

A low-angle, upward-looking photograph of several modern skyscrapers. The buildings are dark, with a grid-like pattern of windows. Some windows are illuminated from within, showing a warm orange glow. The sky is a pale, overcast grey. The perspective creates a sense of height and scale.

Portfolio Re-Optimization

MGSC 416 — Group 11

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Background & Problem Identification

Background

- Fama French Three Factor Pricing Model (FF3) is a forecasting model developed to predict a firm's returns by adding size and value variables to the CAPM single factor model
- Its prediction will be compared with the firm's realized returns to identify idiosyncratic premium returns and effectively capitalize on them

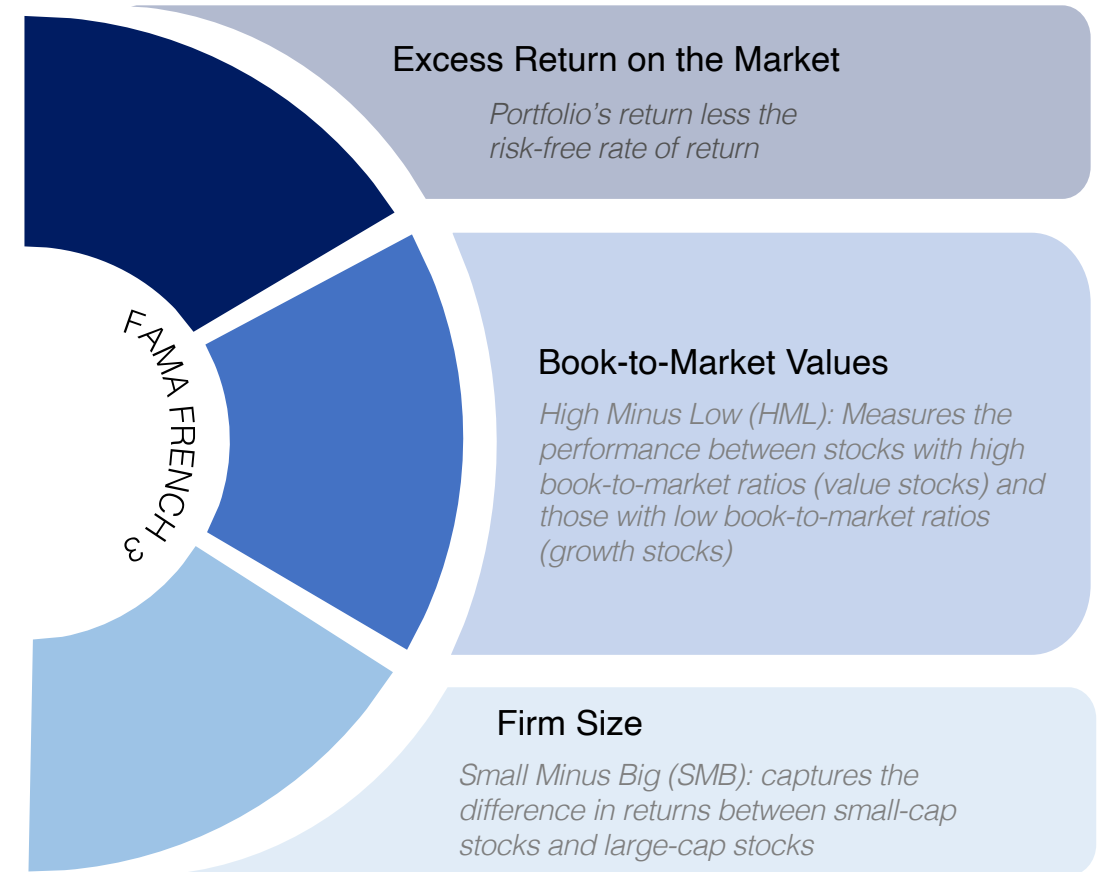
Problem

- Traditional models do not fully address rapid market shifts and investor diversity
- Current investment strategies lack adaptability to real-time data and risk management

This project seeks to perform a **weekly portfolio re-optimization** to **maximize and capture excess returns** for enhanced portfolio performance, taking into consideration investor preferences and risk considerations

Benchmarks

- Extent to which the portfolio outperforms the market regarding excess returns (alpha) while adhering to investor preferences
- Consistency of the forecasting model, validated through comparisons between predicted and realized returns



Data Overview & Preparation

Data Overview

- 749 daily observations of the adjusted price of 500 stocks in the S&P 500 and the market itself between 2021 and 2023 (total of 375,972 entries)
 - 451 assets remaining post sector information integration
- To be used in the linear optimization model and serve as the test sample to determine the results of the investment strategy
- Calculated metrics included daily returns, weekly returns, weekly volatility, weekly 5% value-at-risk (VaR), weekly downside deviation, and idiosyncratic variance

Summary Statistics of the Market

2021	2022	2023
21.67% return	-20.36% return	18.4% return

- 2022 Fears of an economic “hard landing” and stubborn inflation led to a 20.36% loss
- 2023 Strong year of returns despite the collapse of Silicon Valley Bank and Credit Suisse

Average Weekly Return
11.99 BPS

Average Annual Volatility
17.88%

Mean Weekly Value-at-Risk (VaR)
-3.904%

Mean Weekly Downside Deviation
3.305%

Data Preparation

- 1 Calculate arithmetic daily returns of each stock
 - Used to calculate the weekly variables for the constraints
- 2 Identify which week each daily observation belonged to
 - Allows for reconstruction of the portfolio on a weekly basis
- 3 Group data by the “Week” variable and calculate the necessary features for the model
 - Features include daily returns, weekly returns, weekly volatility, weekly 5% value-at-risk (VaR), weekly downside deviation
 - Weekly VaR is a measure of the potential loss in value of an investment over one week at a 5% confidence level
 - Downside deviation measures the volatility of returns below 0
 - The idiosyncratic variance is calculated by subtracting the systematic variance of a stock from it's realized variance

Dataset Variables				
Week	MktPrem	SMB_Vol	Mkthml	HMLBeta
Symbol	SMB	HML_Vol	hmlsmb	alpha
Weekly_Returns	HML	ddev	Sector	ExpectedReturn
Weekly_LogReturns	RF	Weekly_VaR	MarketBeta	SharpeRatio
Weekly_Volatility	MktPrem_Vol	mktsmb	SMBBeta	IdioVol

Methodology & Analysis — Predictive Analysis

Employing the FF3 Model for Enhanced Stock Return Predictions

- 1
- Historical data (2017-2019) of 452 assets were collected to avoid COVID-19 impacts on the training data, ensuring robust factor analysis
- Daily metrics for FF3 factors were collected:
 - Market premium measures the performance of the market against the risk-free rate provided by the US 1-Month T-Bill
 - HML (High Minus Low) provides insights into the profitability of investing in consistent stocks versus stocks you believe have large growth potential
 - SMB (Small Minus Big) highlights the profitability of investing in smaller companies relative to larger ones

- 2
- Regressed the daily premium return of each stock on the daily market premium, HML, and SMB metrics
- Ran individual regressions utilizing all observations across three years for each stock
- 3
- Pulled the coefficients of each factor to estimate each stock's exposure to the three different macro-market risk factors
- Macro-market risk exposures are known as the “betas” to each factor

$$R_{t,i} - RF_{t,i} = \alpha_{t,i} + \beta_{i,M}MktPrem_t + \beta_{i,SMB}SMB_t + \beta_{i,HML}HML_t + \epsilon_{t,i}$$

Observations

Observation Set No. 1	Observation Set No. 2	Observation Set No. 3
<ul style="list-style-type: none">Average Adjusted R-Squared is 30.75%<ul style="list-style-type: none">High for stock returns that are typically difficult to forecast due to unpredictable market newsStandard deviation is 16.62%4% improvement in predictive performance over the single-factor CAPM model	<ul style="list-style-type: none">Weekly expected returns calculated by applying betas to the corresponding factor's weekly performanceExcess returns (alpha) determined by comparing expected returns to actual target stocks that outperform benchmarks<ul style="list-style-type: none">Serves as a main metric to maximize for each weekly portfolio	<ul style="list-style-type: none">Calculated expected weekly volatility of each stock using the market premium factor (FF3)<ul style="list-style-type: none">Subtracting this from the realized volatility of each stock left the idiosyncratic volatility to later be used in constraintsRepresents the fluctuations in a stock's returns that are firm-specific and not attributable to the volatility of their market

Methodology & Analysis — Linear Optimization

Objective Function

- Determine the optimal portfolio based on an investor's preferences each week in our sample
- Maximize excess returns

$$\sum_{i=1}^{452} (w_{ti} * \alpha_{ti}), \forall t$$

Decision Variables

w_{ti}	Stock's weight in the portfolio (<i>continuous</i>)
x_{ti}	Chosen stock (<i>binary</i>)
t	List of weeks over three years
i	452 stocks in the S&P 500

Given Variables

α	Alpha
v	Value-at-risk
d	Downside deviation
p	CAPM expected volatility
s	Sector

We assume a single factor CAPM where the expected variance of a stock is a function of its exposure to market variance in addition to its idiosyncratic variance.

Restrictive Constraints

- Value-at-risk** was never binding at the mean level.
- Downside deviation** constraint was sometimes binding.
- CAPM** expected portfolio volatility was binding for almost every week and can be adjusted for investors seeking increased or decreased risk and return.
- Sector** constraint was consistently binding can be changed to adjust sector diversity.

Constraints

$$x_{ti} \in \{0, 1\}$$

A stock is either chosen or not.

$$\sum_{i=1}^{452} x_{ti} = 25, \forall t$$

In total, 25 stocks must be chosen.

$$\sum_{i=1}^{452} w_{ti} = 1, \forall t$$

The **total weight** for the chosen stocks in the portfolio must equal to 100%.

$$0.01 \leq w_{ti} \leq 1$$

Each chosen stock's **weight** in the portfolio must be between 1% and 100% (both inclusive)

$$\sum_{i=1}^{452} (w_{ti} * v_{ti}) \geq -3.904, \forall t$$

The total **value-at-risk** of the portfolio must be greater than or equal to -3.904%.

$$\sum_{i=1}^{452} (w_{ti} * d_{ti}) \leq 3.305, \forall t$$

The total **downside deviation** of the portfolio must be less than or equal to 3.305%.

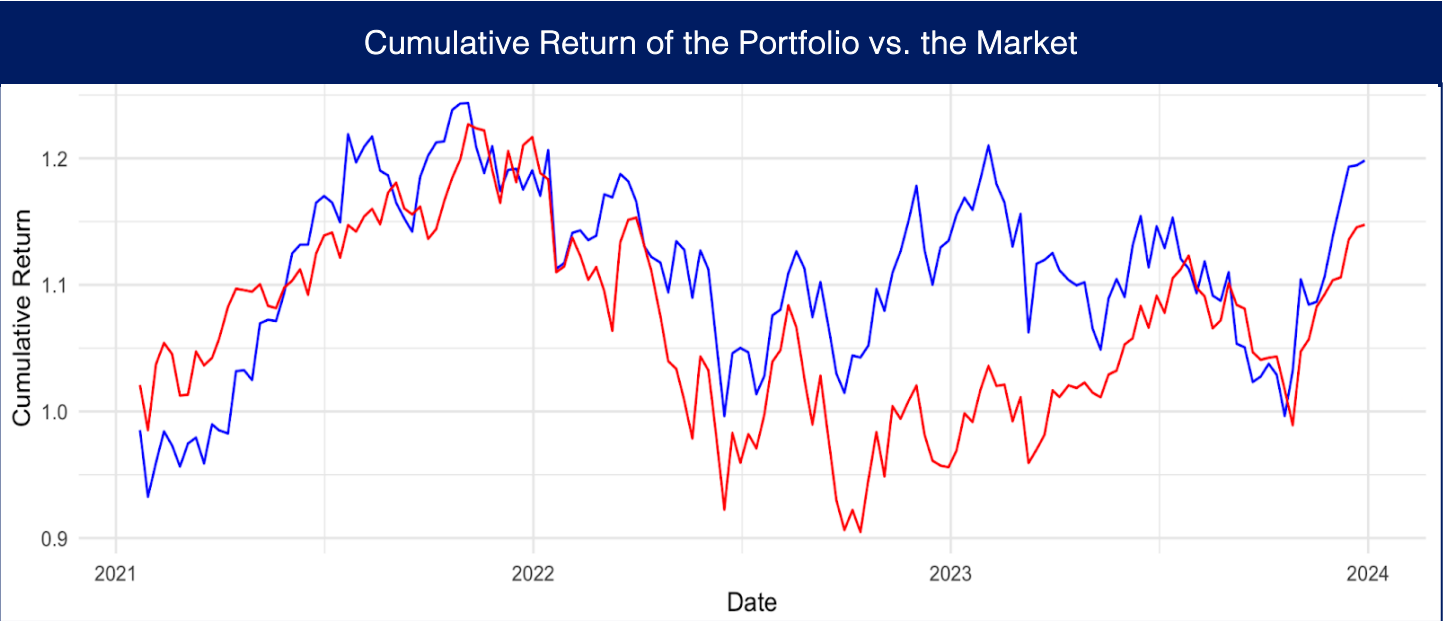
$$\sum_{i=1}^{452} (w_{ti} * p_{ti}) \leq 3.874, \forall t$$

The total **CAPM expected portfolio volatility** must be less than or equal to 3.874%.

$$\sum_{i=1}^{452} (w_{ti} * s_{ti}) \leq 0.2, \forall t$$

The **sector weight** for each sector of the portfolio must be less than or equal to 20%.

Results & Insights



2021	2022	2023
Portfolio: 19.05% Market: 21.67%	Portfolio: -2.95% Market: -20.36%	Portfolio: 6.21% Market: 18.4%

Insights

- Baseline portfolio strategy: average values of each constrained metric to serve as their limit to construct an alpha-maximizing portfolio with average risk preferences
- Total return of **19.83%** versus the market's 14.75%
- “Momentum” strategy: leveraging previous short-term success in hopes of forecasting short-term future returns
- CAPM expected portfolio variance and Sector weight constraints can be effectively adjusted to change the risk profile and sector diversity of the portfolio strategy

On Average		
Volatility 18.42%	Sharpe Ratio 0.3372	7.48bps more than market

Limitations

Baseline Portfolio Strategy

Average values of each constrained metric to serve as their limit to construct an alpha-maximizing portfolio with average risk preferences

FF3 Model

Despite its robustness compared to CAPM by including size and value factors, it still does not capture the full spectrum of variables that can influence asset returns (e.g., momentum, liquidity)

The financial industry’s task will be to further refine these models such as the Fama French Five Factor Model and incorporate a broader set of factors like profitability and investment to quickly adapt to market realities and diverse investor needs

A low-angle, upward-looking photograph of several modern skyscrapers. The buildings are dark, with some windows reflecting light, creating a grid-like pattern against a pale, overcast sky. The perspective makes the buildings appear to converge towards the top of the frame.

Thank you!

Questions?

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Appendix

Related Charts & Tables

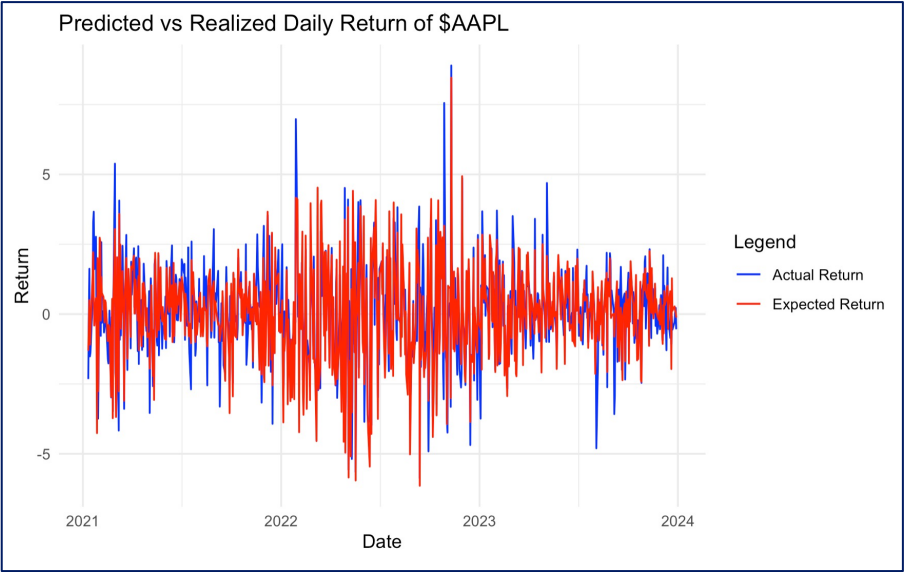
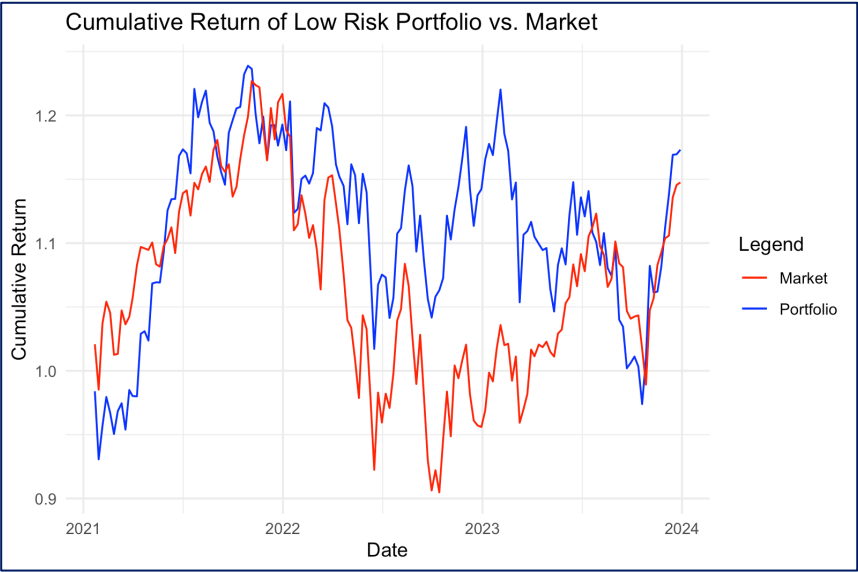
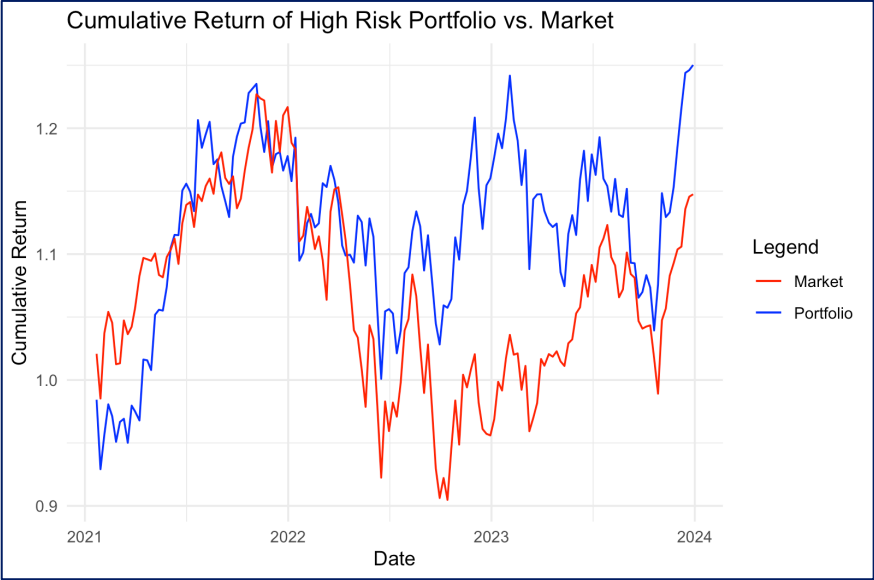


Figure 2: Weights assigned for the 25 stocks in the portfolio for every week.

```
-- CHOSEN STOCKS --  
  
Week: 2021-01-15, Stock: AES, Weight: 0.19  
Week: 2021-01-15, Stock: AMP, Weight: 0.01  
Week: 2021-01-15, Stock: BXP, Weight: 0.01  
Week: 2021-01-15, Stock: CINF, Weight: 0.01  
Week: 2021-01-15, Stock: CTRA, Weight: 0.18  
Week: 2021-01-15, Stock: DHI, Weight: 0.01  
Week: 2021-01-15, Stock: EQR, Weight: 0.066  
Week: 2021-01-15, Stock: FE, Weight: 0.01  
Week: 2021-01-15, Stock: GM, Weight: 0.164  
Week: 2021-01-15, Stock: HPE, Weight: 0.2  
Week: 2021-01-15, Stock: JCI, Weight: 0.01  
Week: 2021-01-15, Stock: KMI, Weight: 0.01  
Week: 2021-01-15, Stock: L, Weight: 0.01  
Week: 2021-01-15, Stock: LH, Weight: 0.01  
Week: 2021-01-15, Stock: LMT, Weight: 0.01  
Week: 2021-01-15, Stock: LOW, Weight: 0.01  
Week: 2021-01-15, Stock: MET, Weight: 0.01  
Week: 2021-01-15, Stock: MRNA, Weight: 0.01  
Week: 2021-01-15, Stock: NOC, Weight: 0.01  
Week: 2021-01-15, Stock: OKE, Weight: 0.01  
Week: 2021-01-15, Stock: PFG, Weight: 0.01  
Week: 2021-01-15, Stock: PRU, Weight: 0.01  
Week: 2021-01-15, Stock: SYF, Weight: 0.01  
Week: 2021-01-15, Stock: TSCO, Weight: 0.01  
Week: 2021-01-15, Stock: UDR, Weight: 0.01  
Week: 2021-01-22, Stock: AAPL, Weight: 0.2  
Week: 2021-01-22, Stock: AEP, Weight: 0.01  
Week: 2021-01-22, Stock: AMGN, Weight: 0.01  
Week: 2021-01-22, Stock: CBRE, Weight: 0.01  
Week: 2021-01-22, Stock: CLX, Weight: 0.01  
Week: 2021-01-22, Stock: EXR, Weight: 0.01
```

Figure 4: Snippet of slack values of the weekly value-at-risk (var), downside deviation (dd), and CAPM expected volatility (pv).

```
-- CONSTRAINT: var --  
2021-01-15: slack = -3.000495292612845  
2021-01-22: slack = -5.4329001968581  
2021-01-29: slack = -3.2345023039591525  
2021-02-05: slack = -6.191376384593591  
2021-02-12: slack = -5.295629259075647  
2021-02-19: slack = -5.439409724018589  
2021-02-26: slack = -3.4556959660374567  
2021-03-05: slack = -4.911142755474794  
2021-03-12: slack = -4.524691404008865  
  
-- CONSTRAINT: dd --  
2021-01-15: slack = 1.4496093960526366  
2021-01-22: slack = 3.178155149094428  
2021-01-29: slack = 2.0019015755690415  
2021-02-05: slack = 3.2291291363145653  
2021-02-12: slack = 2.8462139010192495  
2021-02-19: slack = 3.186567715727197  
2021-02-26: slack = 2.440275286530934  
2021-03-05: slack = 2.708757528738735  
2021-03-12: slack = 2.7714167528140057  
  
-- CONSTRAINT: pv --  
2021-01-15: slack = -4.440892098500626e-16  
2021-01-22: slack = -4.440892098500626e-16  
2021-01-29: slack = 0.0  
2021-02-05: slack = -4.440892098500626e-16  
2021-02-12: slack = 8.881784197001252e-16  
2021-02-19: slack = 1.7763568394002505e-15  
2021-02-26: slack = 4.440892098500626e-16  
2021-03-05: slack = -8.881784197001252e-16  
2021-03-12: slack = 4.440892098500626e-16
```