Probability

2.1 Intro to Discrete Probability

Joshua Fan
Alex Tsun

Agenda

- Definitions
- Axioms
- Equally Likely Outcomes

Definitions

Sample Space: The set Ω of all possible outcomes of an experiment.

- Single coin flip: $\Omega = \{H, T\}$
- Two coin flips: $\Omega = \{HH, HT, TH, TT\}$
- Roll of a die: $\Omega = \{1,2,3,4,5,6\}$

Event: Any subset $E \subseteq \Omega$.

- Getting at least one head in two coin flips: $E = \{HH, HT, TH\}$
- Rolling an even number: $E = \{2,4,6\}$

Mutually Exclusive: Events E and F are mutually exclusive if $E \cap F = \emptyset$ (i.e., they can't simultaneously happen).

- $E = \{2,4,6\}$ and $F = \{1,3\}$, then $E \cap F = \emptyset$.

Example: weird dice (Sample Space)

Suppose I roll two 4-sided dice. What's the sample space (set of possible outcomes)?

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Let D1 be the value of the blue die, and D2 the value of the red die.

What outcomes match these events?

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A. D1 = 1

B. D1 + D2 = 6

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Are A and B mutually exclusive?

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Example: weird dice (mutually exclusive)

Are A and B mutually exclusive?

Yes. $A \cap B = \emptyset$ (no overlap)

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Are B and C mutually exclusive?

No. B and C could happen at the same time (4, 2)

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Random Picture

Axioms of Probability & Their Consequences

Let Ω denote the sample space and $E, F \subseteq \Omega$ be events.

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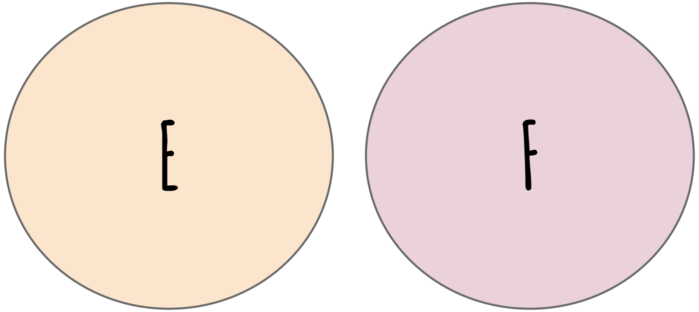

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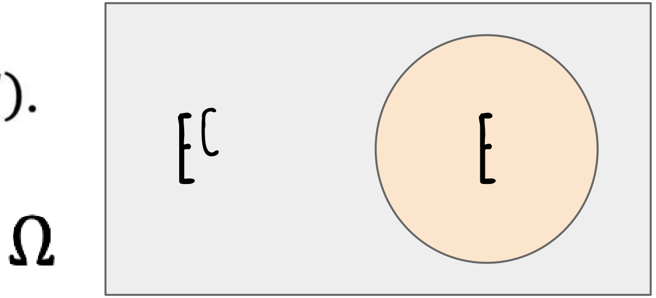

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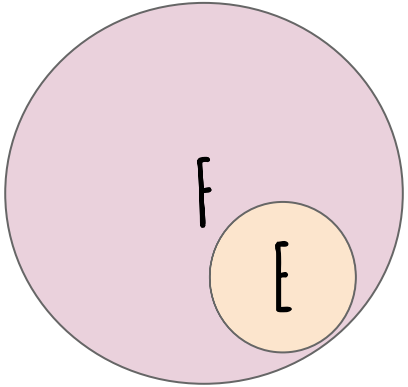

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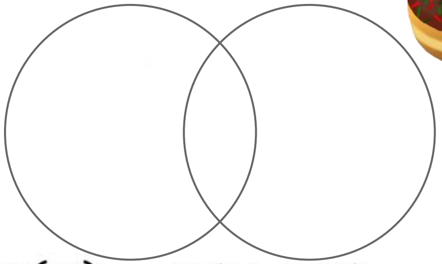


Corollary 1 (Complementation): $P(E^C) = 1 - P(E)$.
Corollary 2 (Monotonicity): If $E \subseteq F$, $P(E) \leq P(F)$.
Corollary 3 (Inclusion-Exclusion): $P(E \cup F) = P(E) + P(F) - P(E \cap F)$.

Example: weird dice (Events)

Think back to the 4-sided dice. Suppose each die is fair.

Intuitively, what is the probability that the two dice sum to 6? $(D1 + D2 = 6)$

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Each of the 16 outcomes is equally likely.

3/16.

Equally Likely Outcomes

If Ω is such that outcomes are equally likely, then for any event $E \subseteq \Omega$,

$$P(E) = \frac{|E|}{|\Omega|}$$

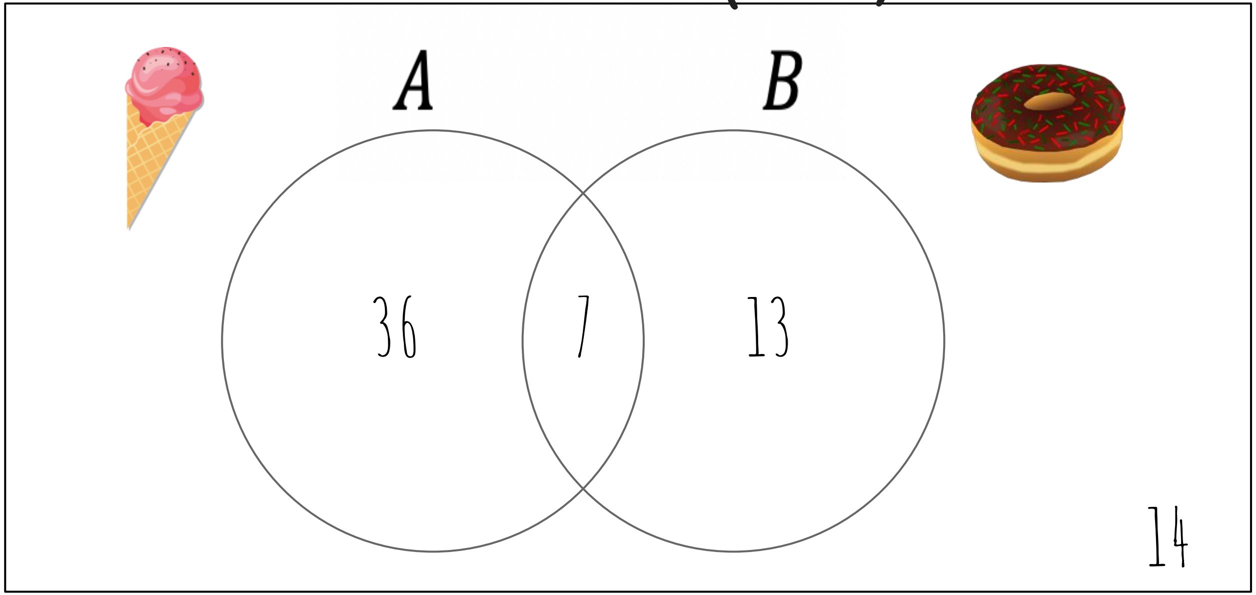
Probability
2.2 Conditional Probability

Alex Tsun

Agenda

- conditional probability
- Bayes Theorem
- Law of Total Probability (LTP)
- Bayes Theorem + LTP

Conditional Probability (idea)



What's the probability that someone likes ice cream **given** they like donuts?

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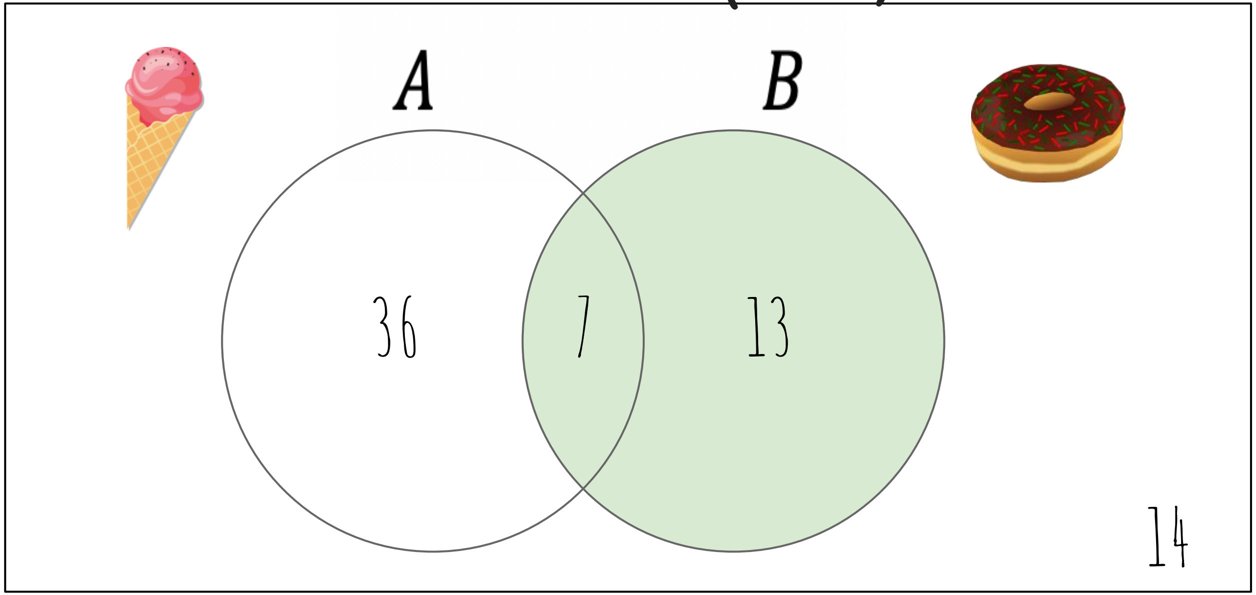


What's the probability that someone likes ice cream **given** they like donuts?

$$P(A|B) = \frac{7}{20} = \frac{|A \cap B|}{|B|} = \frac{|A \cap B|/|\Omega|}{|B|/|\Omega|} = \frac{P(A \cap B)}{P(B)}$$

Conditional Probability

<u>Conditional Probability</u>: The (conditional) probability of A given an event B happened is

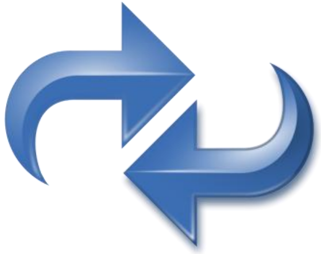
$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

An equivalent and useful formula is $P(A \cap B) = P(A|B)P(B)$.

Conditional Probability (Reversal)

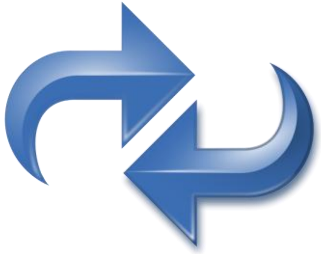
Does $P(A|B) = P(B|A)$?

Conditional Probability (Reversal)



Does $P(A|B) = P(B|A)$? No!!

Conditional Probability (Reversal)



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Let A be the event you are wet.
Let B be the event you are swimming.

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Does $P(A|B) = P(B|A)$? No!!

Let A be the event you are wet.
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$$P(A|B) = 1$$

$$P(B|A) \neq 1$$

Bayes Theorem

Bayes Theorem: Let A, B be events with nonzero probability. Then,

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Allows us to "reverse" the conditioning!

$P(A)$ is called the **prior** (our belief without knowing anything), and
$P(A|B)$ is called the **posterior** (our belief after learning B).

Bayes Theorem (Proof)

By definition of conditional probability,

$$P(A \cap B) = P(A|B)P(B)$$

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But $P(A \cap B) = P(B \cap A)$, so

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But $P(A \cap B) = P(B \cap A)$, so

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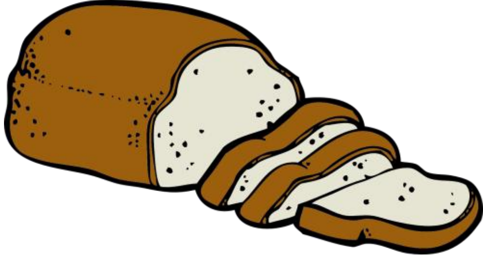
Dividing both sides by $P(B)$ gives

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

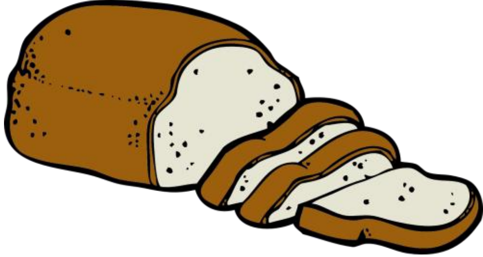
Random Picture

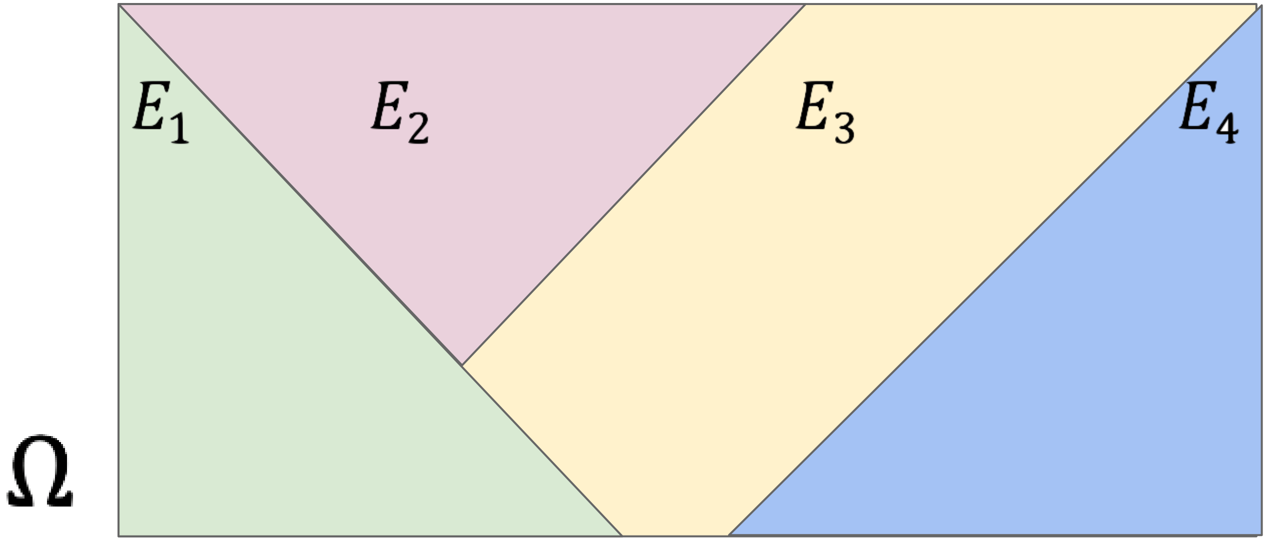
Cutting up a Sample Space

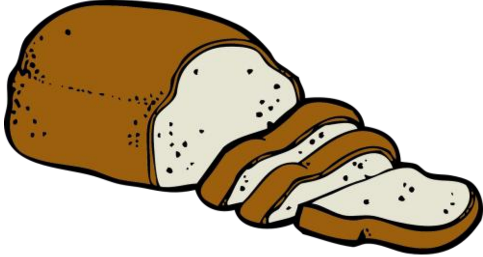


Ω

Cutting up a Sample Space

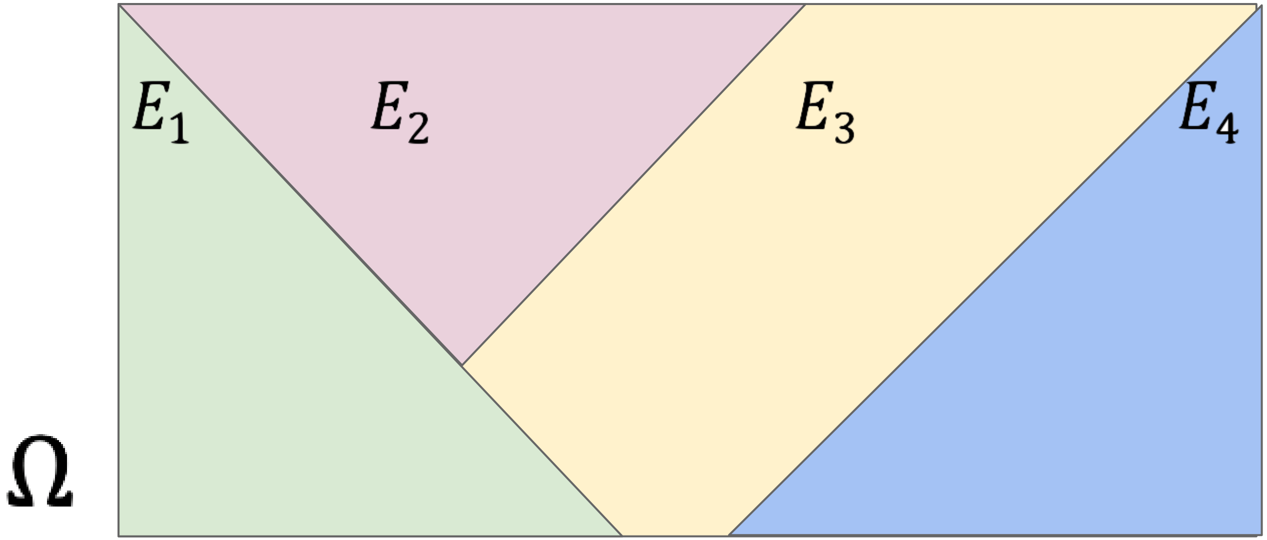






Cutting up a Sample Space

What single English word would you use to describe what the following events do to the sample space?



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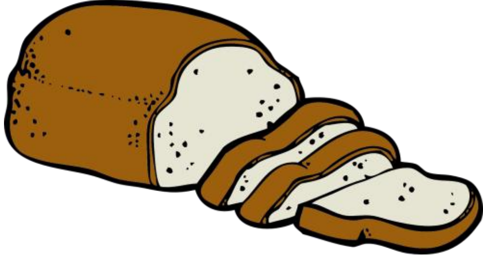
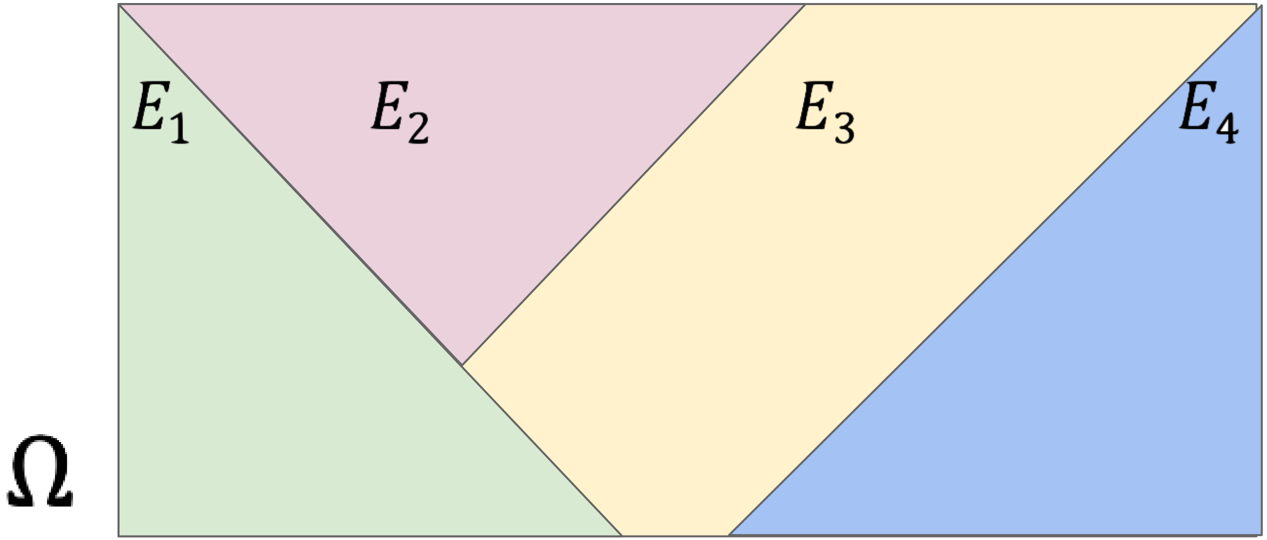


What single English word would you use to describe what the following events do to the sample space? PARTITION



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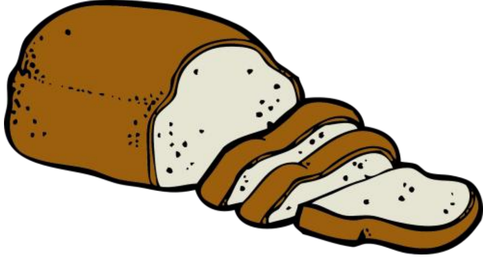
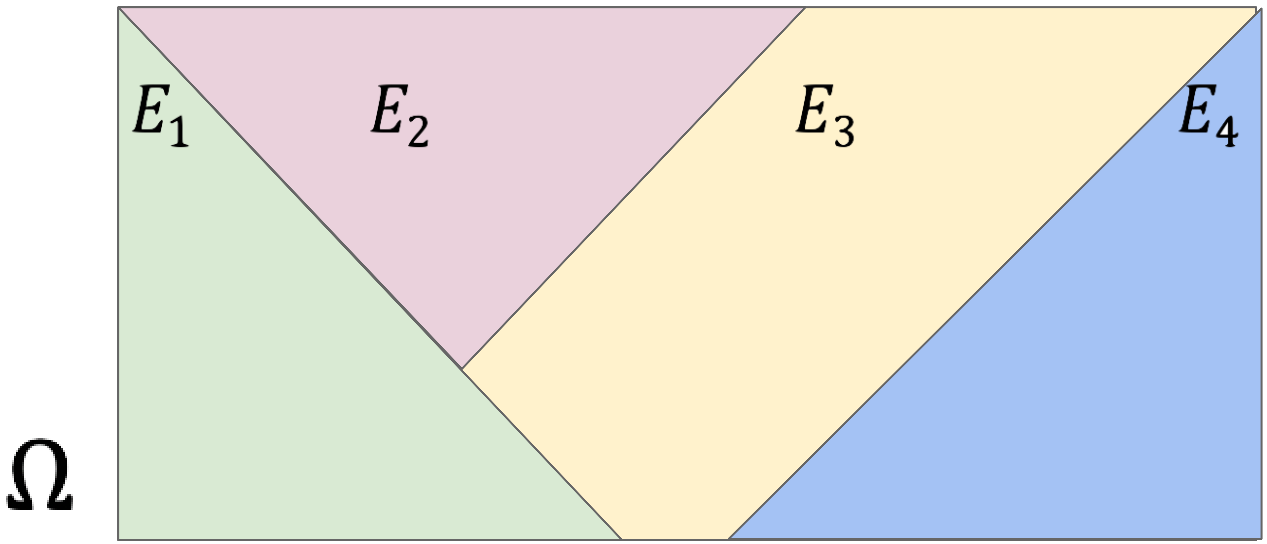


What are two properties of the "partition"?



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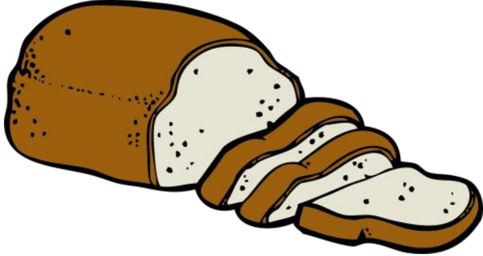
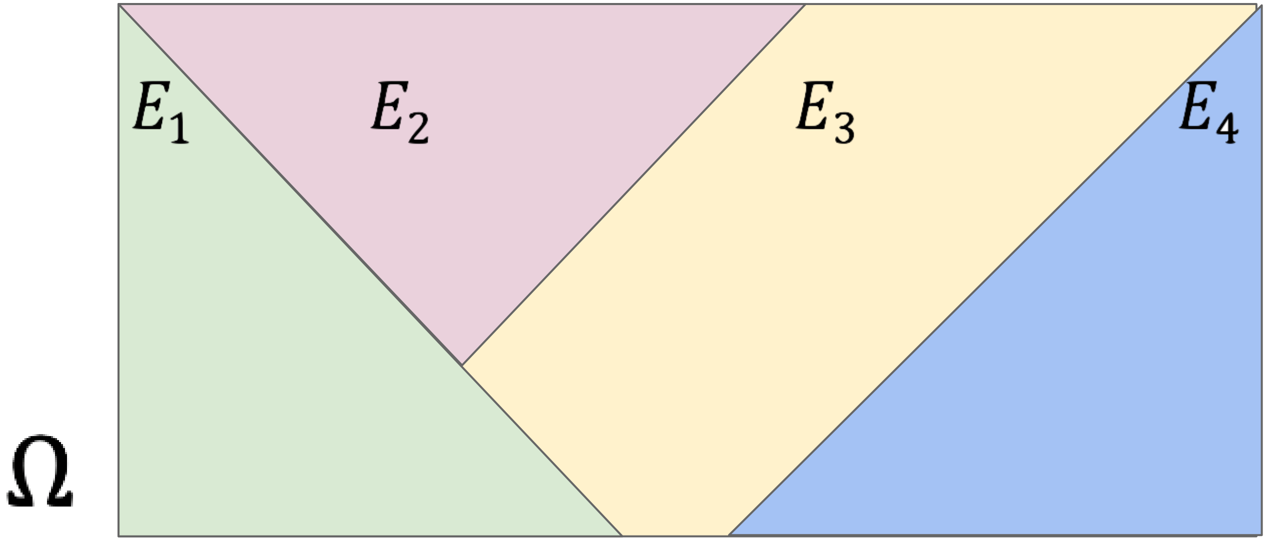


What are two properties of the "partition"?

1. They "cover" the whole space.



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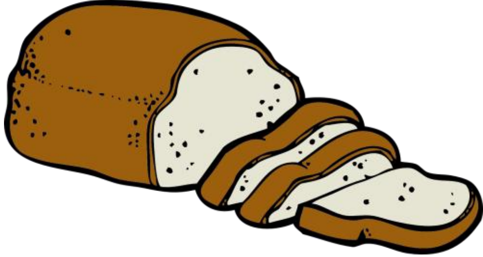
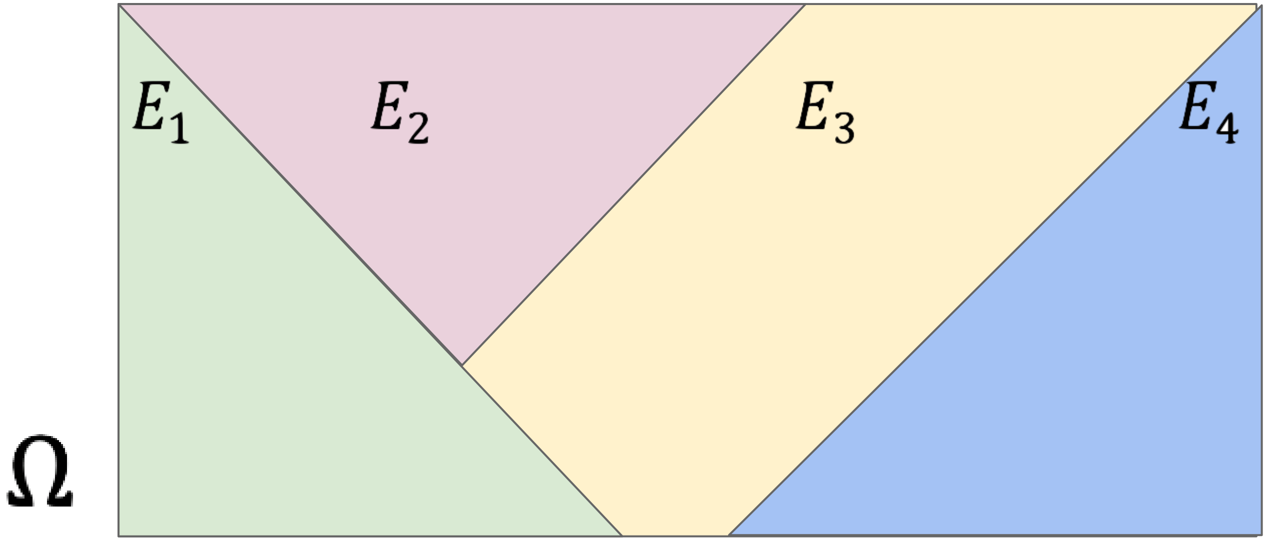


What are two properties of the "partition"?

1. They "cover" the whole space.

2. They don't overlap.



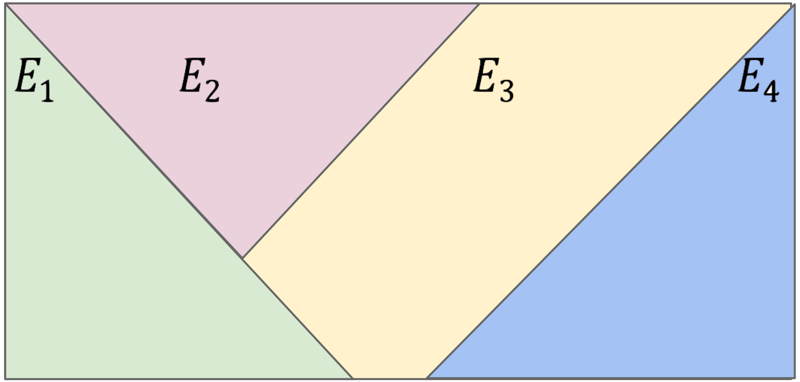
Partitions

<u>Partition:</u> Non-empty events $E_1, \ldots, E_n$ partition the sample space Ω if

- (**<u>Exhaustive</u>**) $E_1 \cup E_2 \cup \ldots \cup E_n = \bigcup_{i=1}^{n} E_i = \Omega$.
- (**<u>Pairwise Mutually Exclusive</u>**) For all $i \neq j$, $E_i \cap E_j = \emptyset$.

Notice for any event E: E and E^C always partition Ω.

Ω



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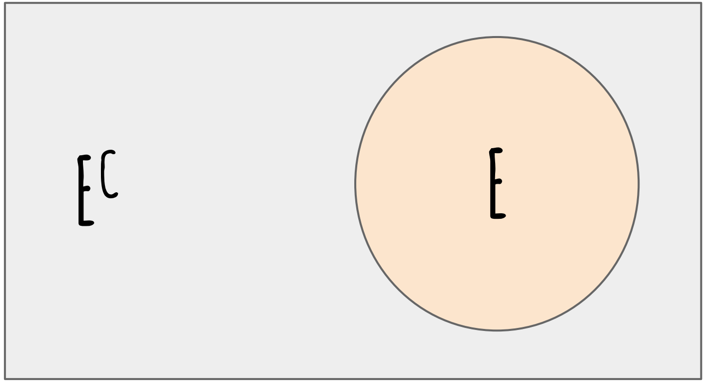
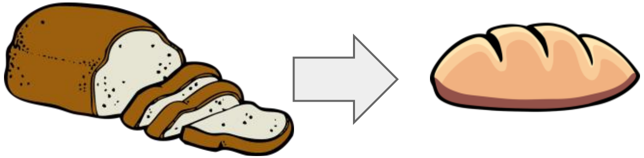


(The Picture) Law of total probability



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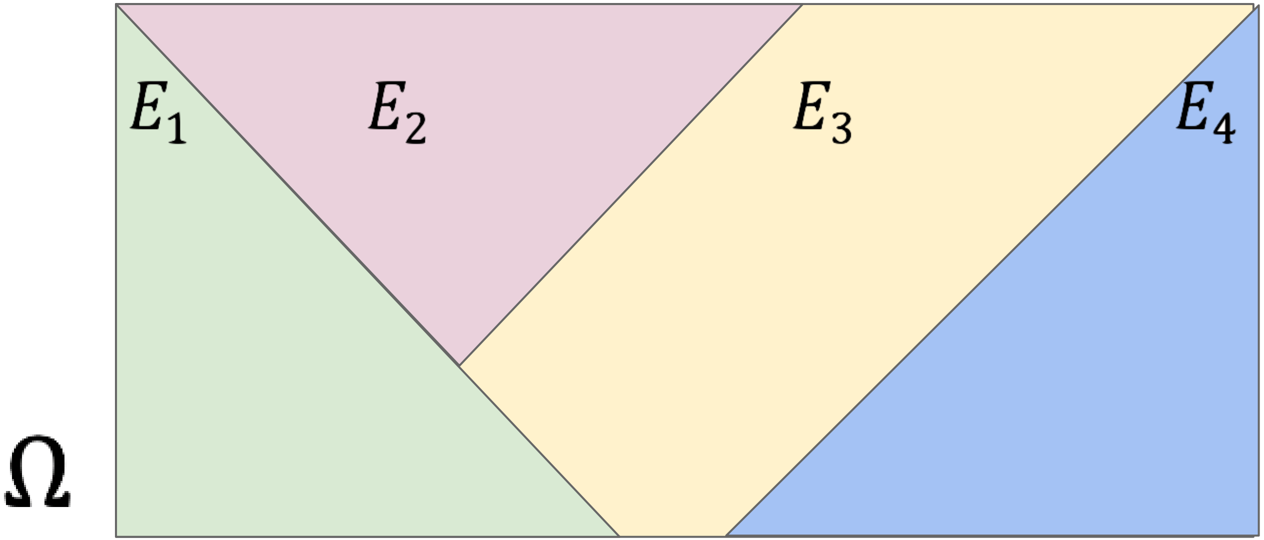

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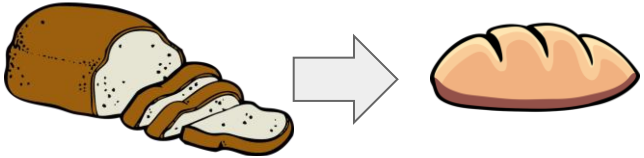
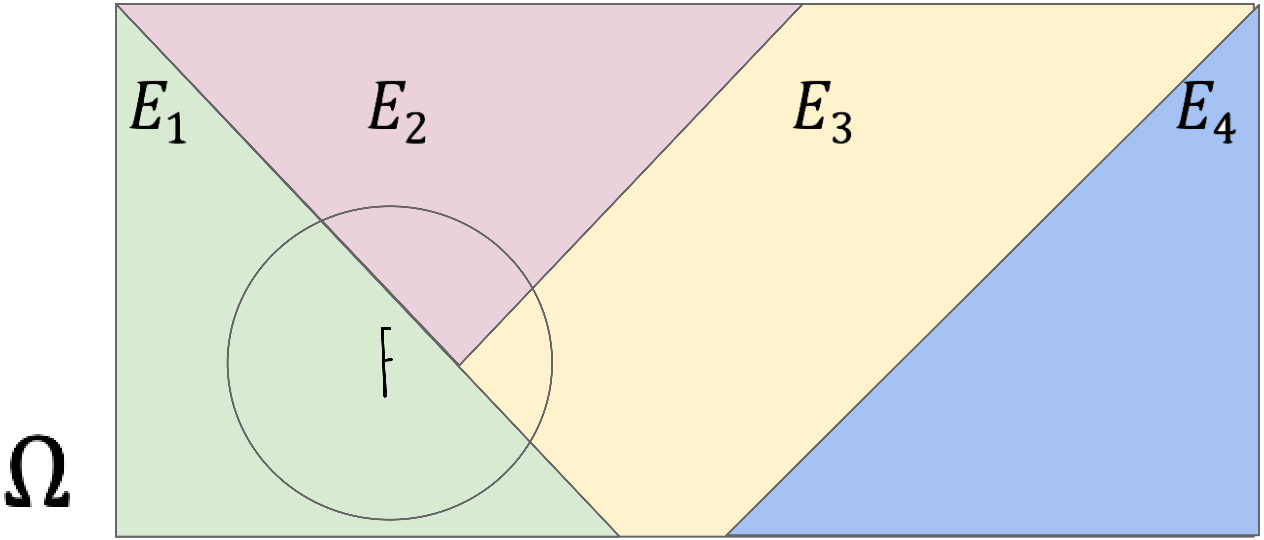


Back to the old picture. How can we decompose event F?

$$P(F) =$$



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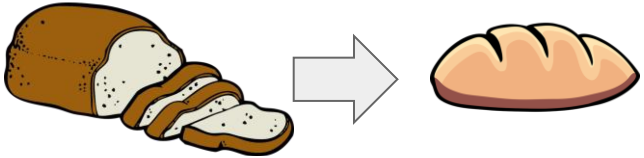
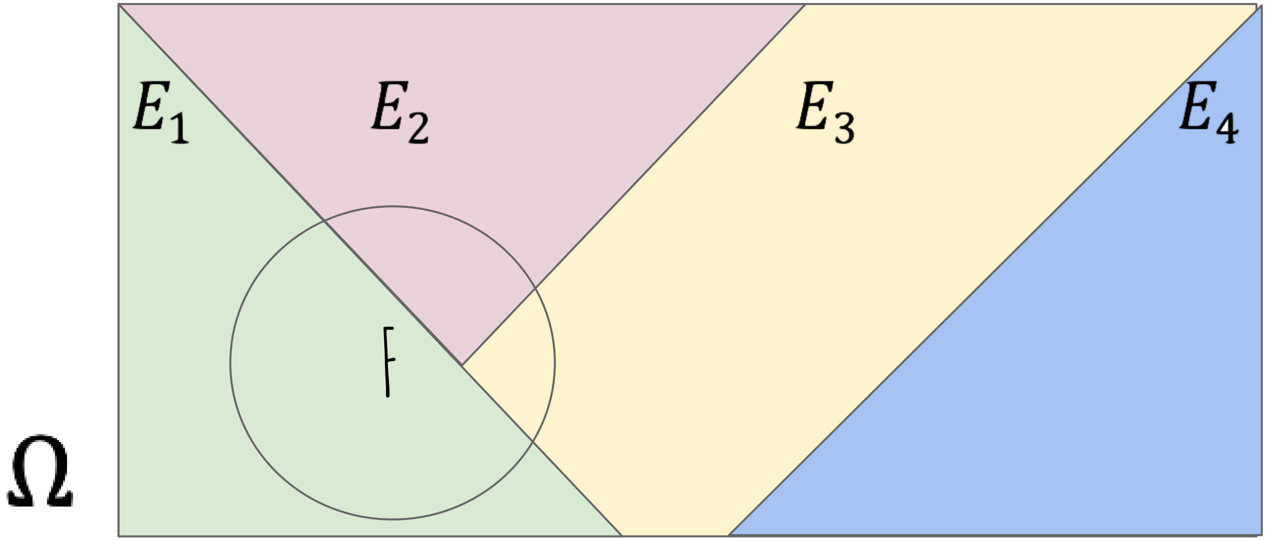


Back to the old picture. How can we decompose event F?

$$P(F) = P(F \cap E_1) + P(F \cap E_2) + P(F \cap E_3) + P(F \cap E_4)$$

For Completion



Law of Total Probability (LTP)

<u>Law of Total Probability</u>: If events $E_1, \ldots, E_n$ partition Ω, then for any event F,

$$P(F) = P(F \cap E_1) + \cdots + P(F \cap E_n) = \sum_{i=1}^{n} P(F \cap E_i)$$

Using the definition of conditional probability ($P(F \cap E_i) = P(F|E_i)P(E_i)$), we get an alternate (more useful) form

$$P(F) = P(F|E_1)P(E_1) + \cdots + P(F|E_n)P(E_n) = \sum_{i=1}^{n} P(F|E_i)P(E_i)$$

Intuition (LTP)

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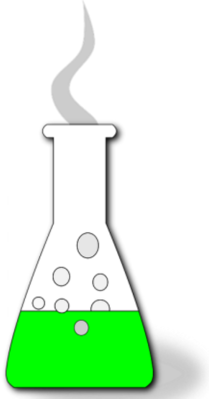

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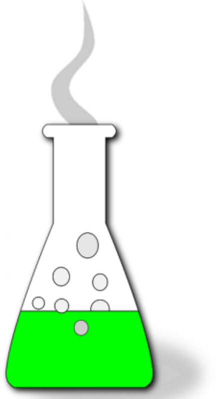

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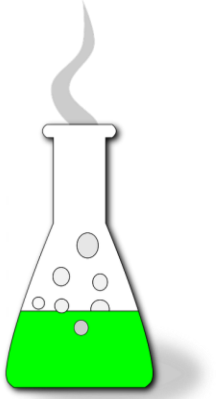

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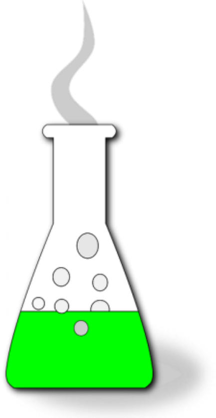

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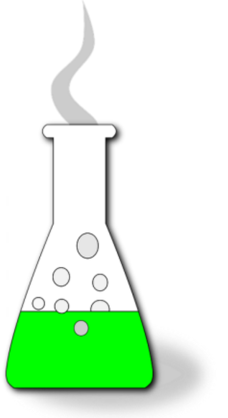

- In chemistry class, you want to know the probability you fail.
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- First, compute the probability of failing in each of the 3 cases.

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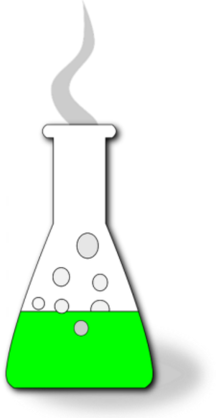
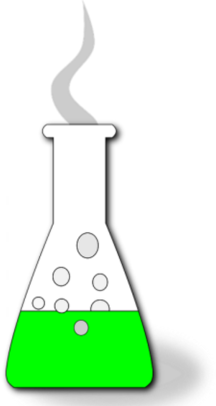


- In chemistry class, you want to know the probability you fail.
- But you are randomly assigned one of 3 teachers. What to do?
- First, compute the probability of failing in each of the 3 cases.
- Then, weight those by the probability of getting that teacher.

Example (LTP)



$$P(F) = P(F|E_1)P(E_1) + \cdots + P(F|E_n)P(E_n) = \sum_{i=1}^{n} P(F|E_i)P(E_i)$$

	Mrs. Mean (E_1)	Mr. Nice (E_2)	Ms. IDC (E_3)
Probability of Teaching	6/8	1/8	1/8
Probability of Failing You			

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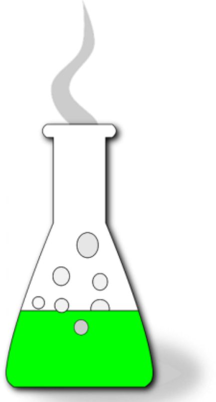

	Mrs. Mean (E_1)	Mr. Nice (E_2)	Ms. IDC (E_3)
Probability of Teaching	6/8	1/8	1/8
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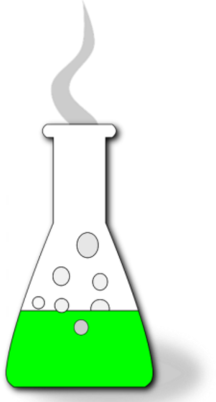

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Probability of Teaching	6/8	1/8	1/8
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How are you liking your chances???

Example (Reversal)

	Mrs. Mean (E_1)	Mr. Nice (E_2)	Ms. IDC (E_3)
Probability of Teaching	6/8	1/8	1/8
Probability of Failing You	1	0	1/2

$$P(F) = P(F|E_1)P(E_1) + P(F|E_2)P(E_2) + P(F|E_3)P(E_3) = 1 \cdot \frac{6}{8} + 0 \cdot \frac{1}{8} + \frac{1}{2} \cdot \frac{1}{8} = \frac{13}{16}$$

What's the probability that you had Ms. IDC, given that you failed?

(High or Low)?

Example (Reversal)

	Ms. IDC (E_3)
Probability of Teaching	1/8
Probability of Failing You	1/2

$$P(E_3|F) =$$

$$P(F) = P(F|E_1)P(E_1) + P(F|E_2)P(E_2) + P(F|E_3)P(E_3) = 1 \cdot \frac{6}{8} + 0 \cdot \frac{1}{8} + \frac{1}{2} \cdot \frac{1}{8} = \frac{13}{16}$$

What's the probability that you had Ms. IDC, given that you failed?

Bayes Theorem with Law of Total Probability

Bayes Theorem with LTP: Let $E_1, \ldots, E_n$ be a partition of the sample space, and F an event. Then,

$$P(E_1|F) = \frac{P(F|E_1)P(E_1)}{P(F)} = \frac{P(F|E_1)P(E_1)}{\sum_{i=1}^{n} P(F|E_i)P(E_i)}$$

(Simple Partition) In particular, if E is an event with nonzero probability, then

$$P(E|F) = \frac{P(F|E)P(E)}{P(F)} = \frac{P(F|E)P(E)}{P(F|E)P(E) + P(F|E^C)P(E^C)}$$

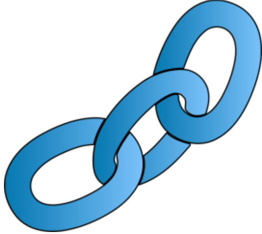
Probability

2.3 Independence

Agenda

- Chain Rule
- Independence
- Conditional Independence

Chain Rule (Idea)



Have a Standard 52-Card Deck.

- 4 Suits (Clubs, Diamonds, Hearts, Spades)
- 13 ranks (A, 2, 3, …, 9, 10, J, Q, K)

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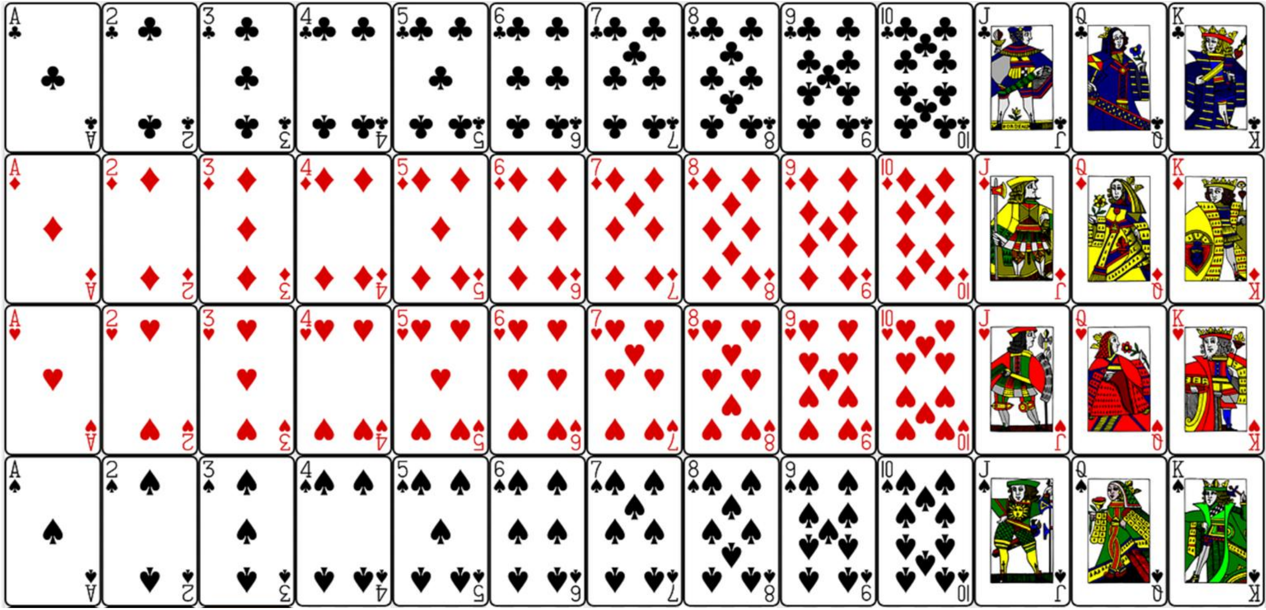
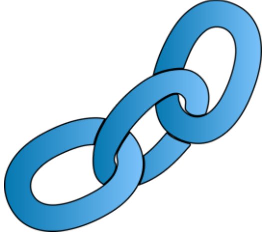
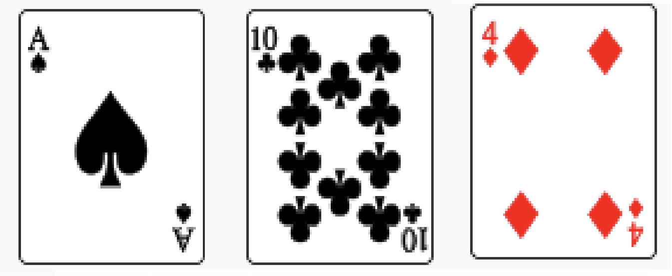




Chain Rule (Idea)

Have a standard 52-card deck. Shuffle it, and draw the top 3 cards.

Forget everything you've learned so far about probability.

What is P () = P(A, B, C)?

A: Ace of Spades First

B: 10 of Clubs Second

C: 4 of Diamonds Third

$$\frac{1}{52} \cdot \frac{1}{51} \cdot \frac{1}{50}$$

$$P(A) \cdot P(B|A) \cdot P(C|A,B)$$

Chain Rule

Chain Rule: Let $A_1, \dots, A_n$ be events with nonzero probability. Then,

$$P(A_1, \dots, A_n) = P(A_1)P(A_2|A_1)P(A_3|A_1, A_2) \dots P(A_n|A_1, \dots, A_{n-1})$$

In the case of two events A, B,

$$P(A, B) = P(A)P(B|A)$$

An easy way to remember this formula: we need to do n tasks, so we can perform them one at a time, conditioning on what we've done so far.

The need for independence

Quick question: In general, is

$$P(A,B) = P(A)P(B)?$$

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$$P(A, B) = P(A)P(B)?$$

The chain rule says

$$P(A, B) = P(A)P(B|A)$$

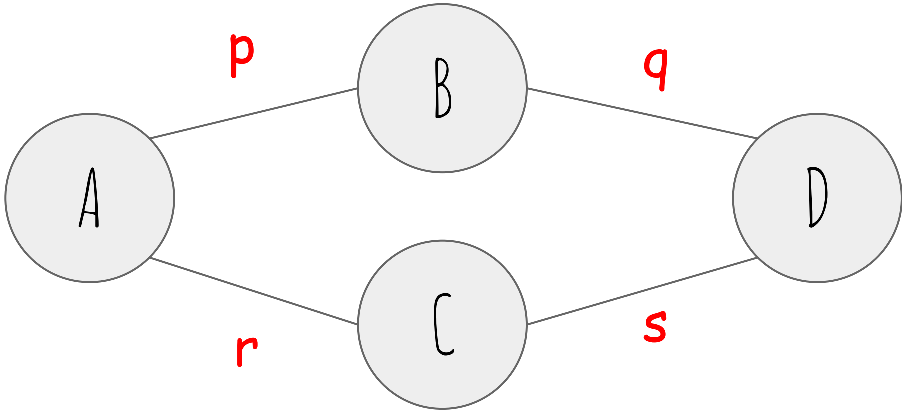
So no, unless the special case when $P(B|A) = P(B)$. This case is so important it has a name.

Independence

<u>Independence:</u> Events A, B are independent if any of the three equivalent conditions hold:

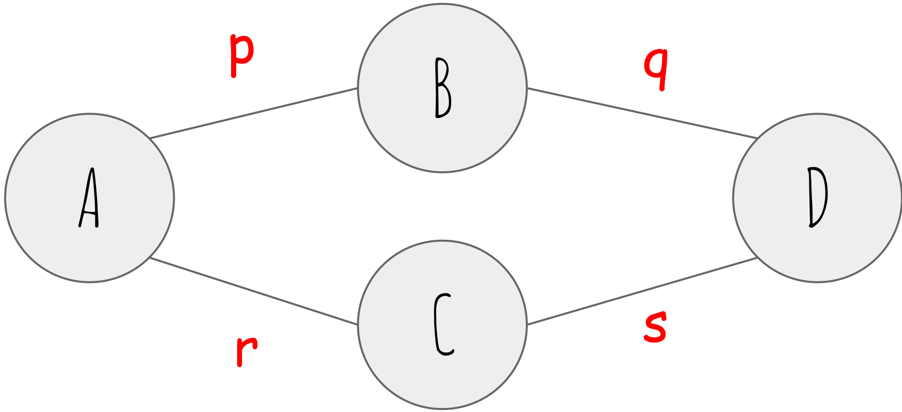
1. $P(A|B) = P(A)$
2. $P(B|A) = P(B)$
3. $P(A, B) = P(A)P(B)$

Network Communication



Network Communication

Each link works with the probability given, **independently**. What's the probability A and D can communicate?



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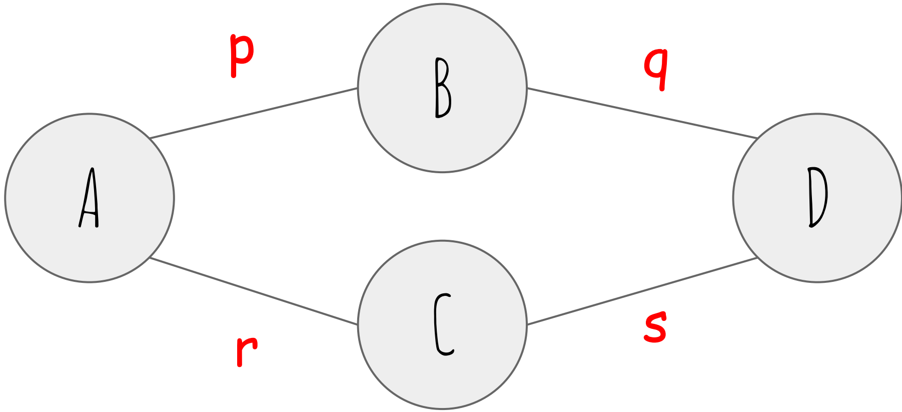

$$P(top) = P(AB \cap BD) =$$

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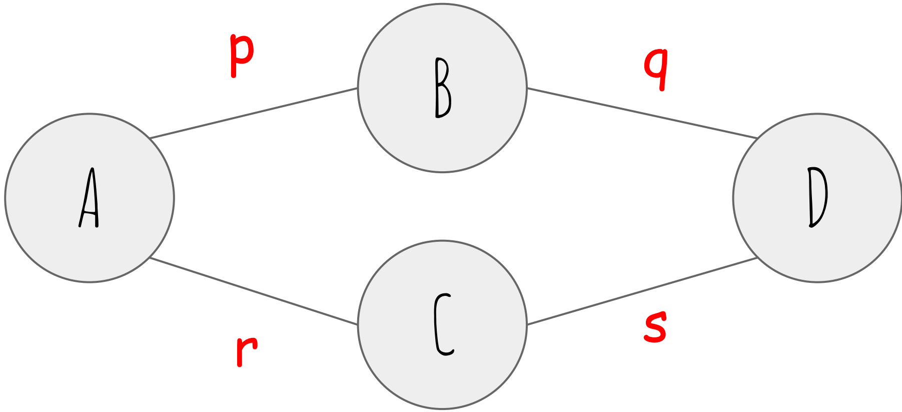

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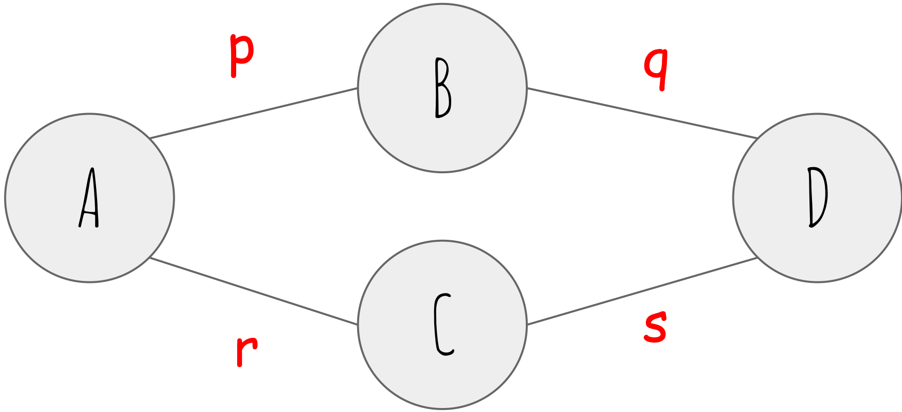

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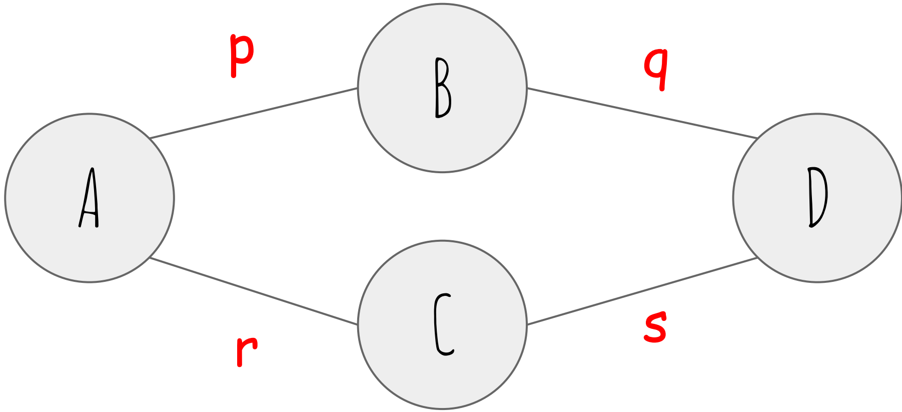
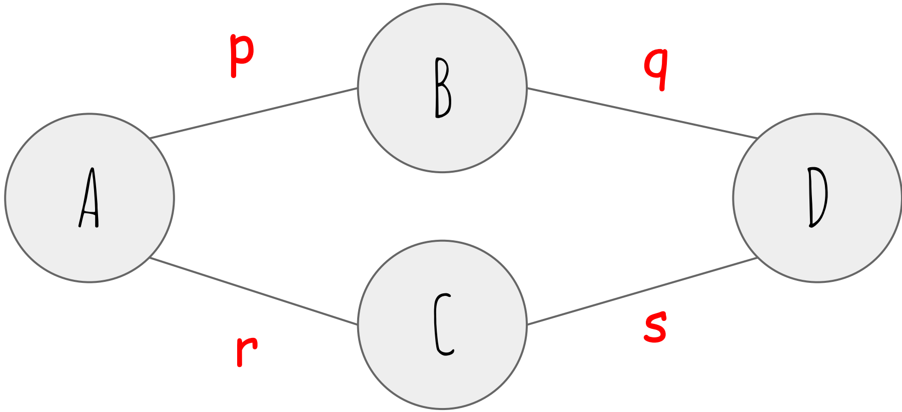


$$P(top) = P(AB \cap BD) = P(AB)P(BD) = pq$$

$$P(bottom) = P(AC \cap CD) =$$

Network Communication



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Independence

Network Communication

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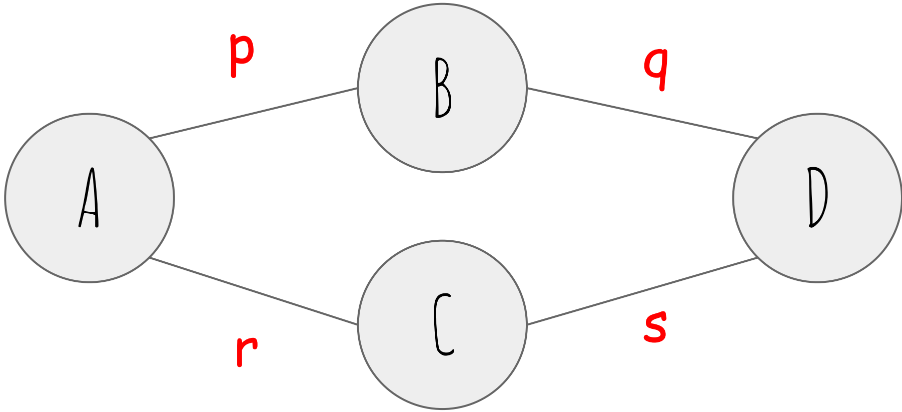

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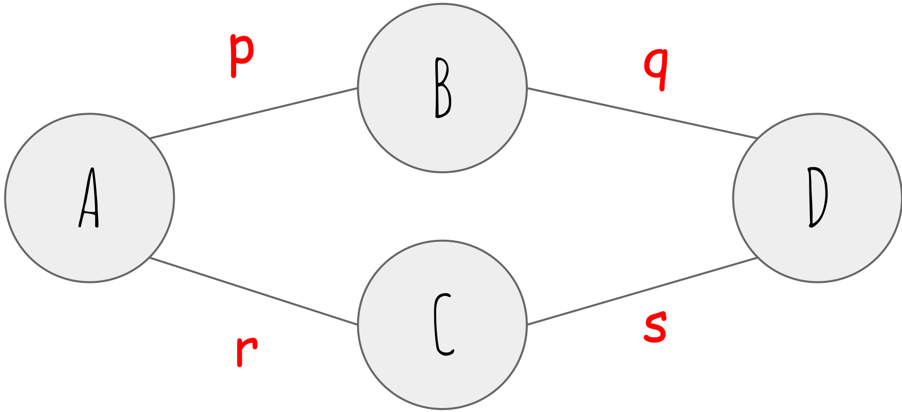
$$P(top \cup bottom) = P(top) + P(bottom) - P(top \cap bottom)$$

Network Communication



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$$= P(top) + P(bottom) - P(top)P(bottom)$$

Independence

Network Communication

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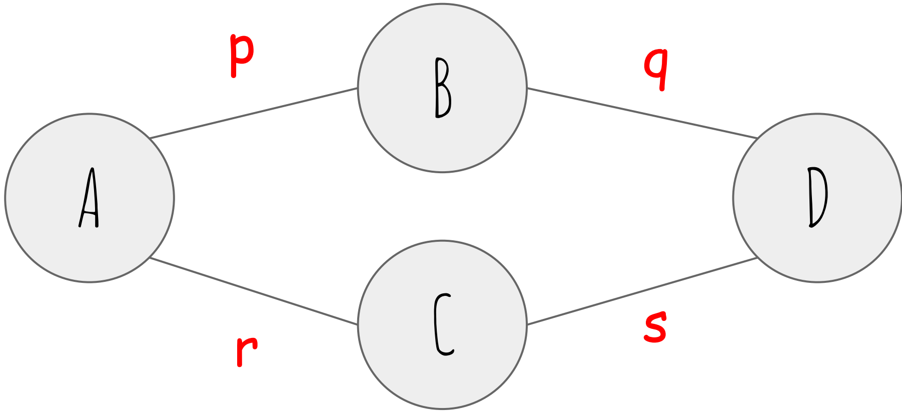

$$P(top) = P(AB \cap BD) = P(AB)P(BD) = pq$$
$$P(bottom) = P(AC \cap CD) = P(AC)P(CD) = rs$$

$$\begin{aligned} P(top \cup bottom) &= P(top) + P(bottom) - P(top \cap bottom) \\ &= P(top) + P(bottom) - P(top)P(bottom) \\ &= pq + rs - pqrs \end{aligned}$$

Conditional Independence

Conditional Independence: Events A, B are (conditionally) independent given C if any of the three equivalent conditions hold:

1. $P(A|B, C) = P(A|C)$
2. $P(B|A, C) = P(B|C)$
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Coin Flipping

Suppose there is a coin C_1 with $P(head) = 0.3$ and a coin C_2 with $P(head) = 0.9$. We pick one randomly with equal probability and flip that coin 3 times independently. What is the probability we get all heads?