Proposal: AGT for facility location

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1 Topic

Our topic of the project is the facility location games with opening costs and payments. And all the agents play a dual role of facility and customer.

The classic facility location games concern little about opening costs and payments, but only the customers are strategic players. In this scenario, the government plans to build some public facilities in a street or a general metric space, and each customer has a service cost corresponding to the distance to the nearest reachable facility.

The customers strategically report their locations to the government to minimize their own service cost, and the government decides some locations for the facilities with the received information.

In our project, the agents act both as facility and customer, thus we need to take opening costs and payments into consideration.

Each agent has his/her own location, and the facilities are only allowed to open at the location of some agents. If a facility opens at an agent's location, this agent incurs a opening cost, which is kept as private information, and can receive payments from the government at the same time.

All the agents also act as customers, and if a facility is not opened at an agent's own location, he/she needs to pay a service cost.

The agents' goal is to maximize his/her utility: the difference between the payment he received and the cost he incurred. We want to design a truthful mechanism with good performance and ensures all the agents benefit from participating this game.

2 Motivation

We choose this topic due to several reasons.

Firstly, it is related to the facility location problem imparted from the algorithmic game theory class. But it further extends to a more general and real scenario, where the location of the facility incurs opening cost, and the government needs to make the payment.

Moreover, It is of practical meanings. If we successfully design a mechanism with good performance, we can apply the method to the facility arrangement of our own campus.

For instance, we can solve problems like if we view each layer of the dormitory building as an entity, and we only has limit numbers of microwave, which locations are the best position to allocate them?

We can also solve questions like if we view each dormitory as an entity, and we only want to buy one printer between 3 or 4 dorms, which location is the best position to allocate it?

With the application of the mechanism, we can better allocate the limited resources and satisfy the students' needs.

3 Plan

We are inspired from the algorithmic algorithm class and the paper reviewed by the team members.

First, we just want to extend the facility location problem from a single street in class to a metric space.

We also concern that it is better to make this project practical and serve for the real settings. So we will add a role to the customers as a facility location provider, and think about the payments instead of pure costs.

In this scenario, we will consider a truthful mechanism that realize minimum social cost, which is the idea to have the least overall cost in our application. We will also consider a truthful mechanism that realize minimum bottleneck cost, which is the idea to minimize the cost for the person who would pay most.

At last, we will design a small game among the students to test our mechanism.

4 Basic Model

$N = \{1, 2, \cdots, n\}$: a set of agents (each agent takes a dual role of both a customer and a facility)
(Ω, d) : metric space, where $d : \Omega \times \Omega \to \mathbb{R}_+$ is the metric.
Each agent $i \in N$ is located at $l_i \in \Omega$ and has a facility

opening cost f_i.
$d(i,j) := d(l_i, l_j)$: the distance between any two agents.
$\mathbf{l} = (l_1, l_2, \cdots l_n)$: the profile of locations
$\mathbf{f} = (f_1, f_2, \cdots, f_n)$: opening costs (private information)
b_i : the bid that each agent strategically report
W : a subset of winners $W \subseteq N$
p_i : payment to each $i \in N$
a mechanism $\mathcal{M} = (s, p)$
selection function $s \ : \ \mathbb{R}^n_+ \to 2^N$
payment function $p \ : \ \mathbb{R}^n_+ \to \mathbb{R}^n_+$
$s(\mathbf{b}) = W$, where $\mathbf{b} = (b_1, \cdots, b_n)$
$p(\mathbf{b}) = (p_1, p_2, \cdots, p_n)$

Given a winner set $W \subseteq N$, each agent $i \in N$ bears a cost $c_i(W) = I_W(i) \cdot f_i + d(i, W)$, $I_W(i) = \begin{cases} 1, & \text{if } i \in W \\ 0, & \text{otherwise} \end{cases}$. $d(i, W) = min_{j \in W} d(i,j)$ is the distance between agent and W. ($d(i,) := Q$, where Q is a big constant).

Each agent wishes to maximize his utility: $p_i - c_i(W)$.

Objective function $C : \ 2^N \to \mathbb{R}_+$
Social cost:

$$C(W) = \sum_{i \in N} c_i(W) = \sum_{i \in N} d(i, W) + \sum_{i \in N} f_i$$

Bottleneck cost:

$$C_B(W) = max_{i \in N} c_i(W) = max_{i \in N} (I_W(i) \cdot f_i + d(i, W))$$

5 Reference Mechanism

Mechanism 1 Given bid vector $\mathbf{b} = (b_1, \cdots, b_n)$

(i) The winner set is $W = s(\mathbf{b})$

(ii) For each agent i, payment
$p_i(\mathbf{b}) = (\sum_{j \in N \setminus \{i\}} d(j, S_i) + \sum_{j \in S_i} b_j)$
$- (\sum_{j \in N \setminus \{i\}} d(j, W) + \sum_{j \in W \setminus \{i\}} b_j)$

Mechanism 2 Sort the bids in non-decreasing order: $b_1 \leq b_2 \leq \cdots \leq b_n$. Set $i := 1, W := \{1\}$

(i) **while** $b_{i+1} \leq max\{max_{j \in N} d(j, W), b_i\}$ **do**
$W := W \cup \{i+1\}$; $i := i + 1$;
the winner set $s(\mathbf{b})$ is $W^* := W$

(ii) For each $i \in W$, let r_i^* be the optimal value of the program: minimize r, subject to $max_{j \in N} \leq r$, where $S_r = \{j | b_j \leq r, j \in N \setminus \{i\}\}$; the payment to i is $p_i(\mathbf{b}) := r_i^* - d(i, S_{r_i^*})$. for each $i \notin W$, $p_i(\mathbf{b}) = 0$

Mechanism 3 Given input: bid vector $\mathbf{b} = (b_1, \cdots, b_n)$ and budget B, for each agent $i \in N$, define $\tilde{C}(\{i\}) = \sum_{j \in N} d(j, i) + b_i$. Sort all m agents with bids no more than B as $\tilde{C}(\{1\}) \leq \tilde{C}(\{2\}) \leq \cdots \leq \tilde{C}(\{m\})$.

- If m = 0, the selection function gives $s(\mathbf{b}) = \emptyset$, and there is no payment. Else, $s(\mathbf{b}) = \{1\}$.
- If m = 1, the payment to agent l is $p_l(\mathbf{b}) = B - Q$. If $m \geq 2$, $p_l = min\{\tilde{C}(\{2\}) - \tilde{C}(\{1\}) + b_l, B\} - d(1, 2)$. For each other agent, the payment is 0.

6 Game Design

Considering the demand of printing, we plan to simulate a situation that

1. Each agent gets a room (location) and private information of the printer price (opening cost)
2. There is a limitation of printer number or the total budget for printers are set.
3. Each agent can strategically report bid.
4. The service cost is related to the distance between rooms.

Then we will run the situation between the students and test our mechanism.