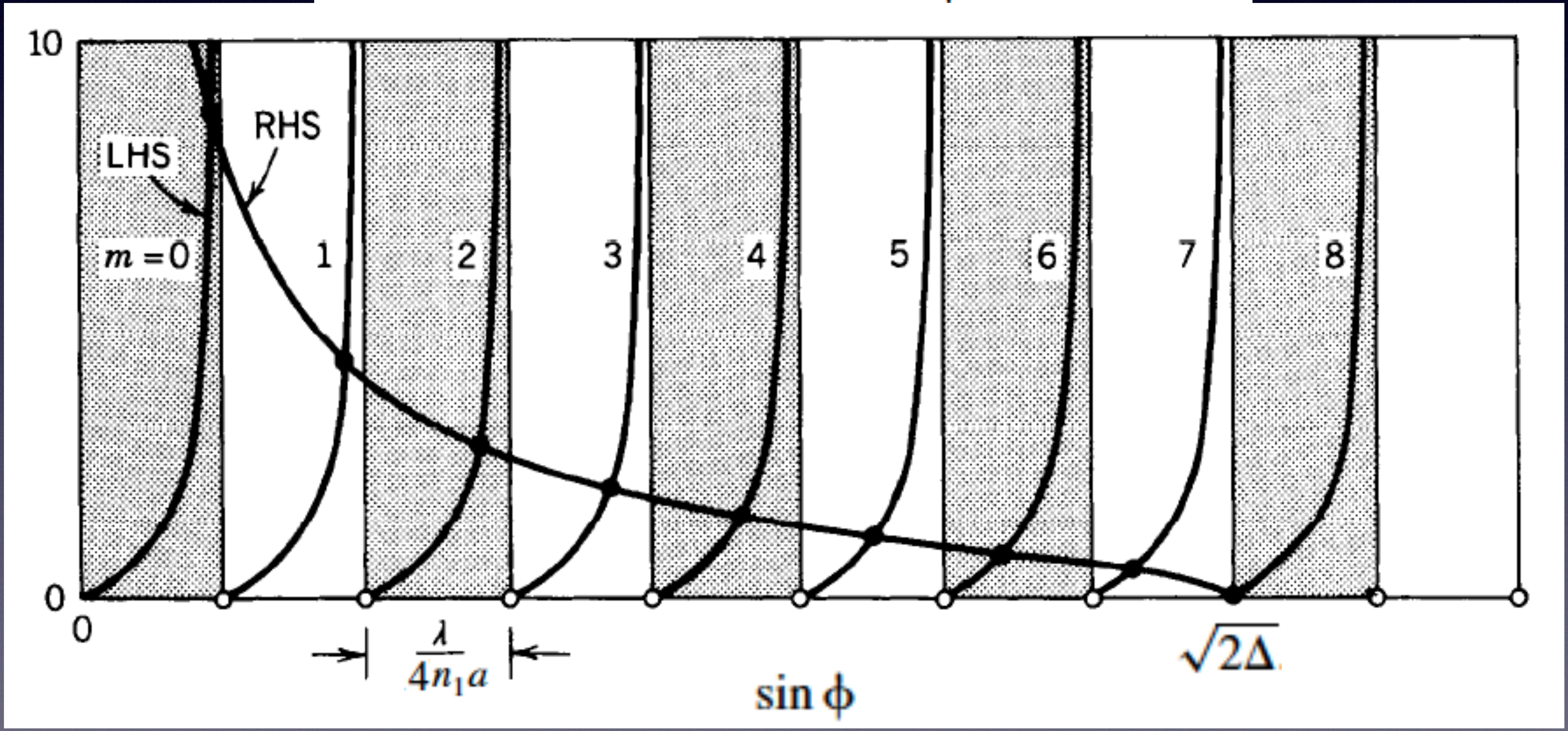
propagation angle is <u>discrete</u>

β's are discrete (called eigenvalues)

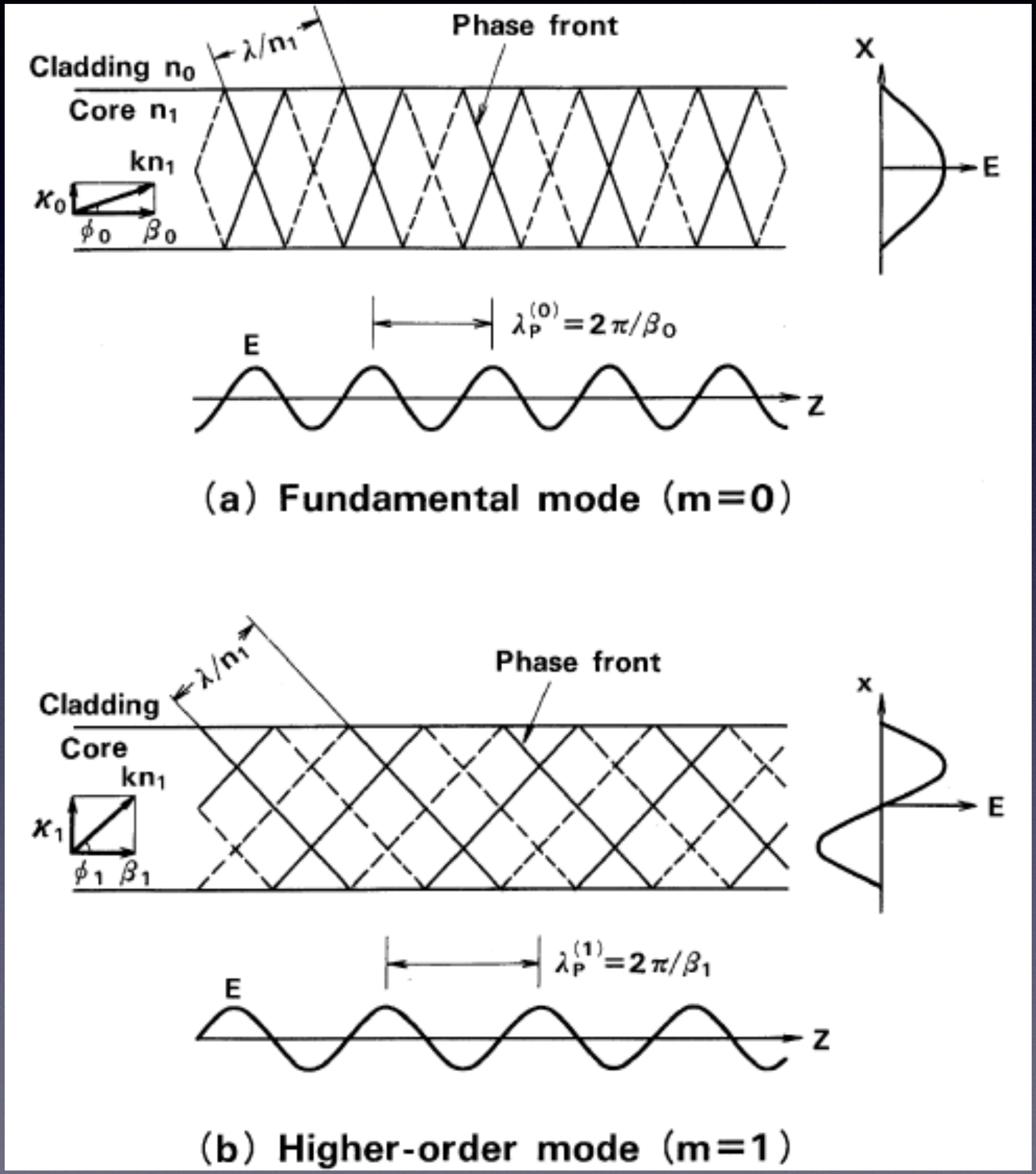
Graphical solution

$$\tan\left(kn_1 a \sin\phi - \frac{m\pi}{2}\right) = \sqrt{\frac{2\Delta}{\sin^2\phi} - 1}.$$



number of modes

Field distribution



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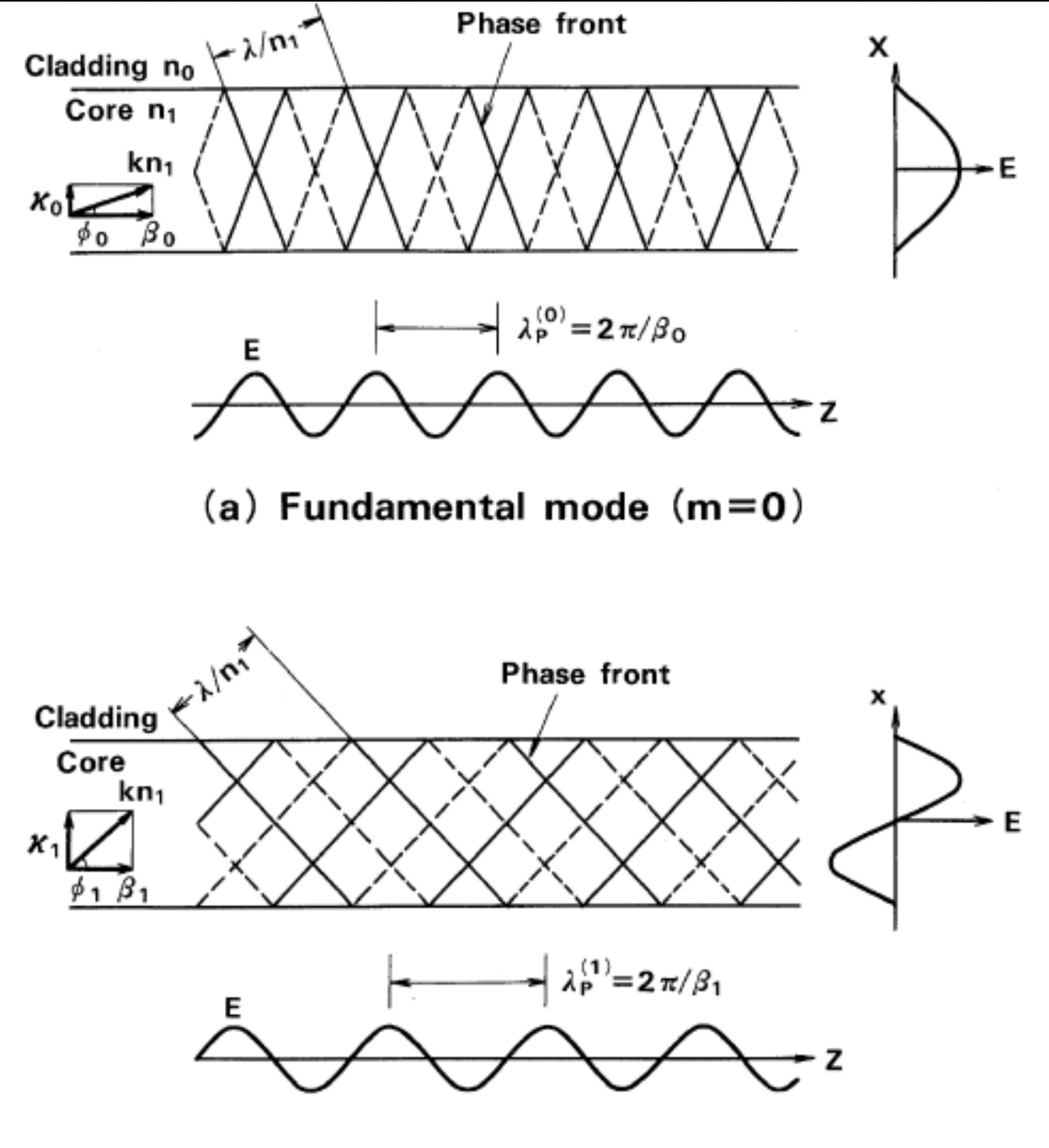
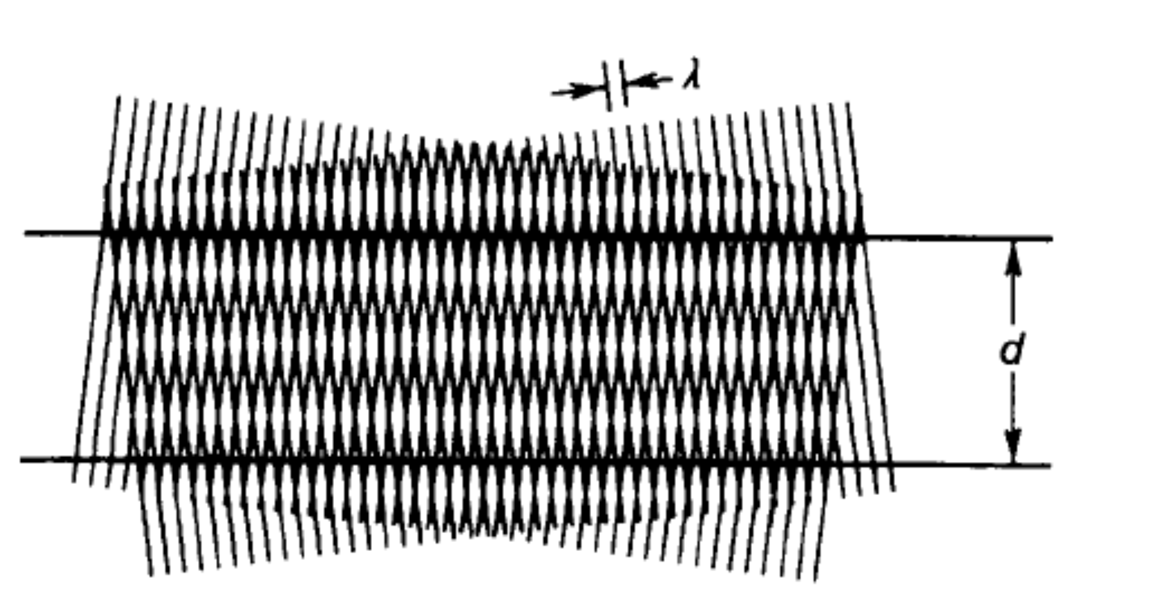


(a) Fundamental mode (m=0)

(b) Higher-order mode (m=1)



Normalized frequency

We know $\sin\phi \leqslant \sqrt{2\Delta}$

Define normalized parameter $\xi = \dfrac{\sin\phi}{\sqrt{2\Delta}}$

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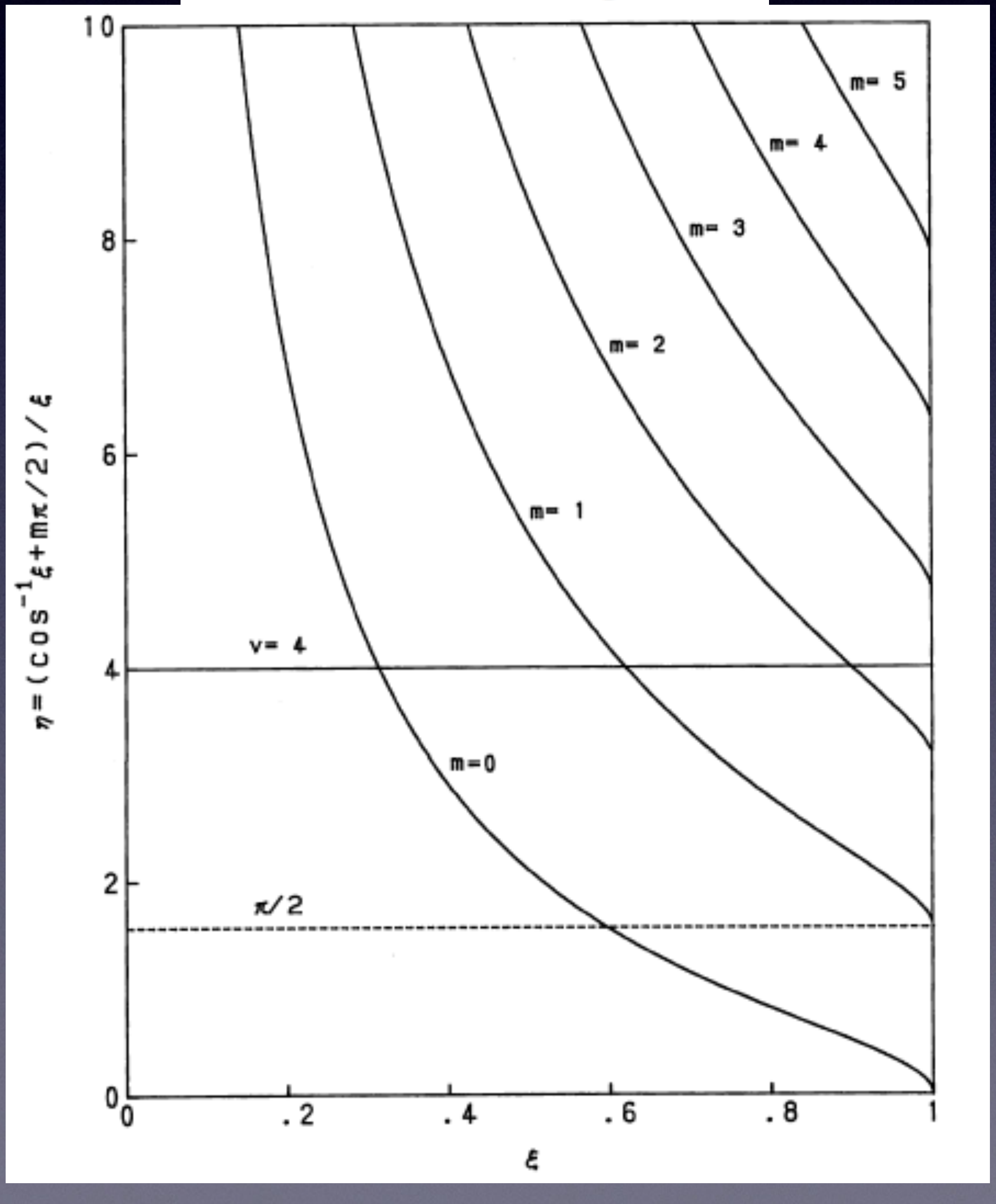
Normalized frequency

$$v = kn_1 a\sqrt{2\Delta}.$$

Dispersion curves

$$kn_1 a\sqrt{2\Delta} = \frac{\cos^{-1}\xi + m\pi/2}{\xi}$$



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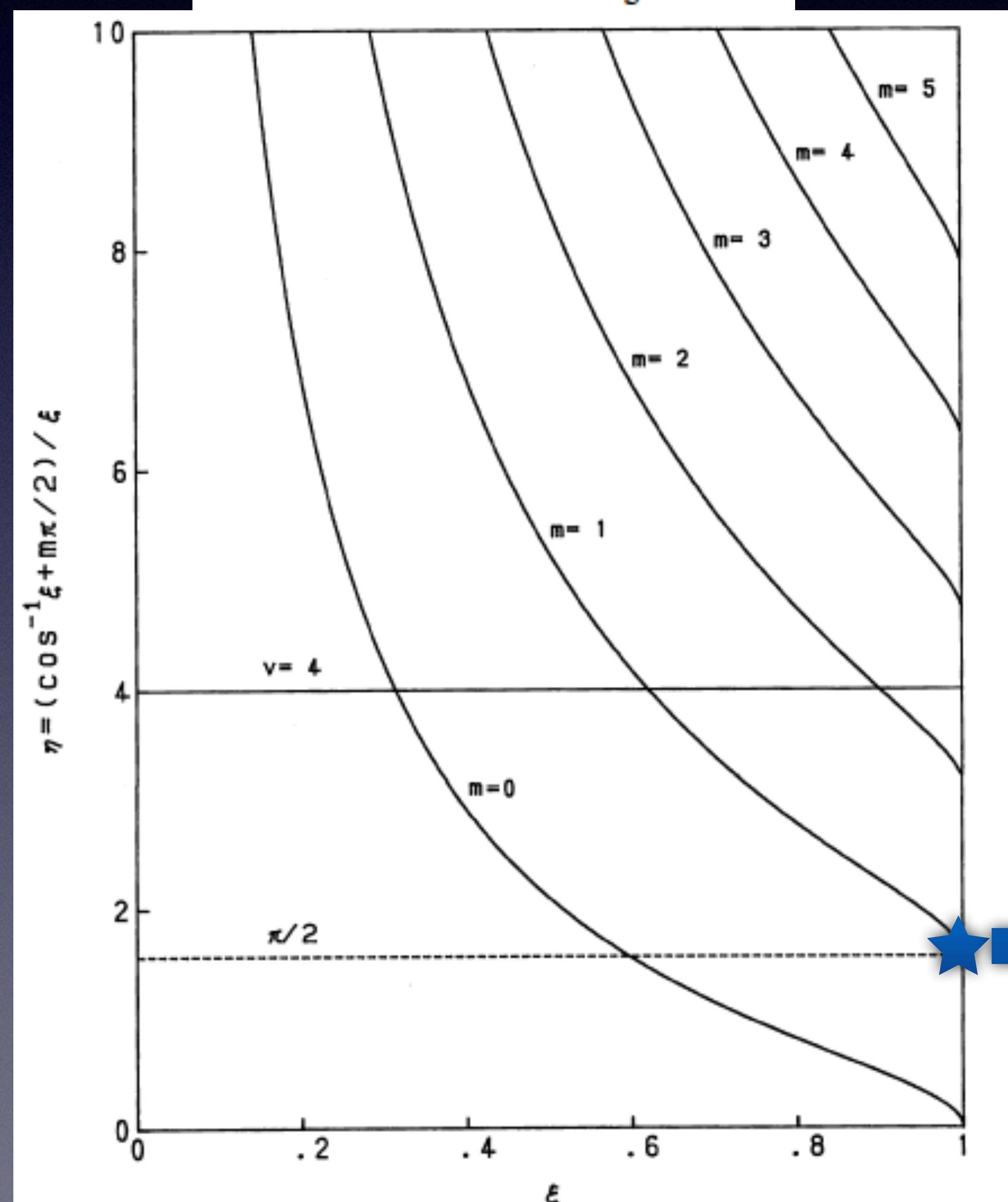
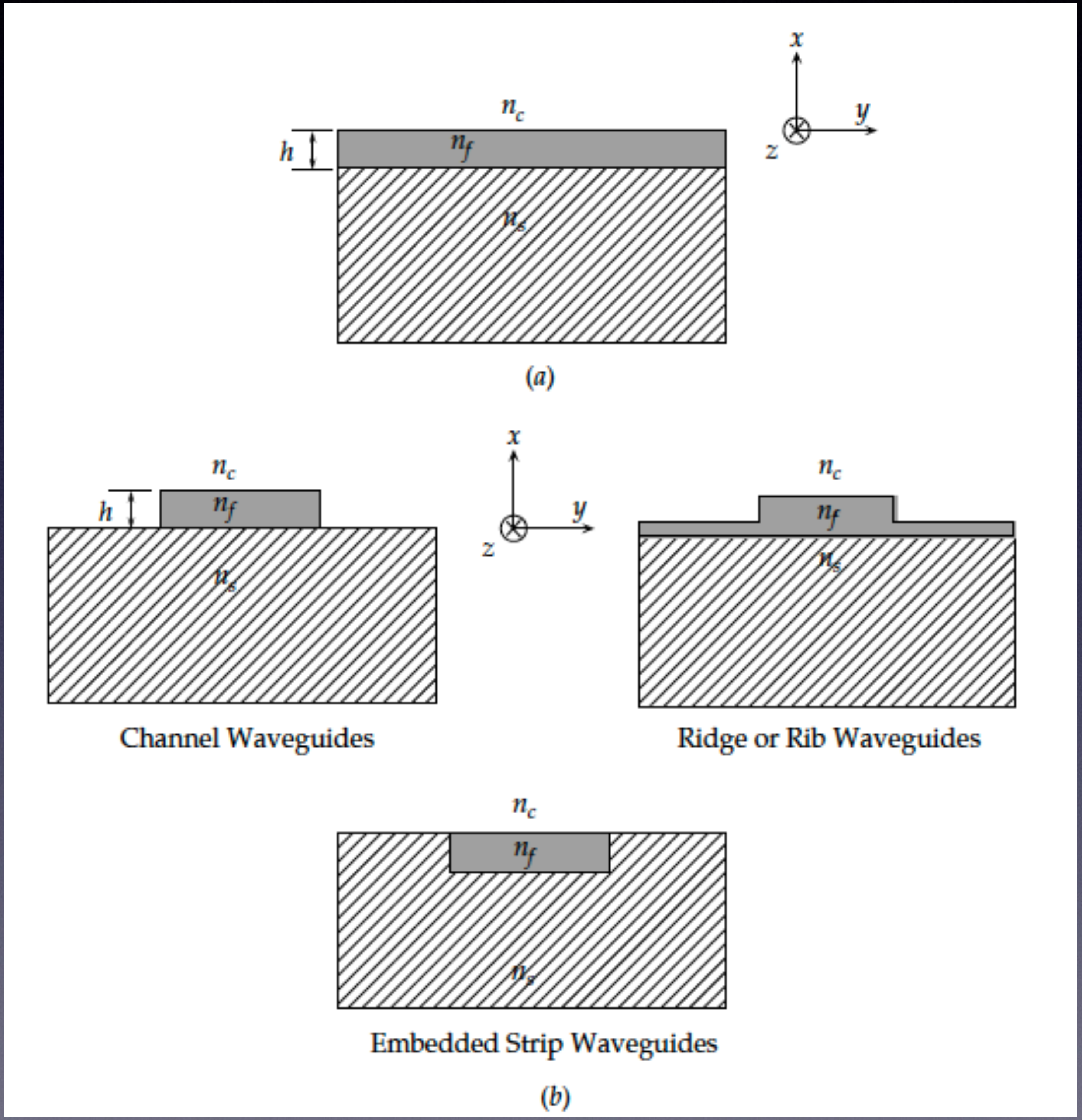


cutoff wavelength

$$\lambda_c = \frac{2\pi}{v_c} a n_1 \sqrt{2\Delta}.$$

- Slab waveguides

Waveguide cross sections



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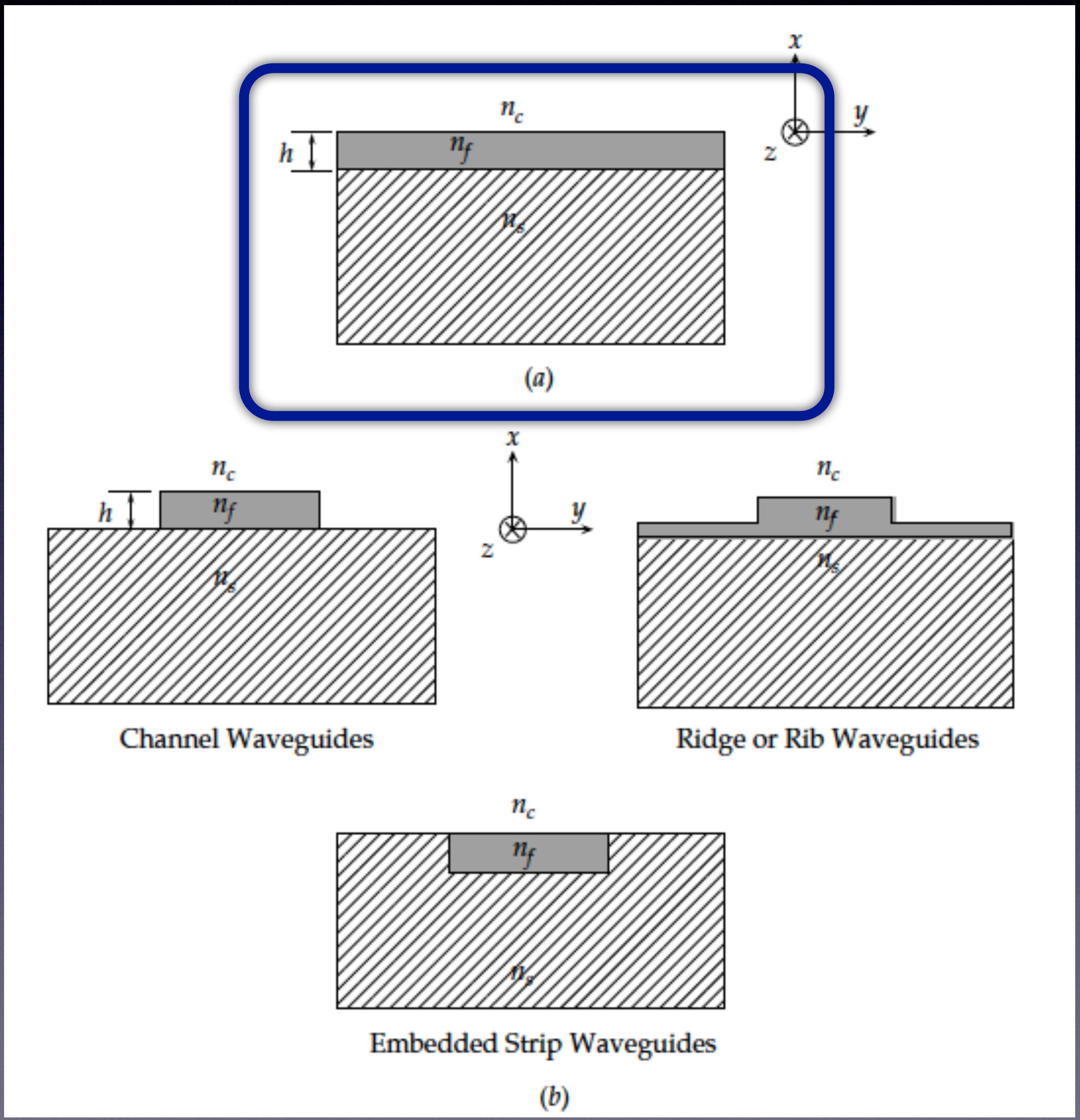

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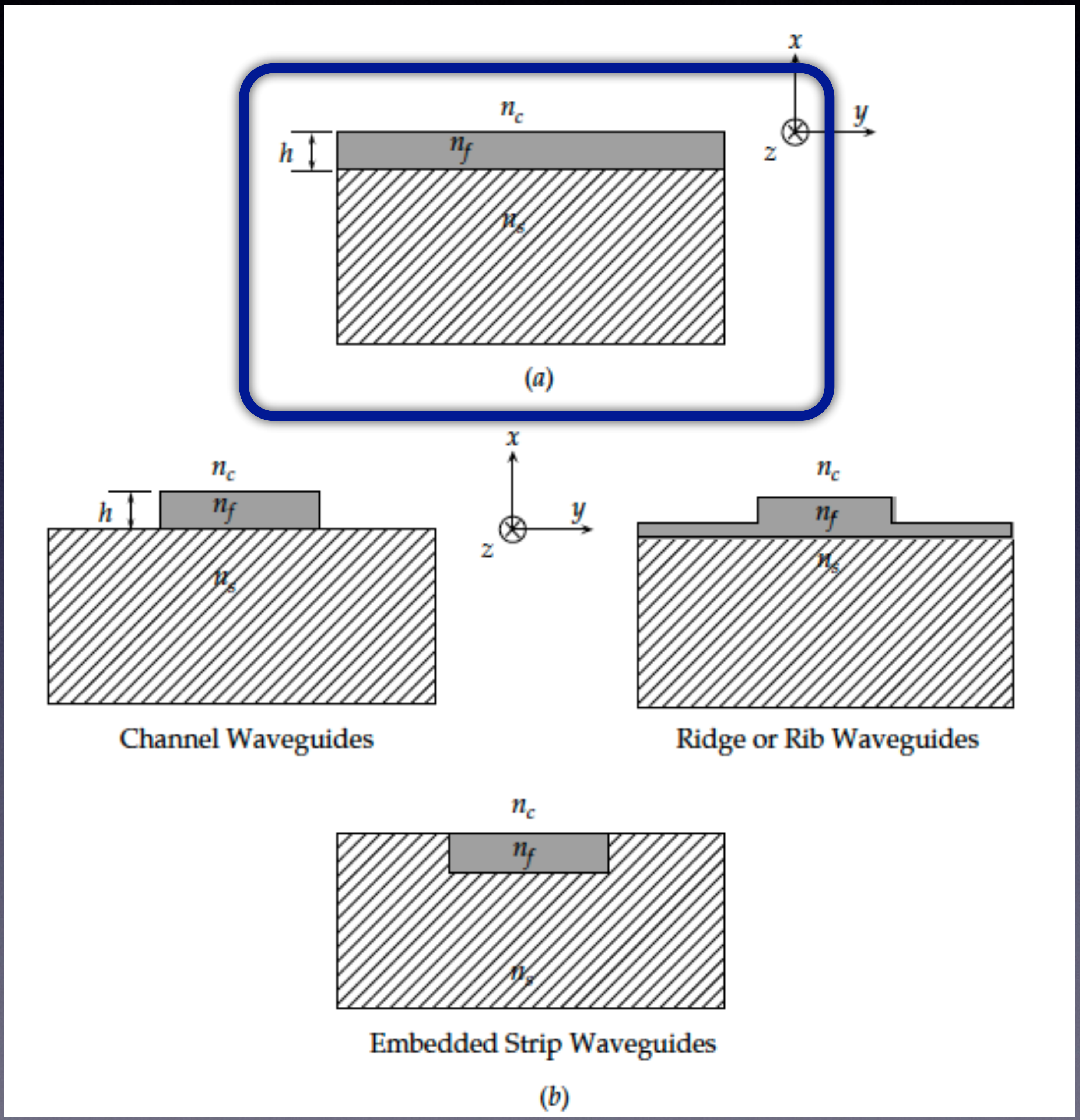

Dimension in the y direction >> h, λ

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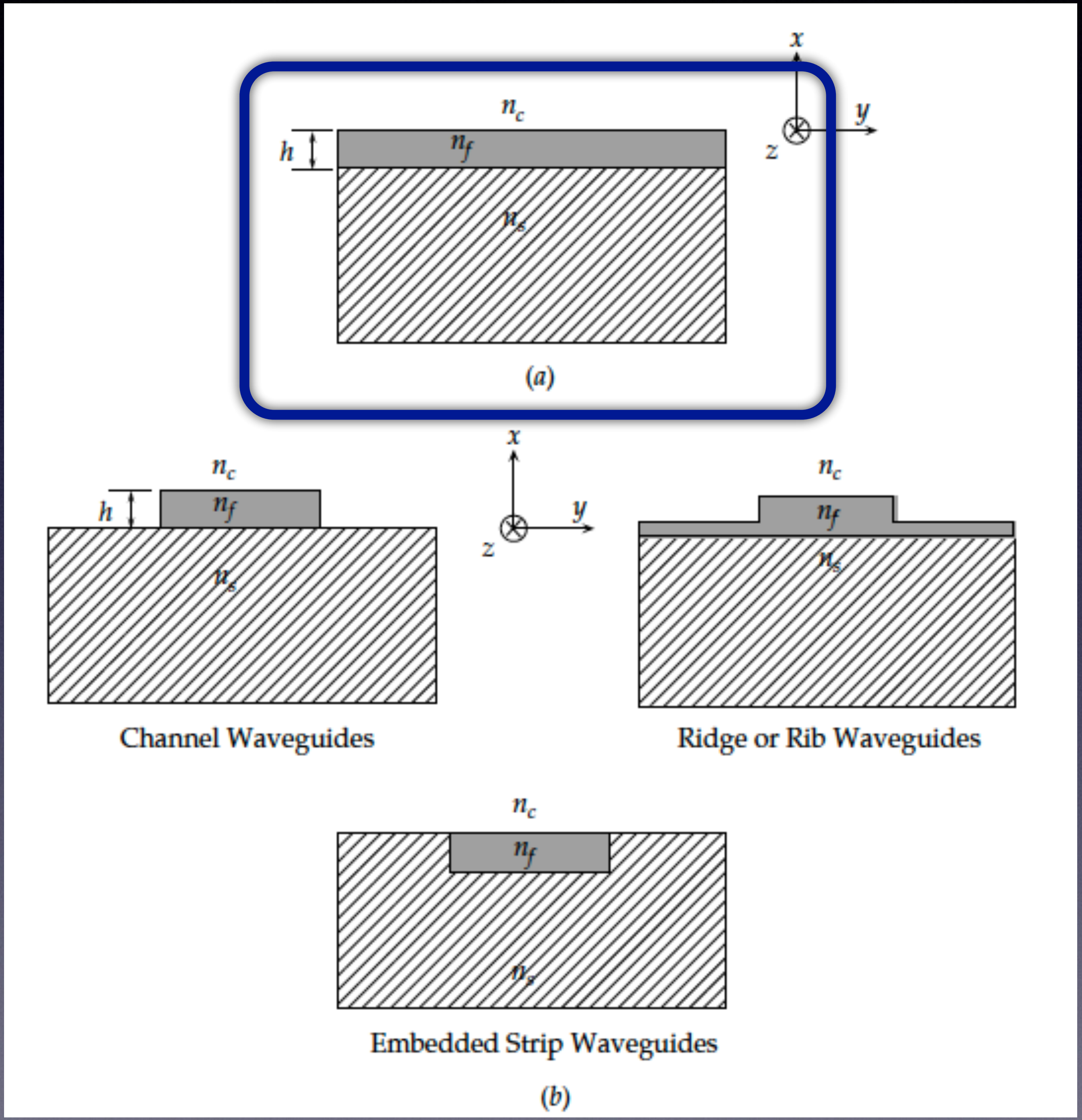

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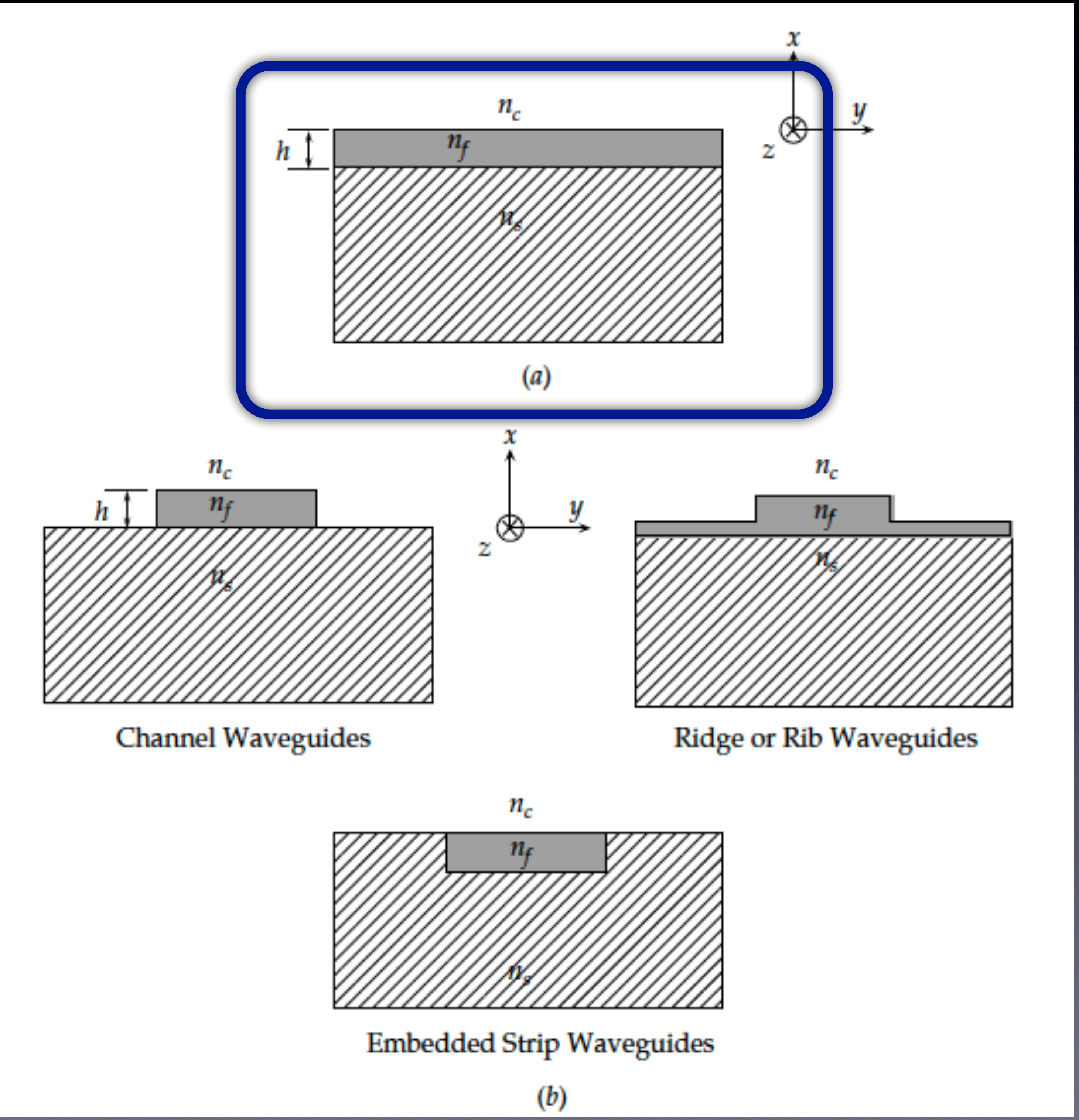

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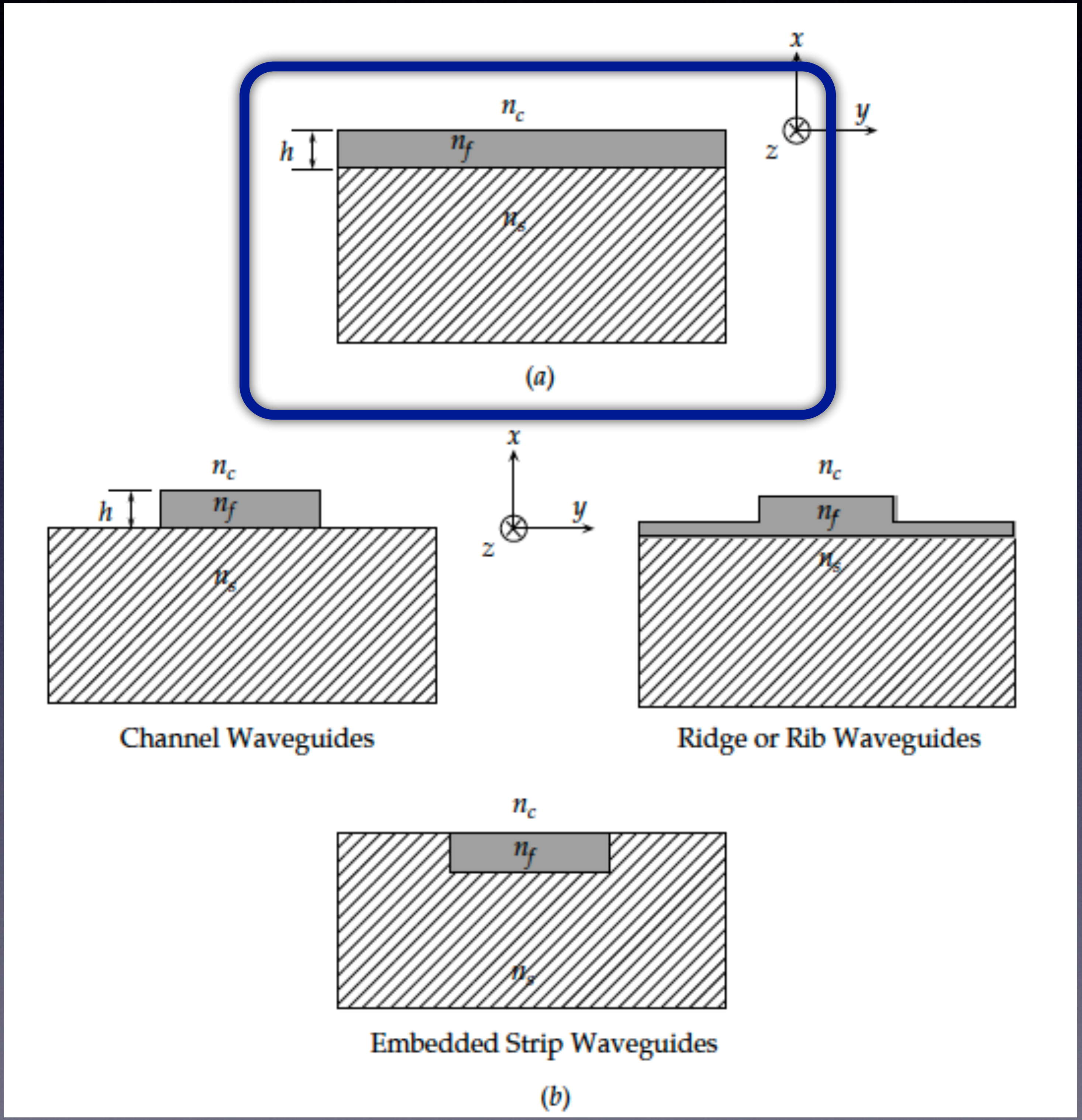

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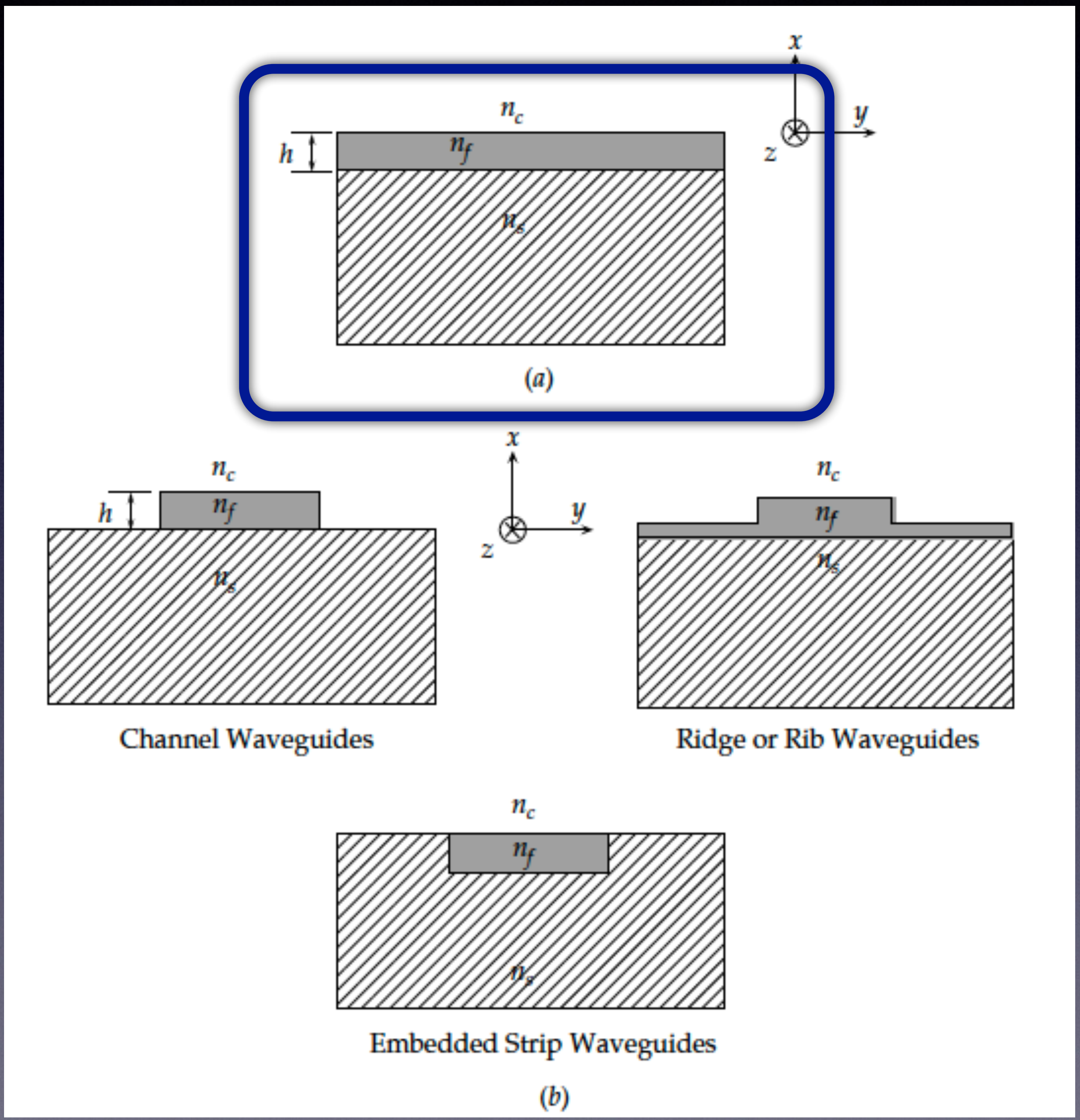

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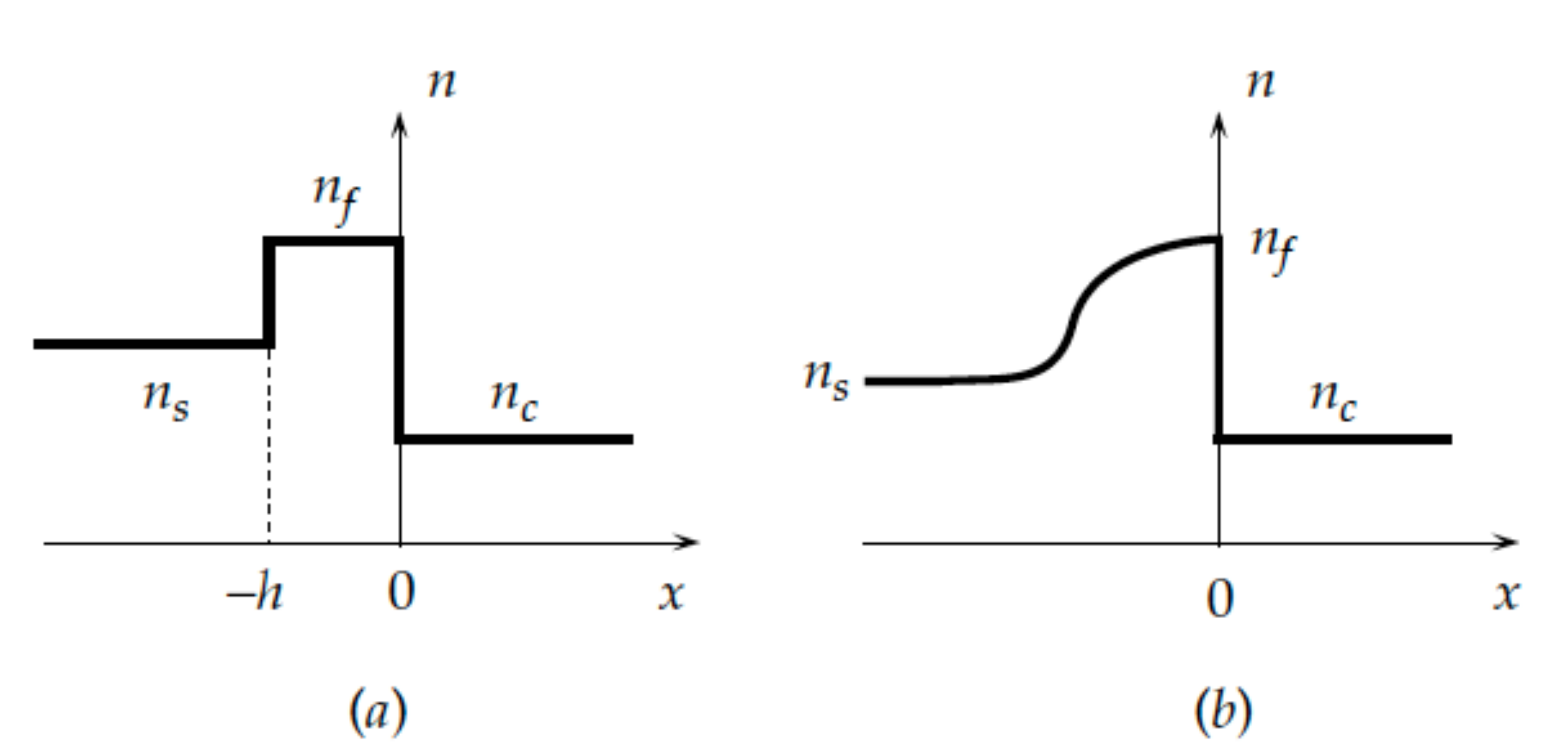
Dielectric slab waveguides

Index profiles



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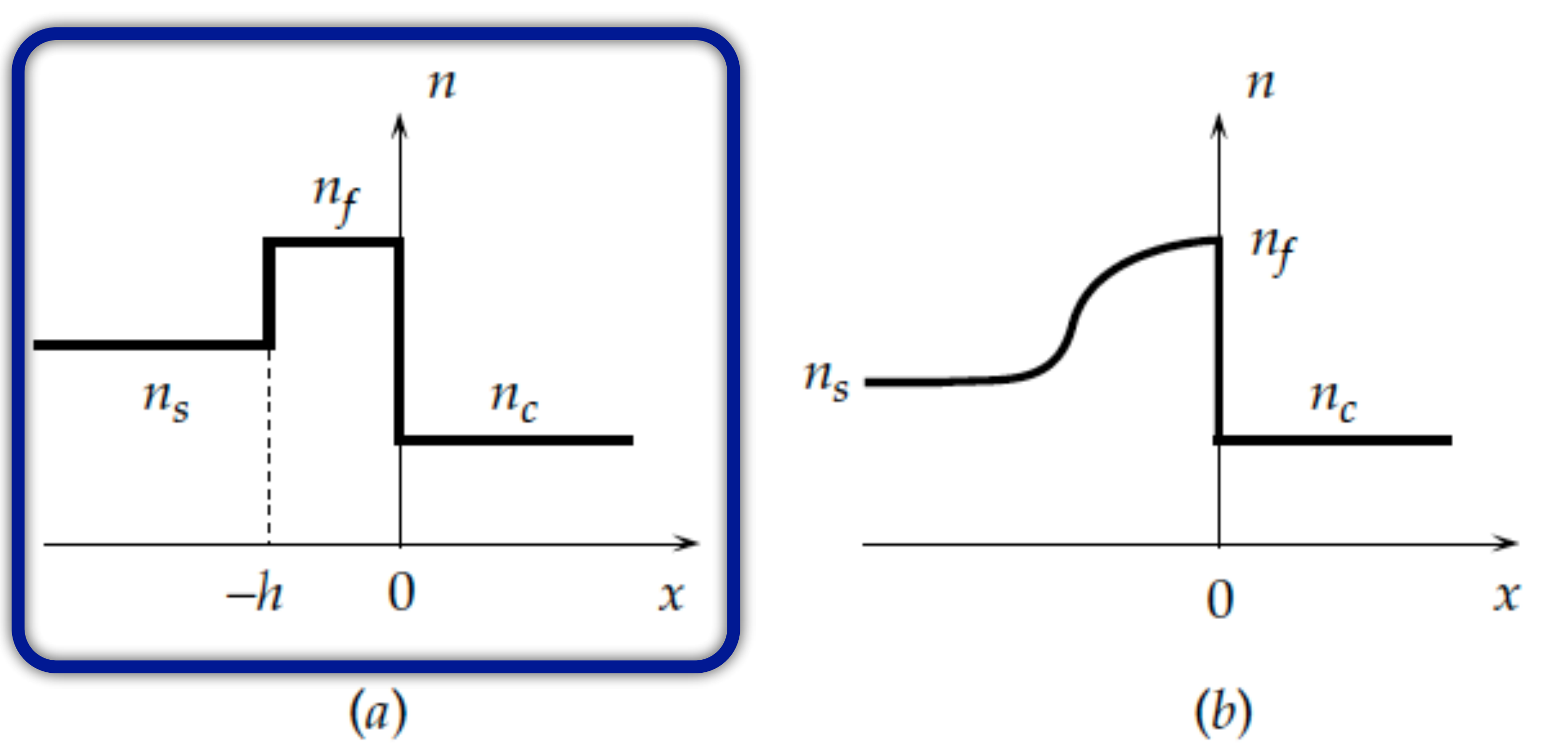
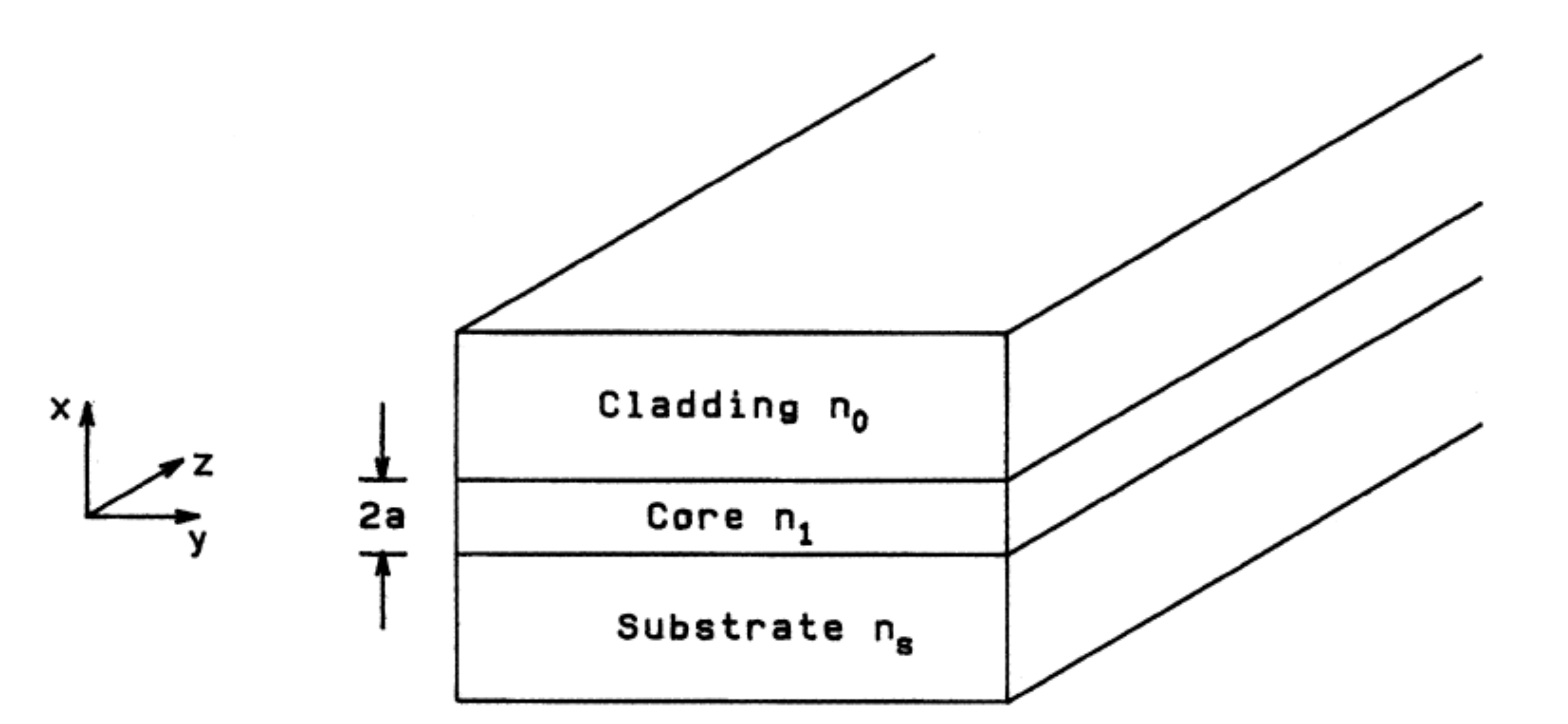


Step-index slab waveguides



Maxwell's equations

$$\nabla \times \mathbf{e} = -\mu \frac{\partial \mathbf{h}}{\partial t},$$

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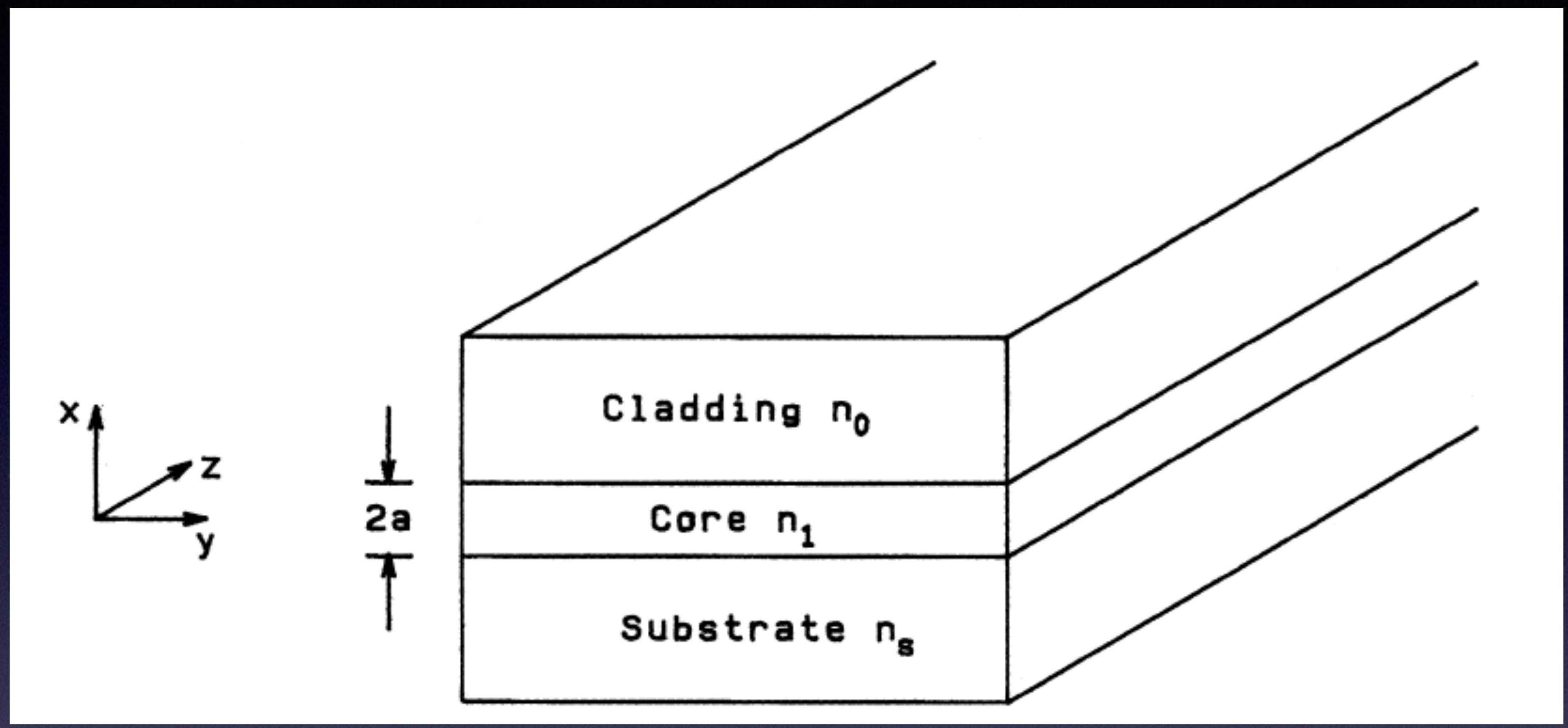

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dielectric medium
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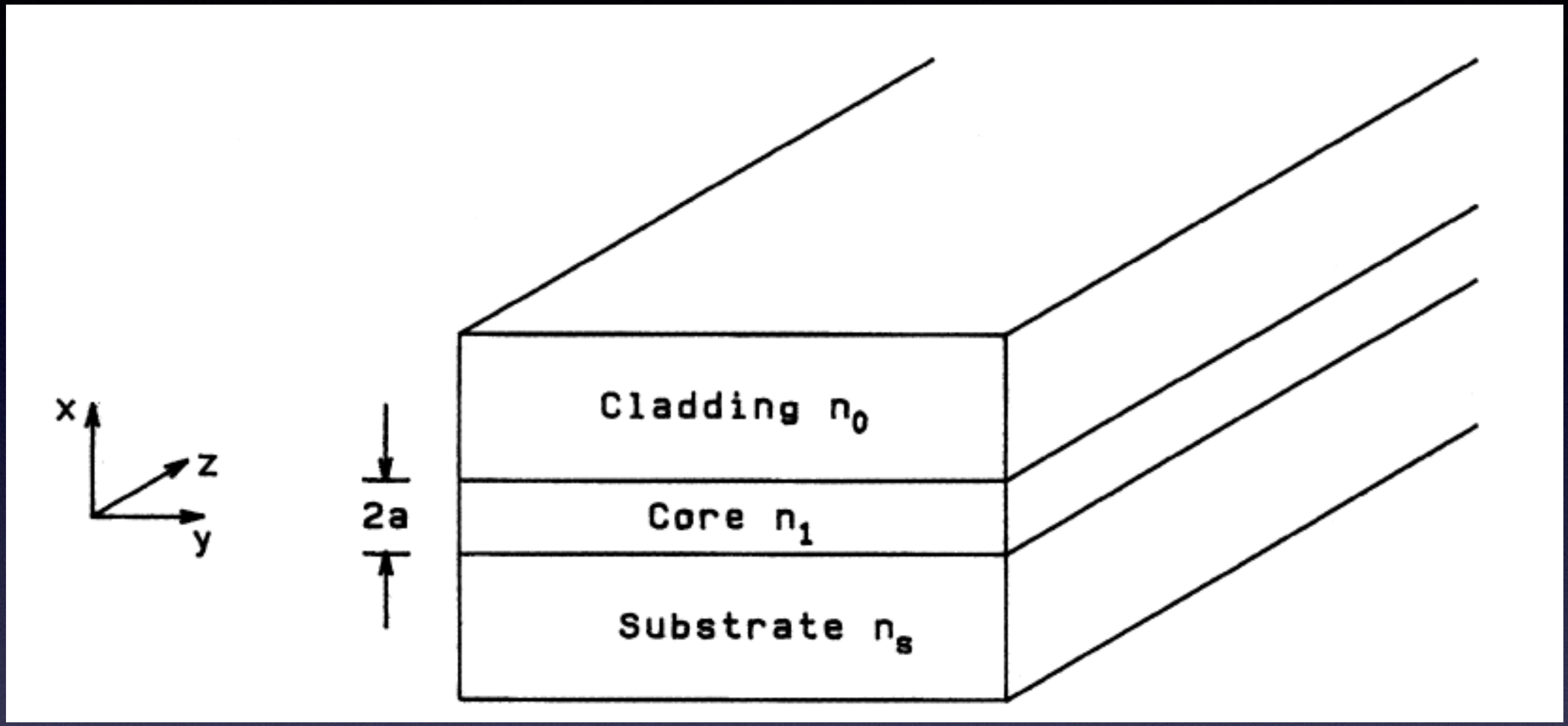

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dielectric medium with index n

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Plane-wave solutions

$$\tilde{\mathbf{E}} = \mathbf{E}(x, y)e^{j(\omega t - \beta z)},$$

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$\partial \mathbf{E}/\partial y = 0$ and $\partial \mathbf{H}/\partial y = 0$

TE modes

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$$H_x = -\frac{\beta}{\omega\mu_0} E_y,$$

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Wave equation for TE modes

$$\frac{d^2 E_y}{dx^2} + (k^2 n^2 - \beta^2) E_y = 0,$$

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TM modes

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$$\begin{cases} \cancel{\dfrac{\partial H_z}{\partial y}} + j\beta H_y = j\omega\varepsilon_0 n^2 E_x \\ -j\beta H_x - \dfrac{\partial H_z}{\partial x} = j\omega\varepsilon_0 n^2 E_y \\ \dfrac{\partial H_y}{\partial x} - \cancel{\dfrac{\partial H_x}{\partial y}} = j\omega\varepsilon_0 n^2 E_z. \end{cases}$$

$$E_x = \frac{\beta}{\omega\varepsilon_0 n^2} H_y,$$

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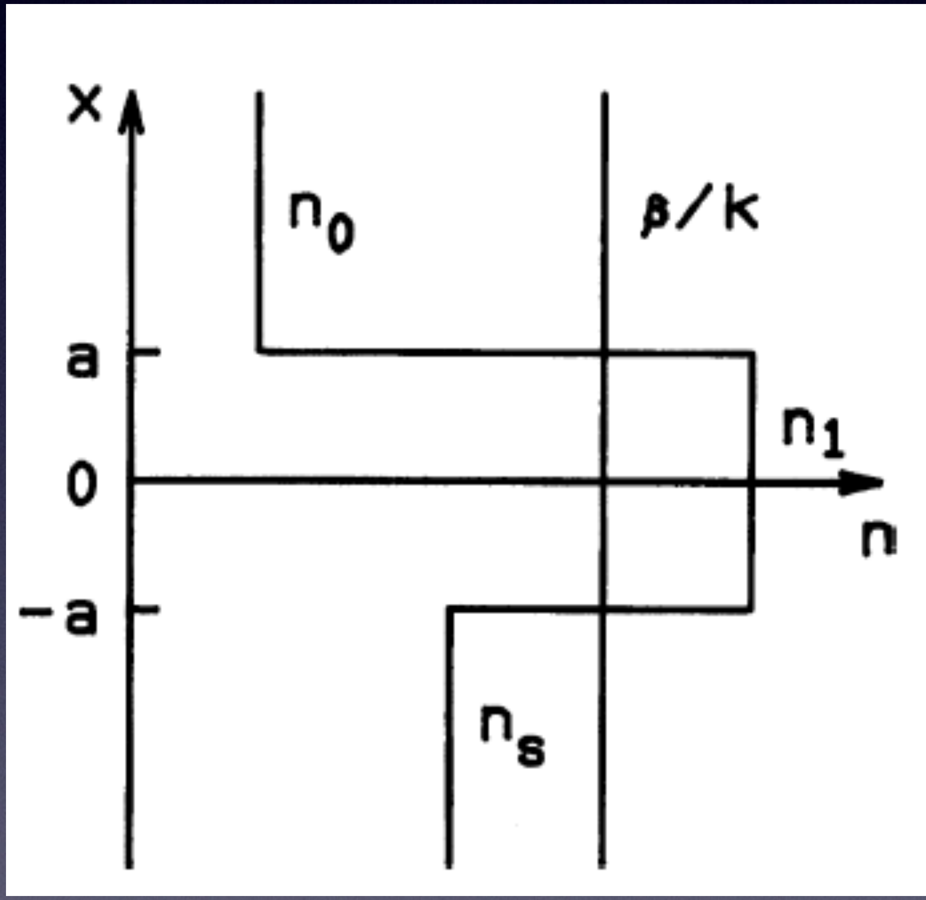
Wave equation for TM modes

$$\frac{d}{dx}\left(\frac{1}{n^2}\frac{dH_y}{dx}\right) + \left(k^2 - \frac{\beta^2}{n^2}\right) H_y = 0,$$

Dispersion relation for TE modes



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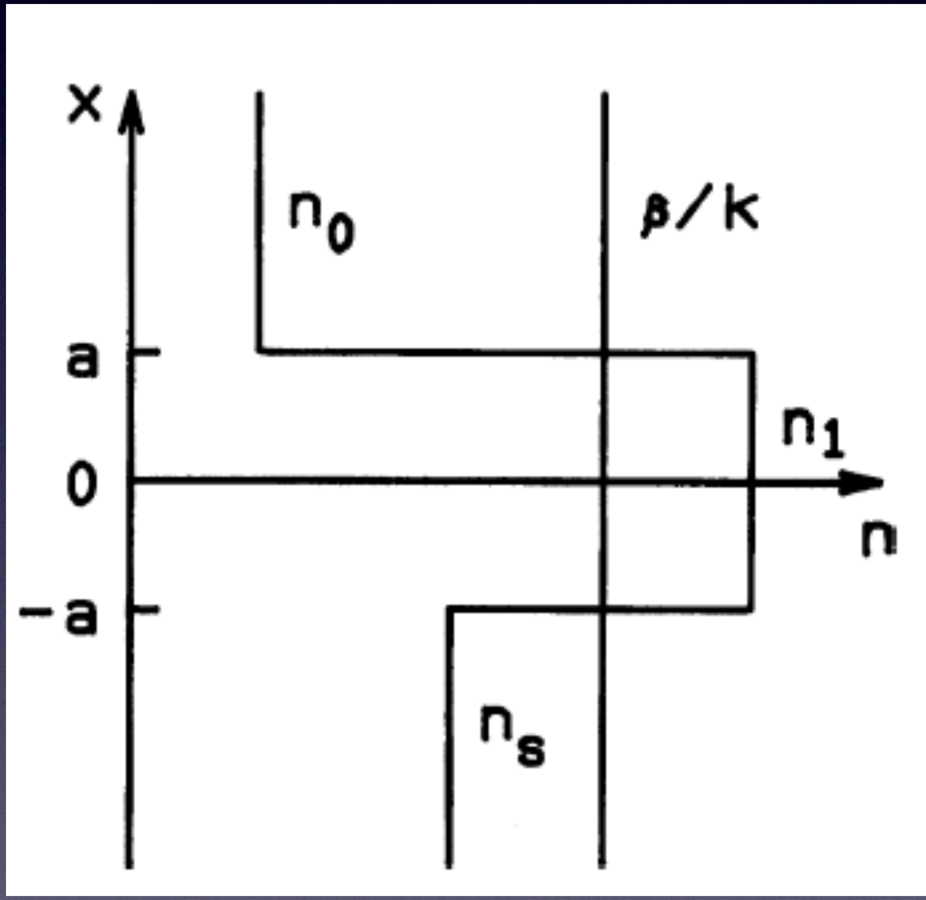

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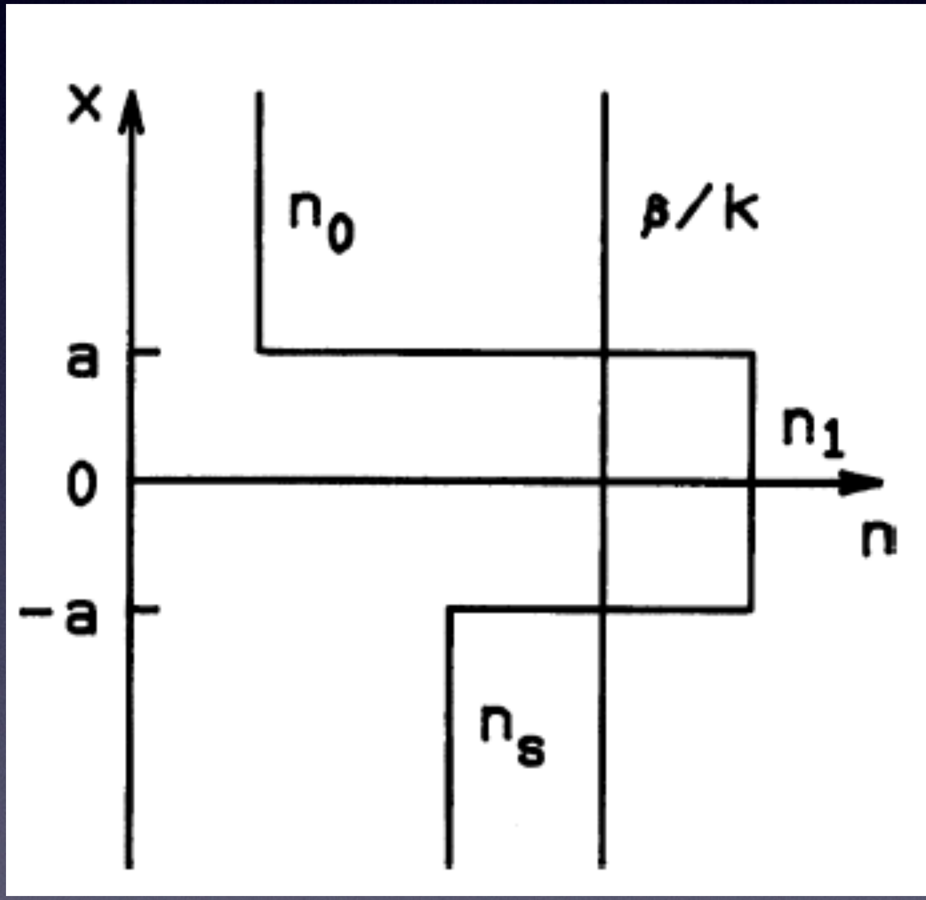

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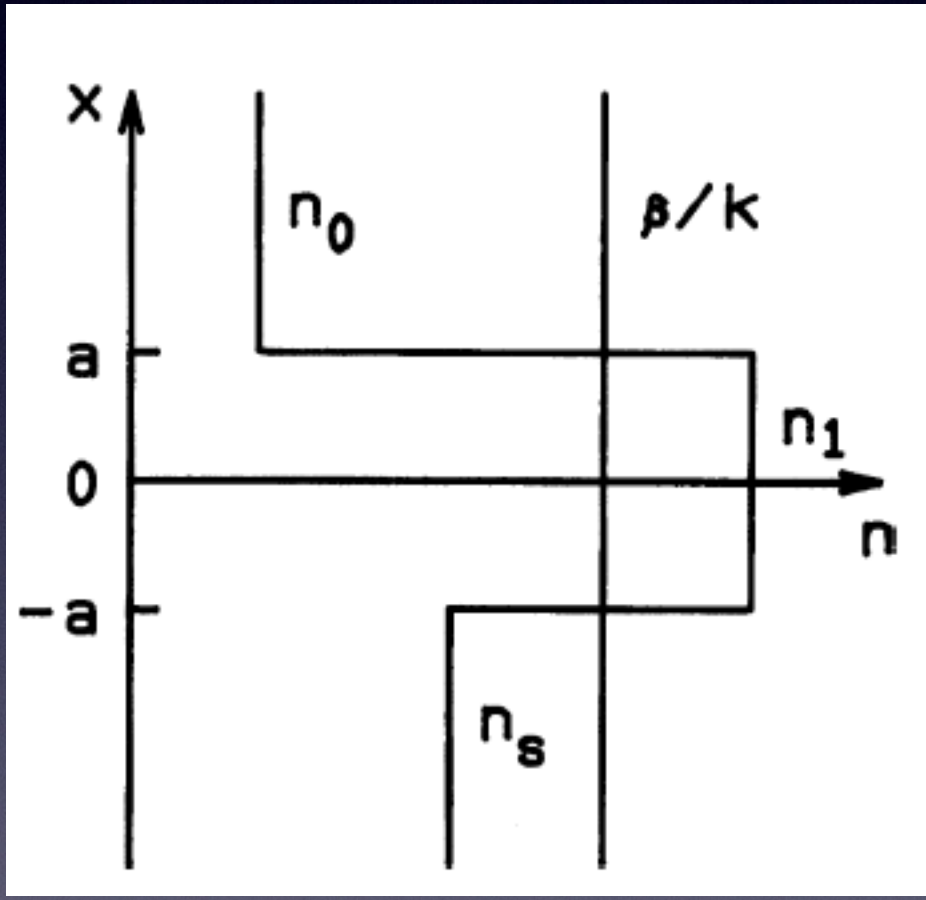

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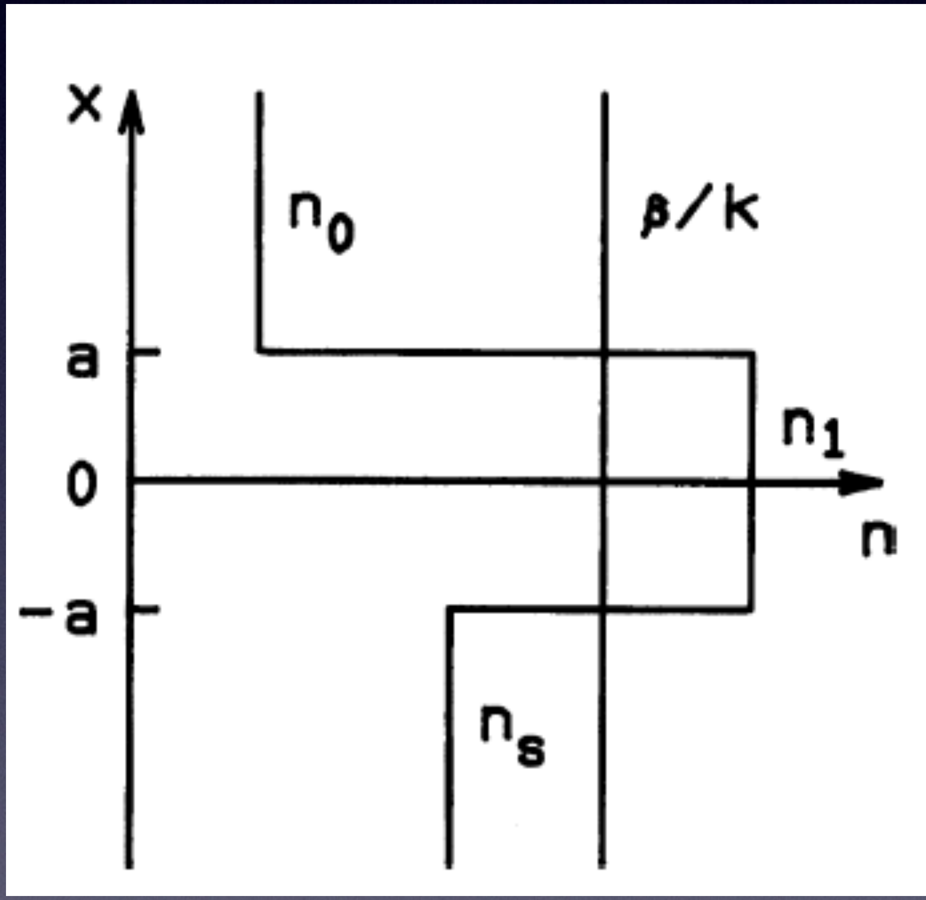

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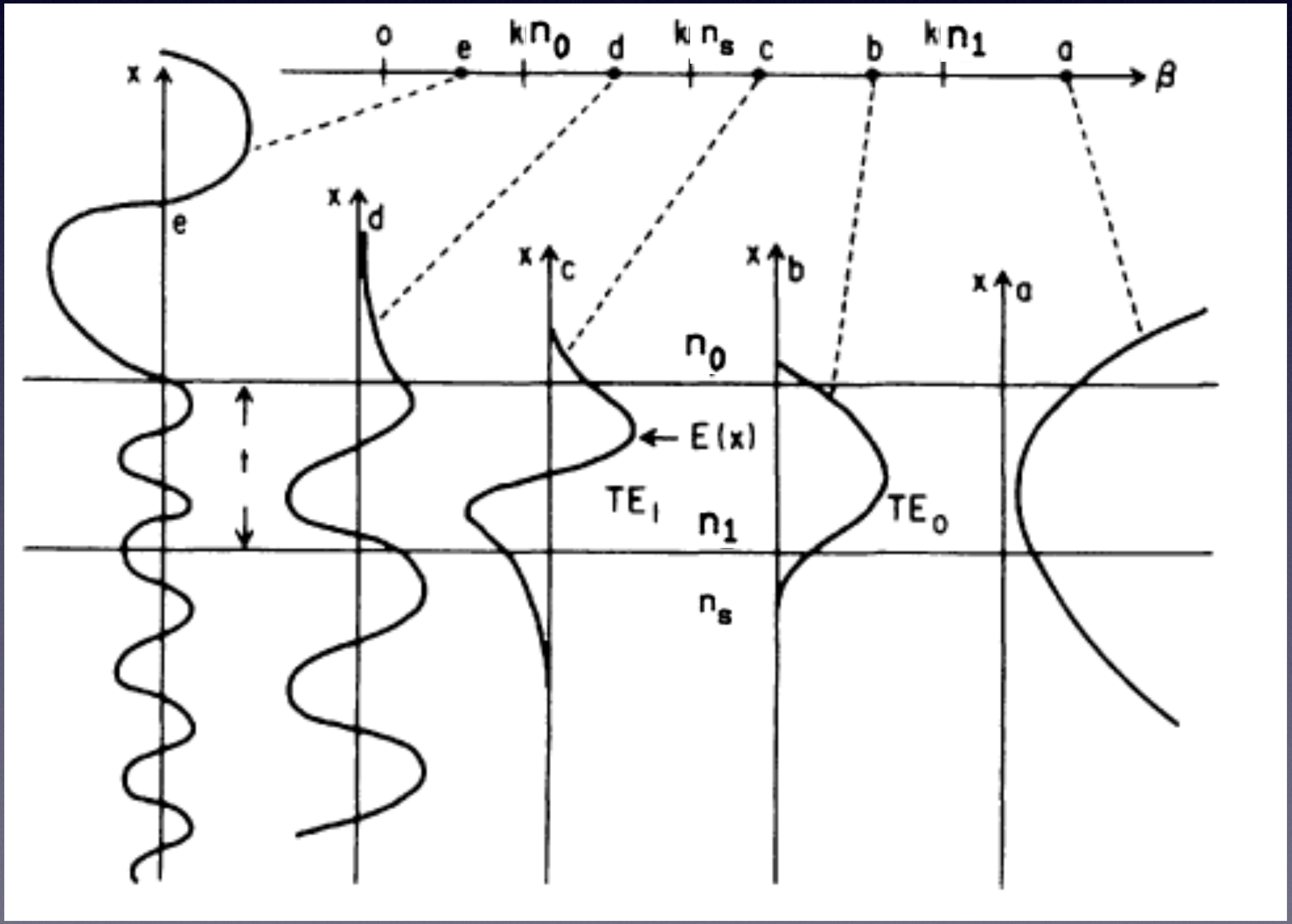
$$\frac{d^2E_y}{dx^2} + (k^2n^2 - \beta^2)E_y = 0,$$

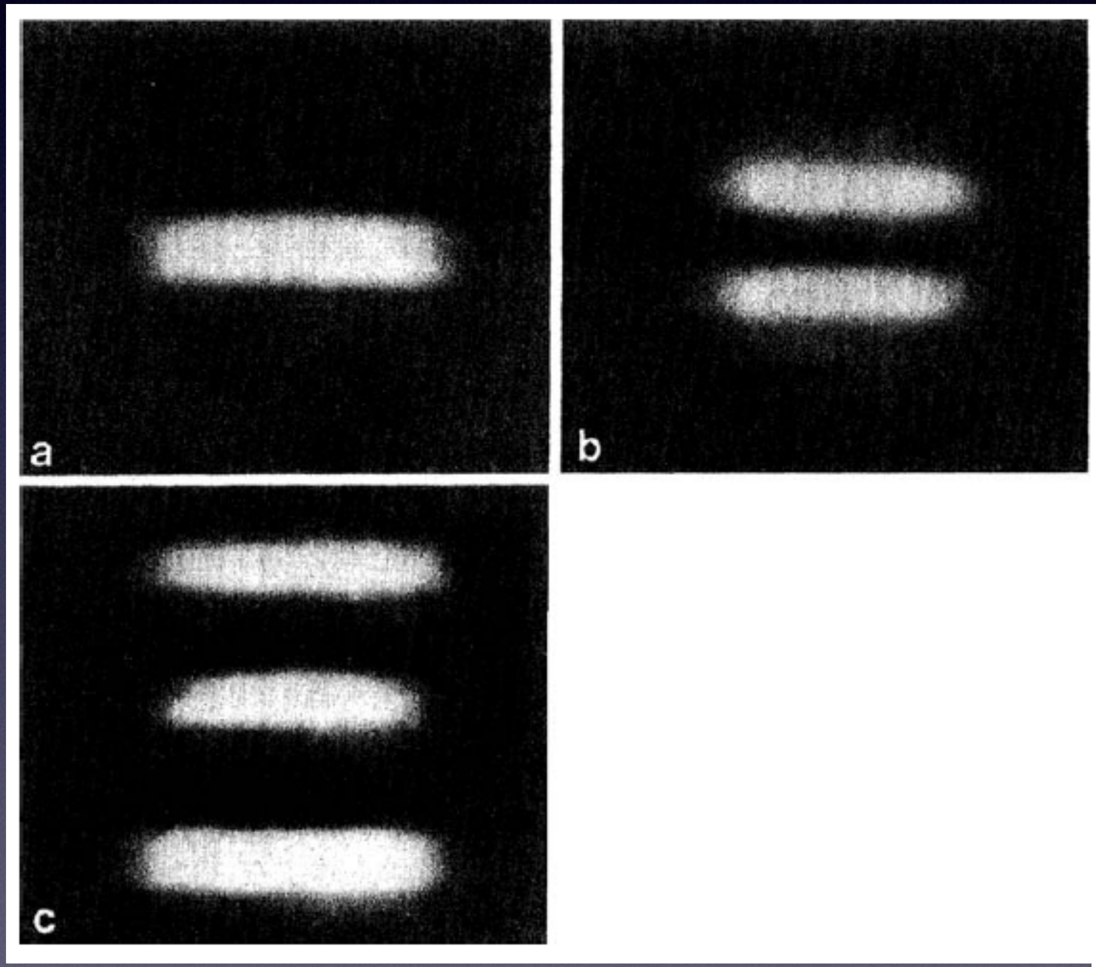
Boundary conditions:
Continuity of Ey and Hz across the boundaries

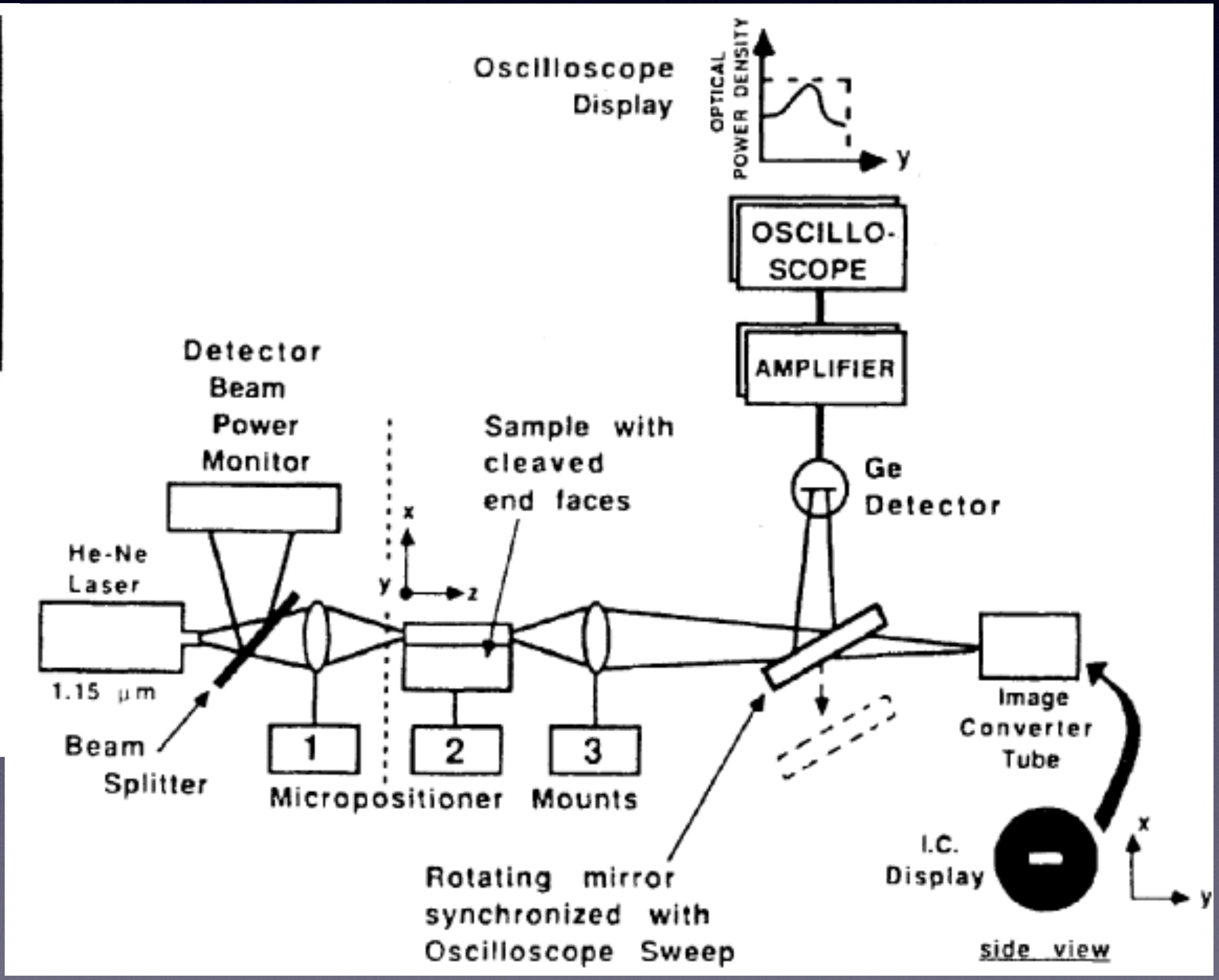
Solutions of the wave equation

$$\frac{d^2 E_y}{dx^2} + (k^2 n^2 - \beta^2) E_y = 0,$$



Observation of waveguide modes





Continuity of Ey

$$E_y = \begin{cases} A\cos(\kappa a - \phi)e^{-\sigma(x-a)} & (x > a) \\ A\cos(\kappa x - \phi) & (-a \leqslant x \leqslant a) \\ A\cos(\kappa a + \phi)e^{\xi(x+a)} & (x < -a), \end{cases}$$

$$\begin{cases} \kappa = \sqrt{k^2 n_1^2 - \beta^2} \\ \sigma = \sqrt{\beta^2 - k^2 n_0^2} \\ \xi = \sqrt{\beta^2 - k^2 n_s^2}. \end{cases}$$

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Continuity of Hz

$$\begin{cases} \kappa A\sin(\kappa a + \phi) = \xi A\cos(\kappa a + \phi) \\ \sigma A\cos(\kappa a - \phi) = \kappa A\sin(\kappa a - \phi). \end{cases}$$

$$\tan(u + \phi) = \frac{w}{u},$$

$$\tan(u - \phi) = \frac{w'}{u},$$

$$\begin{cases} u = \kappa a \\ w = \xi a \\ w' = \sigma a. \end{cases}$$

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Eigenvalue equations

$$u = \frac{m\pi}{2} + \frac{1}{2}\tan^{-1}\left(\frac{w}{u}\right) + \frac{1}{2}\tan^{-1}\left(\frac{w'}{u}\right) \qquad (m = 0, 1, 2, \ldots)$$

$$\phi = \frac{m\pi}{2} + \frac{1}{2}\tan^{-1}\left(\frac{w}{u}\right) - \frac{1}{2}\tan^{-1}\left(\frac{w'}{u}\right).$$

Normalized transverse wavenumbers

$$u^2 + w^2 = k^2 a^2 (n_1^2 - n_s^2) \equiv v^2,$$

Normalized frequency

$$w' = \sqrt{\gamma v^2 + w^2},$$

$$\gamma = \frac{n_s^2 - n_0^2}{n_1^2 - n_s^2}$$

measure of asymmetry

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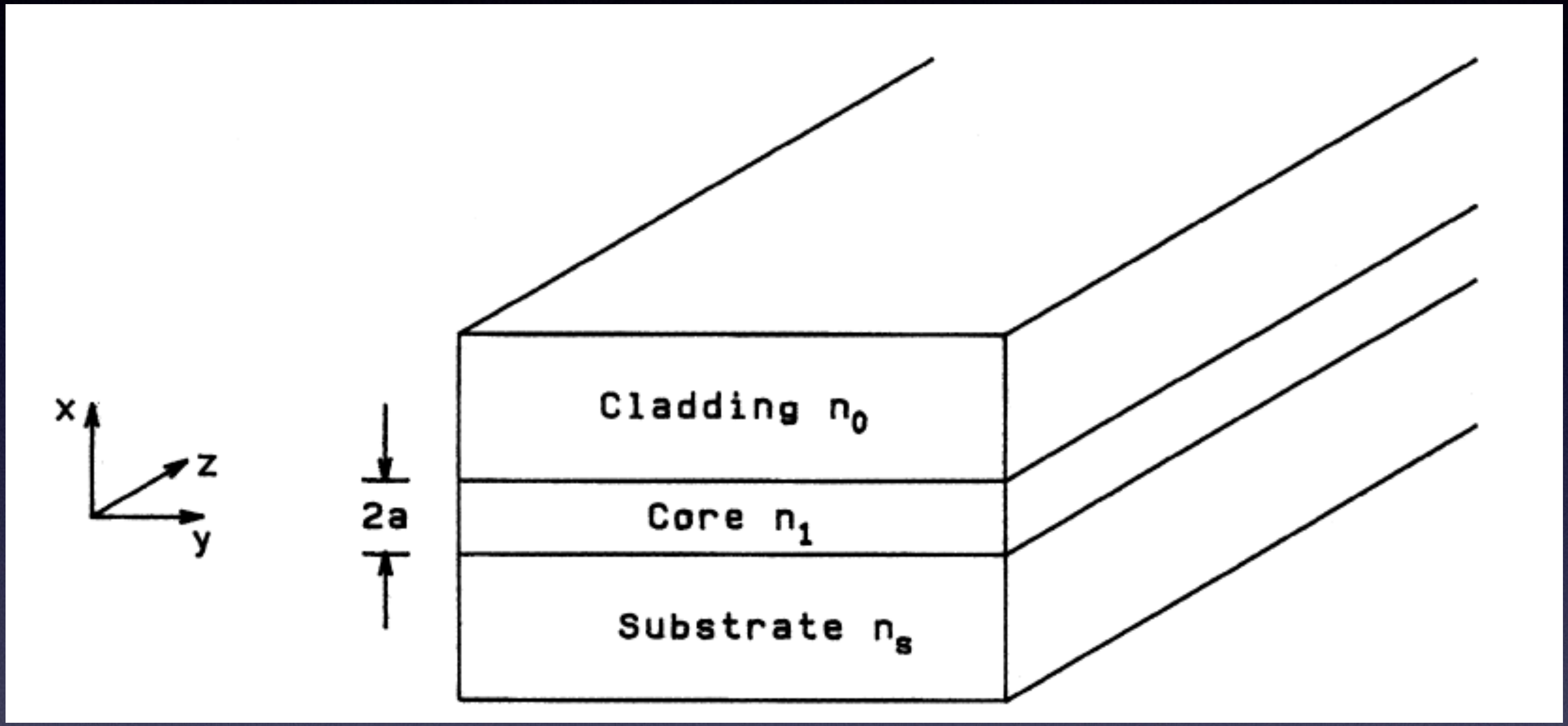
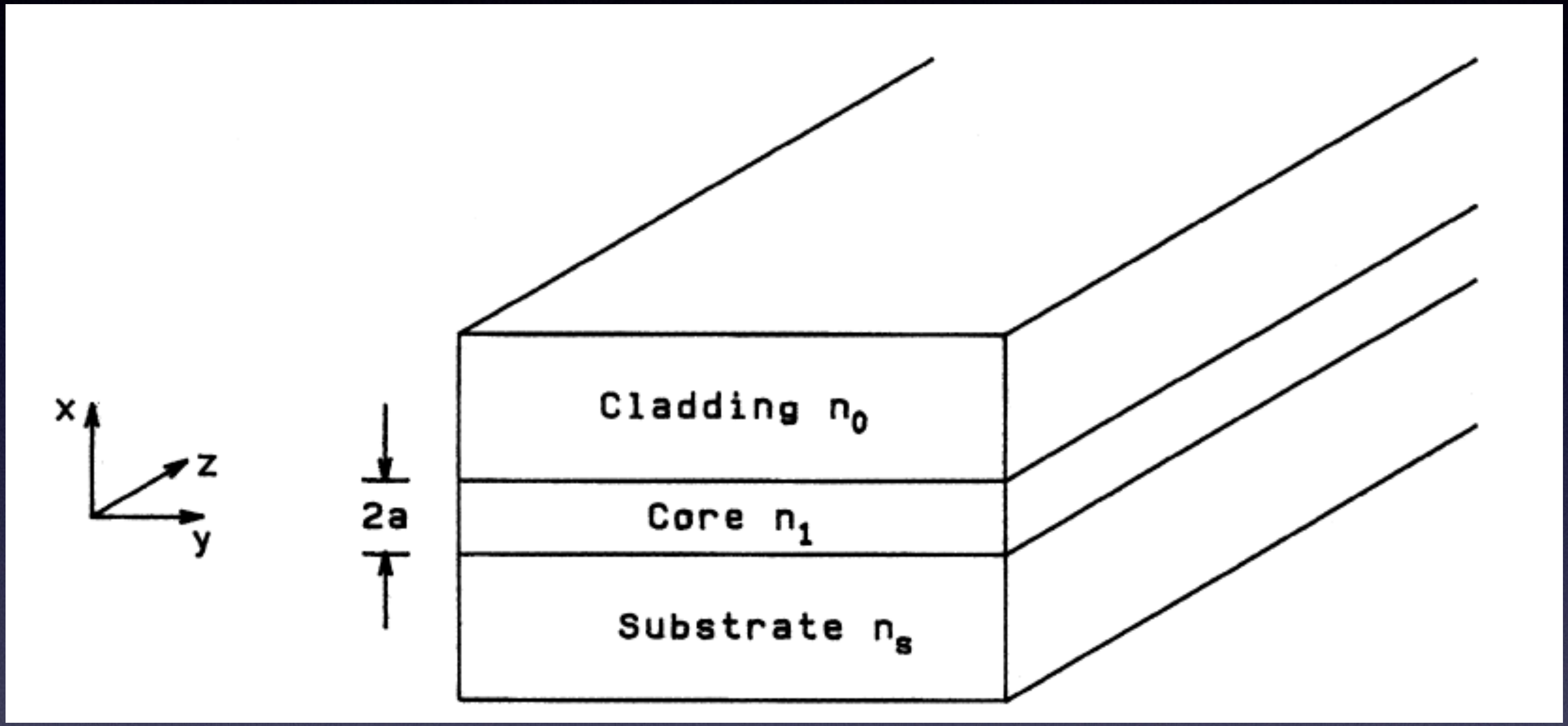
measure of asymmetry

Effective index



$$n_e = \frac{\beta}{k}$$

Effective index



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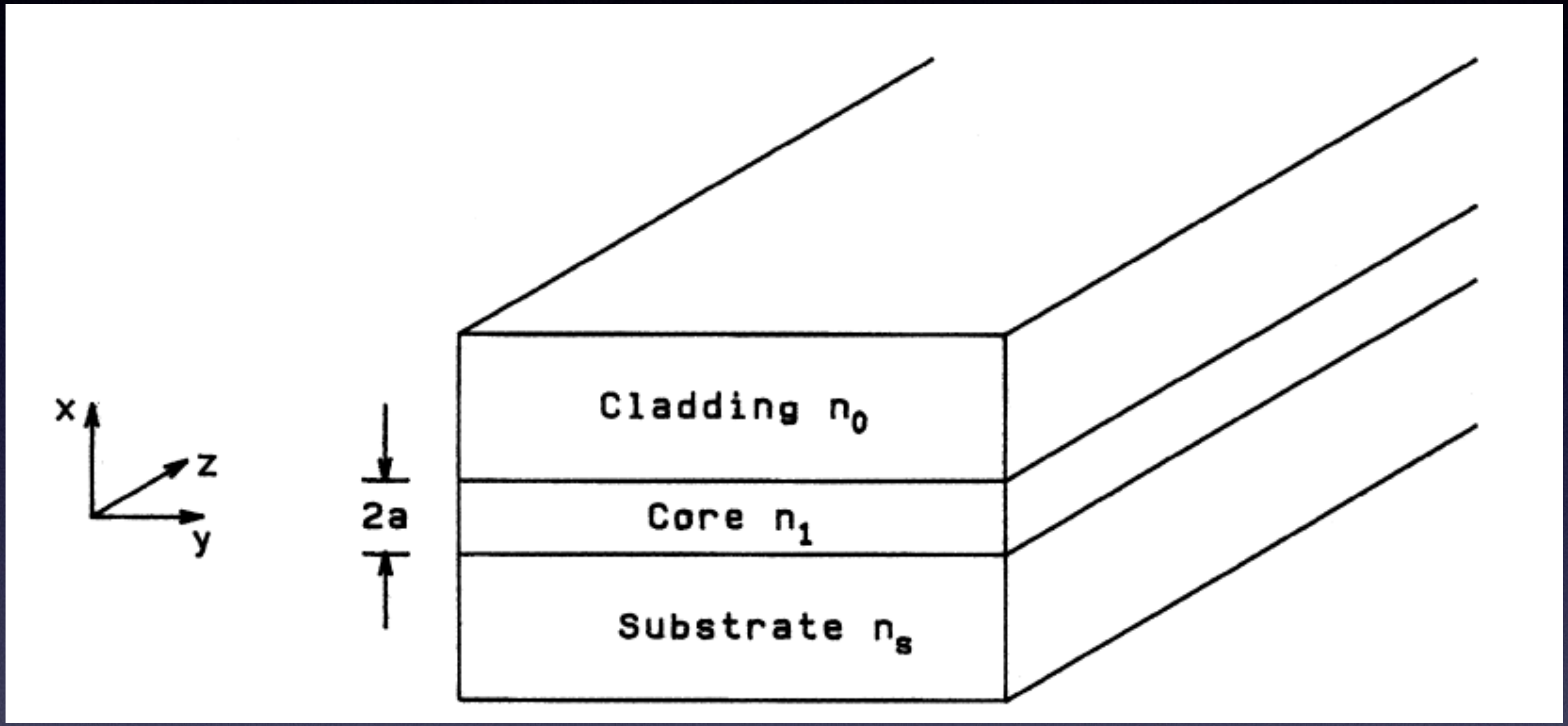
For the optical field to be confined in the core region

Effective index



$$n_e = \frac{\beta}{k}$$

For the optical field to be confined in the core region

$$n_s \leqslant \frac{\beta}{k} \leqslant n_1$$

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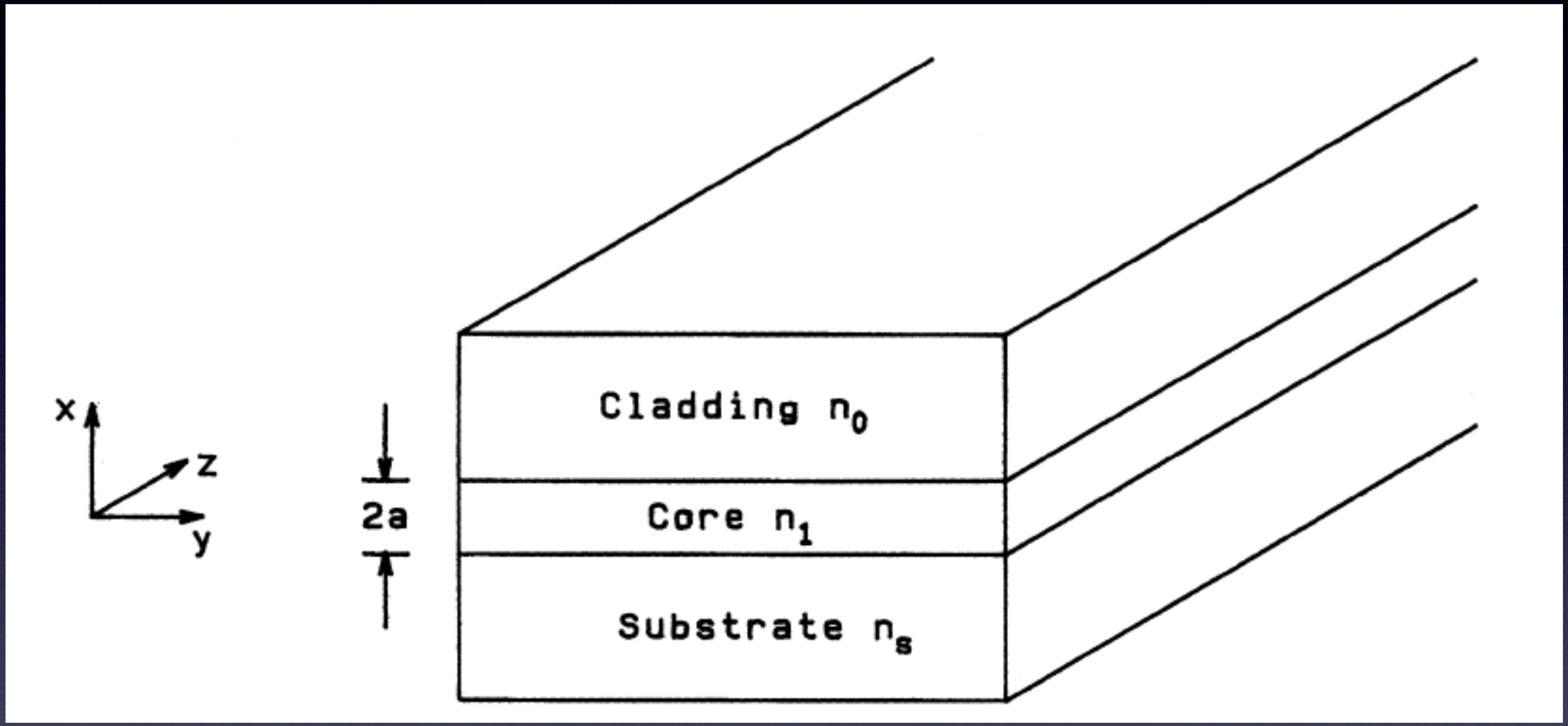


$$n_e = \frac{\beta}{k}$$

For the optical field to be confined in the core region

$$\boxed{n_s \leqslant \frac{\beta}{k}} \leqslant n_1$$

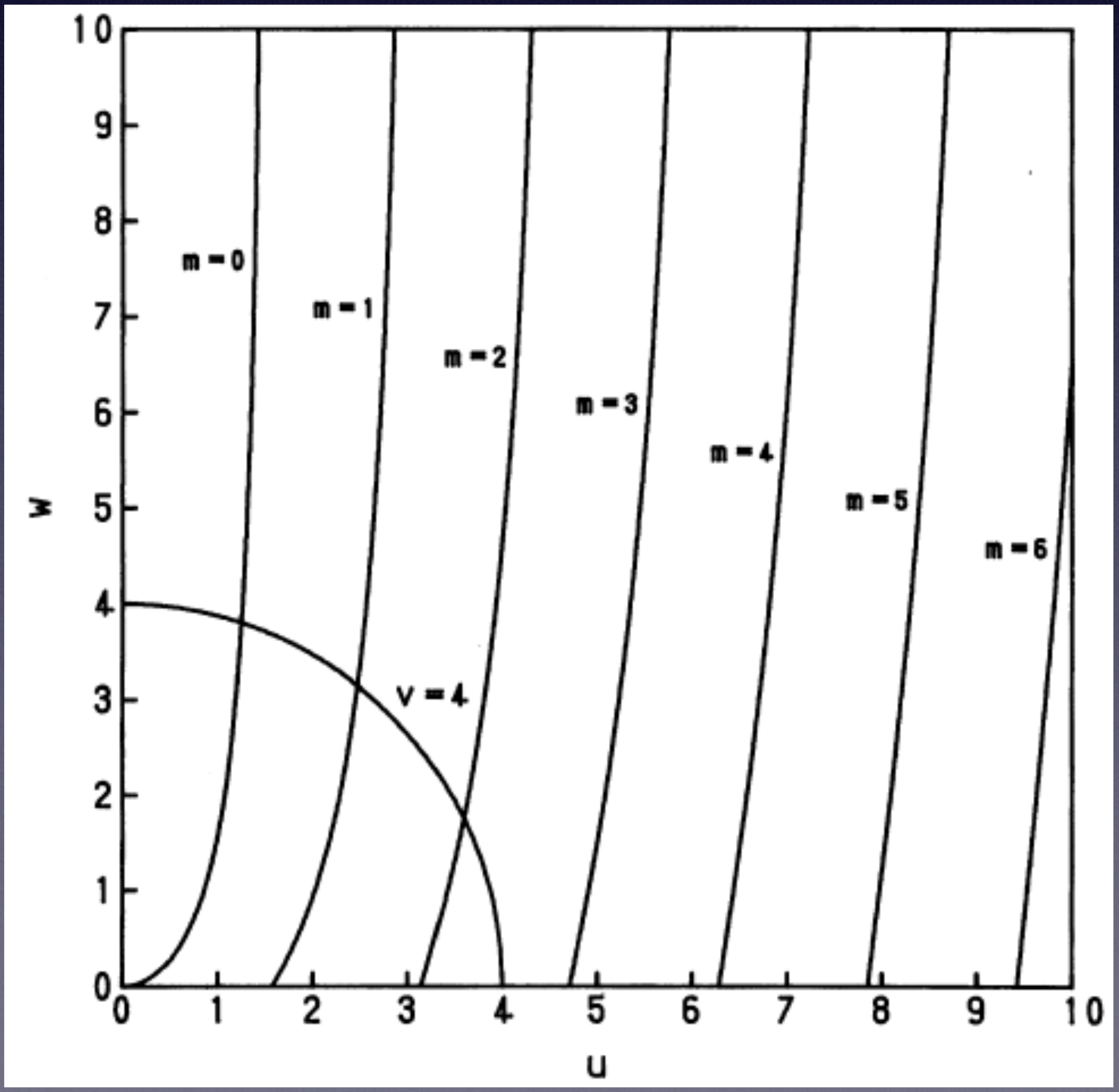
Cutoff condition

Computation of propagation constant: graphical method

For symmetric slab waveguide

$$w = u \tan\left(u - \frac{m\pi}{2}\right)$$

$$u^2 + w^2 = k^2 a^2 (n_1^2 - n_s^2) \equiv v^2$$



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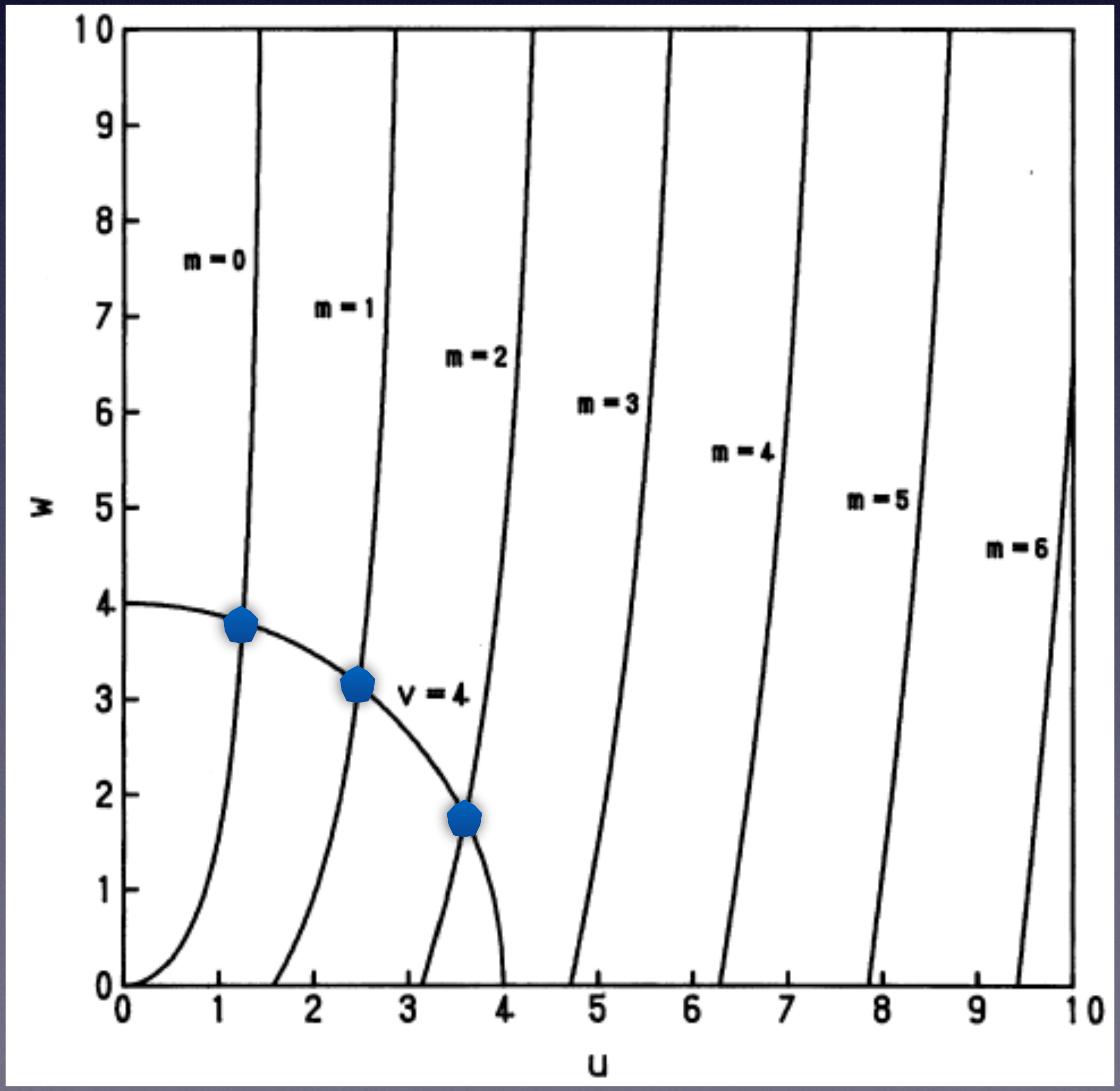

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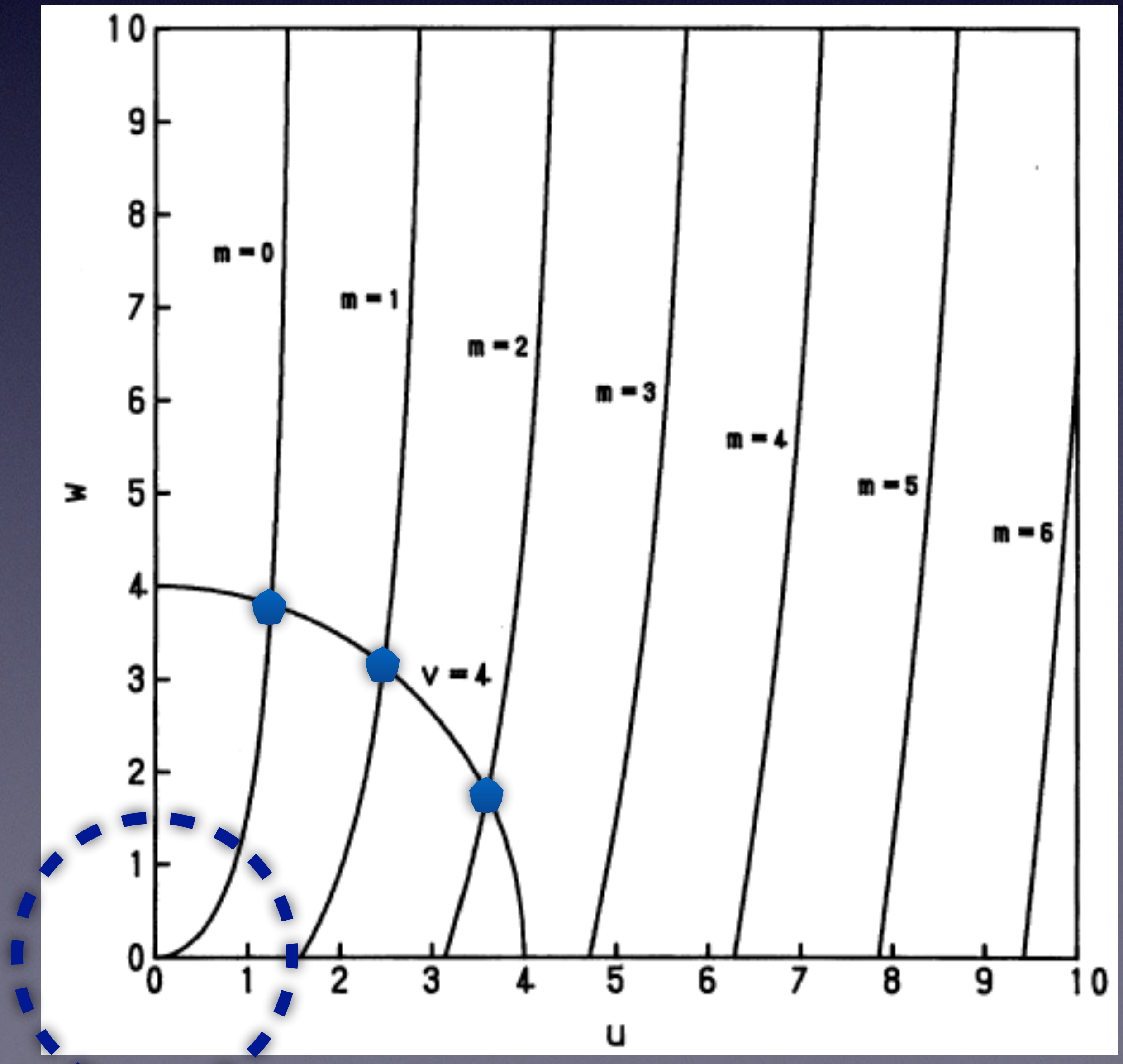

radius: cutoff normalized frequency

Computation of propagation constant: Using generalized parameters

generalized guide index $$b = \frac{n_e^2 - n_s^2}{n_1^2 - n_s^2}$$

asymmetry measure $$\gamma = \frac{n_s^2 - n_0^2}{n_1^2 - n_s^2}$$

generalized frequency $$k^2 a^2 (n_1^2 - n_s^2) \equiv v^2$$

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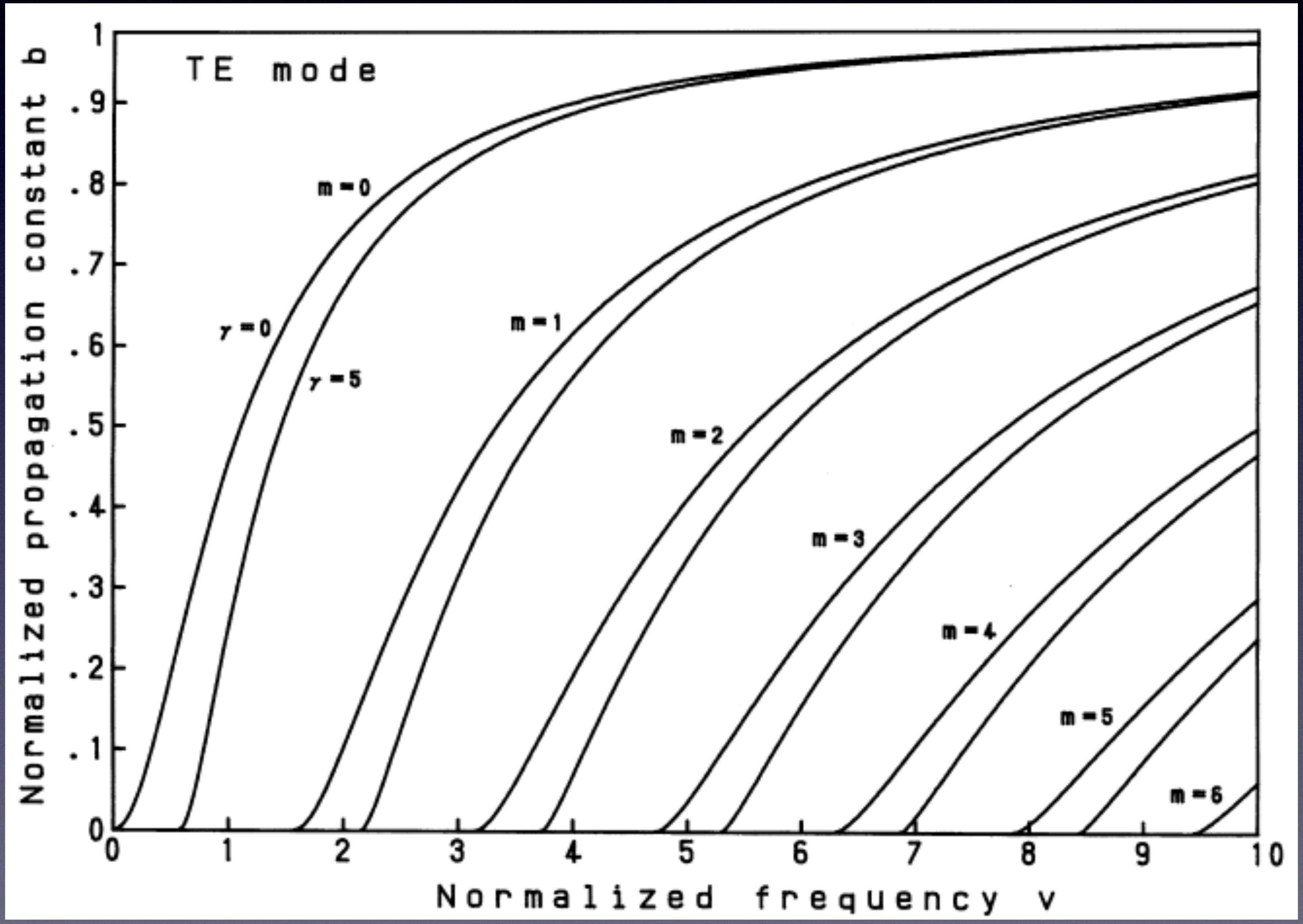
Dispersion relation for TE mode

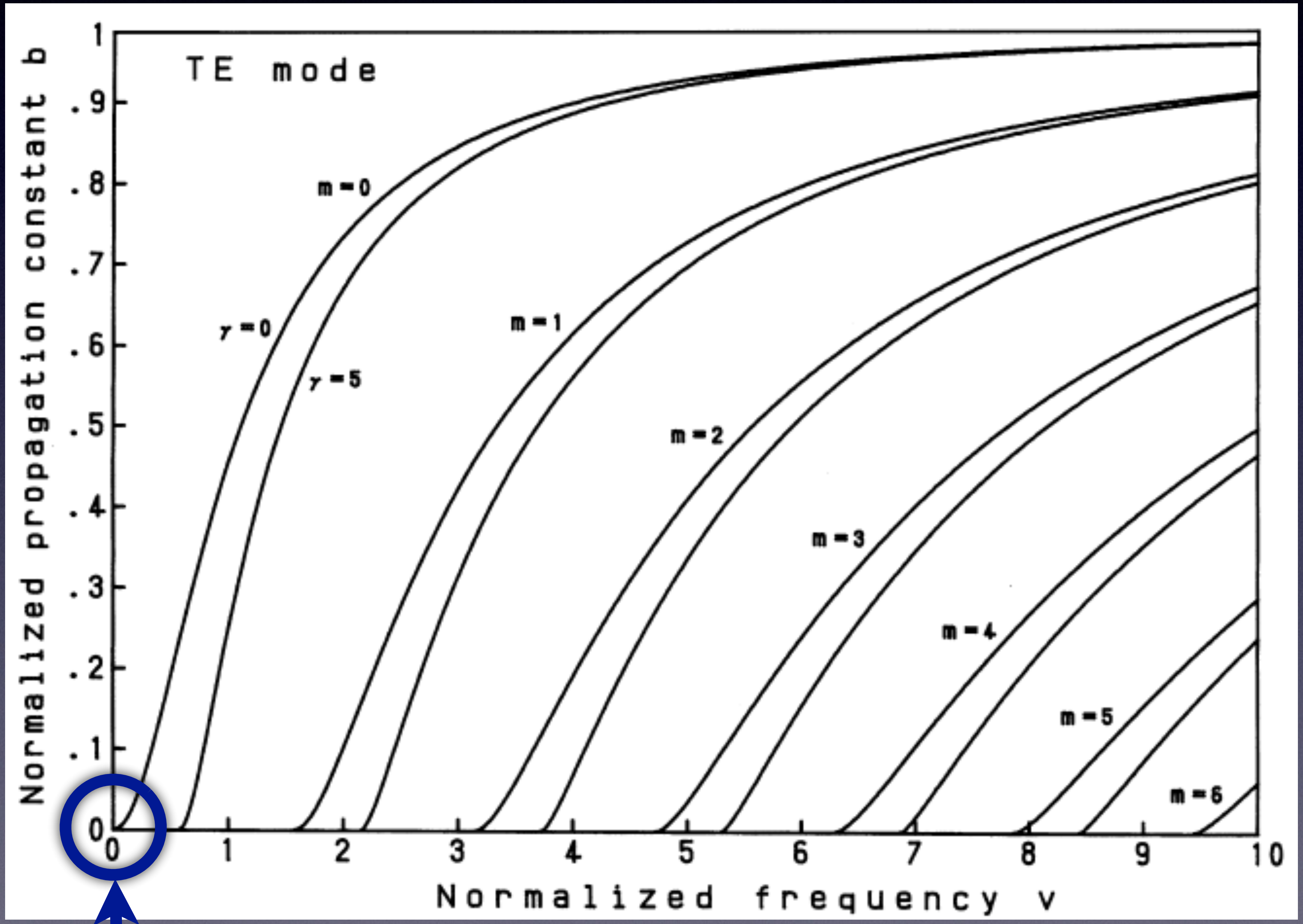
$$2v\sqrt{1-b} = m\pi + \tan^{-1}\sqrt{\frac{b}{1-b}} + \tan^{-1}\sqrt{\frac{b+\gamma}{1-b}}$$

The solution with a specific m gives the propagation constant of TE$_m$ mode

The bV diagram: TE

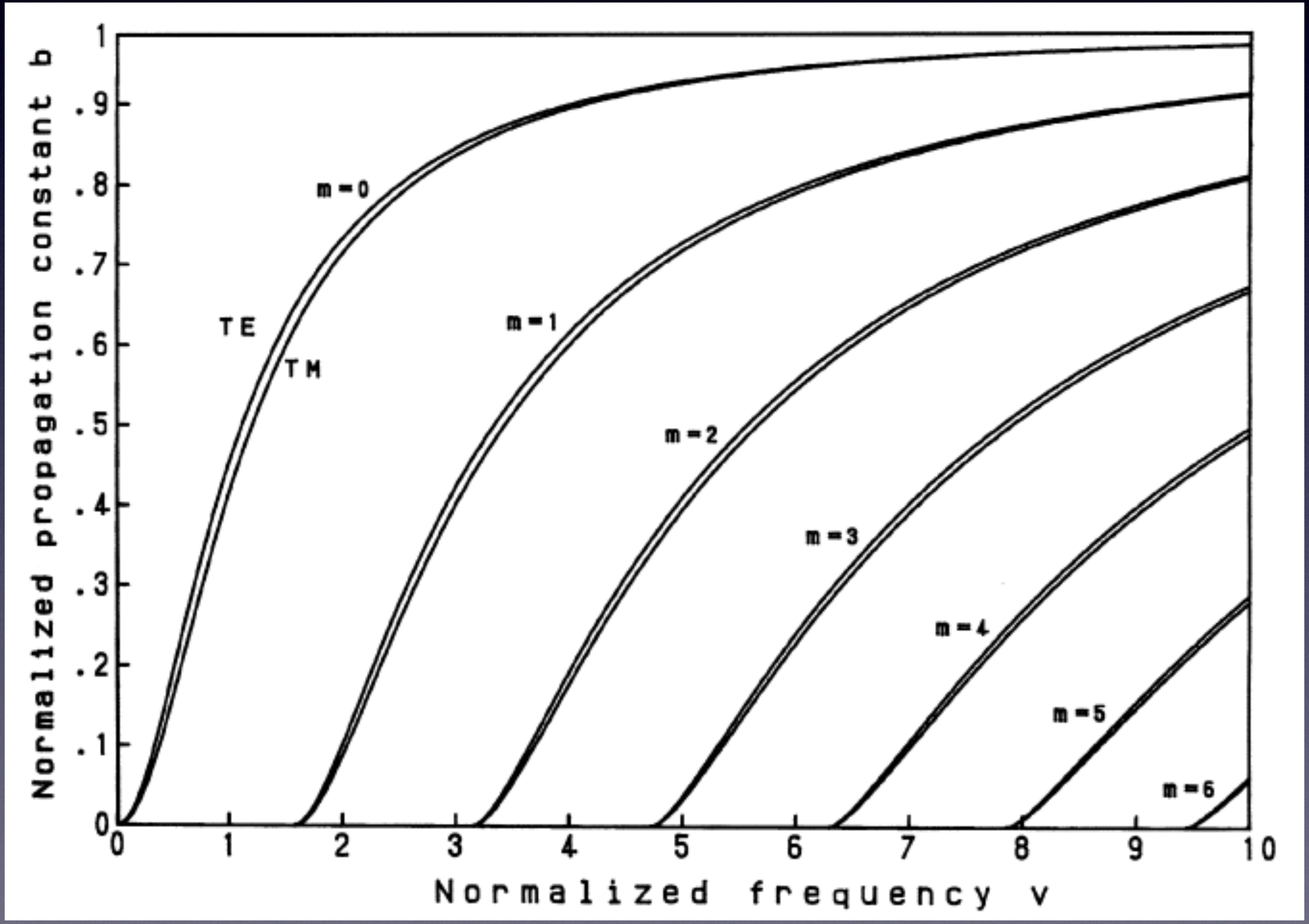


The bV diagram: TE



no cutoff for symmetric waveguide

Birefringence in slab waveguides



curves for TE and TM do not cross or intersect!!

Electric field distribution

$$P = \int_0^1 dy \int_{-\infty}^{\infty} \frac{1}{2}(\mathbf{E} \times \mathbf{H}^*) \cdot \mathrm{u}_z \, dx = \int_{-\infty}^{\infty} \frac{1}{2}(E_x H_y^* - E_y H_x^*) \, dx$$

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for TE modes

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$$E_x=E_z=H_y=0.$$

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$$P=\frac{\beta}{2\omega\mu_0}\int_{-\infty}^{\infty}|E_y|^2\,dx.$$

$$P_{\text{core}} = \frac{\beta a A^2}{2\omega\mu_0}\left\{1 + \frac{\sin^2(u+\phi)}{2w} + \frac{\sin^2(u-\phi)}{2w'}\right\} \quad (-a \leqslant x \leqslant a)$$

$$P_{\text{sub}} = \frac{\beta a A^2}{2\omega\mu_0}\frac{\cos^2(u+\phi)}{2w} \quad (x \leqslant -a)$$

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total power carried by the waveguide

$$P = P_{\text{core}} + P_{\text{sub}} + P_{\text{clad}} = \frac{\beta a A^2}{2\omega\mu_0}\left\{1 + \frac{1}{2w} + \frac{1}{2w'}\right\}$$

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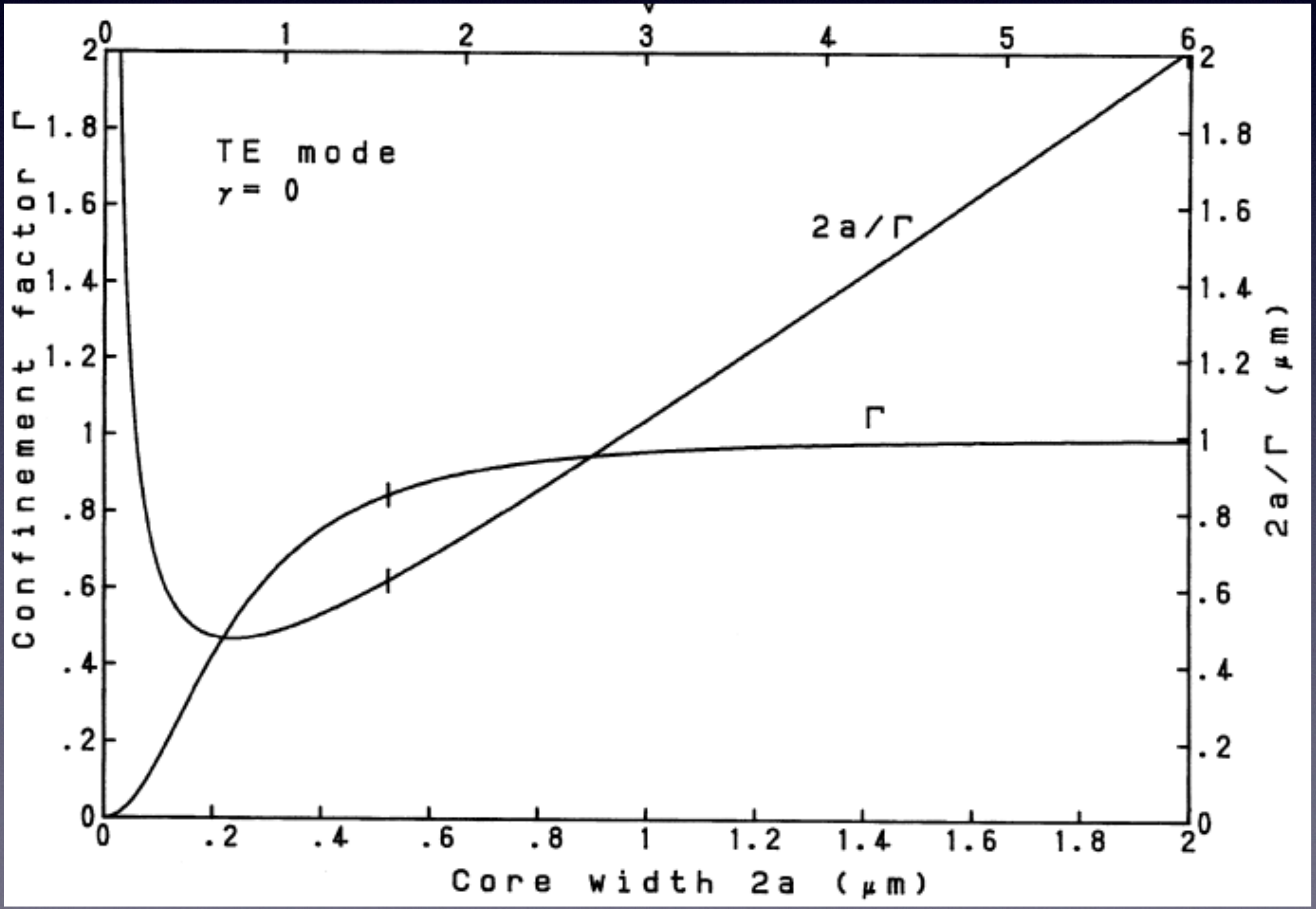
power confinement factor

$$\Gamma = \frac{P_{\text{core}}}{P} = \frac{1 + \dfrac{\sin^2(u+\phi)}{2w} + \dfrac{\sin^2(u-\phi)}{2w'}}{1 + \dfrac{1}{2w} + \dfrac{1}{2w'}}$$

Confinement factor

$$\Gamma_f = \frac{\text{Time-average power transported in the film region}}{\text{Total time-average power transported by the waveguide}}$$



Group velocity/phase velocity

Consider the propagation of a plane wave

$$\mathbf{E}(\omega, z) = \mathbf{E}_{+}(\omega) \exp\left[-jk(\omega)z\right]$$

whose Fourier amplitudes are appreciable only in a narrow band of frequencies around w_0

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whose Fourier amplitudes are appreciable only in a narrow band of frequencies around w_0

The time-dependent field is obtained by Fourier transform

$$\boldsymbol{E}(t, z) = \int_{-\infty}^{\infty} d\omega \exp\{j[\omega t - k(\omega)z]\}\mathbf{E}_+(\omega)$$

expand k in the neighborhood of w_0

$$k(\omega) = k(\omega_0) + \left.\frac{dk}{d\omega}\right|_{\omega_0} \Delta\omega$$

expand k in the neighborhood of w0

$$k(\omega) = k(\omega_0) + \left.\frac{dk}{d\omega}\right|_{\omega_0} \Delta\omega$$

$$\boldsymbol{E}(t, z) = e^{j[\omega_0 t - k(\omega_0)z]} \int_{\text{band}} \mathbf{E}_+(\Delta\omega) e^{j\Delta\omega[t - (dk/d\omega)z]} \, d\Delta\omega$$

$$+ \text{ complex conjugate}$$

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1. A rapidly varying term, the carrier, that propagates with the *phase velocity* $\omega_0/k(\omega_0)$
2. A slowly varying envelope that proceeds with the *group velocity* $d\omega/dk = 1/(dk/d\omega)$

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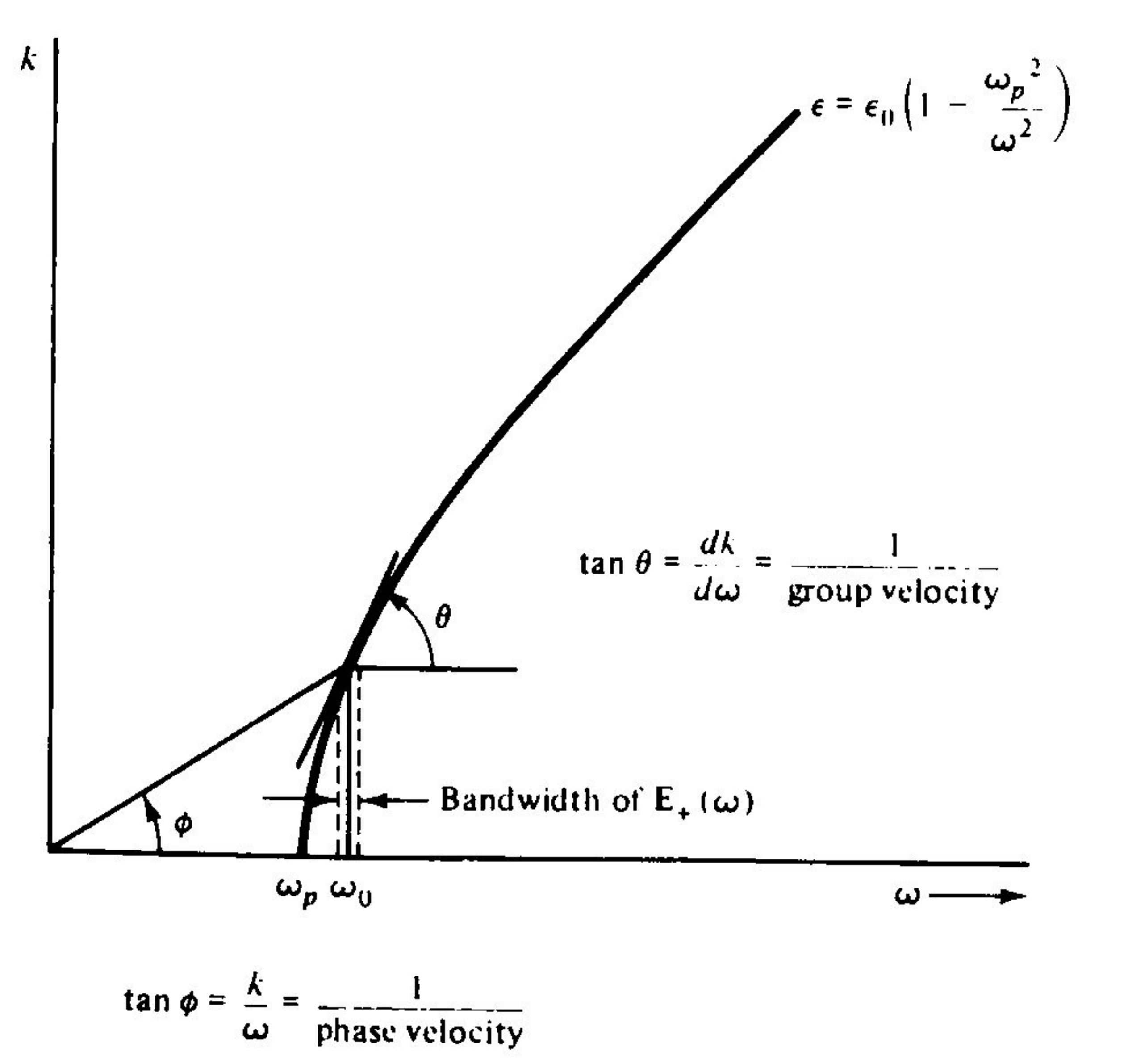
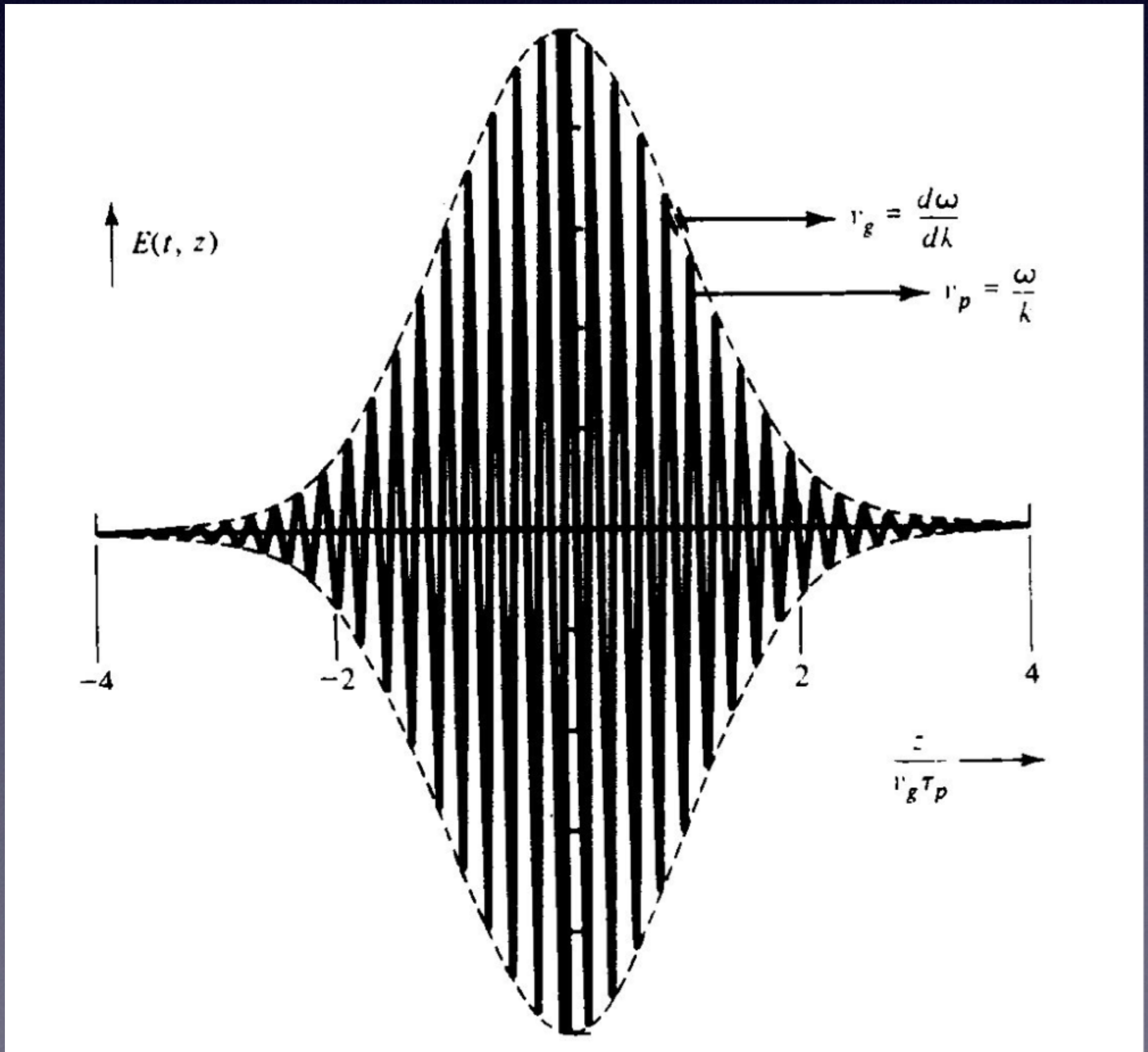
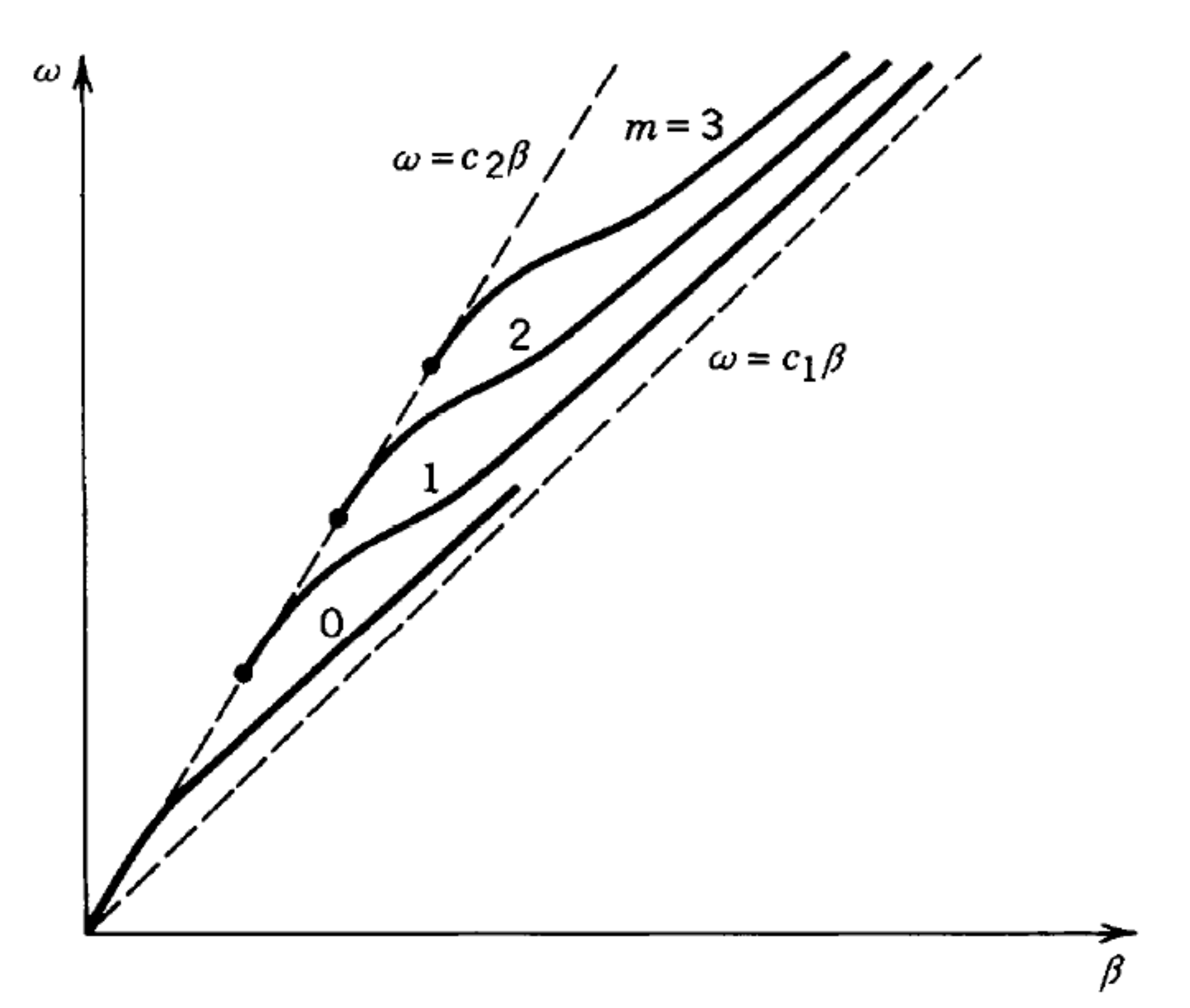




Group velocities

the group velocity $v = d\omega/d\beta$

for a symmetric slab waveguide



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