Your Name: 陈洋 Your ID: 2018533182HW3 Due date: 11:59 PM, September 29, 2019

1 (5') Stack, Queue and Complexity Analysis

Each question has one or more correct answer(s). Select all the correct answer(s). For each question, you get 0 point if you select one or more wrong answers, but you get 0.5 point if you select a non-empty subset of the correct answers.
Note that you should write you answers of section 1 in the table below.

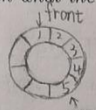
Question 1	Question 2	Question 3	Question 4	Question 5
D	AD	ABCD	A	ABC

C = a/b
r = a - c·b

Question 1. *In the lectures of Week 2, suppose we implement a circular queue by using an array with the index range from 1 to n, then what the size of this queue would be? We assume that the queue is non-empty.*

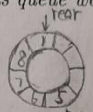
-3/8 = 1 ··· 5

A



5-1+1=5 ✓

B



1-5+1=-3
-3%8=5 ✓

(5-1+8)%8+1=5 D ✓

(A) $rear - front + 1$ ✗

(B) $(rear - front + 1)\%n$ ✗

(C) $(rear - front + n)\%n$ ✗ (+1)

(D) $(rear - front + n)\%n + 1$ ✓

Question 2. *Which of the following is known to be correct?*

(A) Stack is a linear data structure and the operations on stacks are more restricted, the same is true for queue. ✓

(B) Lists store elements in sequential locations in memory. ✗

(C) Both stacks and queues allow us insert or delete an element at the front. ✗

(D) We can use two queues to implement stack. ✓

Question 3. *Which of the following is/are applications of queue and stack respectively?*

(A) ***Queue**: A resource shared by multiple users/processes;* ***Stack**: Handling function calls* ✓

(B) ***Queue**: Loading Balancing;* ***Stack**: Reverse-Polish Notation* ✓

(C) ***Queue**: Handling of interrupts in real-time systems;* ***Stack**: Compilers/Word Processors* ✓

(D) ***Queue**: IO Buffers;* ***Stack**: Arithmetic expression evaluation* ✓

Question 4. *Read the following code, what function does it realize?*

```
void Q4(Queue &Q)
{
  Stack S;
      int d;
      InitStack(S);
      while(!QueueEmpty(Q))
      {
```

```
        DeQueue(Q, d)
        Push(S, d);
    }
    while (!StackEmpty(S))
    {
        Pop(S, d);
        EnQueue(Q, d);
    }
}
```

Q6:

8 2 3 ^ / 2 3 * + 5 1 * -

$2^3 = 8$

8/8 = 1 2*3 = 6

1+6 = 7 5*1 = 5

7-5 = 2

8 / 2^3 + 2*3 - 5*1 = 2

(A) *Use stack to reverse the queue.* ✓

(B) *Use queue to reverse the stack.*

(C) *Use stack to implement the queue.*

(D) *Use queue to implement the stack.*

Question 5. *Which of the following comparison is correct?*

(A) $n^2 + n^3 = O(n^4)$ ✓ $\lim_{n\to\infty} \frac{n^2+n^3}{n^4} = 0$

(B) $\log_2 n = \Theta(\log n)$ ✓ $\lim_{n\to\infty} \frac{\log n}{\log n} = 1$

(C) $\log^2 n = \Omega(\log \log n)$ ✓ $\lim_{n\to\infty} \frac{\log^2 n}{\log(\log n)} = \infty$

(D) $n! = \omega(n^n)$ × $\lim_{n\to\infty} \frac{n!}{n^n} = 0$

$\Omega(g(x))$ $O(g(x))$

$O(g(x))$ $\Theta(g(x))$ $\omega(g(x))$

$\lim_{n\to\infty} \frac{f(x)}{g(x)} =$ 0 $0<c<\infty$ ∞

$\frac{1}{n!} \sim \frac{e^n}{n\sqrt{2\pi n}}$ $(n\to+\infty)$

2 (10') Stack and Queue

Question 6. *(2')The following post-fix expression (Reverse-Polish Notation) with single digit operands is evaluated usng a stack:*

8 2 3 ^ / 2 3 * + 5 1 * -

Note that ^ is the exponentiation operator. Please write down the corresponding in-fix notation A and the final result:

8 / 2^3 + 2*3 - 5*1 = 2 (The process is on the above)

Question 7. *(4')**Describe** how to implement a queue using a singly-linked list. You can use pseudocode or natural language to describe all the operations, especially the key operations.*

First: define a class Single-list, including some functions: bool empty() const ····· determine whether it is empty; Type front() const ····· return the data stored in the first node; void push_back(Type const &) ····· insert a new element at the back of the singly-linked list; Type pop_front() ····· pop the data in the first node.

Then: we can use this singly-linked list to implement a queue.

```
template<typename Type>
class Queue {
  private:
    Single-list list;
  public:
    bool empty() const {
        return the function empty() of the list; };
    Type front() const {
        return the function front() of the list;
    };
    void push(Type const &s)
    {
        return the function push_back(s) of the list;
    };
    Type pop()
    {
        return the function pop_front() of the list;
    };
```

Question 8. *(1')If we use an array with size N to implement a normal queue, it gets full when the index* ***Back*** *pointing to the index =* N-1

Question 9. *(1')By implementing the following operations on stack, the value of x is* a

InitStack(st); Push(st,a); Push(st,b); Pop(st,x); Top(st,x);

Question 10. *(2')What dose "stack overflow" and "stack underflow" mean? (give a short explanation)*

Stack overflow: push something into the stack when it is full

Stack underflow: pop something out of the stack when it is empty

3 (8') Complexity Analysis

Question 11. *(3')Given a fraction of a code as the following, write down the time complexity for each* ***for*** *loop.*

```
for ( i=1; i<n; i*=2) {                    O(log n)
  for( j=n; j>0; j/=2) {                   O(log n)
        for ( k=j; k<n; k+=2) {            O(n)
              sum += (i + j*k)
        }
  }
}
```

Question 12. *(5') Calculate the average processing time $T(n)$ of the following recursive algorithm. Suppose that it takes one unit time for* ***random(int n)*** *to return a random integer which is uniformly distributed in the range [0,n]. Also note that $T(0) = 0$.*
Hints:The equation $\frac{1}{1*2} + \frac{1}{2*3} + ... + \frac{1}{n*(n+1)} = \frac{n}{n+1}$ might be needed.

```
int hw( int n) {
  if ( n <= 0 ) return 0;
    else {
          int i = random( n-1 );
          return hw( i ) + hw( n-1-i );
        }
}
```

$T(n) = \frac{1}{n}\sum_{i=0}^{n-1}(T(i) + T(n-1-i))$ $i \in [0, n-1]$

$nT(n) = n\sum_{m=1}^{n-1} T(m) + n$

$\frac{1}{1\times2} + \frac{1}{2\times3} + \cdots + \frac{1}{n\times(n+1)} = \frac{n}{n+1}$

Guess $T(n) = O(n)$ $\exists c$ s.t. $T(n) \le cn$

$T(n) \le 1 + ci + c(n-1-i)$

$= cn + (1-c)$

$\exists\ c \ge 1$ $T(n) \le cn$

$\therefore T(n) = O(n)$