Problem 1

For a bipartite state ρ^{AB} the geometric entanglement is defined as

$$E_{\mathrm{g}}(\rho) = 1 - \max_{\sigma \in \mathcal{S}} F(\rho, \sigma), \tag{1}$$

with fidelity $F(\rho, \sigma) = (\mathrm{Tr}[\sqrt{\sqrt{\rho}\sigma\sqrt{\rho}}])^2$, and $\mathcal{S}$ denotes the set of separable states. The relative entropy of entanglement is defined as

$$E_{\mathrm{r}}(\rho) = \min_{\sigma \in \mathcal{S}} S(\rho\|\sigma) \tag{2}$$

with the quantum relative entropy $S(\rho\|\sigma) = \mathrm{Tr}[\rho \log_2 \rho] - \mathrm{Tr}[\rho \log_2 \sigma]$. Entanglement of formation is defined as

$$E_{\mathrm{f}}(\rho) = \min \sum_i p_i S(\psi_i^A), \tag{3}$$

where the minimum is taken over all pure state decompositions of ρ such that $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|^{AB}$, and $S(\psi_i^A) = -\mathrm{Tr}(\psi_i^A \log_2 \psi_i^A)$ is the von Neumann entropy.

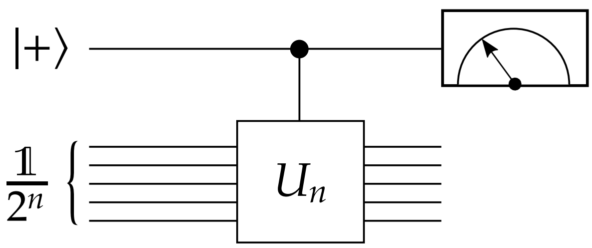
a) For a pure state $|\psi\rangle^{AB}$ show that the geometric entanglement can be expressed as

$$E_{\mathrm{g}}(|\psi\rangle) = 1 - \max_{|\phi\rangle \in \mathcal{S}} |\langle\phi|\psi\rangle|^2. \tag{4}$$

b) For a general bipartite pure state $|\psi\rangle^{AB}$ determine the geometric entanglement as a function of Schmidt coefficients of the state.

c) Using results from scientific literature, plot the geometric entanglement, the relative entropy of entanglement and the entanglement of formation for two-qubit Werner states. In the solution, put references to the articles used to solve this part of the problem.

Problem 2



Consider the following quantum algorithm: a quantum computer has $n + 1$ qubit registers, where the first qubit is initialized in the state

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \tag{5}$$

and n remaining qubits are initialized in the maximally mixed state $\mathbb{1}/2^n$. The overall state of the computer undergoes a controlled unitary operation U_n with the first qubit acting as the control qubit (see the above figure). After this process, a measurement on the first qubit is performed.

Evaluate the expectation values $\langle\sigma_x\rangle$ and $\langle\sigma_y\rangle$ of the first qubit as a function of U_n. What is this quantum algorithm evaluating?