1 Exploratory Random Walk

Definition 1 (Bipartite Graph Instance)**.** Let $RBG(V, W, \mathbf{p})$ be a random bipartite graph, we call an instance of $RBG(V, W, \mathbf{p})$ for a given realization of the family $\xi = \{\xi_{v,w}\}_{v \in V, w \in W}$, a *bipartite instance graph* $BIG(V, W, \xi)$.

Definition 2 (Remainder Graphs)**.** Let $BIG(V, W, \xi)$ be a bipartite instance graph, and let $I \subset W$. The *remainder graph*

$$BIG|_I = BIG(V|_I, W \setminus I, \xi|_I)$$

is the bipartite instance graph on:

- $W \setminus I$
- $V|_I = V \setminus \left(\bigcup_{i \in I} \{v \in V : \xi_{v,i} = 1\} \right)$
- $\xi|_I = \begin{cases} \xi_{u,v} & \text{if } i \in W \setminus I, u \in V|_I \\ 0 & \text{otherwise} \end{cases}$

As is canon in the theory of random graphs[vdH17, Gri10, AS08], we will now define a procedure for revealing the connected component to which a given node belongs.

Our process will explore the bipartite graph whose projection yields the dual random intersection graph. We define the procedure recursively using the remainder graphs defined above.

Definition 3 (Graph Exploration Process)**.** At step $t = 0$, $BIG_0 = BIG(V, W, \xi)$ and:

- j_0: is the node whose connected component we want to explore;
- U_0: is the set of unexplored items, i.e. $W \setminus \{j_0\}$;
- $V(j_0)$: is the set of users who picked j_0, i.e. $V(j_0) = \{v \in V : \xi_{v,j_0} = 1\}$;
- $\mathcal{N}_0$: is the set of items, not including j_0, that the above users picked, i.e. $\mathcal{N}_0 = \bigcup_{u \in V(j_0)} \{j \in U_0 : \xi_{u,j} = 1\}$;
- P_0: is an empty FIFO queue of sets, to which we push $\mathcal{N}_0$;
- R_0: is the set of removed nodes, i.e. $R_0 = \{j_0\}$;
- A_0: is the empty set of active nodes.

At step $t + 1$, $BIG_{t+1} = BIG_t|_{\{j_t\}}$:

- we update the set of actives
- we pick j_{t+1} from the set of actives A_{t+1}

$$A_{t+1} = \{1$$

References

[AS08] N. Alon and J. Spencer. *The Probabilistic Method*. Wiley, 2008.

[Gri10] G. Grimmett. *Probability on Graphs*. Cambridge University Press, 2010.

[vdH17] R. v. d. Hofstad. *Random Graphs and Complex Networks, Volume One*. Cambridge University Press, 2017.