Portfolio Re-Optimization

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1. Introduction

This project seeks to perform a weekly portfolio re-optimization to maximize excess returns for enhanced portfolio performance, considering investor preferences and risk considerations.

This project employs the finance industry standard Fama-French Three Factor Pricing Model (FF3) to forecast a firm's returns as a function of market return, the performance of small versus big firms, and the performance of value versus growth stocks. This prediction will be compared with the firm's realized returns on a weekly basis to identify idiosyncratic premium returns. After that, a linear optimization program will be used to capitalize on them effectively the following week.

The success of this project will be determined by the extent to which our portfolio strategy outperforms the market while offering a superior risk-adjusted performance, as measured by the Sharpe Ratio. Our portfolio strategy is anticipated to yield a greater Sharpe Ratio than the market as maximizing alpha maximizes the return that is premium to what is expected given a level of exposure to systematic risks. Another benchmark will be the consistency of the forecasting model, validated through comparisons between predicted and realized returns.

2. Data Overview

Our original dataset consisted of 749 daily observations of the adjusted price of 500 stocks within the S&P 500 from January 4th, 2021, to December 29th, 2023, for a total of 375,972 entries, including the returns of the S&P 500 itself. Finally, we gathered information on the sectors of each stock in the S&P 500 across this period. After joining, this data left us with 452 remaining assets, including the market itself, which was given the sector identifier "Market." After gathering the data, the first step in data processing was to calculate each stock's percent change daily returns, which is used to calculate the weekly variables for our constraints. After identifying the week that observation belongs to, we grouped our data by the "Week" variable and calculated the necessary features for our model. This included the weekly return, the weekly volatility, the weekly 5% Value-at-Risk, and the weekly Downside Deviation. The weekly return is calculated as the percent difference between Monday's open price and Friday's close price. The weekly volatility is estimated as the standard deviation of the daily returns, scaled by $\sqrt{5}$. The weekly Value-at-Risk uses the daily returns and assumes a normal distribution to estimate the potential loss of an investment in one week, given a confidence level of 5%. Downside Deviation measures the volatility of negative returns, providing insight into the risk associated with negative returns. These metrics are essential in portfolio construction as they help investors quantify and manage the potential downside risk of their investments.

Investigating the summary statistics of the original data reveals the trends and events that have significantly impacted the market's price within the last three years and may impact the performance of our chosen portfolios. After a strong return of 21.67% in 2021, fears of an economic "hard-landing" and stubborn inflation led to a loss of 20.36% in 2022. Despite a turbulent start to 2023 due to the collapse of Silicon Valley Bank and Credit Suisse, resilient growth in the American economy drove a strong year of returns of 18.4%. Overall, the last three years showed an average annual return of 4.69% and an average annual volatility of 17.88% for a total 3-year return of 14.75%. While a turbulent market typically presents opportunities to make superior returns, the effect of unpredictable exogenous market events may make it difficult to forecast stock returns and negatively impact the success of our portfolio.

3. Methodology & Model

3.1 Predictive Analysis

We decided to employ the Fama-French Three-Factor model (FF3) to predict a stock's weekly return. By expanding upon the single-factor Capital Asset Pricing Model (CAPM), the FF3 model incorporates two additional factors beyond market risk, size (SMB) and value (HML), to better capture variations in systematic risk across stocks. Market risk is measured as a stock's exposure to the market premium, which measures the superior performance of the market against the risk-free rate provided by the US 1-Month T-Bill. The remaining two factors, High Minus Low (HML) and Small Minus Big (SMB), measure the performance of two long-short portfolios. HML measures the performance difference between stocks with high book-to-market ratios (value stocks) and those with low book-to-market ratios (growth stocks), providing insights into the profitability of investing in consistent stocks versus stocks you believe have considerable growth potential. SMB captures the difference in returns between small and large-cap stocks, highlighting the profitability of investing in smaller companies relative to larger ones. By breaking down a stock's performance into its exposure to three market factors, the FF3 model offers an interoperable framework for identifying superior idiosyncratic returns.

To apply the FF3 model to conduct our analysis, we first needed to collect data outside of our testing sample. For this, we gathered the daily adjusted price of the same 451 stocks plus the market that we had established in the previous step from the start of 2017 through the end of 2019 to avoid the impacts of COVID-19 on our training data. Furthermore, we needed to collect the daily metrics of each FF3 factor. These factors are updated in daily, weekly, and monthly intervals and are provided for free on Kenneth French's website.

Once we collected the necessary factors, we regressed each stock's daily premium return on the daily Market Premium, SMB, and HML metrics, utilizing all 752 observations across three years for each stock individually. Following the regressions, we pulled the coefficients of each factor to estimate each stock's exposure to the three different macro-market risk factors, otherwise known as the "Betas" to each factor.

$$R_{t,i} - RF_t = \alpha_{t,i} + \beta_{i,M} MktPrem_t + \beta_{i,SMB} SMB_t + \beta_{i,HML} HML_t + \epsilon_{t,i} \tag{1}$$

We find that the average Adjusted R-squared of the regressions we run in this sample is 30.75%, which is quite high for stock returns that are typically incredibly difficult to forecast due to the significance of unpredictable market news. The standard deviation of these results was 16.62%. In comparison to the single factor CAPM model, the FF3 model improves in-sample performance by about 4%. The average out-of-sample performance of our model is 25.68% with a larger standard deviation of these results of 20.27%, in part driven by some stocks having superior out-of-sample performance compared to in-sample performance. This can be explained as, for some firms, in some years, their returns may be more driven by idiosyncrasies, like new products, while returns are more driven by the market in other years. An example of our regression on Google's daily stock performance shows an in-sample Adjusted R-squared of 59.42%. As an example, investigating the residuals in Figures 9 and 10 reveals that the errors are not auto-correlated in our daily returns regression for Google, implying that the FF3 sufficiently explains the variance in stock returns.

We then combine these Betas with weekly observations of the FF3 factors in our test sample to estimate each stock's expected return each week. Using each stock's estimated weekly return, we compare its realized return with what can be explained by its exposure to systematic risks to isolate its excess returns, known as "Alpha." Positive Alpha signals a stock's superior return compared to what is expected given its level of systematic risk. It serves as the primary metric to maximize for each of our weekly portfolios.

3.2 Linear Optimization

A linear programming approach was used to determine the optimal portfolio that maximizes excess return based on an investor's preferences for each week in our sample. An overview is described below; please refer to the code for further details.

a. Decision Variables

Let w_{ti} be the weight, in percent, of the chosen stock i in the portfolio for week t and let x_{ti} serve as a signal variable that indicates if stock i is given a positive weight in week t, where i corresponds to a list of 451 stocks in the S&P 500 plus the market itself and t corresponds to a list of weeks over the three years in our sample.

$$w_{ti} \in [0.0, 1.0] \qquad x_{ti} = \begin{cases} 1 & \text{if } w_{ti} \text{ is greater than 0} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

b. Objective Function

The objective is to maximize the excess returns of each weekly portfolio for enhanced risk-adjusted return.

$$\sum_{i=1}^{452} (w_{ti} * \alpha_{ti}), \ \forall t \tag{3}$$

c. Given Variables

The given and defined variables that are used in building the model's constraints have been constructed previously when creating the dataset and conducting the predictive analysis; These include a stock's alpha (α), value-at-risk (v), downside deviation (d), CAPM expected volatility (p), and sector (s).

To control the volatility of each week's portfolio, a matrix of 452x452 covariances each week would be required to evaluate each stock's variance contribution perfectly. To simplify this, we assume a single factor CAPM where the expected variance of a stock is a function of its exposure to market variance in addition to its idiosyncratic, or "unexplained" variance. Additionally, this model posits that the idiosyncratic variance of a stock is uncorrelated to that of other stocks and instead is only correlated through their exposures to market risk. This allows us to circumvent the need for complex covariance matrices, and instead, we only need to calculate each stock's explained and unexplained variances. The expected volatility of each portfolio is then determined by the following formula:

$$p = E[\sigma_P] = \sum_{i=1}^{25} (w_{ti} * \sqrt{\beta_i^2 \sigma_{M_t}^2 + \sigma_{e_{ti}}^2}) \tag{4}$$

d. Constraints

The model operates with several constraints, as denoted in the code and seen in Figure 11, with two key ones shaping its framework. It mandates exactly 25 stocks in the portfolio, aligning with industry standards to achieve diversification. Secondly, threshold values for essential metrics, including the 5% value-at-risk, downside deviation, and CAPM expected volatility, are determined based on their mean values in the dataset to achieve a portfolio with an average risk profile.

The value-at-risk constraint limits the extent of potentially large losses within the portfolio, and the downside deviation constraint limits the risk associated with negative deviations from the portfolio's expected return. The CAPM expected volatility constraint bears a lot of importance, aiding in adjusting the portfolio's risk profile as it is expected to be consistently binding. As higher risk induces higher returns, this volatility constraint, in particular, can be adjusted for investors seeking increased or decreased risk and return.

We expect that without additional constraints, the optimizer will want to place 99.99% of a portfolio's weight into one stock with the highest alpha. The sector and individual stock weight constraints are implemented to prevent this. These constraints ensure that each chosen stock's weights are between 1% and 100% (inclusive) and that the weight for each sector in the portfolio must be at most 20%. In doing so, stock and sector diversification is encouraged while managing sector-specific risk. Note that given this sector constraint, a stock's weight will never exceed 20%.

4. Results & Insights

Inputting our completed dataset of weekly returns from 2021-2023 with the necessary variables for our constraints into our linear optimization model outputted the decisive weights for the 25 chosen stocks each week. Because the weekly data is only realized at the end of the week, we need to shift our decided stocks and their respective weights by one week. This way, the model uses the previous week's data, selects the 25 stocks to build the portfolio that would have maximized alpha that week, and then invests in this portfolio the following week in hopes of continued success. This type of strategy in finance is known as a "momentum" strategy, leveraging previous short-term success to forecast short-term future returns. Charting the cumulative product of weekly returns of our shifted portfolio, we can compare our optimization's performance against the market from 2021 through 2023. As seen in Figure 5 of the appendix, our weekly re-constructed portfolio manages to outperform the market by 5.08%, achieving a total return of 19.83%. After maintaining pace with the market in the first year of our sample, earning a return of 19.05% compared to the market's 21.67%, the real difference in performance was made in 2022, where our portfolio was far more resilient to turbulent economic conditions relative to the total market, vy only losing 2.95% compared to the market's loss of 20.36%. While the market grew 18.4%, recovering from its loss the previous year, our portfolio strategy grew 6.21%, partially driven by a sharp increase in performance in late 2023, where our portfolio was able to maximize the growth promoted by a booming period in the market. Overall, our portfolio exhibited an average volatility of 18.42% and a Sharpe Ratio of 0.3372, which offers 7.48bps more return than the market for each percent of increased volatility.

We were then able to determine which constraints were consistently binding by investigating the slacks of each constraint each week. We found that the expected CAPM portfolio volatility was consistently binding for almost every week the portfolio was built, while the downside deviation constraint was sometimes binding. In contrast, the value-at-risk was never binding at the mean level which may be explained by its assumption of a normal distribution of returns, meaning that this constraint was likely absorbed in the expected CAPM volatility constraint. As seen in Figures 7 and 8, adjusting the CAPM volatility constraint can allow a higher-risk portfolio that induces increased volatility and a higher total return or a lower-risk portfolio with decreased volatility and a lower total return.

Furthermore, our sector weight constraint was consistently binding and can be changed to effectively adjust the sector diversity of the calculated portfolios. Investors inclined towards lower sector diversity may have a deep understanding or strong beliefs about certain industries, including confidence in the growth prospects or a desire for investments that align with personal values. The maximum sector weight constraint could be increased for chosen sectors, such that investors can concentrate more heavily on areas they believe will outperform or align with their personal preferences. To exclude a sector entirely from the portfolio, like excluding energy to avoid oil and gas for sustainability reasons, a constraint could be explicitly added to set the investment weight for that undesirable sector equal to 0%.

5. Conclusions & Limitations

The project has shown that a weekly re-optimized portfolio under the Fama French Three Factor Model can outperform the market by adapting to changing conditions on short-term trends. The portfolio's outperformance after the first year displays the importance of adaptability and risk management.

Our approach is not without assumptions that present certain risks and limitations. We assume the possibility of owning partial stocks, the absence of trading expenses, the ability to hold the market without ETF expenses, and the assumption of unlimited drawdown. This presents a model that is proven effective in theory but may differ in practical application. These assumptions do not necessarily reflect all investor types, particularly those with lower risk tolerance. Investor profiles were crucial to our constraint considerations as aggressive investors might require looser constraints on volatility and potential losses. In contrast, investors with specific sector preferences might want tailored sector weight constraints.

The limitations of the FF3 model itself should also be acknowledged. Despite its robustness compared to CAPM by including size and value factors, it still does not capture the full spectrum of variables that can influence asset returns such as momentum and liquidity. Moreover, although historical performance can offer valuable insights, this reliance on strictly price data makes it challenging for portfolio optimization, as future market conditions or economic variables may diverge from past patterns, requiring insight into the firms themselves to make informed investment decisions.

In conclusion, the portfolio strategy from this project illustrates the power of weekly re-optimization as a powerful tool to gain excess returns and a superior Sharpe Ratio. While the strategy successfully outperformed the market within the test period, it is crucial to consider the underlying assumptions and limitations. The following steps in this project would be to refine these models further, such as implementing the Fama-French Five Factor Model and incorporating a broader set of factors to adapt to market realities.

6. Appendix

Figure 1: Dataframe snippet with a multi-index setup.

Week	symbol	Weekly_Returns	Weekly_LogReturns	Weekly_Volatility	MktPrem	SMB	HML	RF	MktPrem_Vol	SMB_Vol	HML_Vol	...	mkthml	hmlsmb
2021-01-15	A	-0.189797	-0.671220	1.611180	-1.03	2.33	2.15	0.001	1.089105	2.744194	2.228206	...	0.449237	2.201666
	AAL	6.342780	4.079556	8.346176	-1.03	2.33	2.15	0.001	1.089105	2.744194	2.228206	...	0.449237	2.201666
	AAPL	-1.586812	-3.789162	3.437362	-1.03	2.33	2.15	0.001	1.089105	2.744194	2.228206	...	0.449237	2.201666
	ABBV	3.222188	4.147547	2.896805	-1.03	2.33	2.15	0.001	1.089105	2.744194	2.228206	...	0.449237	2.201666
	ABT	0.080932	0.123926	4.540354	-1.03	2.33	2.15	0.001	1.089105	2.744194	2.228206	...	0.449237	2.201666
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
2023-12-29	YUM	0.600554	0.506404	0.639113	0.17	-0.67	0.23	0.107	0.847545	1.728824	0.763108	...	0.288304	0.582198
	ZBH	1.231073	1.140418	1.109352	0.17	-0.67	0.23	0.107	0.847545	1.728824	0.763108	...	0.288304	0.582198
	ZBRA	1.233329	1.444541	2.927578	0.17	-0.67	0.23	0.107	0.847545	1.728824	0.763108	...	0.288304	0.582198
	ZTS	1.277704	1.218310	0.632183	0.17	-0.67	0.23	0.107	0.847545	1.728824	0.763108	...	0.288304	0.582198
	^GSPC	0.230522	0.319183	0.652048	0.17	-0.67	0.23	0.107	0.847545	1.728824	0.763108	...	0.288304	0.582198

70060 rows × 23 columns

Figure 2: Weights assigned for the 25 stocks in the portfolio for every week.

```
-- CHOSEN STOCKS --

Week: 2021-01-15, Stock: AES, Weight: 0.19
Week: 2021-01-15, Stock: AMP, Weight: 0.01
Week: 2021-01-15, Stock: BXP, Weight: 0.01
Week: 2021-01-15, Stock: CINF, Weight: 0.01
Week: 2021-01-15, Stock: CTRA, Weight: 0.18
Week: 2021-01-15, Stock: DHI, Weight: 0.01
Week: 2021-01-15, Stock: EQR, Weight: 0.066
Week: 2021-01-15, Stock: FE, Weight: 0.01
Week: 2021-01-15, Stock: GM, Weight: 0.164
Week: 2021-01-15, Stock: HPE, Weight: 0.2
Week: 2021-01-15, Stock: JCI, Weight: 0.01
Week: 2021-01-15, Stock: KMI, Weight: 0.01
Week: 2021-01-15, Stock: L, Weight: 0.01
Week: 2021-01-15, Stock: LH, Weight: 0.01
Week: 2021-01-15, Stock: LMT, Weight: 0.01
Week: 2021-01-15, Stock: LOW, Weight: 0.01
Week: 2021-01-15, Stock: MET, Weight: 0.01
Week: 2021-01-15, Stock: MRNA, Weight: 0.01
Week: 2021-01-15, Stock: NOC, Weight: 0.01
Week: 2021-01-15, Stock: OKE, Weight: 0.01
Week: 2021-01-15, Stock: PFG, Weight: 0.01
Week: 2021-01-15, Stock: PRU, Weight: 0.01
Week: 2021-01-15, Stock: SYF, Weight: 0.01
Week: 2021-01-15, Stock: TSCO, Weight: 0.01
Week: 2021-01-15, Stock: UDR, Weight: 0.01
Week: 2021-01-22, Stock: AAPL, Weight: 0.2
Week: 2021-01-22, Stock: AEP, Weight: 0.01
Week: 2021-01-22, Stock: AMGN, Weight: 0.01
Week: 2021-01-22, Stock: CBRE, Weight: 0.01
Week: 2021-01-22, Stock: CLX, Weight: 0.01
Week: 2021-01-22, Stock: EXR, Weight: 0.01
```

Figure 3: Dataframe snippet after dummifying the categorical industry sectors with two new columns displaying if a stock was chosen or not, along with its weight if it were chosen.

rem	SMB	HML	RF	MktPrem_Vol	SMB_Vol	HML_Vol	...	Financials	Health Care	Industrials	Information Technology	Market	Materials	Real Estate	Utilities	Chosen	Weight
1.03	2.33	2.15	0.001	1.089105	2.744194	2.228206	...	False	True	False	False	False	False	False	False	-0.0	0.0
1.03	2.33	2.15	0.001	1.089105	2.744194	2.228206	...	False	False	True	False	False	False	False	False	0.0	0.0
1.03	2.33	2.15	0.001	1.089105	2.744194	2.228206	...	False	False	False	True	False	False	False	False	0.0	0.0
1.03	2.33	2.15	0.001	1.089105	2.744194	2.228206	...	False	True	False	False	False	False	False	False	0.0	0.0
1.03	2.33	2.15	0.001	1.089105	2.744194	2.228206	...	False	True	False	False	False	False	False	False	0.0	0.0
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
0.17	-0.67	0.23	0.107	0.847545	1.728824	0.763108	...	False	False	False	False	False	False	False	False	0.0	0.0
0.17	-0.67	0.23	0.107	0.847545	1.728824	0.763108	...	False	True	False	False	False	False	False	False	0.0	0.0
0.17	-0.67	0.23	0.107	0.847545	1.728824	0.763108	...	False	False	False	True	False	False	False	False	0.0	0.0
0.17	-0.67	0.23	0.107	0.847545	1.728824	0.763108	...	False	True	False	False	False	False	False	False	-0.0	0.0
0.17	-0.67	0.23	0.107	0.847545	1.728824	0.763108	...	False	False	False	False	True	False	False	False	0.0	0.0

70060 rows × 37 columns

Figure 4: Snippet of slack values of the weekly value-at-risk (var), downside deviation (dd), and CAPM expected volatility (pv).

```
-- CONSTRAINT: var --
2021-01-15: slack = -3.000495292612845
2021-01-22: slack = -5.4329001968581
2021-01-29: slack = -3.2345023039591525
2021-02-05: slack = -6.191376384593591
2021-02-12: slack = -5.295629259075647
2021-02-19: slack = -5.439409724018589
2021-02-26: slack = -3.4556959660374567
2021-03-05: slack = -4.911142755474794
2021-03-12: slack = -4.524691404008865
```

```
-- CONSTRAINT: dd --
2021-01-15: slack = 1.4496093960526366
2021-01-22: slack = 3.178155149094428
2021-01-29: slack = 2.0019015755690415
2021-02-05: slack = 3.2291291363145653
2021-02-12: slack = 2.8462139010192495
2021-02-19: slack = 3.186567715727197
2021-02-26: slack = 2.440275286530934
2021-03-05: slack = 2.708757528738735
2021-03-12: slack = 2.7714167528140057
```

```
-- CONSTRAINT: pv --
2021-01-15: slack = -4.440892098500626e-16
2021-01-22: slack = -4.440892098500626e-16
2021-01-29: slack = 0.0
2021-02-05: slack = -4.440892098500626e-16
2021-02-12: slack = 8.881784197001252e-16
2021-02-19: slack = 1.7763568394002505e-15
2021-02-26: slack = 4.440892098500626e-16
2021-03-05: slack = -8.881784197001252e-16
2021-03-12: slack = 4.440892098500626e-16
```

Figure 5: Cumulative return of the portfolio versus the market.

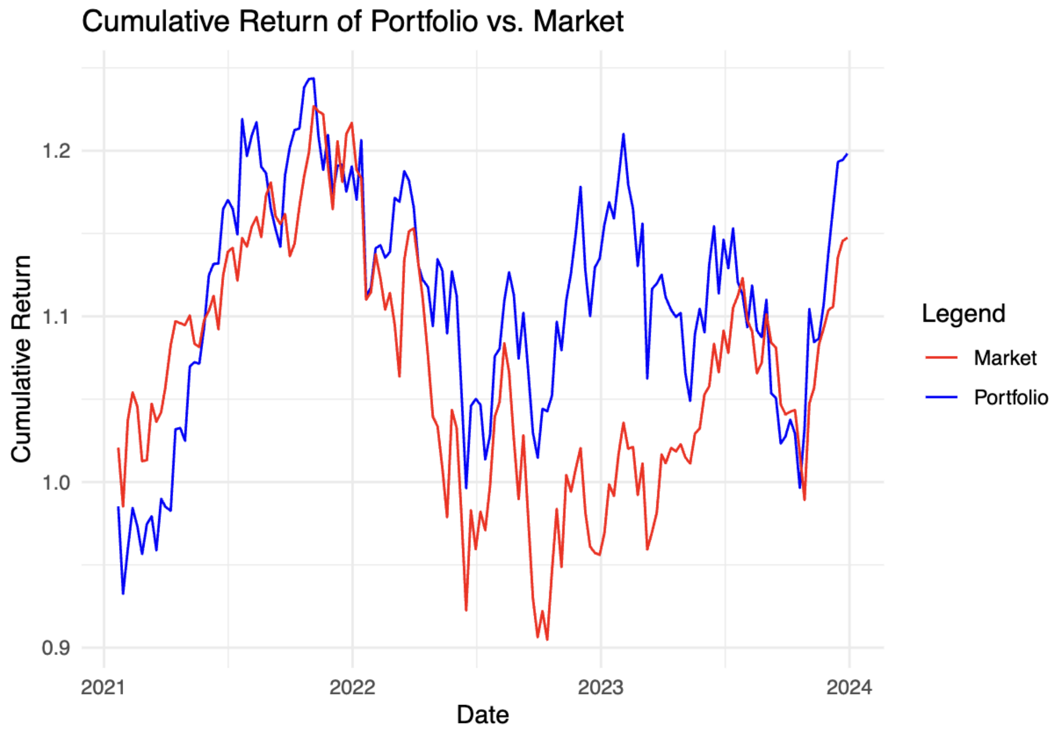

Figure 6: Predicted versus realized daily returns of Google.

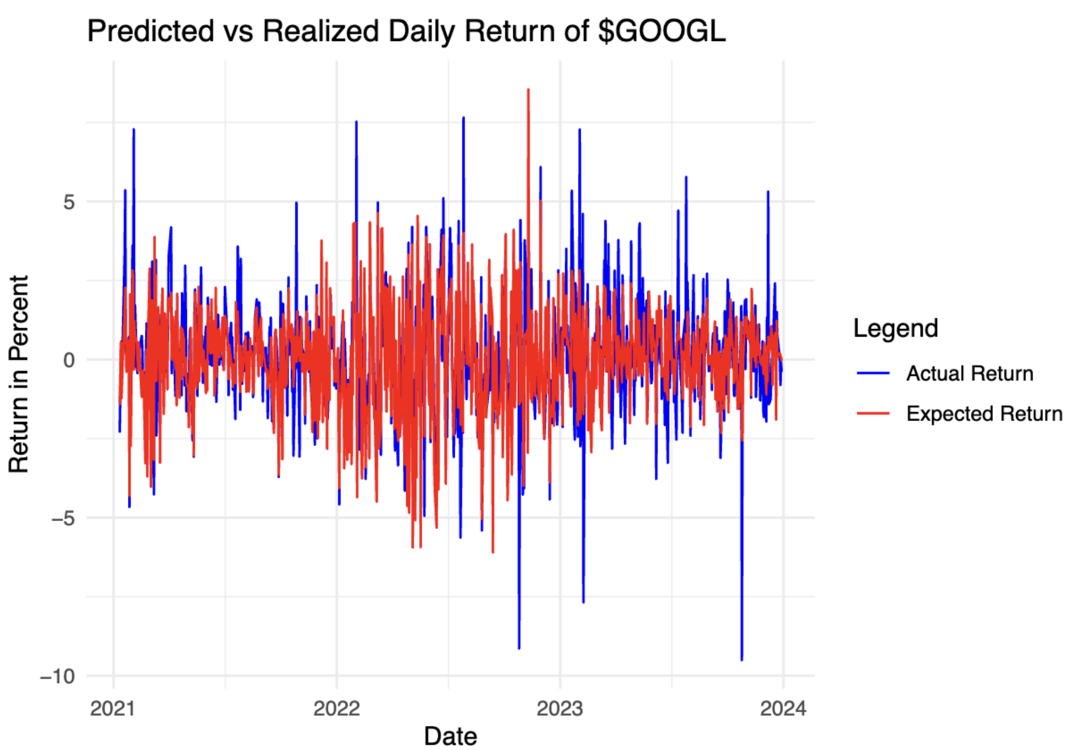

Figure 7: Alternative High Risk Portfolio.

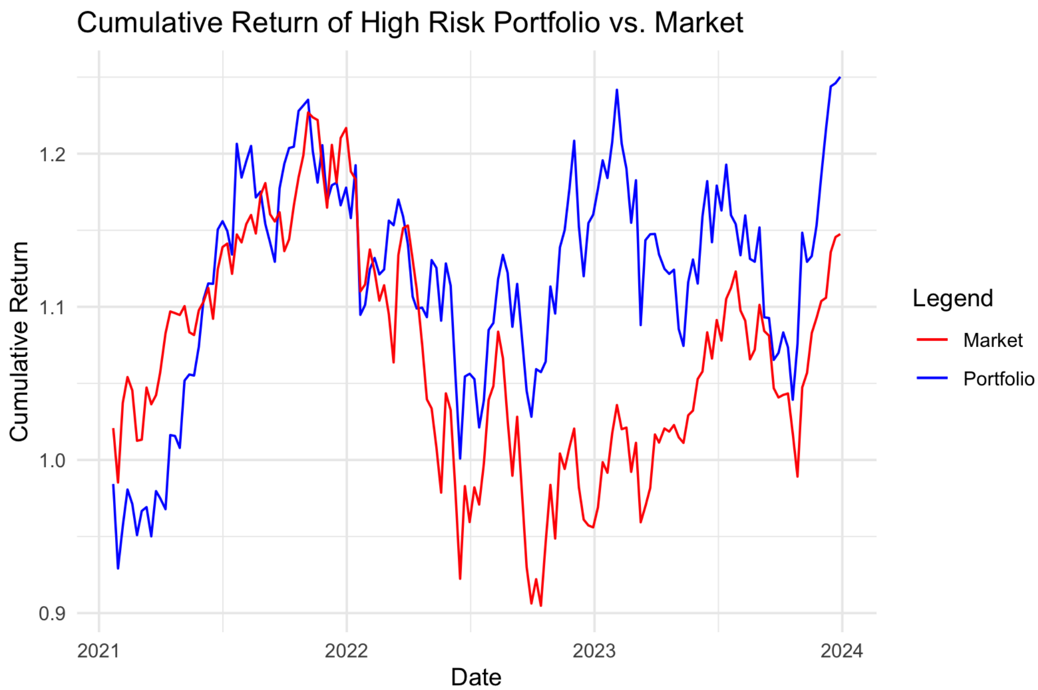

Figure 8: Alternative Low Risk Portfolio.

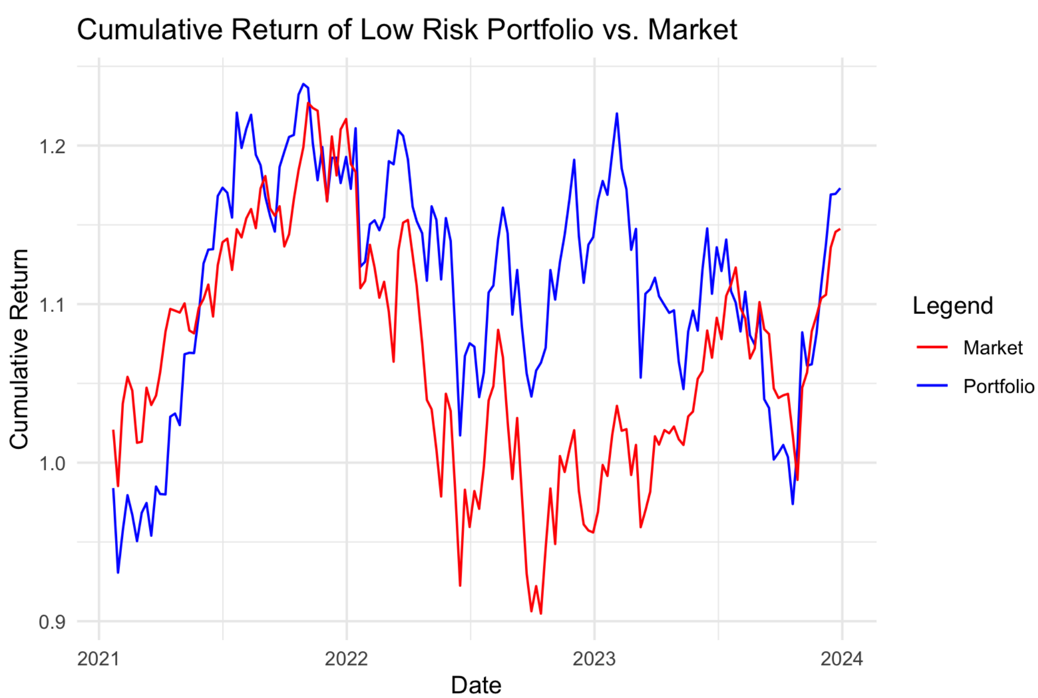

Figure 9: ACF of Google Regression Residuals.

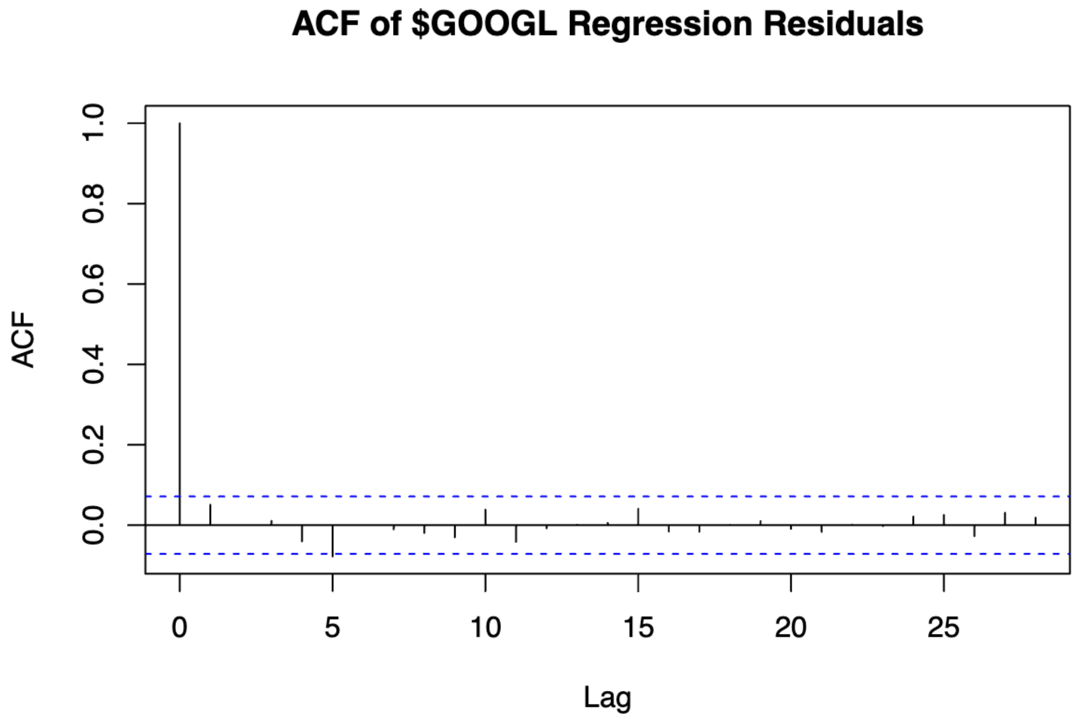

Figure 10: PACF of Google Regression Residuals.

Figure 11: Linear Optimization Constraints.

PROBLEM FORMULATION

DECISION VARIABLES

- w = stock's weight in the portfolio *(continous)*
- x = chosen stock *(binary)*

DEFINED VARIABLES * α = stock's alpha

- v = stock's value-at-risk
- d = stock's downside deviation
- p = stock's CAPM portfolio volatility
- s = stock's sector
- t, i = panel data identifier
- written as "coordinates", where t represents the week and i represents the stock

OBJECTIVE: MAXIMIZE EXCESS RETURNS

$$\sum_{i=1}^{452}(w_{ti} * \alpha_{ti}),\ \forall t$$

SUBJECT TO

1. A stock is either chosen or not.

$$x_{ti} \in \{0, 1\}$$

2. In total, 25 stocks must be chosen.

$$\sum_{i=1}^{452} x_{ti} = 25,\ \forall t$$

3. The total weight for the chosen stocks in the portfolio must equal to 100%.

$$\sum_{i=1}^{452} w_{ti} = 1,\ \forall t$$

4. Each chosen stock's weight in the portfolio must be between 1% and 100% (both inclusive)

$$0.01 \leq w_{ti} \leq 1$$

$$w_{ti} \geq 0.01 * x_{ti}$$

$$w_{ti} \leq 1 * x_{ti}$$

5. The total value-at-risk of the portfolio must be greater than or equal to -3.904%.

$$\sum_{i=1}^{452}(w_{ti} * v_{ti}) \geq -3.904,\ \forall t$$

6. The total downside deviation of the portfolio must be less than or equal to 3.305%.

$$\sum_{i=1}^{452}(w_{ti} * d_{ti}) \leq 3.305,\ \forall t$$

7. The total CAPM portfolio volatility of the portfolio must be less than or equal to 3.874%.

$$\sum_{i=1}^{452}(w_{ti} * p_{ti}) \leq 3.874,\ \forall t$$

Where

$$p_{ti} = \sqrt{\beta_{M,i}^2\, \sigma_{M,t}^2 + \sigma_{e_{ti}}^2}$$

8. The sector weight for each sector of the portfolio must be less than or equal to 20%.

$$\sum_{i=1}^{452}(w_{ti} * s_{ti}) \leq 0.2,\ \forall t$$

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