$|\psi\rangle = \sum_{ij} x_{ij} |i\rangle \otimes |j\rangle$, $\langle\psi|\psi\rangle = 1 \Rightarrow$
bipartite pure state $\sum_{ij} x_{ij}^* x_{ij} = 1$

$|\phi\rangle = |\phi\rangle_A \otimes |\phi\rangle_B$, $|\phi\rangle_A = \sum_i a_i |i\rangle$,
Separable State $|\phi\rangle_B = \sum_j b_j |j\rangle$
S.t: $\langle\phi|\phi\rangle = 1 \Rightarrow \sum_{ij} (a_i^* a_i)(b_j^* b_j) = 1$

→ Distance

$$D^2 = \langle(\phi - \psi)|(\phi - \psi)\rangle = \sum_{ij} (a_i^* b_j^* - x_{ij}^*)(a_i b_j - x_{ij}) = \sum_{ij} a_i^* b_j^* a_i b_j - a_i^* b_j^* x_{ij} - a_i b_j x_{ij}^* + x_{ij}^* x_{ij}$$

→ Extrema of distance

$$\frac{\partial D^2}{\partial a_i} = 0 \Rightarrow a_i^* b_j^* b_j = b_j x_{ij}^* \quad \Big| \quad \frac{\partial D^2}{\partial b_j} = 0 \Rightarrow a_i^* a_i b_j^* = a_i x_{ij}^* \Rightarrow \sum (a_i^* a_i)(b_j^* b_j) = \sum a_i b_j x_{ij}^*$$

$$\text{Distance extreme, } D_{ex}^2 = \sum_{ij} \left(a_i b_j \cancel{x_{ij}^*} - a_i^* b_j^* x_{ij} - \cancel{a_i b_j x_{ij}^*} + 1 \right)$$

$$= 1 - \sum_{ij} a_i^* b_j^* x_{ij} = 1 - \sum_{ij} a_i b_j x_{ij}^*$$

$$1 - \cos^2\theta_{ex} = D_{ex}^2 \Rightarrow \cos^2\theta_{ex} = \sum_{ij} a_i b_j x_{ij}^*$$

$$\rho = |\psi\rangle\langle\psi| \ , \ \rho_A = \sum_{ij} \sum_{i'} x_{i'j}^* x_{ij} |i\rangle\langle i'| \ \text{ or } (\rho_A)_{ii'} = \sum_j x_{i'j}^* x_{ij}$$

$$x_{ij} b_j^* = a_i b_j b_j^*$$

$$\Rightarrow \frac{x_{ij} b_j^*}{b_j^* b_j} = a_i$$

Ref: https://arxiv.org/pdf/1003.4755.pdf