Homework Assignment 1

Tao Wu

2026-01-27

1. Write R code to do the following:

(a) Simulate a random number representing the number of shots that a basketball player makes in 10 tries, if the results of the shots are independent and she has a 40% chance of making each shot.

```r
set.seed(23)
shots_made = rbinom(10, 1, 0.4)
sum(shots_made)
```

```
## [1] 6
```

```r
# alternative
rbinom(n = 1, size = 10, prob = 0.4)
```

```
## [1] 6
```

(b) Simulate a random number representing the number of shots that a basketball player makes in 10 tries, if the results of the shots are independent and her chance of making a shot is 10% for the first shot, 20% for the second shot, 30% for the third, etc.

```r
set.seed(18)
tries = 10
shots_made = rep(NA, tries)
for (i in 1:tries) {
  shots_made[i] = rbinom(1, 1, 0.1*i)
}
sum(shots_made)
```

```
## [1] 6
```

```r
# alternative
probs = seq(0.1, 1, by = 0.1)
shots_made_1 = rbinom(10, 1, probs)
sum(shots_made_1)
```

```
## [1] 5
```

2. (a) Write R code to simulate 100 data points from a linear model with intercept 1, slope 2, and residual standard deviation 3, where the predictors are sampled at random uniformly from the range (0,4). Fit a linear regression to these data using lm, print the fitted model using the display function from the arm package, and report whether the estimates of the intercept and slope fall within 1 standard error of the true parameter values.

```r
#install.packages("arm")
library(arm)
```

```
## Loading required package: MASS

## Loading required package: Matrix
## Loading required package: lme4

##
## arm (Version 1.14-4, built: 2024-4-1)

## Working directory is /Users/tiow/Downloads/1 STAT5293G004 Gelman/STAT_5293/Homeworks
```

```
set.seed(21)
n = 100
beta_0 = 1 # true intercept
beta_1 = 2 # true slope

predictor = runif(n, 0, 4)
residuals = rnorm(n, 0, 3) #residual standard deviation 3
y = beta_0 + predictor * beta_1 + residuals

fitted_linear_model = lm(y ~ predictor)
display(fitted_linear_model)
```

```
## lm(formula = y ~ predictor)
##             coef.est coef.se
## (Intercept) 0.74     0.62
## predictor   2.03     0.25
## ---
## n = 100, k = 2
## residual sd = 2.95, R-Squared = 0.41
```

```
if (abs(coef(fitted_linear_model)[1] - beta_0) <= se.coef(fitted_linear_model)[1]) {
  print("Intercept estimate falls within 1 standard error of the true intercept value.")
}
```

```
## [1] "Intercept estimate falls within 1 standard error of the true intercept value."
```

```
if (abs(coef(fitted_linear_model)[2] - beta_1) <= se.coef(fitted_linear_model)[2]) {
  print("Slope estimate falls within 1 standard error of the true slope value.")
}
```

```
## [1] "Slope estimate falls within 1 standard error of the true slope value."
```

(b) Fit the regression model using *stan_glm* and print the results.

```
#install.packages("rstanarm")
library(rstanarm)
```

```
## Loading required package: Rcpp

## This is rstanarm version 2.32.1

## - See https://mc-stan.org/rstanarm/articles/priors for changes to default priors!

## - Default priors may change, so it's safest to specify priors, even if equivalent to the defaults.

## - For execution on a local, multicore CPU with excess RAM we recommend calling

##   options(mc.cores = parallel::detectCores())

##
## Attaching package: 'rstanarm'

## The following objects are masked from 'package:arm':
##
##     invlogit, logit
```

```r
stan_linear_model = stan_glm(y ~ predictor, data = data.frame(y, predictor), refresh = 0) # add refresh

print(stan_linear_model)
```

```
## stan_glm
##  family:       gaussian [identity]
##  formula:      y ~ predictor
##  observations: 100
##  predictors:   2
## ------
##             Median MAD_SD
## (Intercept) 0.7    0.6
## predictor   2.0    0.2
##
## Auxiliary parameter(s):
##       Median MAD_SD
## sigma 3.0    0.2
##
## ------
## * For help interpreting the printed output see ?print.stanreg
## * For info on the priors used see ?prior_summary.stanreg
```

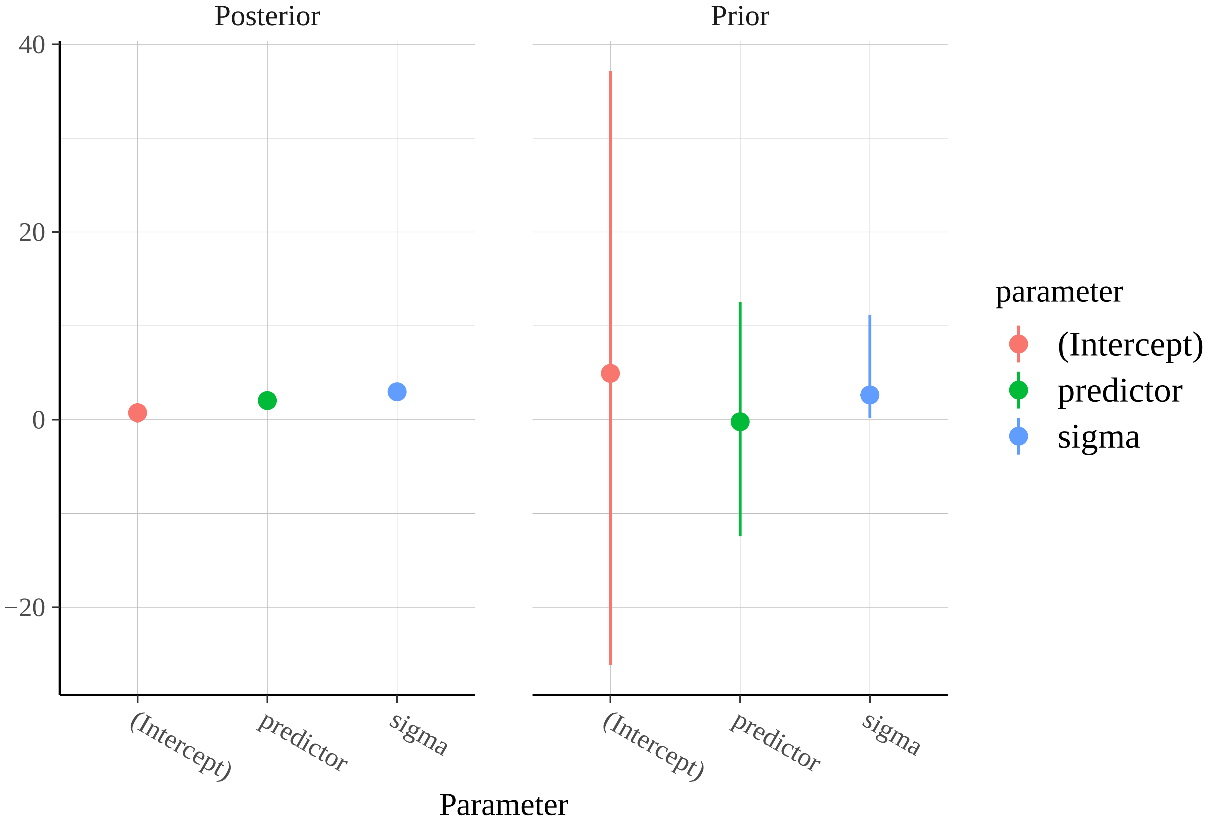
```r
# quick linear/logistic regression bayesian analysis, use stan_glm

prior_summary(stan_linear_model)
```

```
## Priors for model 'stan_linear_model'
## ------
## Intercept (after predictors centered)
##   Specified prior:
##     ~ normal(location = 5.3, scale = 2.5)
##   Adjusted prior:
##     ~ normal(location = 5.3, scale = 9.6)
##
## Coefficients
##   Specified prior:
##     ~ normal(location = 0, scale = 2.5)
##   Adjusted prior:
##     ~ normal(location = 0, scale = 7.9)
##
## Auxiliary (sigma)
##   Specified prior:
##     ~ exponential(rate = 1)
##   Adjusted prior:
##     ~ exponential(rate = 0.26)
## ------
## See help('prior_summary.stanreg') for more details
```

```r
posterior_vs_prior(stan_linear_model)
```

```
##
## Drawing from prior...
```



(c) Write a Stan program, fit the model using the *cmdstanr* package, and print the results.

```
#cmdstanr::install_cmdstan(cores = 4) #install the C++ toolchain and the CmdStan engine

#install.packages("cmdstanr", repos = c("https://mc-stan.org/r-packages/", getOption("repos")))
library(cmdstanr)
```

```
## This is cmdstanr version 0.8.0

## - CmdStanR documentation and vignettes: mc-stan.org/cmdstanr

## - CmdStan path: /Users/tiow/.cmdstan/cmdstan-2.38.0

## - CmdStan version: 2.38.0
```

```
#example(stan_model, package = "rstan", run.dontrun = TRUE)

stan_data = list(n = 100, x = predictor, y = y) #prepare the data
mod = cmdstan_model("linear_model.stan")         #compile the model
```

```
## Warning in readLines(stan_file): incomplete final line found on
## 'linear_model.stan'
```

```
fit_cmdstan = mod$sample(                         #fit model using MCMC
  data = stan_data,
  seed = 21,
  chains = 4,
  parallel_chains = 4,
  refresh = 0
)
```

```
## Running MCMC with 4 parallel chains...
## Chain 4 Informational Message: The current Metropolis proposal is about to be rejected because of the

## Chain 4 Exception: normal_lpdf: Scale parameter is 0, but must be positive! (in '/var/folders/7h/m_g

## Chain 4 If this warning occurs sporadically, such as for highly constrained variable types like cova

## Chain 4 but if this warning occurs often then your model may be either severely ill-conditioned or m

## Chain 4

## Chain 1 finished in 0.1 seconds.
## Chain 2 finished in 0.0 seconds.
## Chain 3 finished in 0.0 seconds.
## Chain 4 finished in 0.1 seconds.
##
## All 4 chains finished successfully.
## Mean chain execution time: 0.1 seconds.
## Total execution time: 0.2 seconds.
```

```
fit_cmdstan$print()
```

```
##   variable    mean  median   sd  mad      q5     q95 rhat ess_bulk ess_tail
##  lp__      -157.81 -157.48 1.25 1.06 -160.31 -156.42 1.00     1430     2411
##  intercept    0.74    0.74 0.65 0.64   -0.33    1.81 1.00     1730     1611
##  slope        2.03    2.03 0.26 0.25    1.60    2.46 1.00     1704     1616
##  sigma        3.00    2.99 0.22 0.21    2.65    3.39 1.00     1796     1804
```

```
# want full control on custom likelihoods, unusual priors, or complex hierarchical structures that aren
```