Keep your enemies closer and be loud about it

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Abstract

Under what conditions can cooperation emerge and how can we sustain it? We build a computer simulation of a multi-agent spatial environment using Prisoner's Dilemma as the principal agent interaction. We expand the model by allowing agents to remember a fixed number of past defectors and abstain from interacting with them again. Agents will be allowed to communicate by openly announcing remembered defectors, warning nearby agents. This way local reputation of each agent is created. We measure how well our model sustains cooperation and how robust it is against environmental harshness. The conclusion we reach is that local reputation works excellent in sustaining cooperation and punishing defection. The length of agent memory and amount of gossip are not important factors, only the range of gossip has to be greater than the agent movement speed.

1 Introduction

How can we encourage and sustain cooperation? Humans dominate their environments thanks to our ability to cooperate flexibly and at scale, as argued by Harari [9]. To study the conditions necessary for cooperation to flourish we need a suitable model of an activity with temptations to defect and punishments for doing so.

In 1950, Albert Tucker named a two-player exchange game "the Prisoner's Dilemma" to attract more research interest [11]. This game elegantly captures the difficulty of the decision between cooperation and defection in a single choice. Despite being so simple compared to the complexity of the problem it is representing, it was used to model many aspects of behaviour in systems of selfish individuals; and, according to Axelrod [1], for "discovery of the precise conditions that are necessary and sufficient for cooperation to emerge".

In the case of a one-off exchange, there being no opportunity for a follow-up punishment, the rational behaviour is defection. (This extends to all rounds for a fixed-length game, inductively [1].) The interesting behaviour arises if there is no end; or, at least, if there is no way for the participants of the game to know when the game ends or even if there is an end. An agents has to expect that even a single defection can be infinitely punished by never again being cooperated with [7]. Such a risk may just not be worth it.

In the next section we provide a short overview of related work and show gaps in knowledge that we aim to fill. Then we define the goals of this paper explicitly and describe the methodology we will use. Afterwards we show the results with short impartial explanations, followed by discussion of the implications of the results obtained; we also connect our results with similar research. We close the paper with a short reflection on the ethical aspects and the reproducibility of this paper, after which we summarize and explicitly state our conclusions with some ideas for further research directions.

2 Related Work

Prisoner's Dilemma is generally used to model agent behaviour when we care about promoting cooperation and all agents are acting selfishly: trying to maximize their personal profit. Defectors can only be punished (or avoided) if they can be identified, this is the main benefit of a reputation system—providing identity and behaviour history for agents. This is why services like Ebay or Airbnb have a rating system in place.

Presence of a reputation system strongly boosts cooperation, as shown by Dong et al. [6], Stahl [17], among others. These studies used different approaches to constructing a reputation system. All of them observed a significant boost in cooperation levels after introduction of the reputation system, which was to be expected.

In another study, De Backer et al. [4] investigated the effects of gossip between PD game participants on the cooperation levels. After each round, players would be allowed to exchange some information about other players. They observed some increased levels of cooperation in the gossip group when compared to the control group, however, the results were inconclusive, mostly because of a small sample size.

Camera and Casari [2] allowed game participants to report information about their opponents, with various amounts of this information then made public to players in the following rounds—ranging from only the latest move of the current opponent, to full histories of all moves taken by every player. They suggest that "[the efficacy] depends on the quality of platforms that store reported information." Similarly, Kagel [10] allowed unrestricted but non-binding communication (cheap talk) between agents. The unrestricted communication performed best and boosted the cooperation levels more than when the amount of information to be exchanged was limited.

All of the studies mentioned above were carried out using human subjects as game participants; the group sizes were also kept relatively small with few rounds. This made the results difficult to analyze and almost impossible to draw strong conclusions from. These studies also used external infrastructure for information passing: eliminating noise, delays, and deliberately wrong information. As shown by Gevers and Yorke-Smith [8], not all strategies that perform well in noiseless environments can do so under the presence of noise. Using external infrastructure for passing information also meant that the transmission speed was uniform for all participants receiving all necessary information in time for their next round of the game. These are non-trivial idealizations and relaxing them would yield a model closer to real-world systems and could change the results dramatically.

We aim to conclusively find out if, and how well, local reputation—built up via gossip—promotes and sustains cooperation and under what parameters does it yield optimal results. We will build a computer simulation for investigating communication between neighbors in a multi-agent spatial environment. Using a computer simulation we isolate the communication mechanism from complex human behaviour: humans learn to anticipate and predict behaviour of other players, making it more difficult to isolate and analyze the effects of the communication mechanism. We hope to provide a definitive evaluation of the effectiveness of local reputation in promoting and sustaining cooperation in Spatial Prisoner's Dilemma and provide a solid foundation for further research on communication in spatial exchange games.

3 Methodology

To evaluate the effectiveness of local reputation in enforcing cooperation we will have to first build a computer simulation of a multi-agent spatial environment. We will use Spatial Iterated Prisoner's Dilemma as the principal exchange game for modelling agent interactions. We hypothesize that local reputation will drive the simulation towards a cooperator–only population and will be able to sustain it indefinitely.

In this section we define the goals of this paper explicitly, explain the design of the model and simulations, and present the measurable properties and evaluation criteria of the model. We end the section with an explanation of how we will connect our simulation results back to the original question and what would constitute a confirmation of our hypothesis.

3.1 Problem Statement

We will use the prisoner's dilemma game to model agent interactions. This is a good choice for modelling behaviour of rational and selfish actors. And will allow us to observe the conditions necessary for cooperation to emerge in the population as well as what makes it sustain itself.

The agents will live in a spatial environment and act independently; the only mutual interactions will be playing the PD game with a neighbor and exchanging gossip with nearby agents. We will vary the range at which the gossip can be exchanges as well as the amount of information which can be included in a single gossip message. We want to determine the effectiveness of the gossip mechanism in promoting and sustaining cooperation.

3.2 Simulation Design

To explore the effects of local reputation, built up via openly gossiping with nearby neighbors, we will use a computer simulation of a multi-agent spatial environment. We will base the simulation on the design of Smaldino et al. [16].

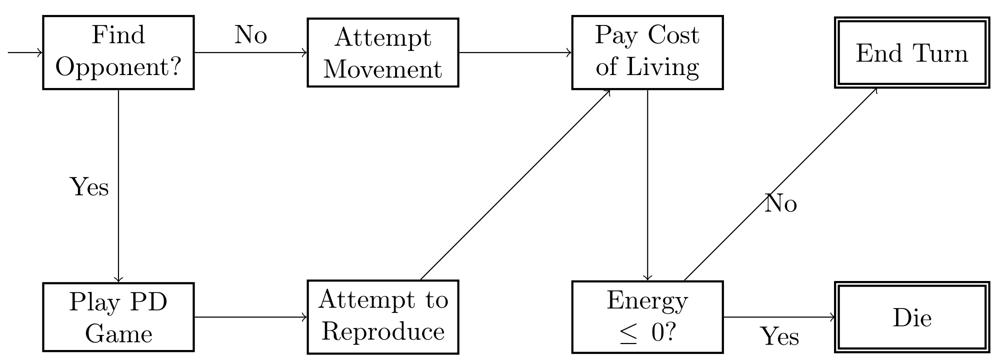

Figure 1: Agent behaviour diagram: showing the decision flow of an agent's single turn

The model consists of a spatial environment: square grid with torus (wrapping) bounds, each cell can be occupied by a single agent. This is a discreet time model; at each time step, every agent takes a single turn. The order in which agents take their turn is randomized in each time step. Agent's turn is defined by the finite state diagram shown in Figure 1. The agents pay a fixed cost to survive ($k = 0.5$) to the next round (agents who deplete their energy die and are removed from the simulation), and try to reproduce once they accumulate enough energy via positive interactions with other agents—PD game wins.

Every agent can play at most a single PD game in each time step. Playing a game in a time step is not guaranteed and depends on the spatial configuration of agents, order in which the agents are scheduled, and randomness in choosing an opponent from agent's neighbors. Similarly, when no opponent is found, movement only happens if an empty cell is found nearby; if there is more than one empty cell, one is chosen at random.

Agents reproduce by creating a new agent with the same parameters as themselves in an empty neighborhood cell (chosen randomly if more than one, no reproduction if there are no empty cells in the neighborhood). Reproduction is only attempted if the agent has accumulated enough energy: at least twice the amount of the cost of reproduction. The cost of reproduction is then subtracted from the parent and the offspring is birthed with this energy level. The cost of reproduction is effectively transferred from the parent to the offspring; reproducing does not change the net amount of energy in the model.

		Opponent's move	
		Cooperate	Defect
Player's move	Cooperate	Player: R Opponent: R	Player: S Opponent: T
	Defect	Player: T Opponent: S	Player: P Opponent: P

Table 1: Payoff matrix

A single round of the game is defined using a payoff matrix as shown in Table 1, with $T > R > P > S$ and $2R > T + S$ [3]. Like in the original model [16], we use $R = 3$ (reward for cooperating), $T = 5$ (temptation to defect), $P = 0$ (punishment for mutual defection). We use a different $S = -1.5$ (sucker's payoff: cooperator got defected) from $S = -1.0$ in the original.

We choose the sucker's payoff value ($S = -1.5$) empirically, by running simulations of the model and taking note of a value which leads to cooperator extinction relatively quickly. This choice for the parameter will ensure that, if we reach cooperation, it was caused by the reputation mechanism; and not because of random interactions in the model.

Environmental harshness of the model is defined by the sucker's payoff S (social harshness) and the cost to survive k (external harshness). We will investigate the effect of varying these parameters on our results and check the robustness of gossip in promoting cooperation across environments of various harshness.

To better suit our needs we will introduce some changes to the original model. We will reduce the spatial grid size from 100x100, as in the original design, to a more manageable 20x20. This will allow us to run more simulations with more complex agent behaviour in a reasonable amount of time. We will also decrease the starting number of agents from 1600 to 64: keeping the same percentage of 16% of the total grid size occupied as in the original model.

This reduction of the environment size (by a factor of 25!) has the effect of significantly increasing the chance of a total extinction of all agents: caused by the random behaviour of agents exploiting each other until all cooperators are dead and the population of pure defectors cannot sustain itself. We disregard runs that end in extinction and increase the number of simulation runs to compensate for this.

We will expand the model by giving the agents a (limited size) memory to keep track of past defectors and later to allow them to actively and freely share this knowledge by gossiping with other agents in a given range. Prior research [12, 13, 14] has already shown that memory can be an effective tool in promoting cooperation.

Another expansion to the model will be the addition of the localized gossip mechanism. This will allow agents to consult nearby peers to try and find out the reputation of agents unknown to the agents themselves. The range at which agents can be contacted and the amount of information which agents provide will be varied. In the results section, an overview of the effects of varying the parameters of the gossip is shown. We believe this extension of the model will have a strong positive effect on sustaining cooperation in the model.

The simulation will be implemented in Python using the Mesa[1] framework.

3.3 Simulation Evaluation

To determine the effectives of gossip in promoting cooperation, we will observe the rate of convergence to a population of cooperators, stopping the simulation once stable equilibrium is achieved. To verify that this is indeed an equilibrium, we let the simulation run longer and check that the general behaviour does not change after some point: we assume that if the behaviour of the simulation only varied in the first 5% of the simulation and then remained unchanged, it has converged. We will also observe the maximal population size of defectors which can sustain itself alongside the cooperators.

The main properties measured about the model will be the saturation of cooperator and defector populations. We will measure the saturation as the fraction of agents of the type alive in the model divided by the total carrying capacity of the population: 50% of all the cells. This differs from the measure of agent type frequency used by Smaldino et al. [16]; we have chosen to use the saturation measurement because with the decreased size model we encounter total extinction more often, and with the gossip extension aim to speed up defector extinction.

We believe the measure of agent type saturation is clearer and easier to understand as the relative population sizes do not influence the measure of the other type (as frequency does). Consider the case of 10 defector and 10 cooperator with maximal carrying capacity of 20 for either type: saturation is 50% for both agent types, and cooperator frequency is also 50%. Now 10 more cooperators are born: defector saturation stays at 50%, cooperator saturation increases to 100%, but the cooperator frequency is now at 33.3%. Using the saturations explains what happened more clearly, at the cost of having to present 2 complementary charts for each measurement: one for cooperator and one for defector saturation.

Since we are simulating stochastic behaviour and we are interested in the converged outcomes, we will run every simulations 30 times for each parameter combination and we will plot the standard deviations of the results. We will drop the 2 most extreme outliers, both positive and negative, from the plots; this allows us to show the actual behaviour of the model, devoid of unusual probabilistic anomalies. To ensure integrity of the results, we check the outliers removed and investigate any unusual interactions for relevance to the results.

We will also record characteristic patterns formed by the populations as influenced by different parameters. We do this, because it was shown by earlier work [15] that in Spatial Prisoner's Dilemma interesting patterns can emerge over time. These patterns are very similar to patterns occurring in nature, which are often created by reaction–diffusion processes.

[1] `https://github.com/projectmesa/mesa`

This suggests a deeper link between our model and biological activity.

Our simulation setup is available (`https://github.com/tinybeachthor/IPD`) for anyone interested in investigating any of these or other effects.

4 Results

Now we present the results of our simulation with commentary on the setup and the meaning of the results. We show the results impartially and let the data do most of the talking. A thorough explanation of the data in the context of similar research with explanation of wider implications will be in the next section.

We start by presenting the behaviour of the baseline (no memory, no gossip) model in the first subsection, then we expand it by adding memory and show how this alters the results obtained, afterwards we enable communication (gossip) between agents and run the simulation once again.

After making the comparisons between the behaviour as influenced by various extensions we take a closer look at the robustness of the gossip model against environmental harshness: both social (sucker's payoff S) and external (cost of living k).

We end this section by comparing the characteristic spatial patterns of the models. This can provide some intuition behind the behaviour of the various models and could be a good starting point for readers less familiar with the topic.

4.1 Baseline model

First we look at the plain model with 0-memory and no communication (gossip) between agents. This is a replication of Smaldino et al. [16]; with some alterations. As explained in the model design subsection, we modify the original model slightly. We decrease the size of the grid from 100x100 to 20x20, and decrease the starting agent count from 1600 to 64. We will keep the following parameters unchanged: starting energy will be randomly chosen for each agent to be between 1 and 49 (inclusive), energy cost to reproduce is 50 (agent has to accumulate at least 100 energy and reproduces by transferring 50 to the offspring), and the maximum energy an agent can hold is 150.

For the PD game payoffs we will use $R = 3$, $T = 5$, $P = 0$, and for the living cost $k = 0.5$. These values were shown to lead to the cooperators struggling to survive and maintain their numbers [16]. We will determine the value for S (sucker's payoff) from the simulations. Unless otherwise noted, we will use these parameters for all simulation runs.

We begin by exploring the robustness of the model against social harshness—represented by the S (sucker's payoff) parameter. In Figure 2 we present the results of running the simulation for $2.5 \leq S \leq 0.0$. We observe that $S = -1.5$ leads to total extinction in the model by step 500; we will use this value for the parameter for further simulations to determine the effectiveness of memory and the gossip mechanism at promoting and sustaining cooperation.

4.2 Remember defectors

Now we expand the model by allowing agents to remember some fixed number of most recent defectors encountered. We will examine how varying the number of defectors which can be remembered (memory size) influences the cooperator and defector saturations.

The effect of various memory sizes on the agent type saturations are shown in Figure 3. Allowing agents to remember defector increases convergence speed to saturated population.

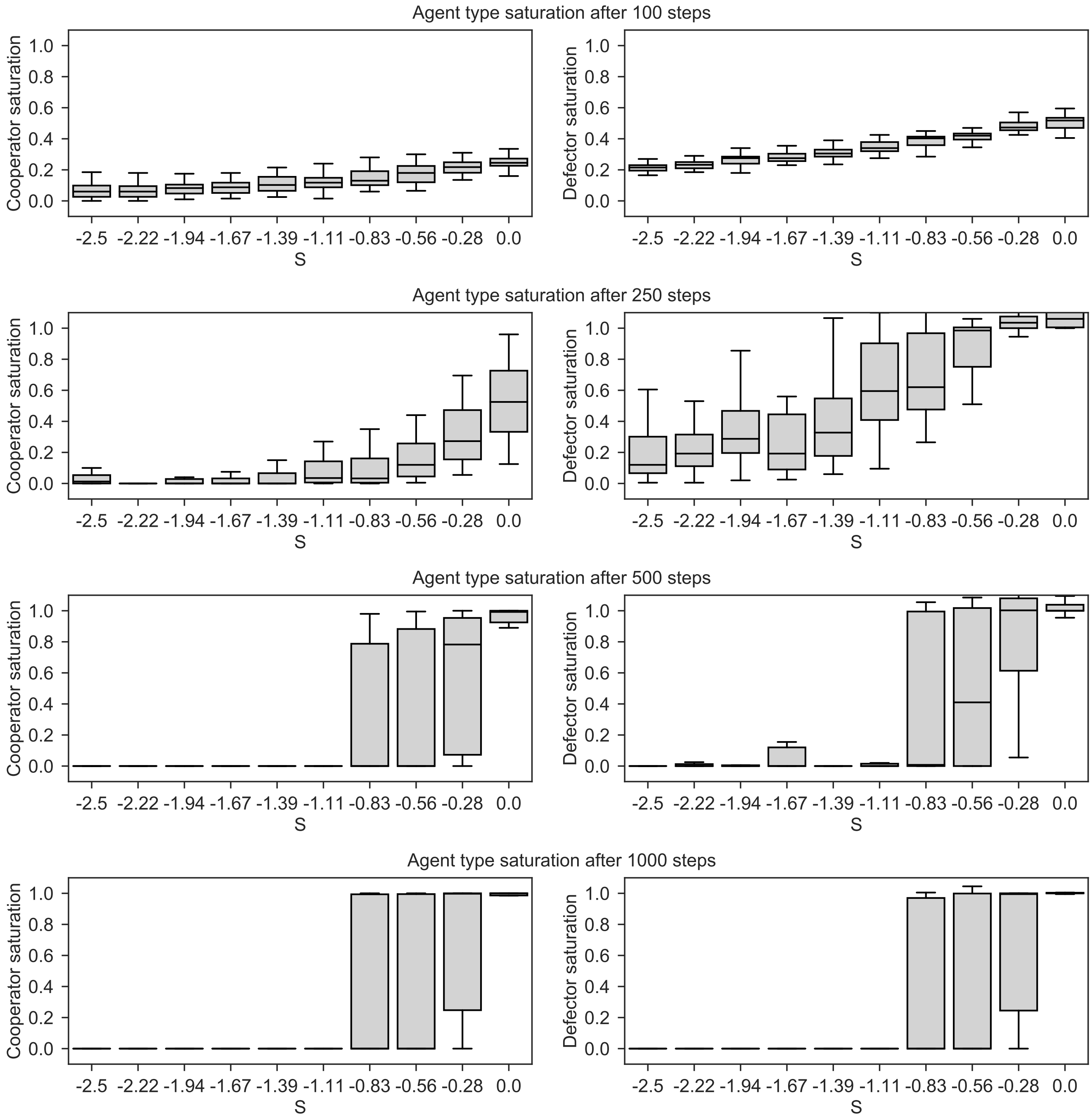

Figure 2: Agent type saturation for various S (sucker's payoff) after 100, 250, 500, 1000 steps; SD of 30 simulation runs, outliers removed

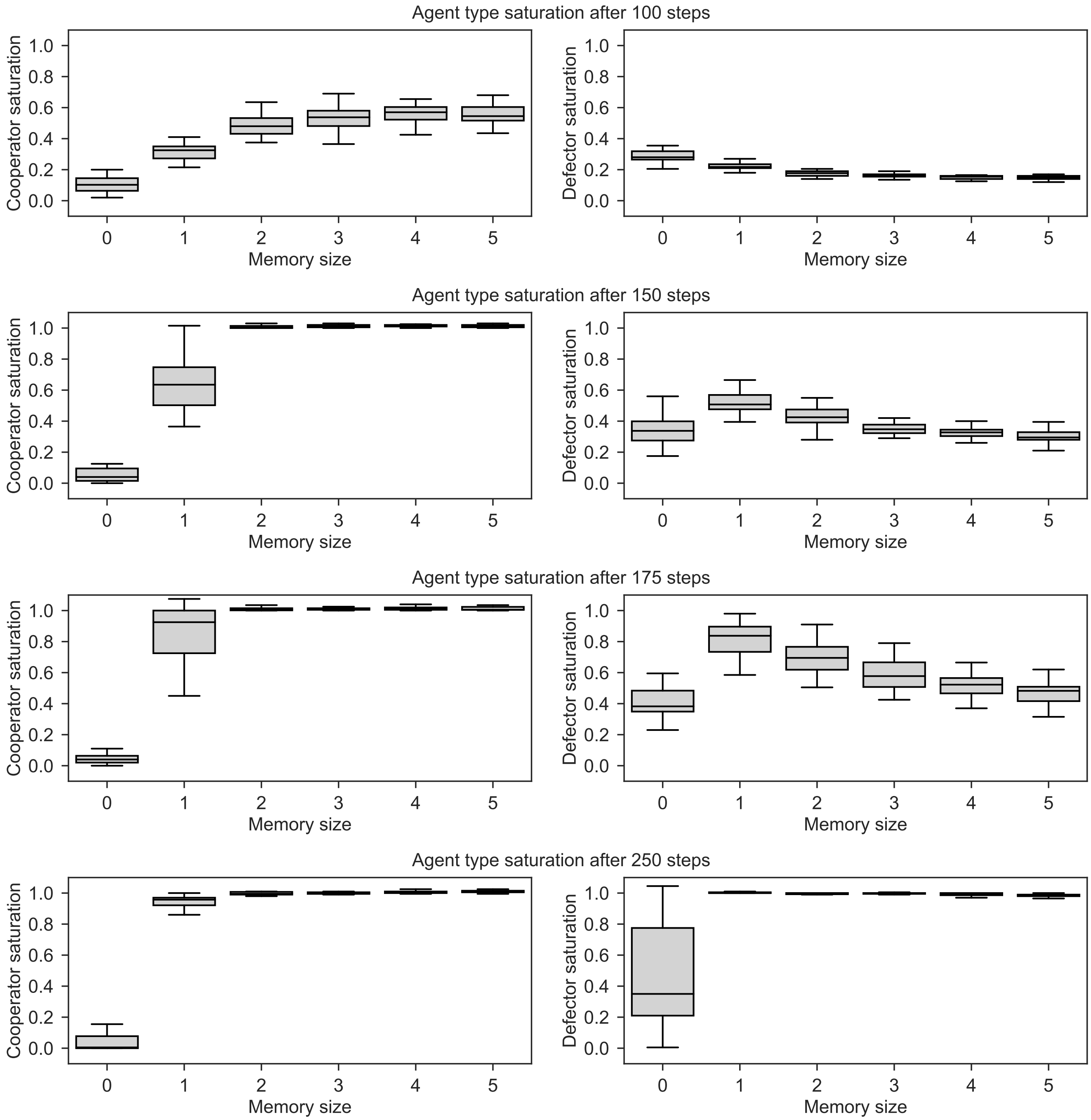

Figure 3: Agent type saturation for various memory sizes after 100, 150, 175, 250 steps; SD of 30 simulation runs, outliers removed

Cooperators benefit from this more as they saturate faster. After cooperators reach full saturations, defectors start to reproduce faster and soon catch up with cooperators, reaching full population saturation.

Increasing the memory size has only a small effect on speeding up convergence to population saturation for cooperators and slowing it down for defectors. This is a desired effect, however, it is very minute and is not worth the extra trouble of managing a large queue-based memory. In the long run, memory size beyond one does not provide additional benefits.

4.3 Gossip about defectors

Next we enable communication between the agents. We will allow agents to ask nearby agents in a Moore neighborhood of radius 1, 2, and 3 to ask others if they remember an agent defecting in a certain number of past encounters (gossip size).

We will set agent memory size to 5. The gossip size will vary between 0 and 5, including both bounds. We run the simulation for 200 and 1000 steps and plot the agent type saturations in Figure 4 and Figure 5 respectively.

We have found that the introduction of gossip is a strong deterrent of defection and leads to cooperator–only populations quickly. The size of the memory and the size of the gossip are not significant factors, only speeding up the convergence slightly. The most important factor in predicting cooperator success is the range of the gossip.

4.4 Robustness of the gossip

To check the effectiveness and robustness of our model against environmental harshness we will use the same simulation setup for determining the S value from the beginning of this section (baseline model). We will plot agent type saturations across various S (sucker's payoff).

To test the gossip model at the extreme, we will use memory size of 1, gossip size of 1 and gossip range of 3. We will run the simulation for 250 and 1000 steps. The final agent type saturations are plotted in Figure 6.

Next we fix $S = -1.5$ again and vary the cost of living $0.0 \leq k \leq 3.5$ instead. In Figure 7 we plot the achieved saturations. With $3.0 \leq k$, which means $R \leq k$, we see that all agents go extinct. Another interesting case is $k = 0.0$, meaning no cost of living is imposed on the agents, there is no counter pressure on the defectors, so they are able to survive; cooperators also manage to reach and sustain fully saturated population. For all values between these extremes the gossip model (with range 3) enforces cooperator-only population.

4.5 Spatial patterns

We include images of the spatial patterns formed by the simulations in Figure 8. Since the model has an inspiration in biology this is an interesting visualization to include. We show 3 patterns formed after simulating for 500 steps for each of the following configurations (these are the same as used in the sections above):

1. **Baseline:** the base model, without any extensions (0-memory, no gossip)
2. **Memory-5:** every agent can remember the 5 most recently encountered defectors
3. **Final:** every agent can remember the single last defector and gossip about it, radius of the gossip is 3 units in a Moore neighborhood

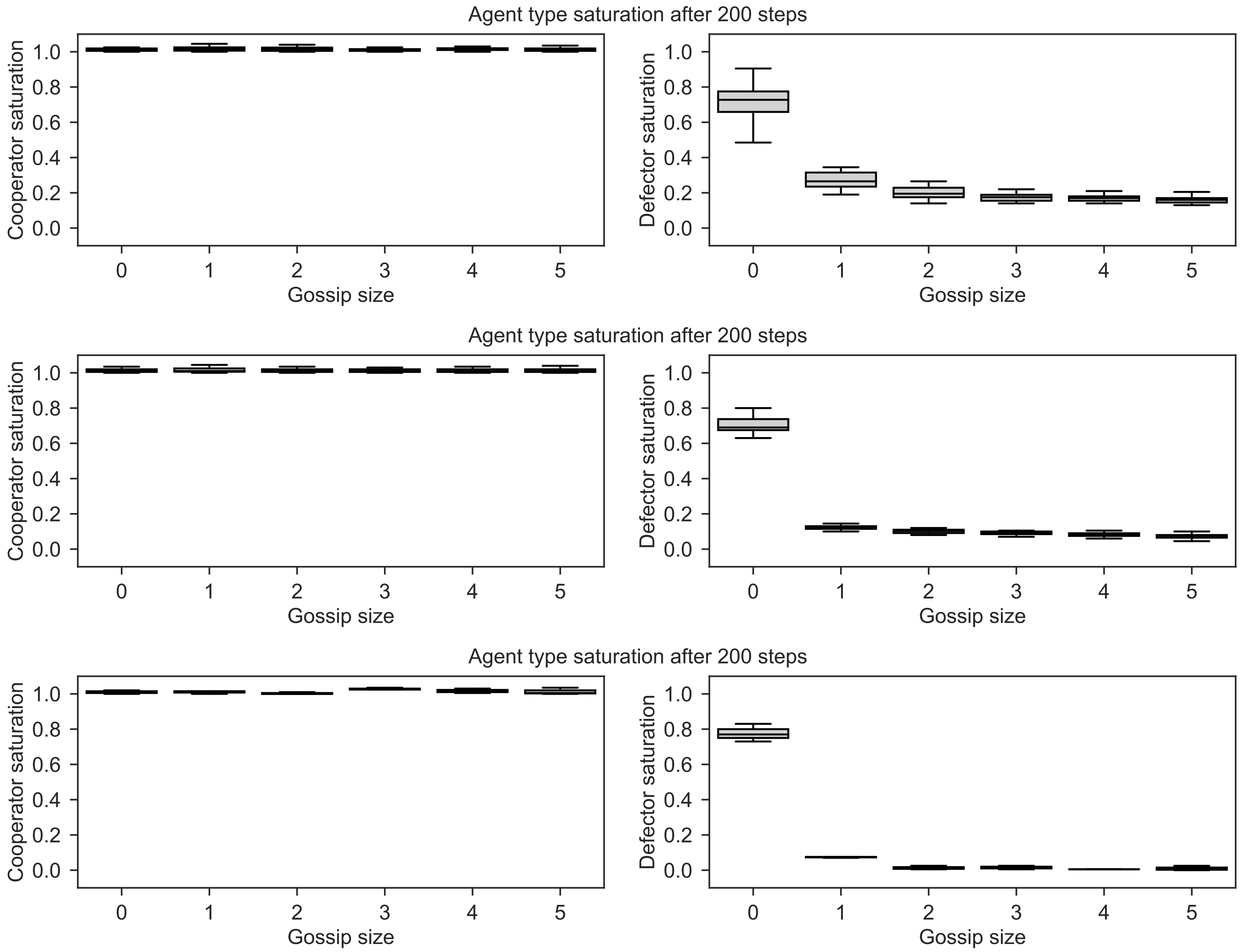

Figure 4: Agent type saturation for various gossip sizes after 200 steps, gossip radii 1, 2, and 3, respectively top to bottom; SD of 30 simulation runs, outliers removed

5 Discussion

Looking back at the results we collected in the previous section; now we present a more in-depth commentary on their meaning and put them in a wider context.

We have seen that the baseline model act as expected and leads to total extinction quickly. This is consistent with the behaviour observed by Smaldino et al. [16]. Agents can prolong their existence if the cooperators are able to form a cluster, but there is no escaping eventual extinction.

By extending the model with memory, we are able to sustain the population. Both cooperators and defectors profit from this equally, with cooperators having a slight edge as memory size increases, although this is almost negligible. In the spatial patterns for the memory-5 model we see more fragmentation: this is because cooperator can survive next to

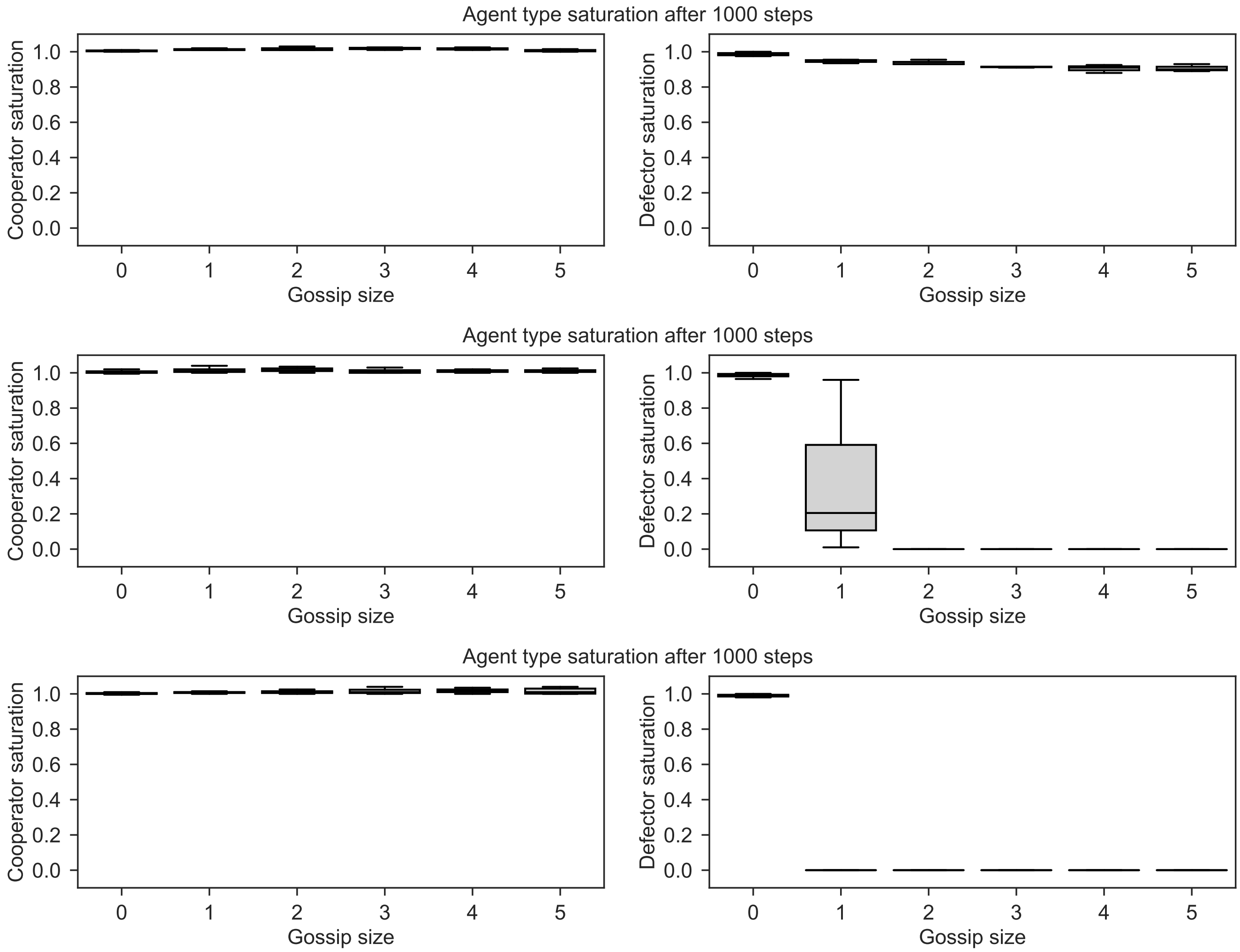

Figure 5: Agent type saturation for various gossip sizes after 1000 steps, gossip radii 1, 2, and 3, respectively top to bottom; SD of 30 simulation runs, outliers removed

defectors by remembering them.

From Camera and Casari [2], Stahl [17] we know that reputation systems promote cooperation. We wanted to see if local reputation system suffices: we let agents communicate what they remember to nearby agents. This works very well, we can consistently achieve cooperator–only population quickly. The amount of the information included in the gossip is not very important, the main parameter is the range of the gossip. By increasing it we can converge to cooperation faster.

Last we looked at the robustness of the local reputation model against environmental harshness. We found it to be almost entirely independent of the environmental parameters. The gossip mechanism is too strong to be influenced by environmental effects. We conclude by certifying the efficacy of the local reputation, confirming our hypotheses.

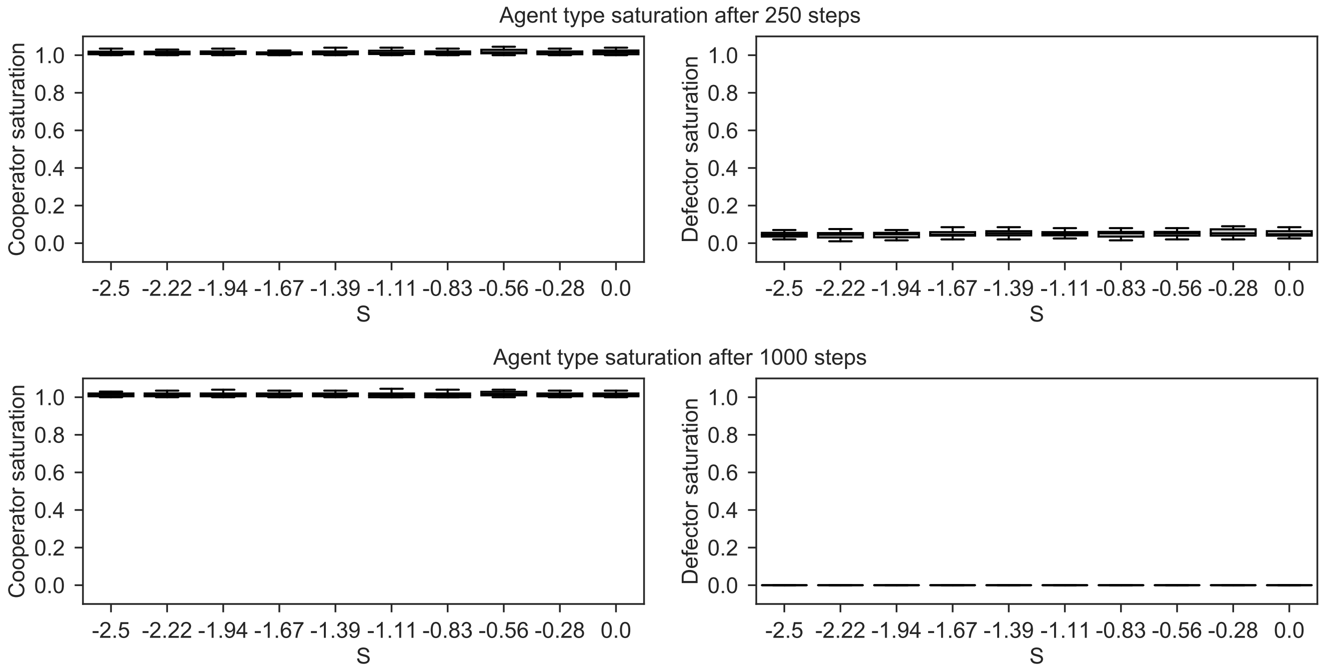

Figure 6: Agent type saturation for various S (sucker's payoff) after 250 and 1000 steps, memory size 1, gossip size 1, gossip range 3; SD of 30 simulation runs, outliers removed

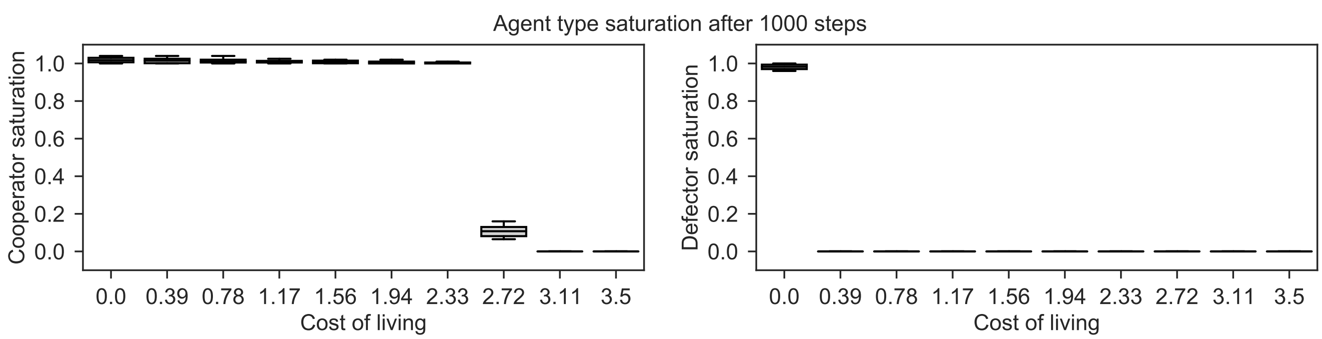

Figure 7: Agent type saturation for various k (cost of living) after 1000 steps, memory size 1, gossip size 1, gossip range 3; SD of 30 simulation runs, outliers removed

6 Responsible Research

We recognize the inherent difficulties in conducting ethical and sustainable research. We take the following actions to ensure this paper is ethically good and can be reproduced by others.

We want to make this paper a good foundation for future work and as such we provide all source materials for this paper: including the code files for running the simulations and evaluating results, the LaTeX files for this paper, and anything else used while conducting this research. Next, we use Nix Flake[2] to capture the exact versions of all software, libraries, and packages used for this research [5]. All of this is captured together in and provided as a fully reproducible environment. We hope that by doing this it becomes easy to reproduce our results and provide a good foundation environment for other researchers to quickly kickstart their research. Our model itself is implemented in a Jupyter Notebook[3] and care was taken to keep the total amount of dependencies as low as possible and to stick to the most standard dependencies wherever possible. We hope others will appreciate this by building upon our research and uncovering wonderful conclusions.

This paper explores the effects of reputation on promoting and sustaining cooperation. We believe this research is ethically good and our conscience is clean about the methods used and conclusions reached; as well as all other aspects of this paper. While we cannot ensure the findings of this paper will not be misused by others, we implore all readers to always strive for the highest ethical ideals. Let's all be excellent.

[2] `https://nixos.wiki/wiki/Flakes`

[3] `https://jupyter.org/`

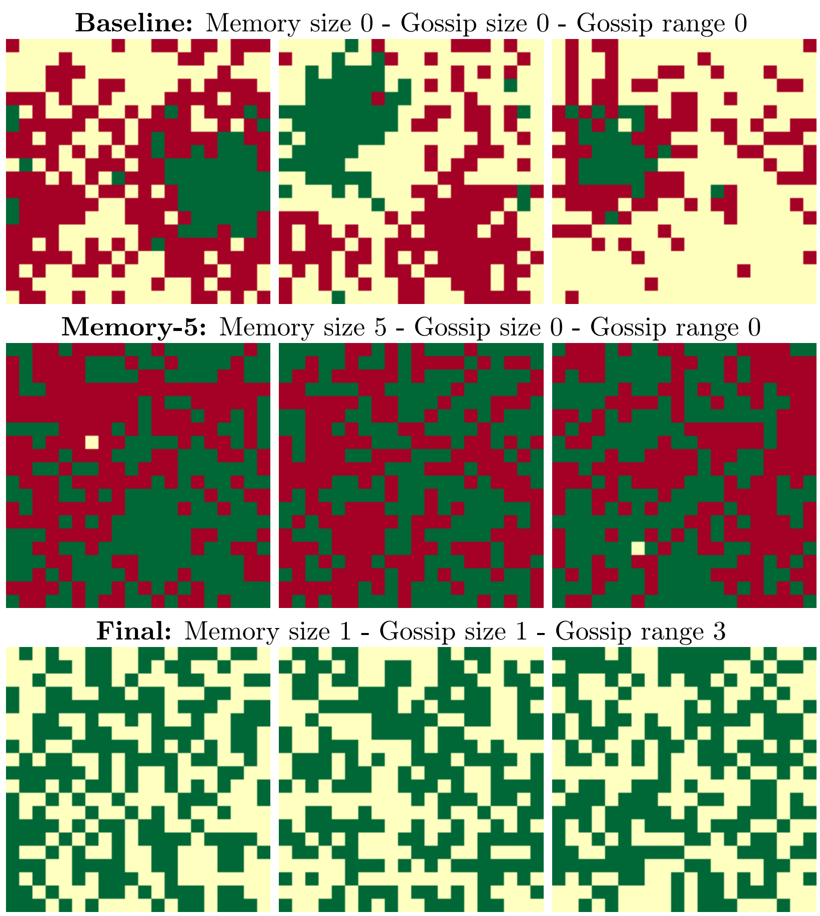

Figure 8: Spatial patterns formed by agents after simulating for 500 steps, defectors are red, cooperators are green

7 Conclusion

In this paper, we wanted to provide a definitive evaluating of the efficacy of local reputation—built up via gossip—in promoting and sustaining cooperation in selfish populations. The inspiration came from many different papers taking various approaches to investigate effects of communication and reputation on cooperation in iterated exchange games. All of these are described in more detail in the related work section at the beginning of the paper. The prior research has used human subjects in relatively small groups with few rounds of the game, the results achieved were difficult to analyze and to draw strong conclusions from. To address this we build a computer simulation for evaluating communication in spatial multi-agent environments; general enough to serve as a solid foundation for further research. We verify it by building a simulation to evaluate effects of local reputation on the cooperation levels.

From the simulation, we find that the most important factor in predicting cooperator success is the range at which gossip can be exchanged; the amount of information included in the gossip has negligible effect. If the gossip can move faster than agents, cooperators will flourish. Otherwise defectors can reach full population saturation. From what we find, the best way to ensure cooperation (and survival of a population) is to keep your enemies close and be loud about it. The louder the better.

Our simulation setup was limited in representing real world conditions. We assumed all information is transferred with 100% fidelity: no noise is present, no information is lost, all agents share information willingly and openly with anyone without any limitations. Introduction of noise to this model could alter the results and show other interesting aspects of cooperation. Next, the population of agents we used was composed of only pure cooperators and pure defectors, and all agents were sharing information openly with all nearby agents. More complex strategies could be defined and perhaps an optimal combined strategy for behaviour–information sharing could be identified. Agents could be given probabilistic behaviour making the classification into cooperator/defector more complex. This would work against the gossip mechanism we used. It could be that the gossip mechanism would not be advantageous if the agent behaviour was random enough, perhaps even favoring the defectors as the gossip mechanism would deter more cooperator–cooperator interactions.

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