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International		IMMC23250946

Multi-Objective Land Use Pattern Planning Model Based on Three-Dimensional Matrix Analysis and Calculus

Summary

Decision makers faced the task of deciding on the best construction plan for a 3-square-kilometer area in Syracuse, NY, USA. The team was required to decide on a series of metrics, collect data and quantitatively evaluate construction plans. In response to the task, the team first decided on geological and human metrics according to data. After this, the team used differential equations to quantify the mutual influence between facilities' activity, geographical conditions and human factors. At last, the team divided the area into a host of grids, and evaluated the suitability of each grid for different facilities in order to give normalized points for different construction plans.

When it comes to the metrics, the team delved into data and brainstormed a series of metrics of geology and human. Geological ones were slope, vegetation coverage, humidity, precipitation, illumination, temperature and so on, while human ones were revenue, maintenance cost, prime cost, employment, visitors flow rate and such. Weights are assigned to each metric, and they are highly changeable according to your demands.

In order to quantify the mutual influence between facilities and the environment, the team took differential equations into consideration. Since the changing rate of metrics was obviously dependent on the activity of facilities, their influence and time, the team could use differential equations to well describe and quantify the relations. By using differential equations, the team can give precise future predictions and better describe the relationship between time and amounts. Based on the differential equations, we have weighted integral equations that can quantify the profits.

Via the model, the team found the best facility construction plan (the highest suitability) for this place and normalized the evaluation results so that the team can better quantitatively assess each grid in the form of scores (out of 100 pts) of a provided construction plan. To apply the model, the team artificially determined two pairs of plans, where each pair included a suitable and an unsuitable arrangements. By substituting these two pairs into the model, the team got the result that the suitable plan scored higher than the unsuitable one among the two pairs.

Our model included good scalability, alongside its well refactoring and decoupling capabilities. After sensitivity and scalability analysis, the team is confident that this model can respond to different construction needs in different regions, while being ready for adjustments according to policy requirements and regional characteristics with a high degree of personalization.

Keywords: Matrix Analysis, Multi-Objective Analysis, Differential Equations and Weighted Integral Operation, Environmental Factors

A Letter To Decision Makers

Dear Community Leaders and Business Planners,

We are writing to provide you with the necessary information regarding the optimal placement of facilities in an area of Red Creek. After careful analysis, our team has developed a mathematical model to address the problem. The model can take into consideration all metrics related to the problem to assess your plans by simulating the interaction between facilities, social and natural conditions. On a greater scale, this not only considers the balance between economic benefits and nature conservation, but also involves the consideration of short-term profit along with long-term sustainability.

Our core idea of the model is the grading system. By evaluating the suitability of each facility on each part of the land, we can evaluate plans and provide the optimal plan for you. Based on the highest score (100 pts) of our best plan, you may provide us with your customized plan so we can assess your plan by Grade A ($85 - 100$ pts), Grade B ($60 - 85$ pts), and Grade C ($0 - 60$ pts) based on our model.

Our model was designed in the following way. We divided the area into nearly 2000 grids (37×54) and set various factors into each grid including topography, climate, environmental impact, economic benefits, social factors, and such. Moreover, our model's algorithm is highly customizable to meet your unique requirements, as we can adjust the weighting of specific metrics during calculations, such as slope, humidity, illumination, temperature, visitors flow rate, and so on.

We applied the model to the land and the result is highly correct. We artificially decided on some parts of the area for four facilities' possibly suitable locations and used the model to calculate the suitability of the plan. By doing so, we can identify the best plan by comparing the suitability scores of the artificial plans with and without location swaps. According to our results, the former is always higher than the latter, which is a testimony to our man-made plan. This can also be applied to your plans as well.

To verify the accuracy and effectiveness of the model, we tested it on a property in Malaysia by comparing the model's results with the actual choice made by real decision-makers. The results showed that the model is highly accurate and effective in determining the optimal location for a sustainable resort.

Our model has several strengths that make it an ideal solution for determining the optimal arrangements of facilities. Firstly, it is highly customizable and can be modified according to specific metrics. Secondly, it can involve every metric you can come up with, making it a highly accurate model.

In conclusion, our team recommends using our mathematical model to evaluate your favored plans. We are confident that our model is accurate and effective in assessing your plans and predicting the optimal location based on various factors. A diagram of our calculated optimal construction plan (Figure 22) is attached to Appendix A .

We look forward to discussing our solution with you further.

Sincerely,
Team IMMC23250946

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1 Introduction

1.1 Background

A decision-maker faced the task of deciding on the best use for a 3-square-kilometer parcel of land located in Syracuse, NY, USA . The property experiences a marine climate with all four seasons, including snowy winters, and has adequate water and power supplies. It is located 50 kilometers away from urban areas of the city, with adequate road and transit systems. In October 2022, the concept of building a semiconductor fabrication facility in Clay, NY, just north of Syracuse, was proposed to increase the employment rate.

1.2 Restatement of the Problem

The problem at hand is to determine the optimal use of a 3-square-kilometer area in Syracuse, taking into account various factors such as geography, climate, business opportunities, community needs, and local culture.

To achieve this, we must first establish a series of metrics in order to give a clear definition of “best use”, which can instill confidence in the decision maker regarding the final use of the land.

Next, we must select at least two options from the list of facilities and evaluate them according to the established metrics of “best use”. We should identify the key factors of the metrics and support their reasoning with specific data. This will allow us to defend their metrics against challenges and ensure that their results are consistent with reality. The introduction of the Micron Technology semiconductor fabrication facility, which is expected to employ up to $9,000$ people with an additional $40,000$ jobs among suppliers, construction firms, and other businesses, is an external factor that must be considered. We should re-evaluate the options after this change and test the sensitivity of our metrics.

Finally, the team should assess the suitability of the model by using it to evaluate a familiar environment and determine how generalizable the model is.

2 Assumptions

To simplify the problem and make it convenient for us to simulate real-life conditions, we make the following basic assumptions, each of which is properly justified.

- **Pollutants all follow the convective diffusion equation.** The convective diffusion equation is stable and the most widely used equation for particles. Considering the pollutants are mostly chemicals, the convective diffusion equation is suitable.

- **All media are isotropic.** The main media in the property are soil and air, which can be seen as equally mixed - thus they are isotropic media. With isotropic media, partial differential equations require less time complexity.

- **If no facilities are built, the population stays stable.** According to Wikipedia, the population growth rate of Syracuse is close to 0% from 2000 to 2020. With an educated guess, the population will not be able to change the overall solution.

- **Short-term benefits are those earned in one year.** If “short-term”is defined to be less than one year, special facilities such as cross-country skiing facility may not be profitable as it opens just for 3 months.

- **Long-term benefits are those benefits earned in ten years.** This allows for sufficient time to recover from market fluctuations and benefit from compound interests, aligning with some common financial goals such as retirement planning, college savings, or a house purchase. This also reduces the impact of taxes and fees on investment returns.
- **The increase in tourists generated by the agglomeration effect is negligible.** In this specific condition, there are only two types of facilities that depend on tourists: an agritourist center and a cross-country skiing facility. These two facilities operate in different seasons, thus they will not interfere with each other by agglomeration effect.
- **Facilities and environment influence each other by causing effect on grids.** We simplify the mutual influence between facilities and environment to the influence on grids. In other words, they affect each other via grids as they depend on conditions of their grids.
- **Geological metrics follow normal distribution.** The probability of most events in nature follow the normal distribution, thus it is rather appropriate to assume that all geological metrics in this case are the same.

With the appropriate assumptions and justifications above, we can make mathematical model more feasible to build while being realistic as much as possible.

3 Notations

Symbol contained in this paper are shown and described in Table1 .

Table 1: Notations contained in this paper

Symbols	Description
M	Number of metrics
N	Number of facilities
Q	Type of facilities
T	Total time of operation
P	Coefficient of profit
α	The value of the metric
$h_c(t)$	The function to describe the relation between the conversion rate and time
S	The function to describe the relation between the profit and time
(x, y)	Position on map
$\text{pos}_k = (x_k, y_k)$	Position of the k^{th} facility
$j = \text{type}_k$	Type of the k^{th} facility
$[f_i]_{i=1}^{n}$	Matrix that contains f_i, $i \in [1, n]$
$f(\cdot)$	A function f where the dot $(\cdot)$ stands for expression

$\begin{bmatrix} A \\ B \\ C \end{bmatrix}$	$A \times B$ matrix with every element a $1 \times C$ matrix
$\begin{bmatrix} A \\ B \end{bmatrix}$	$A \times B$ matrix
$\begin{bmatrix} B \end{bmatrix}$	$1 \times B$ matrix
U	A matrix of $\begin{bmatrix} Q \\ M \end{bmatrix}$ that describes how facilities affect the environment
V	A matrix of $\begin{bmatrix} M \\ Q \end{bmatrix}$ that describes how the environment affects the facilities
A	A value that describes the activity of any facility
W	A matrix of $\begin{bmatrix} 1 \\ M \end{bmatrix}$ that describes the weight of every metrics

4 Model Overview

In our model, we aim at three goals: making the profit as high as possible, lowering the financial risk caused by pollution, and protecting the environment as much as possible.

To solve the problem, we firstly determined metrics which include both geological ones and human ones. Geological metrics are slope, vegetation coverage, humidity, precipitation, illumination, temperature and so on, while human metrics include revenue, maintenance cost, prime cost, employment, visitors flow rate and such. In our model, each metric is assigned with a weight 1; however, the weight is highly changeable according to decision makers' demands. For facilities, we consider cross-country skiing facilities, crop farms, ranches, regenerative farms, and agritourist centers.

Secondly, we get the suitability-time function. By using mathematical calculations, we can get the functions of these metrics changing throughout the time. Then, we multiply each function with certain conversion rate functions $h_c(t)$, and get different values representing how suitable it will be.

After calculating every metrics in discrete time, we get discrete functions of time. Traverse the upper steps for every piece of land property, and we get different matrices from different aspects containing suitability-time function.

Figure 1 is the algorithm of calculating the activity-time $[A(t)]$ function and the metric-time $[\alpha(t)]$ function. in the figure, V is the impact of the metrics on the facility in its own position; U is the impact of the facility on the value of the metrics; W is weight of each metric to the facility.

$$A(t) \xrightarrow[\alpha_{(t)}]{V,W} A(t+\mathrm{d}t)$$

$$\alpha_{(t)} \xrightarrow[A(t)]{U_{\mathrm{type}_k,i}} \alpha_{(t+\mathrm{d}t)}$$

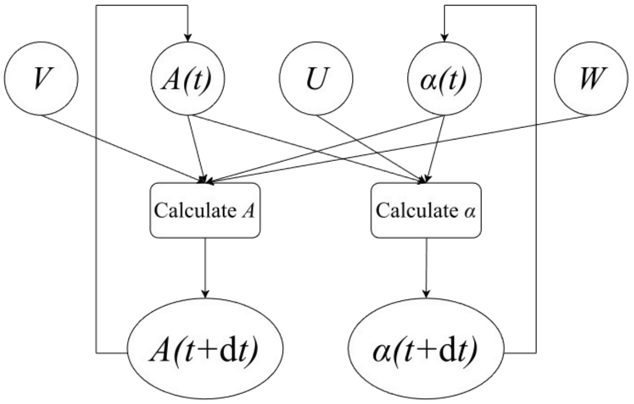

Figure 1: The algorithm for calculating activity and metric function

Overall, the algorithm can be summarized into the flowchart in Figure 2. In Figure 2, A is the "activity"of the facilities, and P is the coefficient of profit.

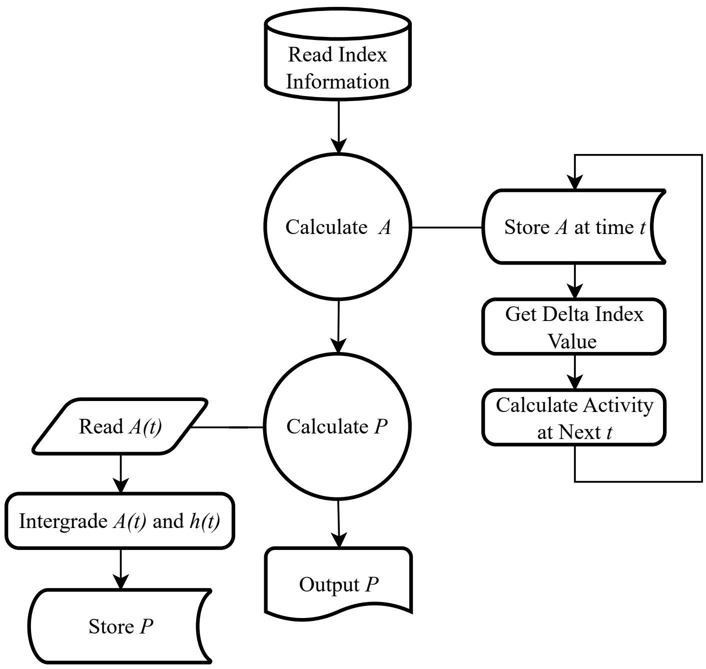

Figure 2: Flowchart of the algorithm

5 Mathematical Model

5.1 Overall Model

5.1.1 Preparatory Model

We start by dividing the land into grids, each of which occupies an area of 61 by 35 meters. Gridlines are shown in Figure 3

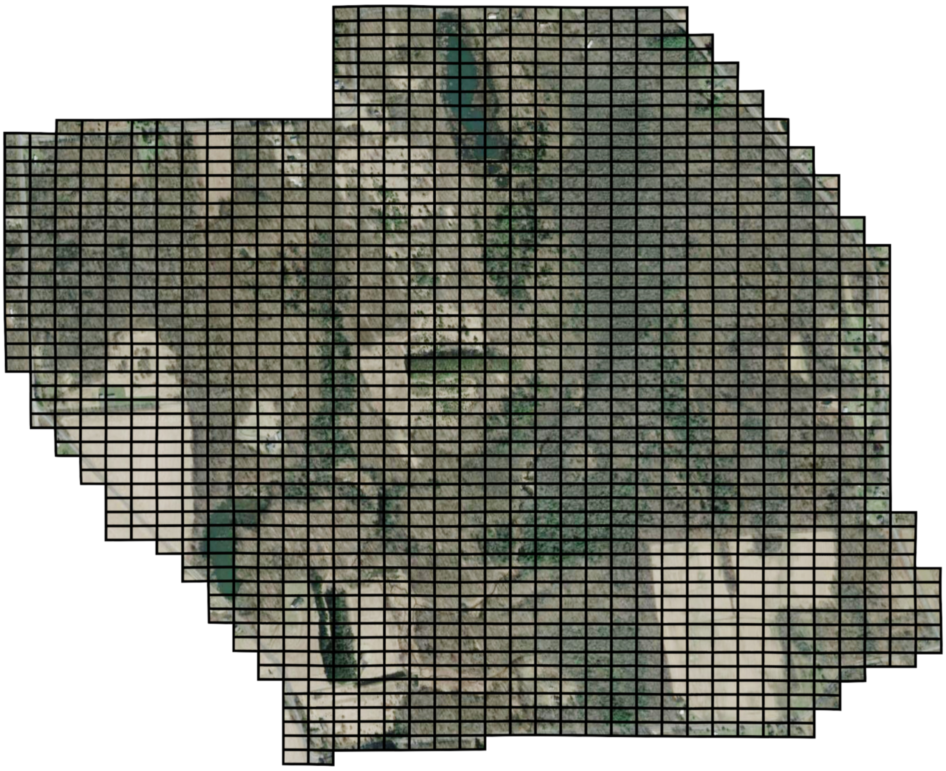

Figure 3: Devision of the the property

Now, the property can be seen as 37×54 grids with some of them discarded since their locations are out of the boundary; thus there are only about 1800 grids in total. By substituting metric values into the grid, we locate data into reality and the data also get discrete. For one of the metrics: slope, see Figure 4 down below.

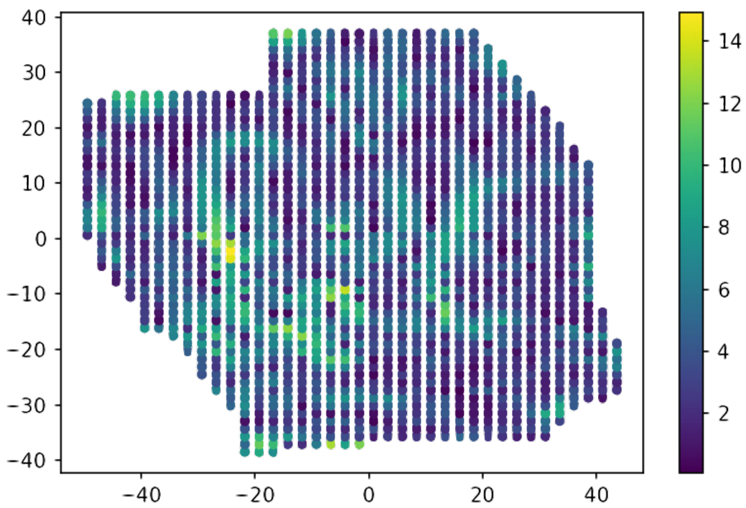

Figure 4: The slope of each grid

With each grid traversed by coordinates, we get a matrix describing all metrics in all grids – see function (1).

5.1.2 Definitions

We first define the following concepts, each of which has no dimension:

Define M as the number of metrics.

Define N as the number of facilities.

Define Q as the type of facilities.

Define $\alpha\left[(x, y), i, t\right]$ as the value of the i^{th} metrics at position (x, y) and time t:

$$\alpha = \begin{bmatrix} \begin{bmatrix} \alpha_{(0,0),0}(t) \\ \vdots \\ \alpha_{(0,0),M-1}(t) \end{bmatrix} & \cdots & \begin{bmatrix} \alpha_{(0,y_{max}-1),0}(t) \\ \vdots \\ \alpha_{(0,y_{max}-1),M-1}(t) \end{bmatrix} \\ \vdots & \ddots & \vdots \\ \begin{bmatrix} \alpha_{(x_{max}-1,0),0}(t) \\ \vdots \\ \alpha_{(x_{max}-1,0),M-1}(t) \end{bmatrix} & \cdots & \begin{bmatrix} \alpha_{(x_{max}-1,y_{max}-1),0}(t) \\ \vdots \\ \alpha_{(x_{max}-1,y_{max}-1),M-1}(t) \end{bmatrix} \end{bmatrix} \Leftarrow \begin{bmatrix} x_{max} \\ y_{max} \\ M \end{bmatrix} \tag{1}$$

which can also be written as:

$$\alpha = \left[\left[\alpha_{(x,y),i}\left(t\right)\right]_{i\in[0,M)}\right]_{x\in[0,x_{max}),y\in[0,y_{max})} \tag{2}$$

The upper matrix is a three-dimensional matrix with the maximum size of $x_{\max}$, $x_{\max}$, M. For each grid there are M metrics in total, as there would be a $\left[M\right]$ matrix to describe the total α of the grade.

For a clearer understanding, matrix α can be visualized as Figure 5: at a certain moment , the x and y axis indicate their position on the map, namely (x, y) in the matrix. The three layers means the figure of grids and two of the M metrics, which respectively are vegetation coverage,grids and slope.

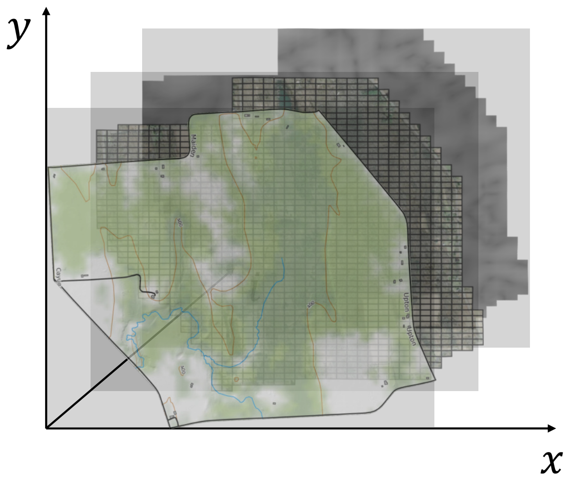

Figure 5: Visulization of different indices conbining

Define $U_{j,i}\left(\text{pos}_k, \text{pos}\right)$ as the impact of the j^{th} kind facility in pos_k (x_k, y_k) on the value of i^{th} metric in pos (x, y). Here is the overall form of U:

$$U = \begin{bmatrix} U_{0,0}(\cdot) & U_{0,1}(\cdot) & \cdots & U_{0,M-1}(\cdot) \\ U_{1,0}(\cdot) & U_{1,1}(\cdot) & \cdots & U_{1,M-1}(\cdot) \\ \vdots & \vdots & \ddots & \vdots \\ U_{Q-1,0}(\cdot) & U_{Q-1,1}(\cdot) & \cdots & U_{Q-1,M-1}(\cdot) \end{bmatrix} = \left[U_{j,i}\left(\cdot\right)\right]_{j\in[0,Q),i\in[0,M)} \Leftarrow \begin{bmatrix} Q \\ M \end{bmatrix} \tag{3}$$

Define $V_{j,i}\left(\alpha\right)$ as the impact of the environment on the facility in its own grid where facility type is j, metric is i and the value of which is α. The overall form of V is:

$$V = \begin{bmatrix} V_{0,0}(\cdot) & V_{0,1}(\cdot) & \cdots & V_{0,Q-1}(\cdot) \\ V_{1,0}(\cdot) & V_{1,1}(\cdot) & \cdots & V_{1,Q-1}(\cdot) \\ \vdots & \vdots & \ddots & \vdots \\ V_{M-1,0}(\cdot) & V_{M-1,1}(\cdot) & \cdots & V_{M-1,Q-1}(\cdot) \end{bmatrix} = \left[V_{i,j}\left(\cdot\right)\right]_{i\in[0,M),j\in[0,Q)} \Leftarrow \begin{bmatrix} M \\ Q \end{bmatrix} \tag{4}$$

Define W as the weight of each metric:

$$W = \begin{bmatrix} w_0 \\ w_1 \\ \vdots \\ w_{M-1} \end{bmatrix} \Leftarrow \begin{bmatrix} M \end{bmatrix} \tag{5}$$

Define $A_k\left(t\right)$ as the activity of the k^{th} facility at moment t, while the information of the k^{th} facility as well contains its position (x_k, y_k) and its type type_k.

The overall matrix form of $A\left(t\right)$ is:

$$A(t) = \begin{bmatrix} A_0(t) \\ A_1(t) \\ \vdots \\ A_{N-1}(t) \end{bmatrix} \Leftarrow \begin{bmatrix} N \end{bmatrix} \tag{6}$$

whose calculation formula is

$$A(t) = \left[\frac{\left[V_{i,\text{type}_k}\left(a_{\text{pos}_k,i}(t)\right)\right]_{i\in[0,M)} \times W^T}{\sum W}\right]_{k\in[0,N)} \tag{7}$$

Define S as the function to describe the relation between the profit and time for every kind of facility, and can present as a matrix:

$$S = \begin{bmatrix} S_0(t) \\ S_1(t) \\ \vdots \\ S_{Q-1}(t) \end{bmatrix} \tag{8}$$

Define $h_c\left(t\right)$ as the function describing the relation between the conversion rate and time, where $c = 1$ or 2; when $c = 1$, $h_c\left(t\right)$ stands for long-term function and when $c = 2$, $h_c\left(t\right)$ stands for short-term one. For instance, we can design the $h - t$ function like the sigmoid function σ in Figure 6 below.

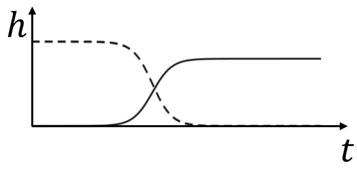

Figure 6: An example for the $h-t$ function (h_1-t is the solid line; h_2-t is the dotted line)

As we see, the value of $h_1(t)$ in the beginning is low, while that of the $h_2(t)$ is comparatively high. After crossing the defined boundary of "short-term "and "long-term ", the size relation of those two value interchange. The figure of $h_c(t)$ can well describe the trend of long-term and short-term profit.

Define $P(r, t_{total}, \alpha_0, W)$ as the coefficient of profit; where r is the weight of long-term profit and correspondingly $(1-r)$ is the weight of short-term profit, t_{total} is the total time the decision-maker considers and α_0 is the value of α when $t=0$.

5.1.3 Differential and Simulation

In order to quantify the mutual influence between facilities and the environment, we take differential equations into consideration. Since the changing rate of α is dependent on the activity of facilities, their influence, and time, we decided to use differential equations to describe and quantify the relations. The equations are:

$$\frac{\mathrm{d}\alpha_{\mathrm{pos},i}(t)}{\mathrm{d}t} = A(t)^T \times \left[U_{\mathrm{type}_k,i}\left(\mathrm{pos}_k, \mathrm{pos}\right)\right]_{k\in[0,N)} \tag{9}$$

$$A(t) = \left[\frac{\left[V_{i,\mathrm{type}_k}\left(a_{\mathrm{pos}_k,i}(t)\right)\right]_{i\in[0,M)} \times W^T}{\sum W}\right]_{k\in[0,N)} \tag{10}$$

$$\alpha(0) = \alpha_0 \tag{11}$$

For the differential equation, the right part of it conveys the idea mentioned above. The result of matrix of the overall activity cross matrix of the overall mutual influence on position (x, y) reflects the total absolute impact on (x, y) 's $\alpha_{\mathrm{pos},i}$. (10) means that the activity of a facility equals to the sum of all products of the impact times corresponding weight and divide by the sum of all weights. Function (11) stands for the initialization of α

5.1.4 Integral and Profit Calculating

In order to quantify the profit, we have to integrate all the profit-related coefficients and functions mentioned above. We have already defined a function that consists of all these things: $P(r, t_{total}, \alpha_0, W)$; then we analyze the relation between the vital variables t and t_{total}, and sort out the integral equations:

For long-term profit, there is:

$$P_{long}(r_0, T, \alpha_0, W) = \int_0^T r_0 h_1(t)\, A(t)^T \times \left[S_{\mathrm{type}_k}(t)\right]_{k\in[0,N)} \mathrm{d}t \tag{12}$$

For short-term profit, there is:

$$P_{short}(1-r_0, T, \alpha_0, W) = \int_0^T (1-r_0)\, h_2(t)\, A(t)^T \times \left[S_{\mathrm{type}_k}(t)\right]_{k\in[0,N)} \mathrm{d}t \tag{13}$$

where the left parts of the upper equations are the total of coefficient of profit and the right parts are long-term profit or short-term profit. These two profit data are calculated by multiplying activity, the profit and the conversion rate, which accords to the left of the equation and proves the rationality of the integral relation.

Obviously:

$$P_{total} = P_{long} + P_{short} \tag{14}$$

Overall, consider equation (12) (13) (14):

$$P(r_0, T, \alpha_0, W) = \int_0^T (r_0 h_1(t) + (1 - r_0) h_2(t)) A(t)^T \times \left[S_{\text{type}_k}(t)\right]_{k\in[0,N)} \mathrm{d}t \tag{15}$$

5.2 Sub-model I: Diffusion Equation

By taking degradation and absorption out of consideration and viewing the diffusion of the polluted chemical as an one -dimensional dynamic system, we can describe the diffusion of the pollution in the soil with the time fractional diffusion equation:

$$D \cdot \frac{\partial^2 C(x,t)}{\partial x^2} - \frac{\partial C(x,t)}{\partial t} = 0 \tag{16}$$

The equation is a second order partial differential equation, so we need to find two boundary conditions: Semi-infinite boundary and Cauchy boundary. The Semi-infinite boundary refers that the concentration gradient of a place that is infinitely far from the source is 0, which is suitable when the weight of diffusion in the whole spreading process is big.

$$\frac{\partial C(x,t)}{\partial x} = 0,\ x = \infty \tag{17}$$

The Cauchy boundary is a mixture of several kind of condition, which can be written in the form of:

$$\lambda \frac{\partial C(x,t)}{\partial x} + \mu C(x,t) = 0,\ x = H \tag{18}$$

λ and μ are both the constant of the boundary condition, and $\frac{\mu}{\lambda}$ is the boundary parameter.

Then it is able can solve this equation by using these two boundary conditions. The two boundary conditions is involved in integrating of the equation, as well as the original condition.

The solution of the equation is:

$$C(x,t) = \frac{\frac{Q}{K_d} \exp \frac{-x^2}{4D\rho t}}{(4\pi D\rho t)^{\frac{3}{2}}} \tag{19}$$

In the equation, D is the average diffusion coefficient of all the polluted chemicals, Q is the total amount of the chemicals that is polluted from the beginning to the end of the investigation period. ρ is the bulk density of the soil, which measures the mass density and porosity of the soil, both of which are related to the diffusion. K_d is the soil-water partition coefficient, t for time, and x is the distance between the source and the observe spot.

However, in order to better use this sub-model to meet with our overall model, the variable t should be eliminated because the diffusion process spans too short on timescale in comparison with our boundaries for short-term and long-term profit (which are respectively 1 year and 10 years). As we only consider a static state of pollutants being diffused, the impact of the pollution would stay still on timescale and is only related to the distance between the source of pollution and the detecting point.

The result is:

$$C(x) = \frac{Q}{K_d \cdot 4\pi D\rho x} \tag{20}$$

5.3 Sub-model II: Impact Function of Environment to Facility

According to the assumptions, we regard the possibility distribution of geological metrics as normal distribution. By collecting data, we discovered that there is always a proper range of each metric $(\alpha_{min}, \alpha_{max})$, where the most suitable value of the metric α_{suit} also exists in the range. The value of V also has its range as it has positive relation with α. Thus, there should be:

$$V_{i,j}(\alpha) \sim N\left(\mu, \sigma^2\right) \tag{21}$$

$\mu = \frac{\alpha_{max}+\alpha_{min}}{2}$ is the arithmetic mean and σ is the standard deviation.

Then, we define $q(l, r, \mu, \sigma)$ as the shadow area in Figure 7.

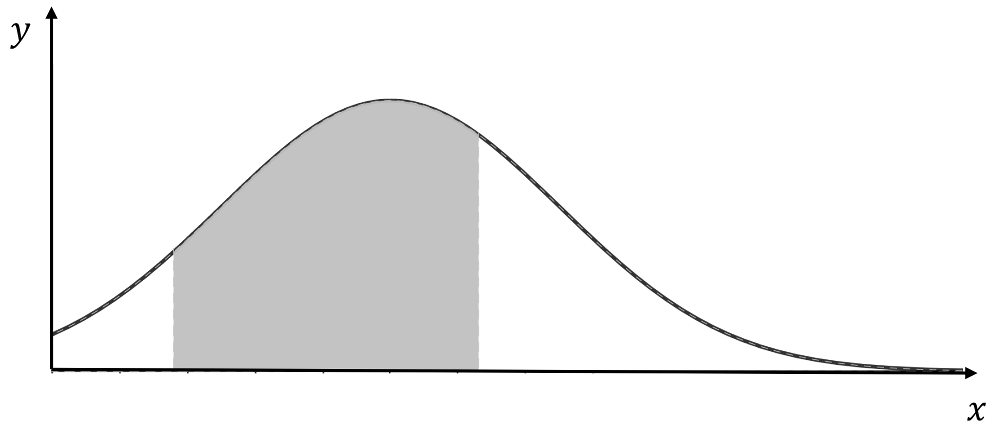

Figure 7: The Actual Meaning of Function q

where l and r respectively refers to the minimum and the maximum of the metric i in a specific position. It is proved that the calculation formula of q is:

$$q(l, r, \mu, \sigma) = \frac{1}{2}\text{erf}\left(\frac{x-\mu}{\sqrt{2}\sigma}\right)\Bigg|_l^r \tag{22}$$

In order to make sure the σ of the normal distribution is reasonable, we need to determine the threshold of the distribution, which can be present as the following form.

$$\frac{q\left(l_{i_{suit}}, r_{i_{suit}}, \mu_i, \sigma\right)}{q\left(\alpha_{i_{min}}, \alpha_{i_{max}}, \mu_i, \sigma\right)} = Ts \tag{23}$$

We artificially determine a reasonable threshold Ts to be 50%. In the equation, l, r, α_{min}, α_{max}, μ and Ts are all known constants, hence we can use the iterative method to solve the standard deviation. The solution of σ helps us to quantify and add up the value of different metric and calculate the overall V.

The equation of V after normalization is:

$$V_{i,j}(\alpha) = \frac{1}{\sqrt{2\pi}q\left(\alpha_{i_{min}}, \alpha_{i_{max}}, \mu_i, \sigma\right)\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \tag{24}$$

6 Application

6.1 Factors of the Model

In our model, there are many factors which needs to be defined when the model is being used to work out real problem.

Firstly, we need to decide the usage of the land. In the given condition, the land can be used to build outdoor sports complexes, cross-country skiing facilities, crop farms, grazing farms/ranches, regenerative farms, solar arrays, agrivoltaic farms and agritourist centers. If the model is used in different condition, other usage such as factories, urban lands, greenbelts, etc. can also be involved.

Then, there are the natural and human influence to be deliberate. What presented in the basic information of problem is geographic factors including elevation, slope, aspect, tree and land cover data.

Additionally, we found the information of rainfall, temperature, soil quality, illumination time, visitors flow rate, population growth, etc. on the government websites and other authorized sources. More factors can be chosen or canceled when encountering different condition with different requirements. The proportion of the natural factors and human factors can also be modified to accord with the need.

6.2 Factor Quantification and Function Design

After considering the factors qualitatively, we need to quantify these factors and connect them to our ultimate goal. In this problem, the ultimate goal is to gain large profit through time, so we need to figure out the conversion relationship between factors mentioned above, time and profit. For example, the pollution factor could gain negative impact to the facility because the damage to environment could decrease the total production of agriculture, push away the tourists and affect the sustainability of the development of the entire property. The academic literature indicates that the damage to the environment and the concentration of pollution are in direct ratio, while the concentration at any position outside the pollution source can be calculated according to the time fractional diffusion equation. Thus, the weakening effect of pollution can be quantified.

When considering other factors like visitors flow rate, which could bring positive effect to the tourism, the corresponding function would change and likely to be a positive ratio function.

Further expansion can be similarly produced according to other specific needs.

6.3 The Verification of Application to the Given Problem

To verify the model, we first decide three part in Red Creek to build two kinds of facilities, and decide the solution according to our common sense. Then, we put the data we found into our algorithm and respectively calculate the absolute profit coefficient P. In contrast to the reasonable solution, we swap the position of the two facilities, which would be very unreasonable, and the output of the algorithm should be considerably lower than the previous result. By comparing this, we could roughly make out the reliability of the algorithm.

The first pair of facility is the cross-country skiing facility and the crop farm. We divide three parts of the land according to its slope. We first put the cross-country skiing facility to the single one with greater slope, while the crop farms were put to gentler lands. Then we swap them. The detailed configurations are as follows:

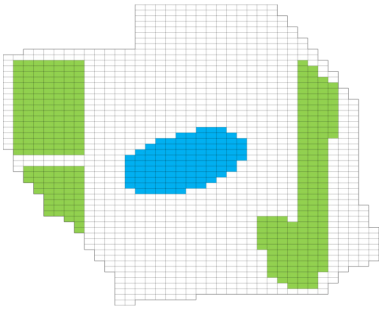
Figure 8: Reasonable Configuration

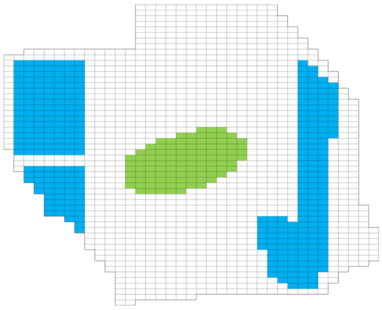
Figure 9: Swapped Configuration

The blue part is the cross-country skiing facility, and the green part is the crop farm. The two outputs of the algorithm are:

Score of the reasonable solution: 28619(69.4 pts)

Score of the swapped solution: 19671(47.7 pts)

The second pair of facility is the regenerative farm and the agritourist center. We divide three parts of the land according to its slope as well. We first put the agritourist center to the single one with greater slope, while the regenerative farm were put to gentler lands, which is obviously better than the swapped ones. Then we swap them. The detailed configurations are as follows:

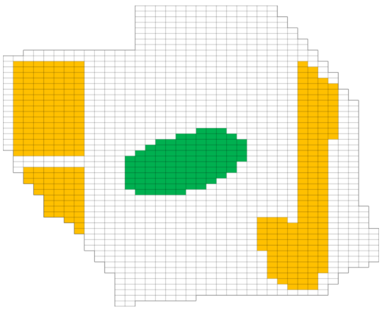
Figure 10: Reasonable Configuration

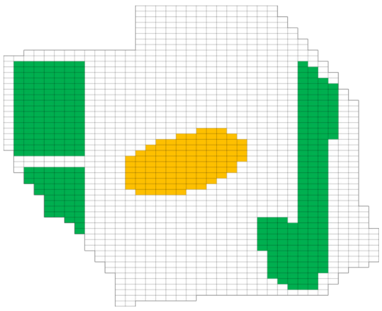
Figure 11: Swapped Configuration

The orange part is the regenerative farm, and the dark green part is the agritourist center. The two outputs of the algorithm are:

Score of the reasonable solution: 26368(64.3 pts)

Score of the swapped solution: 18510(44.9pts)

7 Solution

We calculated each the coefficient of profit of every facility on every piece of land. In Figure 12, greener colors means more suitable configuration, and redder colors means less suitable configuration. However, color difference between different facilities does not necessarily mean a difference in the coefficient of profit between facilities. Please see Appendix A for more information about the optimal solution.

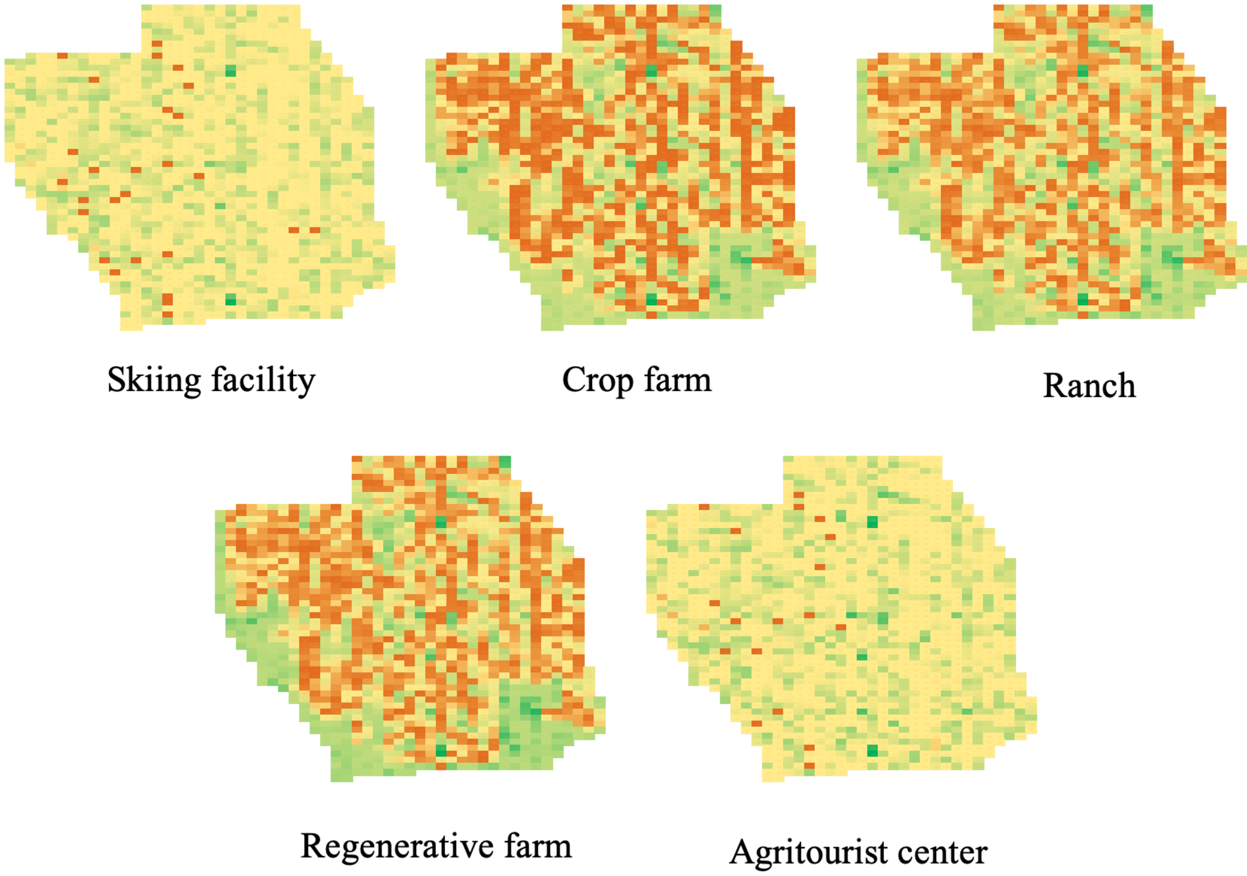

Figure 12: Heatmap of facilities'coefficient of profit

8 Model Verification

To prove the mathematical model's accuracy and effectiveness, we would verify it by selecting another location and put the raw information into the model. Then compare the model's results with the actual choice of the decision makers. If the result is the same or similar, then the model should be accurate and effective.

We choose a property in Malaysia to be the calibration. A satellite view of the property is provided in Figure 13 and Figure 14 The property's boundaries are defined by four roads and a beach:

- Northern boundaries: Jalan Bukit Cheraing
- Western boundaries: Laluan Presekutuan 3 & Jalan Kampung Cherating Lama & Jalan Pantai
- Southern boundaries: Cherating Beach
- Eastern boundaries: Cherating Beach

Figure 13: A satellite image of the parcel of land

Figure 14: Location of the parcel of land

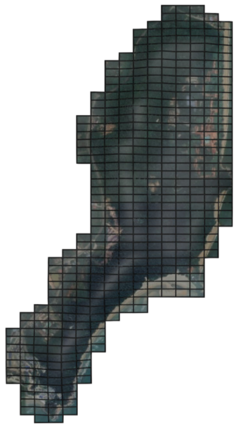

Figure 15: A satellite image of the parcel of land

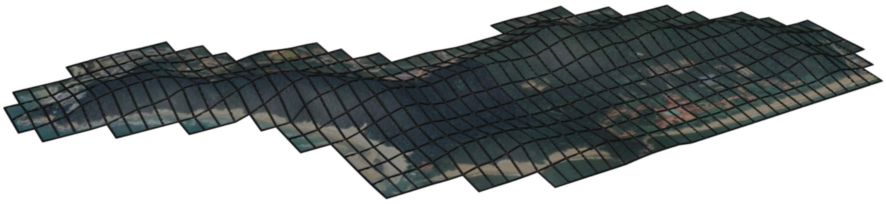

Figure 16: Location of the parcel of land

This property can also be divided into 61 meters by 35 meters grids shown in Figure 15 and Figure 16. The center of the property is located at 4°8′N 103°24′E in a rural tropical rainforest climate zone. The continental shelf is shallow, so it is not suitable for large ports for maritime transport. The river flows fast and the riverbed is meander, so it is also not suitable for either type of waterway transportation. Malaysia was lack of high-tech industries and scientific research capabilities, and cannot afford to build a research center; also, the area is humid and experiment equipments are prone to mold. Although Malaysia have Tin ores, this property is too close to the ocean thus very dangerous to mine , and likely to pollute the ocean. The surrounding area had a low population density in 1990 when the property was first planned. The forest is so dense to build expressways or railroads, thus inconvenient and inefficient to transport large amount of goods by land. For more information about this property, see Appendix A .

In conclusion, it is not suitable for labor-oriented industries, raw-material-oriented industries, energy-oriented industries, power-oriented industries, and technology-oriented industries. The property is surrounded by hills and oceans on west and east side accordingly, so that it is not suitable for agriculture and animal husbandry.

Taking the above into consideration, it is obvious that the property is neither suitable for agricultural nor industrial development. The option left is to develop service industries: including hotels. In reality, the property is currently dominated by Club Med Cherating Beach Hotel owned by the Fosun International.

Our model suggests, when the property is covered with fields, the coefficient of profit P is 37.9 pts in Grade C; when covered with hotels, the coefficient of profit P is 92.2 pts in Grade A .

Our model fits well with geographical analysis and the actual solution, and proved that out model is reasonably accurate and promotable.

9 Sensitivity Analysis

9.1 Impact from the Population

In October 2022, it was announced that a semiconductor fabrication facility 50 kilometers east of the property will be built. It will directly and indirectly create about $49,000$ jobs. Assume 30% of the workers will move to the city and their families will come with them, then there will be about $44,000$ additional people living in the city of Clay.

The fab is closer to the urban center, and shall have more jobs created, so that fewer people will rely on the jobs created by our unplanned property in Red Creek. Meanwhile, more people will live in urban areas, so it will be important for them to relax in the country, thus companies will be more profitable if they develop tourism. So we can reduce the weight for employment and raise the weight for population and visitors flow-rate to get a more suitable solution.

Following the solution in Figure 8, we get Figure 17. It shows the coefficient of profit changes with the addition in weight for population and visitors flow-rate, as well as the subtraction in weight for employment.

"Control" in the figure means the control group, which means it was the same as the original coefficient of profit in Figure 9. As the independent variable of the x-axis increases, the weight for population, visitors flow-rate and the subtraction in weight for employment

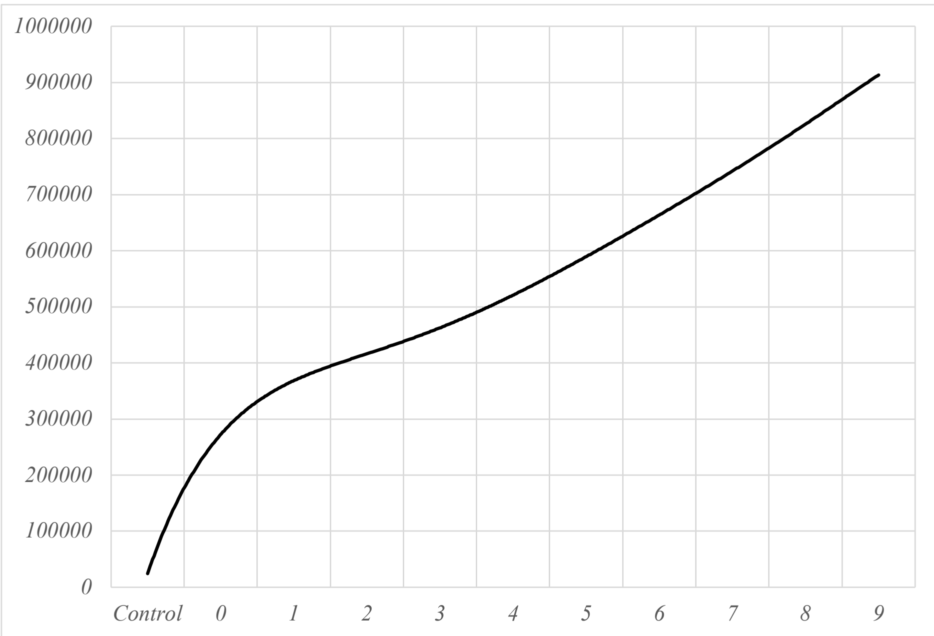

Figure 17: Impact from the population change

With the change, we can see obvious increase in coefficient of profit: increasing from $200,000$ pts to just under $1,000,000$ pts. Therefore, the population does have a large impact on the solution.

9.2 Impact from the Number of Metrics

With fewer metrics, the coefficient of profit will be more sensitive to any change of the solution and have fewer points. While making small changes to the overall solution, we decrease the number of metrics that are taken into consider and get the result in Figure 18

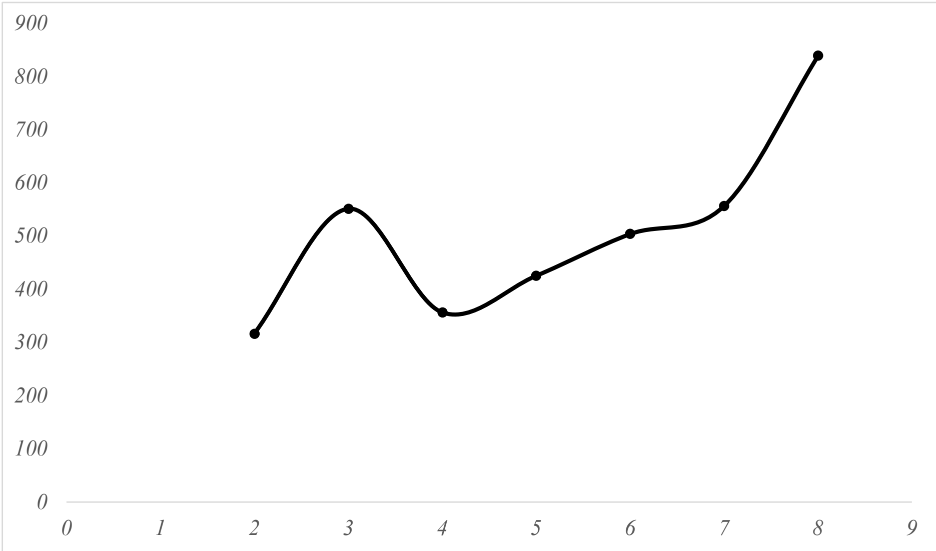

Figure 18: Impact from number of metrics

From Figure 18, we can see a sudden rise at the three metric place, which is in line with our prediction.

9.3 Impact from the Total Time

Accounting prime costs, the longer the total time, the more profitable it would get. Thus, short-term and long-term benefits or costs should vary significantly. We simulate the total time from one year to ten years, with a step size of one year.

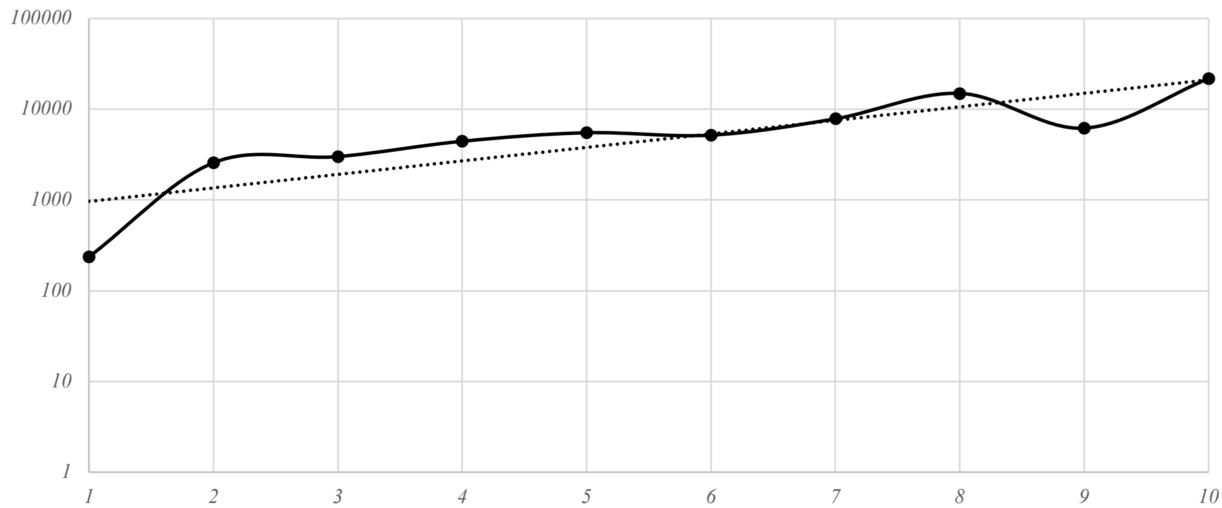

Figure 19: Impact from Operation Time (log-based y-axe)

Shown in Figure 19, the total time has a linear relationship with the logarithm of coefficient of profit P, with a goodness of fit of $R^2 = 0.782$. It shows that out model has a reasonably good sensitivity.

10 Strength and Weakness

10.1 Strength

1. Our model divides the property of the entire Red Creek into just short of 2000 grids, which is able to take the difference in topography and environment of grids into consideration of the whole solution. This model can eliminate some facilities on steeper slopes to make the solution more realistic. Furthermore, the pollution caused by some facilities will distribute in the soil can also be taken into consideration. Generally, the model can be used to analyze complex and varying terrain.

2. The basic structure of our overall model allows highly customized needs and a large range of expansion. The function of customization is developed because the algorithm $\alpha_{pos,i}(t)$ can be modified according to a specific metric i such as slope, humidity or temperature. Similar modification can be made in U and V, which is used to measure the interaction between all the grids to one fixed grid when the range of i is affirmation. Furthermore, the model is expandable for the use of nearly every problem whose pivotal difficulty is to quantify the impact between considered grids. This is because the form of "matrix $\times$ matrix "in the model can reflect all the mutual influence between each grid due to the feature of the cross multiplication.

3. The model has the potential to be very accurate because theoretically it can involve every metric that the problem requires to consider. Also, the sub-function used to quantify the metric is customized, which can also be very accurate if there are sufficient data.

10.2 Weakness

1. The model is simplified. It assumes that each grid within a grid is homogeneous, and does not take into account variations within each grid. This oversimplification may result in inaccurate results in some scenarios.

2. The model is difficulty to calibrate. The model requires calibration to ensure accurate results, which can be a time-consuming and challenging process.

3. The model is relatively complex and may require significant technical expertise to develop and operate, making it less accessible to those without specialized training.

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A Optimal Solution and Regional Information

A.1 Regional Information of Chertaing Beach

Figure 20: Regional information of the property celected in Malaysa – medium

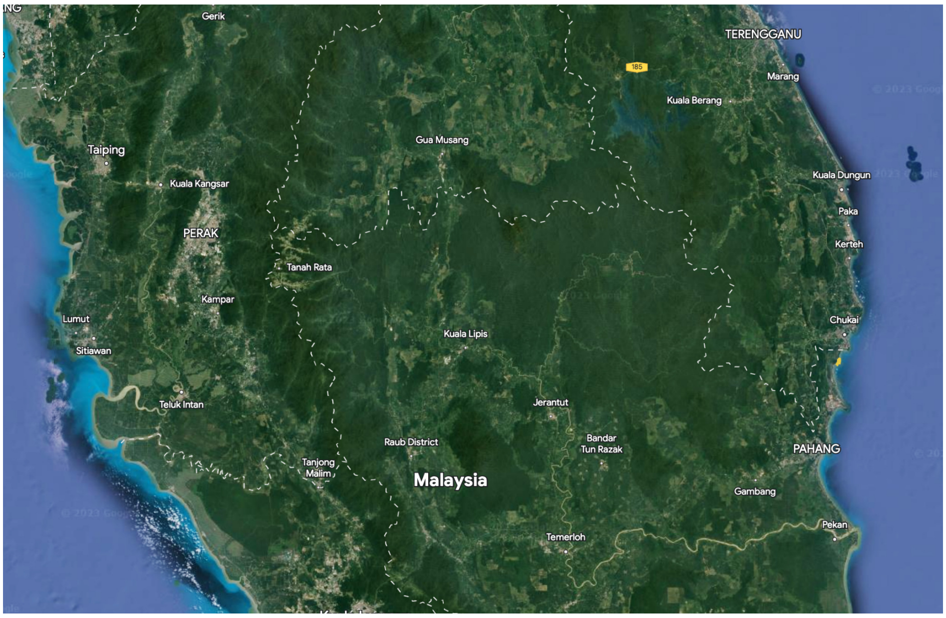

Figure 21: Regional information of the property celected in Malaysa – large

A.2 The Optimal Solution

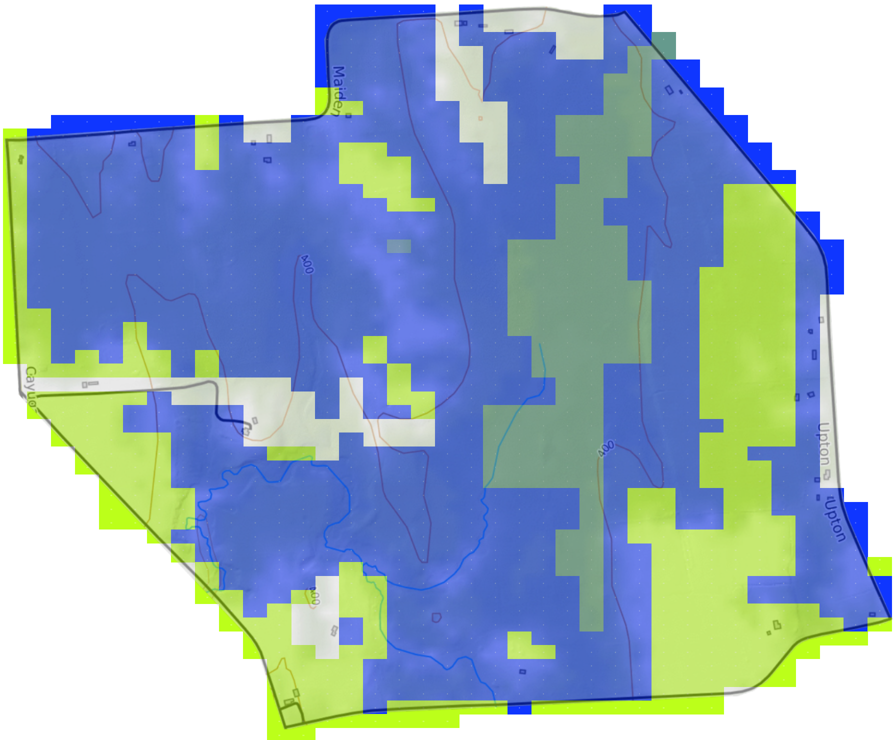

Figure 22: The optimal solution (blue represents the cross-country skiing facility; light green represents the ranch; dark green represents the agritourist center)

B Data

Table 2: Data of cross-country skiing facilities

Item	Cross-country skiing resort
Slope/°	3 – 11
Coverage	35%
Percentage reduction of vegetation coverage	5 – 20% per year in the range of moderate pollution
Humidity/%	15
Standard Deviation of Humidity	0.058
Precipitation/cm	-
Standard Deviation of Precipitation/cm	-
Illumination/(h/day)	6.7
Standard Deviation of Illumination/h	0.989
Temperature/°C	−4
Standard Deviation of Temperature/°C	2.56
Visitors Flow Rate	1.46 people/(m^2·day)
Standard Deviation of Population	1000 people/(^{2}·day)
Pollution	Moderate pollution with max distance of 1km
Financial Impact on facility	10 – 30% lower revenue (moderate pollution)
Revenue/(m^2·year)	13.59$
Maintenance cost/(m^2·ear)	3.22$
Prime cost/(m^2year)	32.16$
Employment	150 people per facility

Table 3: Data of crop farm

Item	Crop farm
Slope/°	0 – 8.53(1.43 the best)
Coverage	-
Percentage reduction of vegetation coverage	5 – 20% per year in the range of moderate pollution
Humidity/%	70
Standard Deviation of Humidity	0.045
Precipitation/cm	70
Standard Deviation of Precipitation/cm	8.22
Illumination/(h/day)	12
Standard Deviation of Illumination/h	2.55
Temperature/°C	21
Standard Deviation of Temperature/°C	5.24
Visitors Flow Rate	-
Standard Deviation of Population	-
Pollution	Moderate pollution with max distance of 0.5-1km
Financial Impact on facility	50% lower revenue (moderate pollution)
Revenue/(m^2year)	2.59$
Maintenance cost/(m^2year)	0.11$
Prime cost/(m^2year)	0.18$
Employment	0.000001 people/m^2

Table 4: Data of ranch

Item	Ranch
Slope/°	< 9
Coverage	-
Percentage reduction of vegetation coverage	5 – 20% per year in the range of moderate pollution
Humidity/%	60
Standard Deviation of Humidity	0.047
Precipitation/cm	76.6
Standard Deviation of Precipitation/cm	9.62
Illumination/(h/day)	7.3
Standard Deviation of Illumination/h	0.76
Temperature/°C	20
Standard Deviation of Temperature/°C	3.14
Visitors Flow Rate	-
Standard Deviation of Population	-
Pollution	Moderate pollution with max distance of 0.5 – 1km
Financial Impact on facility	10 – 30% lower revenue (moderate pollution)
Revenue/(m^2year)	0.675$
Maintenance cost/(m^2year)	0.41$
Prime cost/(m^2year)	0.075$
Employment	0.000001 people/m^2

Table 5: Data of regenerative farm

Item	Regenerative farm
Slope/°	< 10
Coverage	-
Percentage reduction of vegetation coverage	5 – 20% per year in the range of moderate pollution
Humidity/%	55
Standard Deviation of Humidity	0.1
Precipitation/cm	70
Standard Deviation of Precipitation/cm	9.2
Illumination/(h/day)	6.9
Standard Deviation of Illumination/h	0.83
Temperature/°C	18
Standard Deviation of Temperature/°C	5.75
Visitors Flow Rate	-
Standard Deviation of Population	-
Pollution	No pollution if operated properly
Financial Impact on facility	10 – 20% lower revenue (moderate pollution)
Revenue/(m^2year)	1.97$
Maintenance cost/(m^2year)	0.06$
Prime cost/(m^2year)	3.71$
Employment	0.000001 people/m^2

Table 6: Data of agriculturist center

Item	Agriculturist center
Slope/°	3 – 9
Coverage	30%
Percentage reduction of vegetation coverage	5 – 20% per year in the range of moderate pollution
Humidity/%	55
Standard Deviation of Humidity	0.086
Precipitation/cm	67
Standard Deviation of Precipitation/cm	2.93
Illumination/(h/day)	8
Standard Deviation of Illumination/h	0.295
Temperature/°C	16
Standard Deviation of Temperature/°C	4.29
Visitors Flow Rate	0.84 people/(m^2day)
Standard Deviation of Population	1500 people/(m^2day)
Pollution	Moderate pollution with max distance of 1km
Financial Impact on facility	20 – 30% lower revenue (moderate pollution)
Revenue/(m^2 year)	64.26$
Maintenance cost/(m^2year)	3.71$
Prime cost/(m^2year)	185.35$
Employment	0.00001 people/m^2

Table 7: Data of Club Med Cherating Beach Hotel

Item	**Club Med Cherating Beach Hotel**
Slope/°	0 – 45
Coverage	50%
Percentage reduction of vegetation coverage	0%-3% per year in the range of moderate pollution
Humidity/%	80 – 90
Standard Deviation of Humidity	0.037
Precipitation/cm	300
Standard Deviation of Precipitation/cm	2.77
Illumination/(h/day)	7
Standard Deviation of Illumination/h	0.008
Temperature/°C	27
Standard Deviation of Temperature/°C	2.04
Visitors Flow Rate	-
Standard Deviation of Population	-
Pollution	Little pollution if operated properly
Financial Impact on facility	10 – 30% lower revenue (moderate pollution)
Revenue/(m^2year)	240$
Maintenance cost/(m^2year)	4.36$
Prime cost/(m^2year)	143.67$
Employment	0.0005 people/m^2

C Source Code

C.1 Main function

```
import matplotlib.pyplot as plt
import math

dt=0.1

M=8
Q=7
maxx=54
maxy=37

class Pos:
    x=0
    y=0
    def __init__(self,xx,yy):
        self.x=xx
        self.y=yy
    def __neg__(self):
        return Pos(-self.x,-self.y)
    def __add__(self,other):
        return Pos(self.x+other.x,self.y+other.y)
    def __pow__(self,p):
        return self.x**p+self.y**p
    def __eq__(self,other):
        return self.x==other.x and self.y==other.y

pos=[Pos(50,10),Pos(10,30),Pos(20,10),Pos(50,20),Pos(10,10)]

typ=[1,1,1,1,1]

N=pos.__len__()

typName=[["Cross-country Skiing facility"],["Crop Farm"],["Rench"],["
    Regenerative Farm"],["Agrivoltaic Farm"]]

aInfo=[
    ["Slope",[
        [7,    7.31925913496,   1.204006794416 ],
        [1.43, 11.82242338696,  1.812524150414 ],
        [0,    13.3434199704295, 2             ],
        [0,    14.82602218506,  2              ],
        [6,    5.164802761524,  1.139830742189 ]]],
    ["Vegetation Cover",[
        [0.35, 0.0448725256473],
```

```
        [],
        [],
        [],
        [0.3,  0.0383879784927]]],
    ["Humidity",[
        [15,   0.058],
        [70,   0.045],
        [60,   0.047],
        [55,   0.1],
        [55,   0.086]]],
    ["Rainfall",[
        [],
        [70,8.22],
        [76.6,9.62],
        [70,9.2],
        [67,2.93]]],
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        [12,2.55],
        [7.3,0.76],
        [6.9,0.83],
        [8,0.295]]],
    ["Temperature",[
        [-4,2.56],
        [21,5.24],
        [20,3.14],
        [18,5.75],
        [16,4.29]]],
    ["Population",[
        [],
        [],
        [],
        [],
        []]],
    ["Pollution",[
        [-0.2],
        [-0.5],
        [-0.2],
        [-0.15],
        [-2.5]]]]

def norm(u,sig,x):
    return math.exp((-(x-u)**2)/(2*(sig**2)))/(math.sqrt(2*math.pi)*sig)

cx=61.00
cy=34.60
def dis(pos1,pos2):
    d=pos1+(-pos2)
```

```
    return math.sqrt((d.x*60.74)**2+(d.y*36)**2)

def U(j,i,posk,pos):

    if(i==0 or i==2 or i==3 or i==4 or i==5):
        return 0
    if(i==6):
        if(j==0):
            return 0.004*365
        if(j==4):
            return 0.0023*365
        return 0
    if(i==7):
        d=dis(posk,pos)
        if(((j==0 or (j==4)) and d<=1000)or((j==1 or j==2) and d<=750)):
            return 1
        return 0

def V(i,j,a):

    temp=aInfo[i][1][j]
    if(i<=6):
        if(temp.__len__()==0):
            return 1
        if(temp.__len__()==2):
            return norm(temp[0],temp[1],a)
        if(temp.__len__()==3):
            return norm(temp[0],temp[1],a)*temp[2]
    elif(i<=7):
        return temp[0]*(a!=0)

def dif(T,a0,W):

    a=np.zeros([maxx,maxy,M,int(T/dt)])

    a[:,:,:,0]=a0

    A=np.zeros([N,int(T/dt)])

    for t in range(1,int(T/dt)):
        for k in range(N):
            # print(a[1:maxx+1,1:maxy+1,1,t])
            sum=0
            for i in range(M):
                sum+=V(i,typ[k],a[pos[k].x,pos[k].y,i,t-1])*W[i]
            sum/=M
            A[k,t]=sum
        da=np.zeros([maxx,maxy,M])
```

```
        for k in range(N):
            for x in range(maxx):
                for y in range(maxy):
                    for i in range(M):
                        if(i==7 and U(typ[k],i,pos[k],Pos(x,y))):
                            da[x,y,i]=1
                        elif(i==1):
                            if(a[x,y,i,t-1]):
                                da[x,y,i]=-0.125
                        else:
                            da[x,y,i]+=A[k,t]*U(typ[k],i,pos[k],Pos(x,y))
        a[:,:,:,t]=a[:,:,:,t-1]+da*dt
    return A

s=[13.59-3.22,2.59-0.11,0.675-0.41,1.97-0.06,64.26,3.71]

c=[32.16,0.18,0.075,3.71,185.35]

def S(j,t):
    if t==0:
        return -c[j]
    return s[j]

def h(flag,t,T):
    if(flag==1):
        return 1/(math.exp(t-1)+1)
    else:
        return 1-1/(math.exp(t-1)+1)

def rate(r,t,T):
    return r*h(1,t,T)+(1-r)*h(2,t,T)

def P(r,T,a0,W):
    A=dif(T,a0,W)
    p=0
    for t in range(0,int(T/dt)):
        dp=0
        for k in range(N):
            if t==0:
                dp+=S(typ[k],t)*rate(r,t,T)
            dp+=A[k,t]*S(typ[k],t)*rate(r,t,T)
        p+=dp*dt
    return p

a0=np.zeros([maxx,maxy,M])

W=np.ones([M])

```

```
with open("...","rb") as f:
    s = f.read()
    l=s.splitlines()
    for i in range(l.__len__()):
        ii=l[i].split()
        for j in range(ii.__len__()):
            a0[j,i,0]=float(ii[j])

a0[:,:,2]=0.719
a0[:,:,3]=104.1
a0[:,:,4]=5.5
a0[:,:,5]=9.3
a0[:,:,6]=0.01
a0[:,:,7]=0

print(P(0.2,10,a0,W))
```

C.2 Code for Visualizing Altitude and Slope

```
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

with open("./3Dmap.stl","rb") as f:
    s = f.read()

xyz = s.splitlines()

start=[]

end=[]

for inf in range(4,1596):
    start.append([float(xyz[inf].split()[1]),float(xyz[inf].split()[3]),
        float(xyz[inf].split()[2])])

for inf in range(1596,3188):
    end.append([float(xyz[inf].split()[1]),float(xyz[inf].split()[3]),float
        (xyz[inf].split()[2])])

fig = plt.figure()

ax = fig.add_axes(Axes3D(fig))

for i in range(1596-4):
    ax.quiver(start[i][0],start[i][1],start[i][2],end[i][0],end[i][1],end[i
        ][2],arrow_length_ratio=0.1)
```

```
plt.show()
```

C.3 Code for Vacation Coverage Input

```
import numpy as np

data = xlrd.open_workbook("tree.xls")

table = data.sheets()[0]

nrows=table.nrows

t=[]

with open('...', 'a+') as ff:

    for i in range(nrows):
        row=table.row(i)
        t.append([])
        for j in range(row.__len__()):
            t[i].append(table.cell_value(i,j))
            if(t[i][j]==''):
                t[i][j]=-1
            print(t[i][j],end='␣',file=ff)
        print('',file=ff)
```