UNIVERSIDADE EDUARDO MONDLANE
FACULDADE DE CIÊNCIAS
DEPARTAMENTO DE MATEMÁTICA E INFORMÁTICA
2° Teste de Análise Matemática III

Data: 17 de Junho de 2022 **Duração: 110 minutos**

1. Integre as seguintes equações diferenciais:

   a) **[2.5]** $e^{2y}(1-\sin x)+y'\cos^2 x=0$

   b) **[2.5]** $y''+y=\dfrac{1}{\sin x}$

2. **[3.0]** Aplicando a tabela de transformada de Laplace, ache a imagem de $F(p)=\dfrac{e^{-p}}{p^3}$.

3. Dado o sistema $\begin{cases} x'=y+\sin t \\ y'=x+2\cos t \end{cases}$, $x(0)=2,\ y(0)=0$. Resolva-o:

   a) **[4.0]** Aplicando o método de substituição.

   b) **[4.0]** Aplicando o método operacional.

4. Seja $f:[0,1]\to R$, definida por $f(x)=x-x^2$.

   a) **[3.0]** Desenvolva a função dada em série de Fourier incompleta, de senos, no intervalo dado.

   b) **[1.0]** Use o resultado da alínea a) para calcular o valor de $\sum_{n=0}^{\infty}\dfrac{(-1)^n}{(2n+1)^3}$.

Bom Trabalho!
O grupo da disciplina

TESTE 2 - AM III - 2022

① a) $e^{2y}(1-\operatorname{sen} x) + y'\cos^2 x = 0$; $y' = \dfrac{dy}{dx}$

$e^{2y}(1-\operatorname{sen} x)\,dx + \cos^2 x\,dy = 0 \Rightarrow$ Variáveis separáveis

$$\left(\frac{1-\operatorname{sen} x}{\cos^2 x}\right)dx = -\frac{dy}{e^{2y}}$$

$$\int\left(\frac{1}{\cos^2 x} - \frac{\operatorname{sen} x}{\cos^2 x}\right)dx = -\int e^{-2y}\,dy$$

$$\boxed{\operatorname{tg} x - \frac{1}{\cos x} = \frac{1}{2}e^{-2y} + C}$$

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b) $y'' + y = \dfrac{1}{\sin x}$; $y = y_0 + y_p$

y_0: $y'' + y = 0$

$\lambda^2 + 1 = 0$

$\lambda = \pm i$

$$\boxed{y_0 = C_1\cos x + C_2\operatorname{sen} x}$$

$y_p = C_1(x)\cos x + C_2(x)\sin x$

$$\begin{cases} C_1'(x)\cos x + C_2'(x)\sin x = 0 & \times \operatorname{sen} x \\ C_1'(x)(-\operatorname{sen} x) + C_2'(x)\cdot\cos x = \dfrac{1}{\operatorname{sen} x} & \times \cos x \end{cases}$$

$$\begin{cases} C_1'(x)\cdot\operatorname{sen} x\cos x + C_2'(x)\operatorname{sen}^2 x = 0 \\ -C_1'(x)\operatorname{sen} x\cos x + C_2'(x)\cos^2 x = \dfrac{\cos x}{\operatorname{sen} x} \end{cases}$$

$$C_2'(x) = \frac{\cos x}{\operatorname{sen} x} \Rightarrow C_2(x) = \int\frac{\cos x}{\operatorname{sen} x}\,dx = \ln|\operatorname{sen} x|$$

$$C_1'(x) = -\frac{C_2'(x)\cdot\sin x}{\cos x} = -1 \Rightarrow C_1(x) = -x$$

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$$\boxed{y_p = -x\cos x + \sin x\cdot\ln|\operatorname{sen} x|}$$

Solução: $\boxed{y = C_1\cos x + C_2\sin x - x\cos x + \sin x\cdot\ln(\operatorname{sen} x)}$

② $e^{-pt_0}F(p) \to f(t-t_0)$

$$\frac{e^{-p}}{p^3} = \frac{1}{2}\,\frac{e^{-p}\,2}{p^3} \Rightarrow f(t) = \frac{(t-1)^2}{2}$$ 30

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③ $\begin{cases} x' = y + \sin t \\ y' = x + 2\cos t \end{cases}$; $x(0)=2$; $y(0)=0$

a) Método de substituição:

$\begin{cases} x'' = y' + \cos t \\ x'' - \cos t = x + 2\cos t \end{cases} \Rightarrow \boxed{x'' - x = 3\cos t}$ (1)

x_0 : $x'' - x = 0$

$\lambda^2 - 1 = 0$

$\lambda = \pm 1$, $\boxed{x_0 = c_1 e^{t} + c_2 e^{-t}}$

x_p : $x_p = A\cos t + B\sin t$

$x'_p = -A\sin t + B\cos t$

$x''_p = -A\cos t - B\sin t$, substituindo em (1):

$-A\cos t - B\sin t - A\cos t - B\sin t = 3\cos t$

$\begin{cases} -2A = 3 \\ -2B = 0 \end{cases} \Leftrightarrow \boxed{A = -3/2}$ e $\boxed{B = 0}$; $\boxed{x_p = -\frac{3}{2}\cos t}$

$\boxed{x = c_1 e^{t} + c_2 e^{-t} - \frac{3}{2}\cos t}$; $x' = c_1 e^{t} - c_2 e^{-t} + \frac{3}{2}\sin t$

Do sistema: $y = x' - \sin t \Rightarrow \boxed{y = c_1 e^{t} - c_2 e^{-t} + \frac{1}{2}\sin t}$

Das condições iniciais, temos:

$\begin{cases} c_1 + c_2 - 3/2 = 2 \\ c_1 - c_2 = 0 \end{cases} \Leftrightarrow 2c_1 = 7/2 \Rightarrow \boxed{c_1 = 7/4}$; $\boxed{c_2 = 7/4}$

Solução: $\boxed{\begin{cases} x = \frac{7}{4}e^{t} + \frac{7}{4}e^{-t} - \frac{3}{2}\cos t \\ y = \frac{7}{4}e^{t} - \frac{7}{4}e^{-t} + \frac{1}{2}\sin t \end{cases}}$ 40

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b) $\begin{cases} x' = y + \sin t \\ y' = x + 2\cos t \end{cases}$ $\quad x(0) = 2;\ y(0) = 0$

Método operacional:

$$\begin{cases} pX - 2 = Y + \frac{1}{p^2+1} \\ pY = X + \frac{2p}{p^2+1} \end{cases} \Leftrightarrow \begin{cases} pX - Y = \frac{1}{p^2+1} + 2 = \frac{2p^2+3}{p^2+1} \\ -X + pY = \frac{2p}{p^2+1} \end{cases} \times(p)$$

$$+\begin{cases} pX - Y = \frac{2p^2+3}{p^2+1} \\ -pX + p^2 Y = \frac{2p^2}{p^2+1} \end{cases}$$

$$(p^2-1)Y = \frac{4p^2+3}{p^2+1}$$

$$\rightarrow Y = \frac{4p^2+3}{(p^2+1)(p^2-1)} = \frac{4p^2+3}{(p-1)(p+1)(p^2+1)}$$

$$= \frac{A}{p-1} + \frac{B}{p+1} + \frac{Cp+D}{p^2+1}$$

$$4p^2+3 = A(p^2+1)(p+1) + B(p-1)(p^2+1) + (Cp+D)(p^2-1)$$

se $p = 1 \Rightarrow 7 = A \cdot 4 \Rightarrow \boxed{A = 7/4}$

se $p = -1 \Rightarrow 7 = B \cdot (-4) \Rightarrow \boxed{B = -7/4}$

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se $p = 0 \Rightarrow 3 = 7/4 + 7/4 - D \Rightarrow D = 7/2 - 3 = \boxed{1/2}$

se $p = 2 \Rightarrow 19 = 7/4 \cdot 15 - 7/4 \cdot 5 + (2C + 1/2) \cdot 3 \Rightarrow C$

$19 = \frac{35}{2} + 6C + \frac{3}{2} \Rightarrow 19 = 6C + 19 \Rightarrow \boxed{C = 0}$

$$Y = \frac{7/4}{p-1} - \frac{7/4}{p+1} + \frac{1/2}{p^2+1} \rightarrow \boxed{y = \frac{7}{4}e^{t} - \frac{7}{4}e^{-t} + \frac{1}{2}\sin t}$$

Pode-se calcular x de forma análoga ou substituir em uma das equações do sistema:

$$\boxed{x = 7/4\, e^{t} + 7/4\, e^{-t} - 3/2 \cos t}$$

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④ $f:[0,1]\to\mathbb{R};\ f(x)=x-x^2$

a) $b_n=\frac{2}{1}\int_0^1 (x-x^2)\,\text{sen}\, n\pi x\,dx$ $\quad\left|\begin{array}{l} u=x-x^2,\ du=(1-2x)dx \\ dv=\text{sen}\,n\pi x\,dx \\ v=\frac{-\cos n\pi x}{n\pi}\end{array}\right.$

$=2\left[-\frac{(x-x^2)\cos n\pi x}{n\pi}\Big|_0^1+\int_0^1\frac{(1-2x)\cos n\pi x}{n\pi}dx\right]$ $\quad\left|\begin{array}{l} u=1-2x \\ du=-2dx \\ dv=\cos n\pi x\,dx \\ v=\frac{\text{sen}\,n\pi x}{n\pi}\end{array}\right.$

$=\frac{2}{n\pi}\left[\frac{(1-2x)\,\text{sen}\,n\pi x}{n\pi}+\frac{2}{n\pi}\int_0^1\text{sen}\,n\pi x\,dx\right]$

$=\frac{-4}{n^2\pi^2}\,\frac{\cos n\pi x}{n\pi}\Big|_0^1=\frac{4}{n^3\pi^3}\left(1-\cos n\pi\right)=\begin{cases}0, & n \text{ par}\\ \frac{8}{n^3\pi^3}; & n \text{ impar}\end{cases}$

$$\boxed{f(x)=x-x^2=\sum_{n=0}^{\infty}\frac{8}{(2n+1)^3\pi^3}\sin(2n+1)\pi x}$$

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b) Se $x=\frac{1}{2}\Rightarrow \frac{1}{2}-\left(\frac{1}{2}\right)^2=\sum_{n=0}^{\infty}\frac{8}{(2n+1)^3\pi^3}\sin\left(\frac{2n+1}{2}\cdot\pi\right)$

$$\frac{1}{4}=\frac{8}{\pi^3}\cdot\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^3}$$

$$\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^3}=\frac{\pi^3}{32}\quad\#$$

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