Unsupervised Deformable Image Registration with Structural Nonparametric Smoothing

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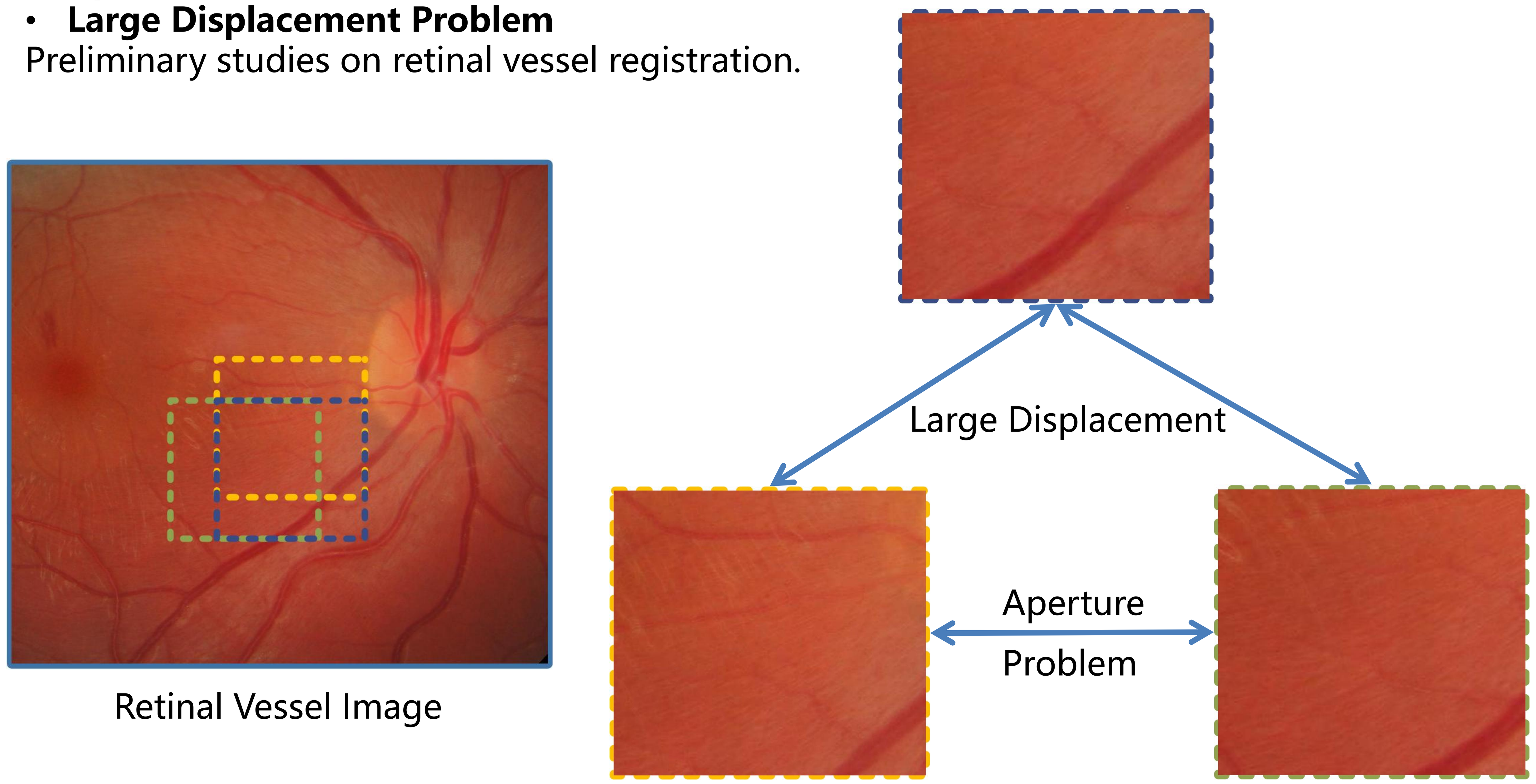
Objective

Tackle challenges in unsupervised learning-based registration, focusing on:

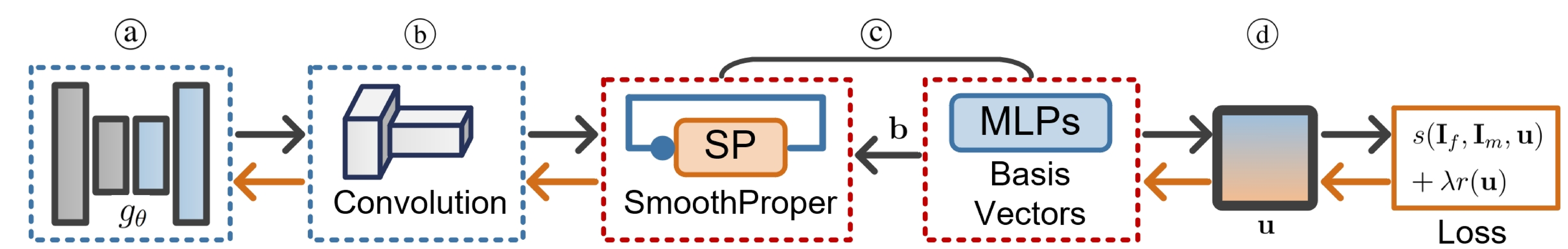
- **Aperture Problem**
- **Large Displacement Problem**

Preliminary studies on retinal vessel registration.



Retinal Vessel Image

Pairwise Regularization Has Been Largely Overlooked



$$\min_{\theta} \mathbb{E}_{(\mathbf{I}_f,\mathbf{I}_m)\sim D}[s(\mathbf{I}_f, \mathbf{I}_m, \mathbf{u}) + \lambda r(\mathbf{u})]$$
$$\text{s.t.} \quad \mathbf{u} = g_\theta(\mathbf{I}_f, \mathbf{I}_m)$$

Role of the Pairwise Term:
1. Enforces deformation smoothness regularization
2. Enables message passing across the field

$$\frac{\partial \mathcal{L}}{\partial \theta} = \sum_{f,m} \frac{\partial s}{\partial g_\theta} \frac{\partial g_\theta}{\partial \theta} + \lambda \sum_{f,m} \frac{\partial r}{\partial g_\theta} \frac{\partial g_\theta}{\partial \theta}$$

The 1st Jacobian provides no more information than **raw intensity-label mutual information**.

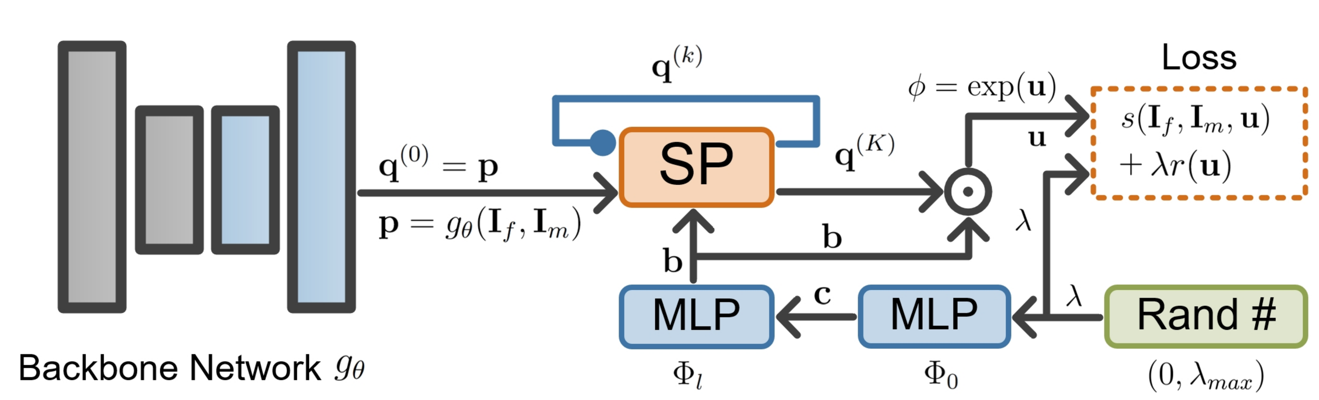
In contrast, the 2nd Jacobian, often overlooked by existing methods, is crucial for addressing the aperture and large displacement problems. It not only enforces smoothness but also enables message passing, propagating registration signals from strong to weak regions.

Introducing SmoothProper (SP)

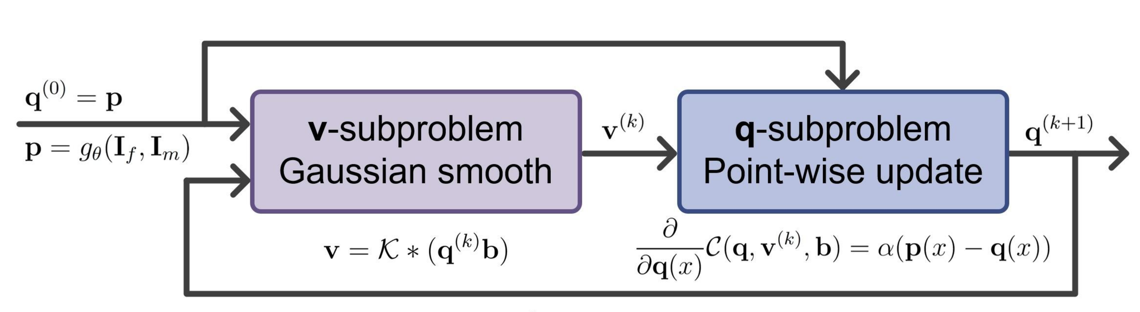
Unrolling the regularization term inside network forward pass via bi-level optimization.

$$\min_{\theta} \mathbb{E}_{(\mathbf{I}_f,\mathbf{I}_m)\sim D}[\mathcal{L}(\mathbf{I}_f, \mathbf{I}_m, \mathbf{u}^*(\theta))],$$
$$\text{s.t. } \mathbf{u}^*(\theta) = \arg\min_{\mathbf{u}} \mathcal{S}(\mathbf{I}_f, \mathbf{I}_m, \mathbf{u}, \theta).$$

$$\min_{\mathbf{q},\mathbf{v}} \sum_{x\in\Omega} \|\mathbf{p}(x) - \mathbf{q}(x)\|^2 + \sum_{x\in\Omega}\sum_{i=1}^{m} \frac{1}{2\alpha}\mathbf{q}_i(x)\|\mathbf{v}(x) - \mathbf{b}_i\|^2$$
$$+ \sum_{x\in\Omega} \frac{1}{2\alpha}\|\mathbf{q}(x)\mathbf{b} - \mathbf{v}(x)\|^2 + \beta\mathbf{r}(\mathbf{v}).$$



$$\mathbf{q}^{(k)} = \arg\min_{\mathbf{q}} \frac{1}{2\alpha}\mathcal{C}(\mathbf{q}, \mathbf{v}^{(k-1)}, \mathbf{b}) + \sum_{x\in\Omega} \|\mathbf{p}(x) - \mathbf{q}(x)\|^2;$$
$$\mathbf{v}^{(k)} = \arg\min_{\mathbf{v}} \frac{1}{2\alpha}\mathcal{C}(\mathbf{q}^{(k)}, \mathbf{v}, \mathbf{b}) + \beta\|\nabla\mathbf{v}\|^2.$$



– **Message Passing**: The term $\|\nabla\mathbf{v}\|^2$ encourages gradual changes in $\mathbf{v}$, smoothing $\mathbf{q}$ as $\mathbf{qb}$ aligns with $\mathbf{v}$. This results in $\mathbf{q}$ serving as a smooth approximation of $\mathbf{p}$, effectively passing flow signals across regions.

– **Structural Consistency**: As $\mathbf{q}(x)$ and $\mathbf{v}(x)$ iteratively align with $\mathbf{p}(x)$ and the basis $\mathbf{b}_i$, the equation ensures that regions with strong flow signals (large $\mathbf{q}(x)$) remain anchored to their representative basis patterns $\mathbf{b}_i$.

Smooth and reinforce loop for The deformation field

Quantitative & Qualitative Results

Category	Methods	TRE ↓	AUC ↑			Group-wise AUC@25 ↑		
			mAUC@15	mAUC@25	mAUC@50	mAUC@$\mathcal{A}$	mAUC@$\mathcal{P}$	mAUC@$\mathcal{S}$
Detector-based Methods	XFeat [CVPR'24] [53]	10.858	0.560	0.637	0.794	0.853	0.102	0.915
	R2D2 [NeurIPS'19] [56]	7.926	0.553	0.701	0.850	0.813	0.333	0.899
	LightGlue [ICCV'23] [41]	7.802	0.575	0.710	0.855	0.853	0.338	0.904
	SuperPoint [CVPR'18] [14]	6.641	0.612	0.757	0.879	0.813	0.453	0.928
	Glampoints [ICCV'19] [64]	6.608	0.595	0.757	0.879	0.733	0.560	0.880
	SuperRetina [ECCV'22] [43]	6.382	0.622	0.767	0.884	0.813	0.516	0.909
	RetinaIPA [MICCAI'24] [71]	5.750	0.657	0.774	0.885	0.799	0.599	0.886
Detector-free Methods	LoFTR [CVPR'21] [62]	7.638	0.526	0.716	0.858	0.693	0.564	0.811
	DKM [CVPR'23] [17]	6.493	0.610	0.760	0.880	0.800	0.529	0.891
	Aspanformer [ECCV'22] [9]	6.415	0.602	0.761	0.881	0.800	0.556	0.877
	RoMa [CVPR'24] [18]	6.388	0.605	0.763	0.881	0.800	0.565	0.875
	GeoFormer [ICCV'23] [42]	6.201	0.625	0.770	0.887	0.813	0.587	0.925
Learning-based Methods w/o Deep Unrolling	KeyMorph [MIDL'22] [20]	5.947	0.640	0.784	0.892	0.813	0.551	0.917
	C2FViT [CVPR'22] [49]	5.842	0.642	0.786	0.893	0.827	0.547	0.920
	GAMorph [BIBM'24] [44]	3.081	0.825	0.895	0.945	0.906	0.800	0.949
Learning-based Methods w/ Deep Unrolling	GraDIRN [MICCAI'22] [54]	6.344	0.657	0.774	0.885	0.799	0.599	0.886
	PDD-Net [MICCAI'19] [28]	5.765	0.688	0.792	0.893	0.819	0.598	0.915
	VR-Net [TMI'21] [38]	4.974	0.705	0.823	0.911	0.827	0.660	0.931
	SmoothProper (Ours)	**1.879**	**0.920**	**0.951**	**0.974**	**0.937**	**0.920**	**0.974**