半导体中载流子的统计分布

复旦大学 微电子学系

13307130163

李琛

June 12, 2015

Contents

1 状态密度

1.1 三维情况下的自由电子气

$$\varepsilon_{\vec{k}}=\frac{\hbar^2}{2m}k^2=\frac{\hbar^2}{2m}(k_x^2+k_y^2+k_z^2),k_x,k_y,k_z=0;\pm\frac{2\pi}{L};...;\pm\frac{2n\pi}{L}$$

每个量子态占据体积为$\dfrac{(2\pi)^3}{V}$ $\quad\therefore$ 再考虑自旋，k空间能量状态密度为$\dfrac{2V}{(2\pi)^3}$

1.2 状态密度

- 为了描述能带电子状态的分布，引入态密度$g(E)$表示单位能量间隔内的状态数

$$g(E)=\frac{dZ}{dE}=\frac{dZ}{d\Omega*}\frac{d\Omega^*}{dk}\frac{dk}{dE}$$

- 状态密度汇总
  - 一维 $L^*=2k\rightarrow 2\left(\frac{L}{2\pi}dL^*\right)\rightarrow dZ=\frac{2L}{h}\sqrt{\frac{2m_n^*}{E-E_0}}dE$

$$\rightarrow g(E)=\frac{2\pi}{h}\sqrt{\frac{2m_n^*}{E-E_0}}$$

  - 二维 $S^*=\pi k^2\rightarrow dZ=2\left(\frac{L}{2\pi}\right)^2dS^*$

$$\rightarrow g(E)=\frac{4S\pi m_n^*}{h^2}$$

  - 三维 $\Omega^*=\frac{4}{3}\pi k^3\rightarrow dZ=\frac{2V}{(2\pi)^3}\frac{4\pi}{\hbar^3}(2m_n^*)^{3/2}(E-E_0)^{1/2}dE$

$$\rightarrow g(E)=\frac{4\pi V}{h^3}(2m_n^*)^{3/2}\sqrt{E-E_0}$$

- 状态密度有效质量
  - 电子状态密度有效质量 导带极值在$\vec{k}=\vec{k_c}$,等能面为椭球面

$$2m_{dn}^{3/2}=M(8m_x^*m_y^*m_z^*)^{1/2}\Rightarrow m_{dn}=(M^2m_l^*m_t^{*2})^{1/3}$$

– 空穴状态密度有效质量　重空穴与轻空穴在价带顶简并

$$(m_{dV}^*)^{3/2} = (m_{lh}^{3/2} + m_{hh}^{3/2}) \Rightarrow (m_{dV}^*) = (m_{lh}^{3/2} + m_{hh}^{3/2})^{2/3}$$

2　费米能级与载流子的统计分布

2.1　费米分布函数

- **费米分布函数f(E)**　$f(E) = \frac{1}{1+exp\left(\frac{E-E_f}{kT}\right)}$
- **玻尔兹曼分布函数f(E)**$(E-E_f) >> kT$　$f(E) = \frac{1}{exp\left(\frac{E-E_f}{kT}\right)}$
- **费米能级的物理意义**　标志了电子填充水平
- $f_e(E) + f_h(E) = exp\left(-\frac{E-E_f}{kT}\right) + exp\left(-\frac{E_f-E}{kT}\right) = 1$

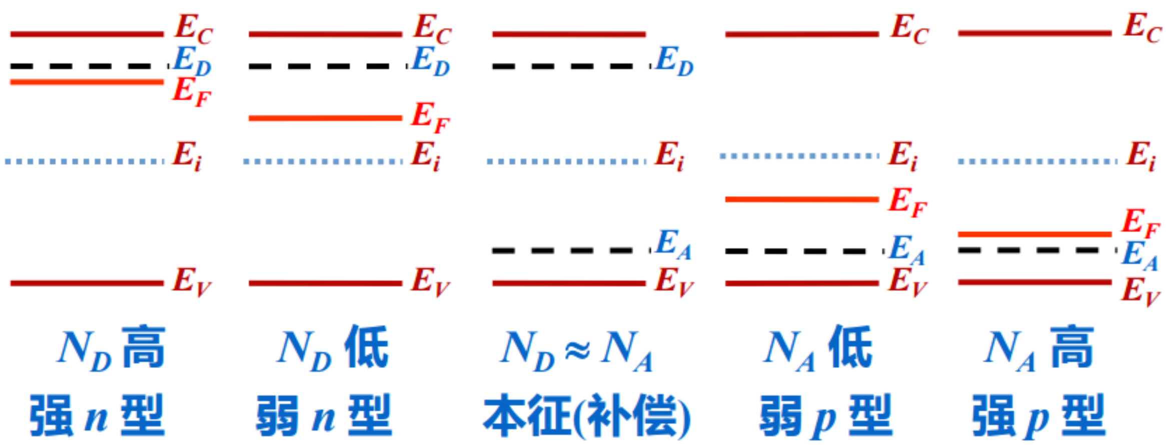

Figure 1: 不同半导体的费米能级

2.2　导带电子和价带空穴浓度

$$导带有效状态密度N_c = \frac{2(2\pi m_{dn}kT)^{3/2}}{h^3}$$

$$导带平衡电子浓度n = N_c exp\left(-\frac{E_c - E_f}{k_B T}\right)$$

$$价带有效状态密度N_v = \frac{2(2\pi m_{dp}kT)^{3/2}}{h^3}$$

$$\text{价带平衡电子浓度} p = N_v exp\left(-\frac{E_f - E_v}{k_B T}\right)$$

$$\therefore np = N_c N_v exp\left(-\frac{E_c - E_v}{k_B T}\right) = N_c N_v exp\left(-\frac{E_g}{k_B T}\right), \text{浓度乘积与} E_F \text{无关}$$

3 本征半导体的载流子分布

热激发产生的载流子

3.1 本征载流子浓度

$$\text{电中性条件：} n \cdot p = n_i^2$$

$$n = p = n_i = \sqrt{N_c N_v} exp\left(-\frac{E_g}{2k_B T}\right)$$

3.2 本征载流子的费米能级

$$E_f = \frac{E_c + E_v}{2} + \frac{k_B T}{2} ln\left(\frac{N_v}{N_c}\right)$$

$$\frac{E_c + E_v}{2} >> \frac{k_B T}{2} ln\left(\frac{N_v}{N_c}\right), \therefore \text{本征费米能级} E_i \text{基本上在禁带中线处}$$

4 杂质半导体的载流子分布

电子占据施主能级的几率

$$f_D(E) = \frac{1}{1 + \frac{1}{2} exp\left(\frac{E_D - E_f}{kT}\right)}$$

空穴占据受主能级的几率

$$f_A(E) = \frac{1}{1 + \frac{1}{2} exp\left(\frac{E_f - E_A}{kT}\right)}$$

4.1 非补偿情形

$$n = p + n_D^+$$

即导带电子浓度= 价带空穴浓度+ 电离施主浓度

$$N_c exp\left(-\frac{E_c - E_f}{k_B T}\right) = N_v exp\left(-\frac{E_f - E_v}{k_B T}\right) + \frac{N_D}{1 + 2exp\left(-\frac{E_D - E_f}{k_B T}\right)}$$

非补偿情形	电中性条件	电子浓度	费米能级
低温弱电离区	$n_0 = n_D^+$	$n_0 = \left(\frac{N_D N_C}{2}\right)^{1/2} exp\left(-\frac{E_C - E_D}{2k_B T}\right)$	$E_F = \frac{E_C + E_D}{2} + \frac{k_B T}{2} ln\frac{N_D}{2N_C}$
强电离区	$n_0 = n_D^+$	$n_0 = N_D$	$E_F = E_C + k_B T ln\frac{N_D}{N_C}$
过渡区	$n_0 = p_0 + N_D$	$n_0 = N_D + \frac{n_i^2}{N_D}$	$E_F = E_i + k_B T sinh^{-1}\left(\frac{N_D}{2n_i}\right)$
本征区	$n_0 = p_0 = n_i$	$n_0 = n_i$	$E_F = \frac{E_C + E_V}{2} + \frac{k_B T}{2} ln\frac{N_V}{N_C}$

4.2 补偿情形

若半导体中同时含有施主、受主杂质，且施主杂质多于受主杂质，低温下，施主杂质将首先填充受主杂质，称为补偿

$N_D > N_A$	电中性条件	电子浓度	费米能级
低温弱电离区	$n_0 = n_D^+ - p_A^-$	$N_A >> n_0$: $n = \frac{N_C(N_D - N_A)}{2N_A}) exp\left(-\frac{E_C - E_D}{k_B T}\right)$ $N_A << n_0$: $n_0 = \left(\frac{N_D N_C}{2}\right)^{1/2} exp\left(-\frac{E_\Delta E_D}{2k_B T}\right)$	$E_F = E_D + k_B T ln\frac{N_D - N_A}{2N_A}$ $E_F = \frac{E_C + E_D}{2} + \frac{k_B T}{2} ln\frac{N_D}{2N_C}$
强电离区	$n_0 = N_D - N_A$	$n_0 = N_D - N_A$	$E_F = E_C + k_B T ln\frac{N_D - N_A}{N_C}$
过渡区	$n_0 = p_0 + N_D - N_A$	$n_0 = \frac{N_D - N_A}{2} +$ $\frac{1}{2}[(N_D - N_A)^2 + 4n_i^2]^{1/2}$	$E_F = E_i + k_B T sinh^{-1}\left(\frac{N_D - N_A}{2n_i}\right)$
本征区	$n_0 = p_0 = n_i$	$n_0 = n_i$	$E_F = E_i$

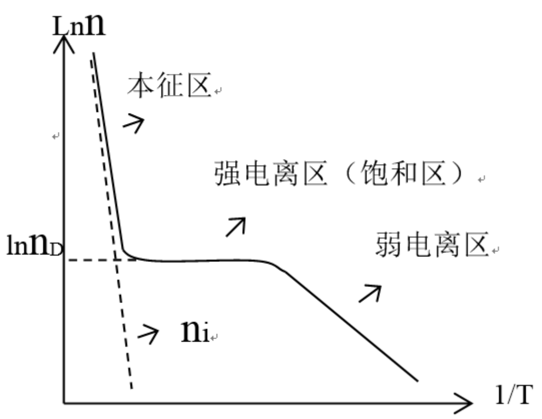

Figure 2: 浓度随温度变化曲线

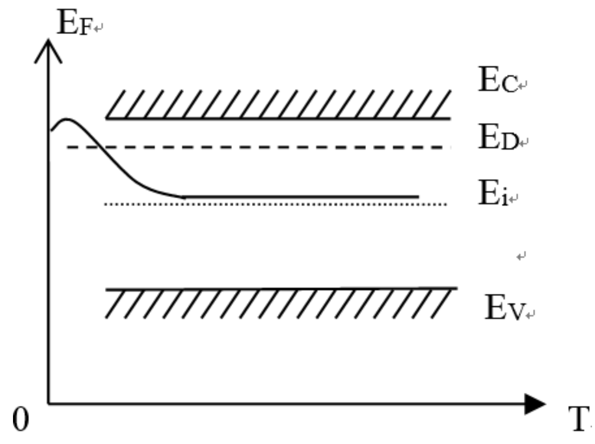

Figure 3: 费米能级随温度变化曲线

5 简并半导体

5.1 简并的出现

E_f在导带底或价带顶附近，或E_f进入导带或价带称为简并情形
此时玻尔兹曼近似不再成立。

5.2 简并半导体的载流子浓度

$$\text{费米积分} F_{1/2}(\xi) = \int_0^{\infty} \frac{x^{1/2}}{1 + exp(x - \xi)} dx$$

$$n_0 = N_C \frac{2}{\sqrt{\pi}} F_{1/2} \left(\frac{E_f - E_c}{k_B T} \right) \quad p_0 = N_V \frac{2}{\sqrt{\pi}} F_{1/2} \left(\frac{E_v - E_f}{k_B T} \right)$$

5.3 简并条件

$$\begin{cases} \text{非简并} \quad E_C - E_f > 2kT \\ \text{弱简并} \quad 0 < E_C - E_f \leq 2kT \\ \text{强简并} \quad E_c - E_f \leq 0 \end{cases}$$