CMSE 802: On the Importance on Partial Differential Equations in Simulating Ultrasound in Nonhomogeneous Fields

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My simulation environment for waves in a nonhomogeneous field will be used in the future to aid in medical research by modeling shear waves movement through the human body. Partial differential equations are critical to describing the motion of waves because a partial differential model can consider changing in acceleration, velocity, position, and time.

Example of the General Wave Equation: $\frac{\partial^2}{\partial t^2} \psi(x,t) = c^2 \frac{\partial^2}{\partial x^2} \psi(x,t)$

In a nonhomogeneous field, a wave will be accelerating due the change in the velocity because of the boundaries between different substances and also from losses of energy in the wave to the environment in travel (but in reality these losses are usually very small and we tend to disregard them). With a partial differential equation form of the wave equation, my simulation can use finite volume or finite difference methods to create more efficient ways to simulate; hopefully, this process overcomes the large amounts of data points that need to be updated to simulate a human body in enough detail.

Simulation is very important to predict the nature of how a potential patient will receive the treatment and help evaluate the benefits and risks on site. The cheapest way to model the human body without clinical trials or computer simulation are tissue phantoms, a manufactured gel similar to ballistics gel which is limited in the detail provided. Ultrasound has been limited to the scope of imaging soft tissue like a fetus in the womb or breast tissue because ultrasound has been tested for the majority of its existence on soft tissue that is usually pretty uniform to spot irregularities that a human eye can see. With computer simulation we also have a way to possibly simulated a whole human body down to the capillaries to test in areas that have not been tested yet due to the limitation of tissue phantoms.
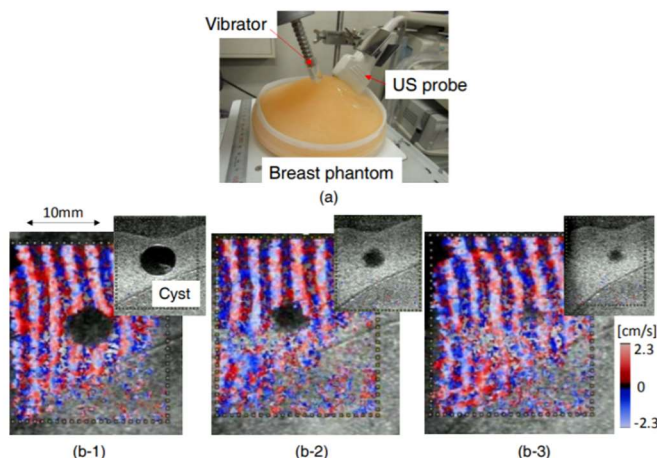

a. example of a tissue phantom

b. imagery of ultrasound moving through the phantom (note: this phantom has complexity because of the model of a cyst on itself)

(images are from [1])

An emerging field that can benefit from this simulation tool is ultrasound therapy. Histotripsy, a new technique in ultrasound therapy, is a process of using ultrasound pulses to destroy cells in the body. A microwave uses a similar concept; everything has a resonant frequency by which waves can transfer energy to a specific

substance. In the microwave's case, that would be water. Microwaves heat food by running waves through food that energizes the water molecules within the food with heat up the food. Now in Histotripsy, cancer cells are targeted by ultrasound pulses that energize the entire cell which produce bubbles within the cell. These bubbles lead to weakening the cell which then breakdown. This method of removal of cells in any body without any need for incisions and can allow for control of cell removal in all three space dimensions. [2]
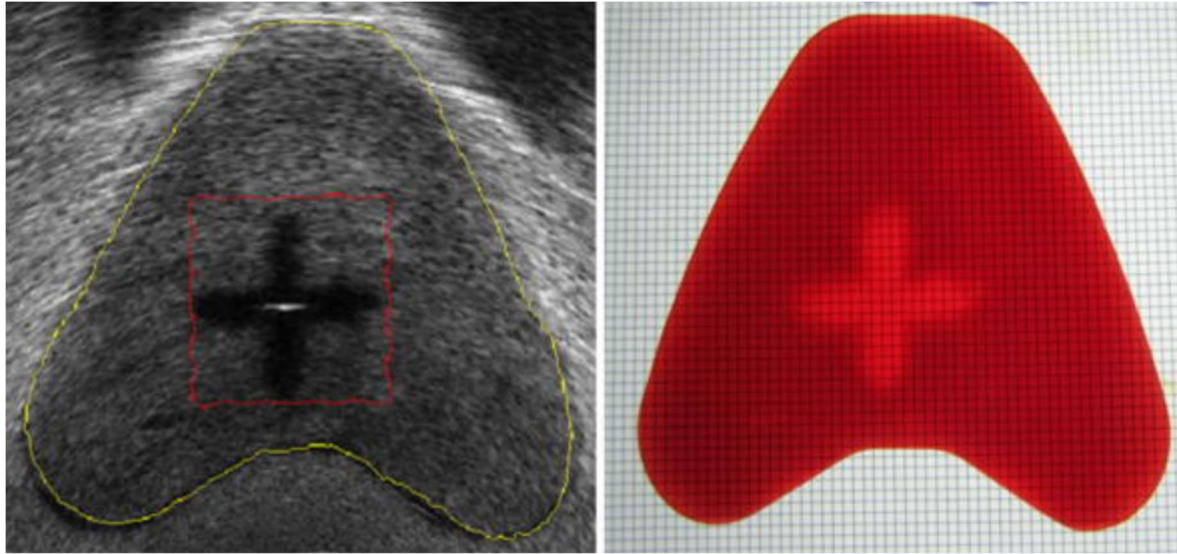
Here a cross shape was removed from a human pelvic phantom with hypotripsy [3]
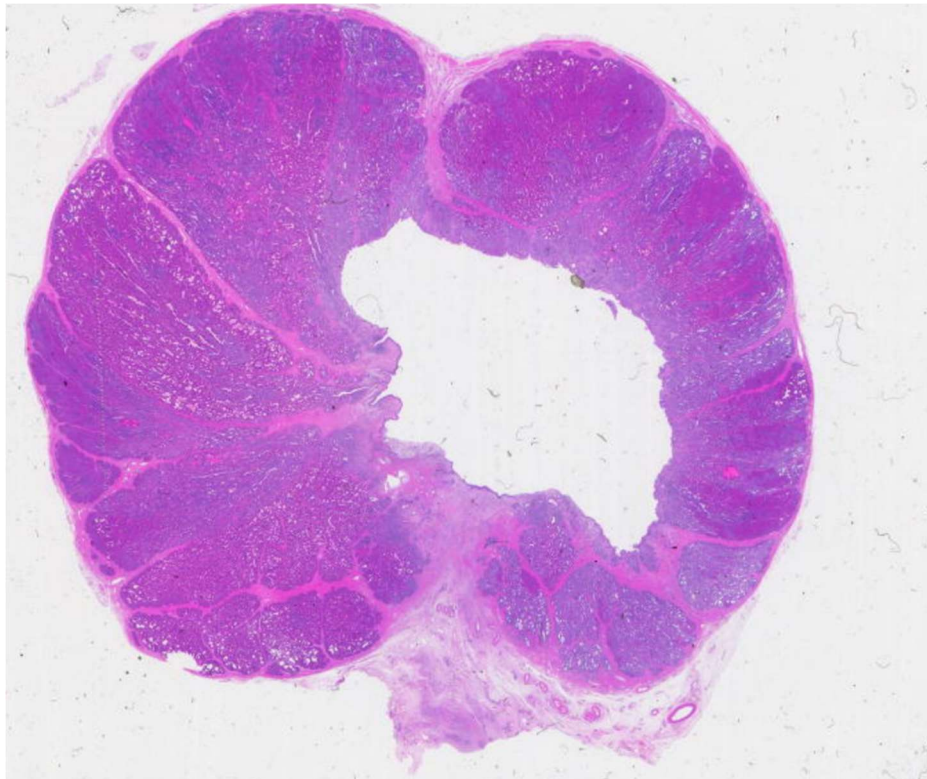
Here a rectangle shape of cells was removed from a prostatic of urethral tissue [3]

[1]   Y. Yamakoshi, A. Yamamoto, and Y. Yuminaka, "Novel imaging method of continuous shear wave by ultrasound color flow imaging," in *2015 IEEE International Ultrasonics Symposium (IUS)*, 2015, pp. 1–4.

[2]   K. Nightingale, S. McAleavey, and G. Trahey, "Shear-wave generation using acoustic radiation force: in vivo and ex vivo results," *Ultrasound Med. Biol.*, vol. 29, no. 12, pp. 1715–1723, Dec. 2003.

[3]   W. W. Roberts, "Development and translation of histotripsy: current status and future directions.," *Curr. Opin. Urol.*, vol. 24, no. 1, pp. 104–10, Jan. 2014.

URLS:

[1] https://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=7329121

[2] https://www.sciencedirect.com/science/article/abs/pii/S0301562903010809

[3] https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3974592/