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Crosscutting Areas

Information Disclosure and Pricing Policies for Sales of Network Goods

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Abstract. We study a two-period model in which a firm faces the problem of deciding whether to commit to sales volume disclosure under market size uncertainty when selling a network good to forward-looking customers who time their purchases. If the first-period sales volume is disclosed, the second-period customers will base their purchase decisions on this information. If the sales volume is not disclosed, all customers will make purchase decisions based on their estimate of market size. We identify two countervailing effects of sales disclosure: (1) a prodisclosure "Matthew effect" (the benefit of a realized large market size tends to outweigh the loss of a realized small one) and (2) an antidisclosure saturation effect (for a sufficiently large market, customers would make a purchase anyway even without knowing the realized market size but might be discouraged if observing a realized small one). With exogenous prices, we show that committing to sales disclosure is a dominating (dominated) policy when the expected network benefit is relatively weak (strong). We also examine three endogenous pricing scenarios. First, under state-independent pricing, committing to sales disclosure is better off if the customer valuation distribution has a high probability of reaching very high values (i.e., a heavy tail). Second, if a firm can credibly preannounce a contingent pricing policy, committing to sales disclosure is always better off. Third, under contingent pricing without commitment, we show that committing to sales disclosure is better off when delaying the purchase decision to the second period does not reduce the value much.

Keywords: information disclosure • pricing • sales information • network effects

1. Introduction

A *network effect* (also called a *network externality*) is the way that the value of a unit of goods or services (collectively referred to as *goods*) increases with the number of units sold (Economides 1996). A good with network effects is called a *network good*. In the presence of network effects, the utility of a good for a (forward-looking) customer depends on the number of other customers who will eventually purchase it. At the time that customers make their own individual purchase decisions, they may be uncertain about how many other potential buyers are out there and what those buyers' purchase decisions might be; hence, they have to cope with uncertainty about the network benefit that they can obtain from the purchase. As a result, the market size information influences each

individual customer's expectation of the network benefit and hence their purchase decisions.

In selling network goods, reporting sales information is one of the best ways for a firm to show not only how many users it has now but also how large the potential market is. Firms often commit to a sales information release policy when selling network goods. For example, regardless of the investors' strong desire and pressure for more transparency, Amazon and Apple, which sell tablet devices, have each adopted a different implicit information policy and have developed distinct "reputations" for releasing information on the sales of their tablets. With Amazon, "even a number as basic, and presumably impressive, as how many Kindle e-readers the company sells is *never* released" (Streitfeld and Haughney 2013). With Apple,

iPhone and iPad sales numbers are *always* released, even if they are disappointing (Elmer-DeWitt 2012).

Sales information voluntarily disclosed by a firm could be a boon for customers by allowing them to make better informed purchase decisions. However, it is not clear whether a commitment to revealing sales information would always benefit a firm that sells network goods under market uncertainty. On the one hand, it could be beneficial if the market reaction is enthusiastic, in which case more customers may be drawn to make a purchase. On the other hand, it could be detrimental if interest is scant, in which case customers may be discouraged.

In this paper, we study the sales information release policy for selling network goods subject to market size uncertainty. Our model tends to be better at capturing new emerging network goods, such as fast-turnover gadgets and mobile games. The sales horizon for these goods is relatively short, sometimes only days, and usually no more than a year. Moreover, *demand uncertainty* is a crucial element in selling these goods because of the fast-changing market conditions, and it is the key feature that differentiates this paper from the prior art on network goods. In these markets, because of the fast turnover, there could be many *repeated interactions* between the seller and consumers. As a result, the seller is likely to develop and sustain a reputation about information disclosure, or alternatively, it may simply commit to an information policy.

To investigate the impact of revealing sales information on a monopolistic firm's profitability, we consider a two-period model in which there is a cohort of customers of a random size arriving at the market in the first period. All customers are forward looking: that is, they can decide to buy the good in the first period or delay their purchase decisions to the second period. A customer valuation has two additive components: a *standalone valuation*, and a *network benefit* that depends on the total sales over the two periods. Instead of studying the general mechanism design problem, which covers the entire policy space, we compare two simple and practically motivated information release policies: sales disclosure versus sales nondisclosure. In the sales nondisclosure setting, the firm commits to not disclosing early sales information, and all customers have to base their purchase decisions on their prior knowledge of the market size. In the sales disclosure setting—with early sales information from the first period disclosed—the customers who delayed their decisions to the second period can learn the realized market size and then make informed purchase decisions.

We identify two opposing effects of sales disclosure. One is a prodisclosure *Matthew effect*. This is a term, coined by Robert K. Merton, to describe how eminent scientists will often get more credit than a comparatively unknown researcher, even if their work is similar, with the *visibility* of the scientists as an implicit driver of the effect (Merton 1968). In our context, this effect refers to the phenomenon, which is intrinsically driven by the positive network externalities, that the benefit of a disclosed large market size potentially outweighs the loss associated with a small one. As a new customer enters the market, existing customers will increase their expectation of the ultimate network size, which determines their expected utility of buying the good. As a result, the new customer is more willing to buy the good. In anticipation of that, other customers will further increase their utility of buying the good. Thus, with some assumptions (Assumption 1) on the distribution of standalone valuations and the form of the network benefit function (i.e., how the network benefit depends on the total sales), the equilibrium adoption rate is more than linearly increasing with the market size. Therefore, the benefit of a realized large market size tends to outweigh the loss of a realized small market size. The other effect is an antidisclosure *saturation effect*. That is, for a sufficiently large expected network benefit owing to a sufficiently large expected market size, customers would buy the good even without knowing the exact market size but might be discouraged by seeing a small realized market size.

The trade-off between these two effects drives the firm's sales information disclosure policy. When the prices of the good are exogenously given, we show that if the *expected* network benefit is sufficiently strong relative to the exogenous prices, the saturation effect dominates the Matthew effect and that it is better to maintain sales nondisclosure. However, if the network benefit is sufficiently weak, the Matthew effect dominates the saturation effect, and committing to sales disclosure is better. These results lend some support to the practices in the competitive online games industry. For example, Blizzard's World of Warcraft, after developing a strong customer base, announced that it would not report the subscription numbers for all of its future releases (Tassi 2015). However, numerous niche online game developers, with small market potential, tend to have their subscription numbers continuously reported by third-party sales watchdogs, such as addictinggames.com.

We then study the situation where the firm can decide on the all-or-nothing information disclosure policy together with endogenized prices. We examine three endogenous pricing policies from static to more contingent ones, all of which are seen in the literature.

1. *State-independent pricing*: that is, the firm decides on and announces a first-period price and credibly commits to a second-period price at the beginning of the first period (Nagle 1984).

2. *Contingent preannounced pricing*: that is, the firm determines its first-period price and credibly commits to a second-period price scheme at the beginning of the first period; in this case, the second-period price scheme is a function of the observed sales volume of the first period (Aviv and Pazgal 2008, Elmaghraby et al. 2008, Correa et al. 2016).

3. *Contingent pricing without commitment* (also known as *dynamic pricing*): that is, the firm determines prices at the beginning of each period contingent on the state that it faces in order to maximize the remaining profits (Bitran and Caldentey 2003).

We obtain the following set of results. First, under a state-independent pricing policy, we show that the optimal information release policy depends on the distribution of customer valuations, the market size distribution, and the network benefit structure. Specifically, committing to sales disclosure dominates committing to sales nondisclosure if the customer valuation distribution has a sufficiently high probability of reaching very high values (i.e., a heavy tail) relative to the expected network benefit at its full strength. However, committing to sales nondisclosure becomes better off if the customer valuation is capped by the expected network benefit (which can be interpreted as the valuation distribution having a relatively light tail) and the market size distribution has a relatively low probability of reaching very high values (i.e., a light tail).

Second, we consider a contingent preannounced pricing scheme. With preannounced pricing flexibility, if the first-period market interest is enthusiastic, then the firm can profit from the expected strong network benefit by setting a high second-period price, and if the first-period market interest is lukewarm, then the firm can set a low second-period price to stimulate demand. Pricing flexibility in the latter scenario not only improves the expected network benefit but also reduces the risk that the ultimate network benefit will be disappointing for first-period customers, thereby driving more customers to buy the good in the first period. We show that under contingent preannounced pricing, committing to sales disclosure always dominates committing to sales nondisclosure. Contingent pricing serves as a defense against the double-edged sword of information disclosure because it can unleash the benefits of information dissemination while minimizing its negative impacts.

Third, we consider contingent pricing without commitment. We show that given the same market condition at the beginning of the second period, the expected second-period profit under sales disclosure always dominates that under sales nondisclosure. However, the first-period demand under sales disclosure may not be higher than that under sales nondisclosure, implying that the first-period profit under sales disclosure

is not always higher than that under sales nondisclosure. Nevertheless, we find that committing to sales disclosure is better than committing to nondisclosure when delaying the purchase decision to the second period does not reduce the value much: that is, the discount factor is sufficiently large. In this case, most customers choose to delay their decisions to the second period, for which committing to sales disclosure results in a higher profit level than committing to nondisclosure.

The rest of this paper is organized as follows. The remainder of Section 1 reviews the relevant literature. In Section 2, we present the model formulation. Section 3 focuses on a preliminary one-shot model and reveals the basic trade-off in market size disclosure mechanisms. Section 4 presents a two-period model and investigates sales information disclosure mechanisms in relation to customers' intertemporal purchase behavior and a variety of pricing schemes. Section 5 concludes the paper and points out future research directions. All the proofs are relegated to the online appendix.

1.1. Literature Review

As the title of this paper indicates, there are at least two related bodies of literature: one on information disclosure and the other on the pricing of network goods. The former has been studied in a variety of sender–receiver games in the economics literature, including auctions (Milgrom and Weber 1982, Bergemann and Pesendorfer 2007, Mezzetti et al. 2008, Board 2009), price discrimination (Ottaviani and Prat 2001), labor market matching (Ostrovsky and Schwarz 2010), and other general settings (Lizzeri 1999, Rayo and Segal 2010, Kamenica and Gentzkow 2011, Ely 2017, Bergemann et al. 2018). These papers consider settings in which a sender can commit to an information disclosure rule that determines the distribution of signals observed by receivers, and receivers choose actions depending on the signals they observe, which, in turn, affect the sender's utility. A basic question, therefore, is which information disclosure rule maximizes the sender's expected utility. Our work builds on this stream of literature by studying the information disclosure problem in the specific setting of selling network goods. There are at least three distinctions. First, a key feature of our setting is that the state to be revealed by the sender (seller) is the number of receivers (buyers). This number affects receivers' actions considerably because of the network effect and hence influences the sender's choice of information disclosure policy. Second, in the literature, an optimal information disclosure policy often involves information distortion. However, in our setting, it could be illegal for the firm to manipulate the sales numbers. As a result, we focus on

two truthful information disclosure policies (i.e., full disclosure and no disclosure; see Section 5 for more complicated information disclosure policies). Third, although the previous studies focus mainly on the design of information disclosure, we also study the interaction between information disclosure and pricing policies in selling network goods to forward-looking customers.

Moreover, our work is related to a large body of existing literature dealing with pricing in the presence of network effects (Dhebar and Oren 1986, Xie and Sirbu 1995, Bensaid and Lesne 1996, Cabral et al. 1999, Arthur et al. 2009, Akhlaghpour et al. 2010, Anari et al. 2010, Cabral 2011, Radner et al. 2014). This stream of literature considers general network benefit functions of a *global* nature (i.e., the network benefit of a customer depends on the behavior of all customers). More recently, there is a growing body of research on pricing over networks with *local* network effects (Candogan et al. 2012, Bloch and Quérou 2013, Campbell 2013, Fainmesser and Galeotti 2015). These models capture the network effect by modeling social interactions among neighbors in a network. Our work differs from these papers because we consider market size uncertainty. The market size in the presence of network effects can influence customers' purchase decisions. Under market size uncertainty, our work focuses on the market size information disclosure policy in conjunction with pricing policies.

There are papers in the operations literature that address information disclosure issues similar to ours. For example, in a group-buying setting where a sufficient amount of signups results in a deal being unlocked, Hu et al. (2013) study a two-period game where cohorts of customers arrive at a deal and make signup decisions sequentially. They show that it is always more beneficial for the firm to disclose to second-period arrivals the number of signups accumulated in the first period than not to do so. In contrast, our work demonstrates that sales volume disclosure may not always be more beneficial under a *general* form of positive network externality. In a service system, Hu et al. (2017) study the optimal information disclosure policy about the queue length. In this setting, when there are *negative* network externalities, it may not be optimal to always reveal the real-time congestion information; instead, a randomized information disclosure policy may be even better. Over a social network, Zhou and Chen (2015, 2016) consider a contribution game with local positive network effects. The authors show that *without* market size uncertainty, it is always beneficial for players to move sequentially by revealing first-period players' actions to second-period players. Again, in contrast, we show that when there is market size uncertainty, it is not always better to reveal first-period customers' purchase decisions.

2. Model Formulation

We consider a monopolistic firm selling a network good. Without loss of generality, we assume that the good has zero marginal cost. At the beginning of the sales horizon, there comes a cohort of potential customers with a random size M. Each customer is interested in buying at most one unit of good. We assume that any single customer is negligible with respect to the total market size. The distribution of M is public information, but its exact realization is not known ex ante to the firm and neither is to customers.

For customers, this network good exhibits *positive* externalities (i.e., a customer's willingness to pay for the good depends positively on the total sales volume). To model such network externalities, we assume that a customer's willingness to pay consists of his or her own valuation and a social utility. Specifically, we write the customer's willingness to pay as $u = v + s(d)$. The first term v represents the standalone valuation, which is the customer's intrinsic valuation of the good. We allow for customer heterogeneity in the intrinsic valuation. More precisely, we assume that v is realized from a random variable V with a cumulative distribution function $G(\cdot)$ and probability density function $g(\cdot)$. The second term $s(d)$ represents the network benefit, where d is the network size of the good at the end of the sales horizon. (In this paper, we consider a homogeneous network benefit function for all customers. We leave the heterogeneity of the network benefit $s(\cdot)$ among customers to future research because it opens up opportunities for more sophisticated information release policies than the simple sales disclosure or nondisclosure ones that we focus on in this paper.) Now we make the following assumption about V and $s(\cdot)$.

Assumption 1. *The density $g(\cdot)$ is continuous on its support. There exist $\underline{v}$ and v^0 such that $g(v) = 0$ for $v < \underline{v}$ and $g(v^0) > 0$, and $g(v)$ is nonincreasing for $v \geq v^0$. The network benefit function $s(\cdot)$ is nondecreasing on $[0, +\infty)$ with $s(0) = 0$. Furthermore, there exists d^0 such that $s(d)$ is continuously differentiable, and $s'(d)$ is nondecreasing for $d \leq d^0$.*

The assumption regarding the valuation distribution accommodates commonly used distributions, such as uniform, exponential, gamma, log-normal, and normal distributions truncated below at $\underline{v}$. The assumption that the network benefit function has a (weakly) convex part includes the linear function as a special case (Bensaid and Lesne 1996, Jing 2007). If the social utility of the good is derived from connecting with other users, then a convex $s(\cdot)$ might be plausible: for example, $s(d)$ can be proportional to $d(d-1)/2$, the number of all possible links in a social network with d users. In addition, a logistic form of network benefit plus a constant also satisfies our assumption.

In this paper, we assume that the network benefit of the good for a customer is determined by the *final* network size, not just the network size at the time of purchase. (This treatment of the network benefit is appropriate when the purchase period is much shorter than the consumption period.) Because the utility of the good for a customer depends on the number of other customers who will eventually purchase it, the market size information influences each individual customer's expectation of the total sales volume of the good and hence influences his or her purchase decision. The firm learns more about the exact market size during the sales process. Customers who strategically wait can make informed purchase decisions if they can observe early sales. Hence, it is a critical decision for the firm whether to report its sales information, which is the focus of this paper. In particular, we focus on ex ante policies for the firm. The firm can commit to implementing either a sales disclosure policy or a sales nondisclosure policy before the sales horizon starts. In the sales disclosure setting or the "full information" setting (*F* setting in short), the firm commits to disclosing sales information. In this case, customers who wait until the second period can observe past sales and make informed purchase decisions. In the sales nondisclosure setting or the "no information" setting (*N* setting for short), the firm commits to sales nondiclosure throughout the whole horizon. Customers in the *N* setting make uninformed decisions because they cannot observe past sales. Because booming sales will potentially attract more customers, whereas slack sales may drive potential customers away, it is not completely clear whether it would be beneficial for the firm to disclose sales information and have customers make informed decisions.

3. Single-Period Model: Matthew vs. Saturation Effect

To gain intuition, we first study a preliminary model, which essentially boils down to a single-period model. This model is used to illustrate the main trade-off in a firm's decision making. We then extend this prototypical model to a more realistic setting in the next section. We assume that in the *F* setting, customers can infer the exact market size M after observing the purchase behavior of a (negligibly) small fraction of customers purchasing at the beginning of the sales horizon. Therefore, almost all customers (except that initial small fraction) are informed of the exact value of M in the *F* setting, but they are not informed in the *N* setting. Hence, we can interpret the sales disclosure as the market size disclosure. In this setting, the firm's problem is equivalent to a problem of whether to commit to reporting the learned market size right after observing the small initial sales. For simplicity,

we assume an exogenous price p in the single-period model. Next, we analyze this single-period model and illustrate the main trade-off in the general model.

3.1. Sales Disclosure: The *F* Setting
In the *F* setting, (almost) all customers can learn the realized market size m before they make their purchase decisions. We consider symmetric strategies among them. Each of those customers believes that an α fraction of customers will buy the good.[1] Each customer will base his or her purchase decision on this belief. One will buy the network good if and only if his or her *expected utility* from buying the good is greater than or equal to the price. We adopt the standard approach of *rational expectations equilibrium* (REE) to study the equilibrium outcome. In an REE, a customer's belief is self-fulfilling, and the resulting outcome is consistent with his or her original belief. More precisely, given the realized and observed market size m, the equilibrium adoption fraction α can be characterized as

$$\alpha = \bar{G}(p - s(\alpha m)), \tag{1}$$

where $\bar{G}(\cdot) = 1 - G(\cdot)$.

Equation (1) is, in fact, a fixed-point problem. In this paper, we formulate several fixed-point problems, like Equation (1). The existence of fixed points for those problems follows from a special version of Tarski's fixed-point theorem, which states that every monotonic function $f : [0,1] \rightarrow [0,1]$ has at least one fixed point.[2] Note that the right-hand side of Equation (1) is (weakly) increasing in α. Thus, there exists at least one solution to (1) for any $m \geq 0$. The following lemma provides sufficient conditions for the uniqueness of the equilibrium. (Recall that v^0 and d^0 are defined in Assumption 1.)

Lemma 1 (Uniqueness of Equilibrium in *F* Setting). *If $p > v^0 + s(m)$ and $m \leq d^0$, then the equilibrium α^* characterized by* (1) *must be unique and $\alpha^* < 1$.*

Lemma 1 shows that when the market size is relatively small, implying that the network benefit at its *full strength* is relatively small or the price is relatively high, the REE characterized by (1) is unique. We note that without the conditions in Lemma 1, Equation (1) has a unique fixed point if V is uniformly distributed and $s(\cdot)$ is linear, which is a special case of Assumption 1.

When the market size is relatively large or the price is relatively small, the uniqueness of equilibrium characterized by (1) may not hold. In this paper, we focus on the *largest* REE if there exist multiple ones. This is a commonly adopted approach in the literature (see Farrell and Saloner 1985, Katz and Shapiro 1986, Fudenberg and Tirole 2000, Wang and Wang 2016). On the one hand, the largest REE is stable to small

Figure 1. (Color online) Matthew Effect vs. Saturation Effect
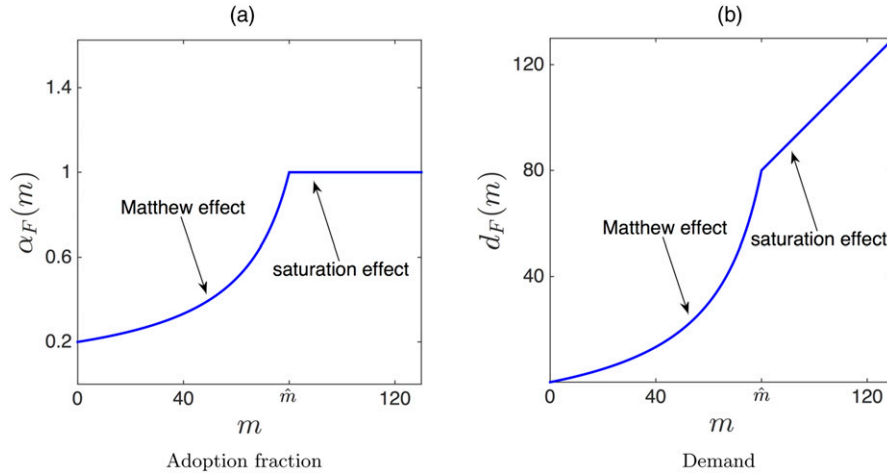
Note. $V \sim U[0,1]$, $s(d) = 0.01d$, and $p = 0.8$.

perturbations. On the other hand, a higher REE leads to higher social welfare (i.e., more purchases, a higher profit level for the firm, and more surplus for the customers at the same price). Therefore, it is reasonable to focus on the largest REE.

In the following, we denote the largest solution to Equation (1) by $\alpha_F(m)$. The demand for the firm in the F setting can be written as $d_F(m) = \alpha_F(m)m$, and the expected profit for the firm is $R_F = p\mathsf{E}d_F(M)$.

Next, we identify two countervailing effects of market size disclosure: a prodisclosure Matthew effect and a prosecrecy saturation effect. To this end, we first illustrate the effects with an example where V is uniformly distributed on $[0, 1]$, $s(d) = 0.01d$, and $p = 0.8$. Figure 1 plots the adoption fraction function $\alpha_F(m)$ and the demand function $d_F(m)$ (in this example, $\alpha_F(m)$ is the unique solution to Equation (1)). Note that in Figure 1, the parameter $\hat{m} = 80$ is the smallest market size such that all customers buy the good in equilibrium.

Figure 1(a) shows that the equilibrium adoption fraction $\alpha_F(m)$ has two pieces: when m is smaller than $\hat{m}$, $\alpha_F(m)$ increases convexly in m; when m is larger than $\hat{m}$, $\alpha_F(m) = 1$. Thus, the demand function has two pieces as well (see Figure 1(b)). These two pieces correspond to the two countervailing effects in the F setting as mentioned: the Matthew effect and the saturation effect.

The first piece with *convexity* implies that the benefit of revealing market size, owing to customers being encouraged to make a purchase when the market turnout is high, outweighs the loss resulting from customers being discouraged when the turnout is disappointing. This phenomenon, which we call the *Matthew effect*, is intrinsically driven by the positive network externalities. As a new customer enters the market, existing customers will increase their expectation of the ultimate network size, which determines their expected utility from buying the good. As a result,

the new customer is more willing to buy the good. In anticipation of this, other customers will further have a higher utility from buying the good. Therefore, the equilibrium adoption fraction is more than linearly increasing in the market size in this range of the first piece. We will show that the firm prefers committing to sales disclosure when the Matthew effect is strong enough (i.e., when the probability that M falls into the region where $\alpha_F(\cdot)$ is convex is high enough).

However, the Matthew effect can be saturated. The adoption fraction $\alpha_F(m)$ is capped by one for $m \geq \hat{m}$. In other words, if the realized market size is sufficiently large, the market is *saturated*. That is, all customers in the market are induced to make a purchase; a fortiori, the entire market of an even larger turnout would all make a purchase as well because of an even stronger network benefit. Hence, the information when the exact market size is more than $\hat{m}$ is redundant. We call this phenomenon the *saturation effect*. With the market is saturated, revealing full information can be detrimental: the benefit of revealing full information when the realized market size is large may be limited, whereas the risk of turning away potential buyers looms in case the realized market size is low. We will show that a firm benefits from the sales nondisclosure policy when the saturation effect is strong enough (i.e., when the probability that $\alpha_F(M) = 1$ is high enough).

To conclude this subsection, we formalize the observations from the example in Figure 1.

Lemma 2 (Matthew Effect vs. Saturation Effect)**.**

a. *(Matthew Effect) If $p > v^0 + s(d^0)$, then both $\alpha_F(m)$ and $d_F(m)$, $m \leq d^0$, are (weakly) increasing and (weakly) convex in m; the increasing and convex properties here are in a strict sense if $\bar{G}(p) > 0$ and $s'(d) > 0$ for $d < d^0$.*

b. *(Saturation Effect) If $p \leq \underline{v} + s(m)$, then $\alpha_F(m) = 1$ and $d_F(m) = m$.*

3.2. Sales Nondisclosure: The *N* Setting

In the *N* setting, each customer only knows the distribution of the market size throughout the sales horizon. Similar to the *F* setting, the equilibrium adoption fraction can be characterized as

$$\alpha = \bar{G}(p - \mathsf{E}s(\alpha M)). \qquad (2)$$

The existence of a fixed point of (2) is again a direct consequence of Tarski's fixed-point theorem. The following lemma provides sufficient conditions for the uniqueness of the equilibrium.

Lemma 3 (Uniqueness of Equilibrium in *N* Setting). *If $p > v^0 + \mathsf{E}s(M)$ and $\mathsf{P}\{M \leq d^0\} = 1$, then the equilibrium α^* characterized by (2) must be unique, and $\alpha^* < 1$.*

Lemma 3 points out that when the market size is (stochastically) small, which implies that the expected network benefit at its full strength is small or the price is relatively high, the equilibrium characterized by (2) is unique. The intuition behind this result is similar to that behind Lemma 1.

As before, if there are multiple fixed points to (2), we focus on the largest one, denoted by α_N. Thus, the demand for the firm in the *N* setting can be written as $d_N = \alpha_N \mathsf{E}M$, and the expected profit for the firm can be written as $R_N = pd_N$.

3.3. Sales Disclosure vs. Nondisclosure

In this single-period model, we focus on the case in which the price $p(\geq 0)$ is exogenous. We will analyze a general case with endogenous prices in a two-period model (see Section 4).

To illustrate better the main trade-off, we assume for the time being that $s(\cdot)$ is linear. Under this assumption, by comparing Equations (1) and (2), $\alpha_N = \alpha_F(\mathsf{E}M)$, which yields $d_N = d_F(\mathsf{E}M)$. To compare R_N with R_F, the key is the second-order property of $d_F(\cdot)$. As shown in Lemma 2 and Figure 1(b), $d_F(m)$ is first convex and then linear. By Jensen's inequality, $\mathsf{E}d_F(M) \geq d_F(\mathsf{E}M)$ if $\mathsf{P}\{p > v^0 + s(M)\}$ is high enough (i.e., the Matthew effect is strong enough), and $\mathsf{E}d_F(M) \leq d_F(\mathsf{E}M)$ if $\mathsf{P}\{p \leq \underline{v} + s(M)\}$ is high enough (i.e., the saturation effect is strong enough).

For a general $s(\cdot)$, we can no longer compare R_N with R_F in such a simple way, but the trade-off between the Matthew effect and the saturation effect persists and plays a key role in the comparison between the two settings. The following proposition provides sufficient conditions for either policy to be better off.

Proposition 1 (Sales Disclosure vs. Nondisclosure).
   a. *(Weak Network Benefit) If $p \geq v^0 + s(\overline{m})$ and $\overline{m} \leq d^0$, then $R_F \geq R_N$, where $\overline{m}$ is the upper bound of the support of M.*
   b. *(Strong Network Benefit) If $p \leq \underline{v} + \mathsf{E}s(M)$, then $R_F \leq R_N$.*

Proposition 1(a) presents a case where the Matthew effect dominates the saturation effect. In particular, when the exogenous price p is high enough or the network benefit at its full strength is relatively weak, the Matthew effect will be the dominant driving force in the *F* setting, and hence, the firm is better off committing to revealing its sales information. In contrast, Proposition 1(b) provides a sufficient condition under which the saturation effect is relatively strong. When the exogenous price p is low enough or the expected network benefit is relatively strong, the firm is better off not committing to revealing the market size because of the dominating saturation effect.

To conclude this section, we provide some remarks on Assumption 1. Under this assumption, the stand-alone valuation V has a lower bound $\underline{v}$. When the expected network benefit is strong enough, any customer in the market will buy the good, even with the lowest valuation. Hence, the adoption rate function achieves one when the market size reaches some threshold, as shown in Figure 1(a). However, the network effect may not always be able to make the market saturated. For example, when the standalone valuation V follows a normal distribution and can take negative values (i.e., some customers need to be paid to "buy" the good), the market is never entirely covered, regardless of how strong the expected network benefit is. In this case, the shape of the equilibrium adoption function of the market size is first convex, then concave, as shown in Figure 2(a). The concave segment limits the benefit of revealing full information when the realized market size is beyond some level; hence, revealing full information is more likely to be detrimental than in the example in Figure 1. This phenomenon can be referred to as the *generalized saturation effect*. Nevertheless, the central trade-off between the *F* setting and the *N* setting in this paper—Matthew effect versus saturation effect—continues to hold.

4. Two-Period Model

In the single-period model, we assume that in the *F* setting, (almost) all customers can learn the exact market size at the very beginning of the sales horizon, which helps us illustrate the trade-off between the two settings *F* and *N*. In this section, we consider a more realistic two-period model in which the first-period customers make uninformed decisions without knowing the exact market size, whereas the second-period customers will learn the market size and make informed decisions if the firm commits to reporting the first-period sales.

In the two-period model, we consider customers' intertemporal purchase behavior. All customers arrive at the beginning of the first period. They recognize that they can learn the market size in the second period if the firm commits to reporting first-period

Figure 2. (Color online) Matthew Effect vs. Saturation Effect
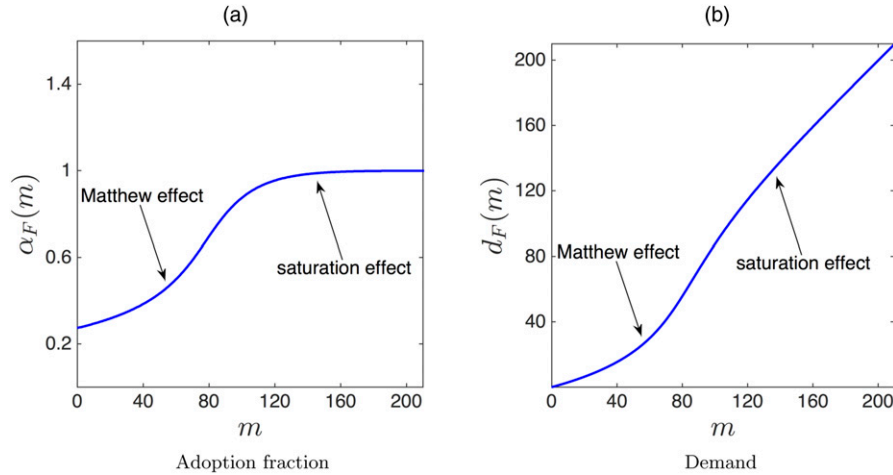
Note. Parameters are the same as in Figure 1 except that $V \sim N(0.5, 0.25)$.

sales. Because the network benefit for a customer depends on the total sales volume across the two periods, to maximize individual expected surplus, a customer chooses between buying immediately in the first period or delaying his or her purchase decision to the second period. For example, after a new gadget is launched, given the option of adopting it now or later, customers take into account the expected future network benefit. With sales disclosure, a customer who buys immediately takes the risk that the ultimate number of adopters may be disappointing, whereas a customer who delays his or her decision will get a discount on the utility via a discount factor δ, $0 < \delta < 1$. Moreover, in the two-period model, the firm can also choose different pricing policies. In this section, we examine four pricing policies: exogenous pricing, state-independent pricing, contingent preannounced pricing, and contingent pricing without commitment with the last three as endogenized pricing policies.

In the next four subsections, we analyze each pricing policy and explore which information policy (i.e., committing to sales disclosure or nondisclosure) is better off in that specific setting.

4.1. Exogenous Pricing
In this case, we assume that the prices for both periods $p_1 (\geq 0)$ and $p_2 (\geq 0)$ are exogenously given. This scenario emerges, for example, when the retailer is unable to set the prices across the two periods. Let $\bar{v}$ be the upper end of the support of V. We assume that $\bar{v}$ is large enough that a positive fraction of customers will always buy the good in the first period in either the F or the N setting.[3] Then we present the model, analyze the REE results, and characterize the optimal policy for the firm.

4.1.1. Sales Disclosure: The *F* Setting. With sales disclosure, customers who delay their purchase decisions to

the second period can observe the first-period sales d_1. At the beginning of the second period, given that an α_1 fraction of the cohort has adopted the good in the first period, customers who delayed their purchase decisions form an identical belief that an α_2 fraction of the cohort will buy the good. The second-period adoption fraction α_2 in an REE can be characterized by

$$\alpha_1 + \alpha_2 = \max\{\alpha_1, \bar{G}(p_2 - s(d_1 + \alpha_2 m \,|\, \alpha_1, d_1))\}, \quad (3)$$

where $(m \,|\, \alpha_1, d_1) = d_1/\alpha_1$. We can see from (3) that α_2 in an REE is a function of α_1 and the realized market size m.

In the first period, all customers identically anticipate that an α_1 fraction of customers will buy the good in the first period and that an $\pmb{\alpha_2}$ fraction of customers will buy the good in the second period, where $\pmb{\alpha_2} = \{\alpha_2(\alpha_1, M)\}$ is a function of α_1 and M. Based on these beliefs, a customer with an intrinsic valuation v will have an expected utility $u_1(v)$ from buying the good immediately and $u_2(v)$ from waiting, where

$$u_1(v) = v - p_1 + \delta \mathsf{E}s((\alpha_1 + \alpha_2(\alpha_1, M))M),$$
$$u_2(v) = \delta \mathsf{E}[v - p_2 + s((\alpha_1 + \alpha_2(\alpha_1, M))M)]^+.$$

Here we assume that the network benefit from the first period is negligible. This assumption is reasonable for group buying and crowdfunding, where the group benefit only kicks in at the end of the pledging horizon. This assumption also holds for other settings, such as launching of gadgets with preordering and regular ordering, in which the selling horizon is relatively short compared with the time that the customers enjoy the product with the network benefit. Even though the customers may enjoy some network benefit when they buy early, that benefit can be negligible compared with benefits from the relatively long product usage period. Our main

results hold qualitatively when the network effects are imposed in the alternative way of taking into account the first-period network benefit.

An individual customer buys the good in the first period whenever $u_1(v) \geq u_2(v)(\geq 0)$. It can be seen that there exists a threshold $v_c(\alpha_1, \boldsymbol{\alpha_2})$ for the intrinsic valuation, above which the customer will buy the good immediately and below which the customer will wait and see. An REE requires that the outcomes in the two periods be consistent with what customers expect. Then we can characterize the REE for the two-period game by the following equations:

$$\alpha_1 + \alpha_2(\alpha_1, m) = \max\{\alpha_1, \bar{G}(p_2 - s(\alpha_1 m + \alpha_2(\alpha_1, m)m))\},$$
$$\forall\, m,$$

$$u_1(v_c(\alpha_1, \boldsymbol{\alpha_2})) = \max\{u_1(\underline{v}), u_2(v_c(\alpha_1, \boldsymbol{\alpha_2}))\},$$
$$v_c(\alpha_1, \boldsymbol{\alpha_2}) = G^{-1}(1 - \alpha_1),$$

where $G^{-1}(x) = \inf_{y \geq 0}\{y : G(y) \geq x\}$.

Let $\alpha(\alpha_1, m) = \alpha_1 + \alpha_2(\alpha_1, m)$ be the total adoption fraction conditional on α_1 and m, and let $\tilde{\alpha}(m)$ be the fixed point of

$$\tilde{\alpha} = \bar{G}(p_2 - s(\tilde{\alpha} m)), \quad \forall\, m. \qquad (4)$$

Then the REE equations can be rewritten as

$$\alpha(\alpha_1, m) = \max\{\alpha_1, \tilde{\alpha}(m)\}, \quad \forall\, m, \qquad (5)$$
$$G^{-1}(1 - \alpha_1) - p_1 + \delta \mathsf{E}s(\alpha(\alpha_1, M)M)$$
$$= \delta \mathsf{E}[G^{-1}(1 - \alpha_1) - p_2 + s(\alpha(\alpha_1, M)M)]^+, \text{ if } \alpha_1 < 1,$$
$$\underline{v} - p_1 + \delta \mathsf{E}s(M) \geq \delta \mathsf{E}[\underline{v} - p_2 + s(M)]^+, \text{ if } \alpha_1 = 1.$$

An REE, as a solution to these equations, has two components: α_1 and $\tilde{\boldsymbol{\alpha}}$, where $\tilde{\boldsymbol{\alpha}} = \{\tilde{\alpha}(M)\}$. The existence of $\tilde{\boldsymbol{\alpha}}$ is derived from Tarski's fixed-point theorem.

If there are multiple equilibria, we select the equilibrium $(\alpha_1, \tilde{\boldsymbol{\alpha}})$ such that

$$\alpha_1 = \sup_{x \in [0,1]} \{x : G^{-1}(1 - x) - p_1 + \delta \mathsf{E}s(\max\{x, \tilde{\alpha}(M)\}M)$$
$$\geq \delta \mathsf{E}[G^{-1}(1 - x) - p_2 + s(\max\{x, \tilde{\alpha}(M)\}M)]^+\}, \qquad (6)$$

and $\tilde{\alpha}(m)$ is the largest fixed point of (4). We write the first-period adoption fraction, the second-period adoption fraction, and the total adoption fraction in the REE as $\alpha_{1,F}(p_1, p_2)$, $\{\alpha_{2,F}(p_1, p_2, M)\}$, and $\{\alpha_F(p_1, p_2, M)\}$, respectively. Furthermore, we let $d_{1,F}(p_1, p_2) = \alpha_{1,F}(p_1, p_2)\mathsf{E}M$, $d_{2,F}(p_1, p_2, M) = \alpha_{2,F}(p_1, p_2, M)M$, and $d_F(p_1, p_2, M) = \alpha_F(p_1, p_2, M)M$ denote the first-period demand, the second-period demand, and the total demand, respectively. We also denote by $\tilde{\alpha}_F(p_2, m)$ the largest fixed point of (4) and $\tilde{d}_F(p_2, M) = \tilde{\alpha}_F(p_2, M)M$. Thus, we have that $\alpha_F(p_1, p_2, M) = \max\{\alpha_{1,F}(p_1, p_2), \tilde{\alpha}_F(p_2, M)\}$. The expected total profit for the firm is $R_F(p_1, p_2) = p_1 d_{1,F}(p_1, p_2) + p_2 \mathsf{E}[d_{2,F}(p_1, p_2, M)]$.

4.1.2. Sales Nondisclosure: The *N* Setting.
Without sales disclosure, customers only know the distribution of market size, even if they delay their purchase decisions to the second period. Similar to the *F* setting, given the first-period adoption fraction α_1, the second-period adoption fraction α_2 in an REE can be characterized by

$$\alpha_1 + \alpha_2 = \max\{\alpha_1, \bar{G}(p_2 - \mathsf{E}s((\alpha_1 + \alpha_2)M))\}.$$

At the beginning of the first period, all customers form an identical belief over an adoption fraction α_1 in the first period and α_2 in the second period. Let $\alpha = \alpha_1 + \alpha_2$ be their belief over the total adoption fraction. With these beliefs in mind, a customer with an intrinsic valuation v will buy the good in the first period if and only if

$$v - p_1 + \delta \mathsf{E}s(\alpha M) \geq \delta[v - p_2 + \mathsf{E}s(\alpha M)]^+.$$

There exists a threshold $v_c(\alpha)$ (with slight abuse of notation) for the intrinsic valuation above which the customer will buy the good in the first period. In an REE, the adoption fractions in the two periods can be characterized as

$$\alpha = \max\{\bar{G}(p_1 - \delta \mathsf{E}s(\alpha M)), \bar{G}(p_2 - \mathsf{E}s(\alpha M))\}, \qquad (7)$$
$$G^{-1}(1 - \alpha_1) - p_1 + \delta \mathsf{E}s(\alpha M)$$
$$= \delta \mathsf{E}[G^{-1}(1 - \alpha_1) - p_2 + s(\alpha M)]^+, \text{ if } \alpha_1 < 1,$$
$$\underline{v} - p_1 + \delta \mathsf{E}s(M) \geq \delta[\underline{v} - p_2 + \mathsf{E}s(M)]^+, \text{ if } \alpha_1 = 1.$$

If there are multiple solutions, similar to the *F*-setting model, we focus on the one that satisfies

$$\alpha_1 = \sup_{x \in [0,\alpha]} \{x : G^{-1}(1 - x) - p_1 + \delta \mathsf{E}s(\alpha M)$$
$$\geq \delta[G^{-1}(1 - x) - p_2 + \mathsf{E}s(\alpha M)]^+\} \qquad (8)$$

and that α is the largest fixed point of (7). We write the first-period adoption fraction, the second-period adoption fraction, and the total adoption fraction in the REE as $\alpha_{1,N}(p_1, p_2)$, $\alpha_{2,N}(p_1, p_2)$, and $\alpha_N(p_1, p_2)$, respectively. Let $d_{1,N}(p_1, p_2) = \alpha_{1,N}(p_1, p_2)\mathsf{E}M$, $d_{2,N}(p_1, p_2) = \alpha_{2,N}(p_1, p_2)\mathsf{E}M$, and $d_N(p_1, p_2) = \alpha_N(p_1, p_2)\mathsf{E}M$ denote the first-period demand, the second-period demand, and the total demand, respectively. The total profit for the firm in the *N* setting is $R_N(p_1, p_2) = p_1 d_{1,N}(p_1, p_2) + p_2 d_{2,N}(p_1, p_2)$.

4.1.3. Sales Disclosure vs. Nondisclosure.
As highlighted in the single-period model, there are two countervailing effects of market size disclosure (i.e., the Matthew effect and the saturation effect). We have shown that in the single-period model, the strengths of the two effects determine whether it is better off to commit to sales disclosure or sales nondisclosure. In the two-period model with strategic customers, these two

effects can still exist in the second period because the second-period customers are informed of the market size and will base their purchase decisions on such information if the firm commits to reporting early sales. The following lemma summarizes the two effects in the two-period model.

Lemma 4 (Matthew vs. Saturation Effect). *Define* $m_0 = \inf\{m : \bar{G}(p_2 - s(\alpha_{1,F}(p_1, p_2)m)) \geq \alpha_{1,F}(p_1, p_2)\}$. *The total adoption fraction* $\alpha_F(p_1, p_2, m)$ *has the following properties:*

a. *(Matthew Effect) If* $p_2 > v^0 + s(d^0)$, *then* $\alpha_F(p_1, p_2, m)$ *and* $d_F(p_1, p_2, m)$, $m_0 \leq m \leq d^0$, *are (weakly) increasing and (weakly) convex in* m; *the increasing and convex properties are in a strict sense if* $\bar{G}(p_2) > 0$ *and* $s'(d) > 0$ *for* $d < d^0$.

b. *(Saturation Effect) If* $p_2 \leq \underline{v} + s(m)$, *then* $\alpha_F(p_1, p_2, m) = 1$, *and* $d_F(p_1, p_2, m) = m$.

By Lemma 4, we find that the comparison between $\mathsf{E}d_F(p_1, p_2, M)$ and $d_N(p_1, p_2)$ depends on the second-order property of $d_F(p_1, p_2, m)$ in m. In fact, from definitions of $\tilde{d}_F(\cdot)$ and $d_N(\cdot)$, we have that if $p_1 \geq p_2$ and $s(\cdot)$ is linear, then $\tilde{d}_F(p_2, \mathsf{E}M) = d_N(p_2)$, which yields

$$d_F(p_1, p_2, \mathsf{E}M) = \max\left\{\alpha_{1,F}(p_1, p_2)\mathsf{E}M, \tilde{d}_F(p_2, \mathsf{E}M)\right\}$$
$$\geq d_N(p_2),$$

where we write $d_N(p_1, p_2)$ as $d_N(p_2)$ in short because $d_N(\cdot)$ depends solely on p_2 when $p_1 \geq p_2$. Analogous to the discussion in the single-period model, the expected total demand in the F setting will be greater than the expected total demand in the N setting if the Matthew effect that exists in the second period is strong enough and vice versa. The two effects still play important roles in the firm's choice between the two settings F and N.

In addition to the Matthew and saturation effects, committing to sales disclosure results in another effect, which we call the *effect of information free-riding*. That is, customers in the F setting delay their purchase not only for a possible lower price in the future but also for a free ride on the information generated by the others' purchase decisions. In contrast, customers in the N setting wait only for a possible lower price. Now we are going to show that the comparison between $\mathsf{E}d_{1,F}(p_1, p_2, M)$ and $d_{1,N}(p_1, p_2)$ depends significantly on the information free-riding effect. Define

$$H_F(x) = G^{-1}(1-x) - p_1 + \delta\mathsf{E}s(\max\{x, \tilde{\alpha}_F(p_2, M)\}M)$$
$$- \delta\mathsf{E}\left[G^{-1}(1-x) - p_2 + s(\max\{x, \tilde{\alpha}_F(p_2, M)\}M)\right]^+,$$
$$H_N(x) = G^{-1}(1-x) - p_1 + \delta\mathsf{E}s(\max\{x, \alpha_N(p_2)\}M)$$
$$- \delta\left[G^{-1}(1-x) - p_2 + \mathsf{E}s(\max\{x, \alpha_N(p_2)\}M)\right]^+.$$

Then we have

$$H_F(x) - H_N(x)$$
$$= \delta\left[-G^{-1}(1-x) + p_2 - \mathsf{E}s(\max\{x, \alpha_N(p_2)\}M)\right]^+$$
$$- \delta\mathsf{E}\left[-G^{-1}(1-x) + p_2 - s(\max\{x, \tilde{\alpha}_F(p_2, M)\}M)\right]^+.$$

When the first-period adoption rate is x, $-H_N(x)$ $(-H_F(x))$ measures how much a customer with valuation $G^{-1}(1-x)$ wants to delay his or her purchase to the second period in the N (F) setting. Accordingly, $H_N(x) - H_F(x)$ measures how much this customer wants to free-ride on the sales information in the second period. By Equations (6) and (8), we have

$$\alpha_{1,F}(p_1, p_2) = \sup_{0 \leq x \leq 1}\{x : H_F(x) \geq 0\},$$
$$\alpha_{1,N}(p_1, p_2) = \sup_{0 \leq x \leq 1}\{x : H_N(x) \geq 0\}.$$

We have the following lemma.

Lemma 5. *For* $p_1 \geq 0$ *and* $p_2 \geq 0$,

a. $\alpha_{1,F}(p_1, p_2) \leq \alpha_{1,N}(p_1, p_2)$, *and*

b. *if* $p_1 \geq p_2$, *then* $\alpha_{1,F}(p_1, p_2) = \alpha_{1,N}(p_1, p_2)$.

Whether a firm should disclose sales information depends on the strengths of the three effects. The following proposition provides sufficient conditions for each policy F or N to be better off in terms of δ, the form of $s(\cdot)$, and the distributions of M and V.

Proposition 2 (Exogenous Pricing).

a. *(Weak Network Benefit) If* $p_1 \geq p_2 \geq v^0 + s(\overline{m})$ *and* $\overline{m} \leq d^0$, *then* $R_F(p_1, p_2) \geq R_N(p_1, p_2)$; *if* $p_1 < p_2$, $p_2 \geq v^0 + s(\overline{m})$, *and* $\overline{m} \leq d^0$, *then there exists* $\delta_c > 0$ *such that when* $\delta < \delta_c$, $R_F(p_1, p_2) \geq R_N(p_1, p_2)$.

b. *(Strong Network Benefit) If* $p_2 \leq \min\{p_1, \underline{v} + \mathsf{E}s(M)\}$, *then* $R_F(p_1, p_2) \leq R_N(p_1, p_2)$.

Proposition 2(a) presents a scenario in which committing to sales disclosure results in a higher expected demand and a higher expected profit. In particular, when the network benefit at its full strength is small enough, the Matthew effect dominates the saturation effect. That is, with sales disclosure, the benefit of reporting large sales outweighs the loss of reporting disappointing early sales in the second period (i.e., the second-period demand under sales disclosure is greater than that under sales nondisclosure). In addition, when the discounting factor δ is small enough (customers experience larger surplus discounting if they wait), the incentive for customers to delay their decision—to reduce the risk of estimating the network benefit—is small enough that the first-period demand is only slightly smaller under sales disclosure than under sales nondisclosure. Note that the effect of information free-riding does not exist when $p_1 \geq p_2$ (see Lemma 5(b)). With all forces combined,

the total profit under sales disclosure is greater than that under sales nondisclosure; in other words, the Matthew effect dominates the sum of the saturation effect and the effect of information free-riding. Therefore, in this case, it is better off to commit to sales disclosure.

In contrast, Proposition 2(b) provides a sufficient condition under which committing to sales nondisclosure makes the seller better off. When the expected network benefit at its full strength is large enough, the saturation effect in the second period dominates: that is, the expectation of the network benefit at the beginning of the selling season is large enough. In anticipation of a large network benefit, almost all customers will buy the good even without knowing the exact market size, whereas a fraction of them will wait and buy the good in the second period under sales disclosure. If the firm commits to reporting sales in this case, then customers who chose to delay their decision to the second period might be discouraged if the first-period sales turn out to be low. Meanwhile, the first-period demand in the F setting is always no greater than that in the N setting. Thus, it can be unprofitable for the firm to commit to sales disclosure.

In the online games industry, the price is often exogenously determined: for example, the subscription fee for a game often follows the competitive market price. Consistent with Proposition 2(a), Blizzard's World of Warcraft, a massively multiplayer online role-playing game, announced on its official website that the subscription numbers for its future releases would not be reported (Tassi 2015). The game had already built a huge customer base and hence a strong network effect thanks to the popularity of its previous versions. However, at the same time, numerous niche online game operators with small market potentials have their subscription numbers reported by third-party sales watchdogs, such as addictinggames.com. These observations are consistent with our results.

4.2. State-Independent Endogenous Pricing

In Section 4.1, we assume that the firm's prices are exogenously given, which is often the case for competitive markets. However, firms can sometimes set prices by themselves. This subsection explores the case in which the firm decides on and commits to not only whether to disclose the early sales information but also how to set (state-independent) prices for the two periods.

The firm's optimal pricing problems in the F and N settings can be formulated as $R_F^* = \max_{p_1, p_2} R_F(p_1, p_2)$ and $R_N^* = \max_{p_1, p_2} R_N(p_1, p_2)$, respectively. We denote the optimal prices by $(p_{1,F}^*, p_{2,F}^*)$ and $(p_{1,N}^*, p_{2,N}^*)$.[4] Unless stated otherwise, other symbols have the same meaning as in the previous sections.

Next, we compare R_F^* and R_N^*. Recall that under exogenous pricing, if $p_1 \geq p_2$, then the firm will gain the same first-period profits in the two settings N and F.

Lemma 6 (State-Independent Pricing). $p_{1,N}^* \geq p_{2,N}^*$.

We can see from Lemmas 5(a) and 6 that $\alpha_{1,F}(p_{1,N}^*, p_{2,N}^*) = \alpha_{1,N}(p_{1,N}^*, p_{2,N}^*)$: that is, the effect of information free-riding does not exist as long as the firm charges $p_{1,F} = p_{1,N}^*$ and $p_{2,F} = p_{2,N}^*$. This is so because under these prices, customers who are willing to buy the good in the first period have sufficiently high standalone valuations that they will also be willing to buy the good even if they choose to wait and observe a small market size in the second period (i.e., $\delta E[-G^{-1}(1 - \alpha_{1,N}^*) + p_{2,N}^* - s(\max\{\alpha_{1,N}^*, \tilde{\alpha}_F(p_{2,N}^*, M)\}M)]^+ = 0$). Furthermore, the firm can also be better off from committing to sales disclosure in the second period just by pricing the same as in the N setting if the Matthew effect under the optimal prices of the N setting is strong enough, which implies that $R_F^* \geq R_N^*$. We formalize this discussion as follows.

Proposition 3 (State-Independent Pricing).
   a. *If there exist r, r_1, r_2 such that $r_1 > r > r_2 > v^0 + s(\overline{m})$, $\overline{m} \leq d^0$, $P\{V \geq r_1\} \geq r/r_1$, $P\{r_2 \leq V \leq (r - \delta r_2)/(1 - \delta)\} \geq [v^0 + s(\overline{m})]/r_2$, then $R_F^* \geq R_N^*$.*
   b. *If $\underline{v} + Es(M) > 0$, $P\{V \leq \underline{v} + Es(M)\} = 1$, and $E[M \cdot 1_{\{s(M) \geq \min\{r(\underline{v} + Es(M)) - v^0, s(d^0)\}\}}] \leq EM/r$ for any $r > 1$, then $R_F^* \leq R_N^*$.*

Proposition 3(a) presents a case in which committing to sales disclosure is better off under endogenous pricing. This case holds when customers' valuation distribution has a heavy tail relative to the expected network benefit at its full strength. In this situation, the optimal prices in the N setting are relatively high in order to extract the surplus from a relatively large fraction of customers who have high standalone valuations. Specifically, the condition $P\{V \geq r_1\} \geq r/r_1$ implies that $p_{1,N}^* \geq r$, and the condition $P\{r_2 \leq V \leq (r - \delta r_2)/(1 - \delta)\} \geq [v^0 + s(\overline{m})]/r_2$ implies that $p_{2,N}^* \geq v^0 + s(\overline{m})$ (see the proof in the online appendix for details). Then, in the F setting, by charging $p_{1,F} = p_{1,N}^*$ and $p_{2,F} = p_{2,N}^*$, the Matthew effect dominates the sum of the saturation effect and the effect of information free-riding because of the high price in the second period; see Proposition 2(a). Therefore, committing to sale disclosure is better off in this scenario.

Proposition 3(b) presents a case in which committing to sales nondisclosure is better off. It requires that (1) the standalone valuation be bounded from above by the expected network benefit and that (2) the market size distribution has a relatively light tail. In this situation, the firm would do reasonably well without information disclosure by pricing at the expected network benefit in the second period: that is,

$p_{2,N} = \underline{v} + \mathsf{E}s(M)$. Because the network benefit would be strong, the firm would saturate the second-period market under nondisclosure and earn a healthy profit by pricing high in the first period. In the F setting, the firm faces the following conundrum: if it charges lower than $\underline{v} + \mathsf{E}s(M)$ in the second period, it cannot gain more demand than in the N setting even if the market size is large because the adoption fraction is already one in the N setting, whereas it can do worse if the realized market is small; hence the expected network benefit is smaller than that in the N setting, implying that its first-/second-period profit is also smaller. Meanwhile, if the firm prices higher (e.g., $p_{2,F} > \underline{v} + \mathsf{E}s(M)$), it cannot do better with information disclosure either. This is so because the gain from sales disclosure is limited owing to the light-tailed market size, whereas the loss in profit if the realized market is small can be much more significant. Overall, the firm does better in the N setting than in the F setting in this case.

When the density of customers' valuation $g(\cdot)$ takes some special form (e.g., nonincreasing in its support), Proposition 3(a) can be rewritten in a much simpler way as follows.

Corollary 1 (State-Independent Pricing). *Suppose that $g(\cdot)$ is nonincreasing on $[\underline{v}, +\infty)$. Then $R_F^* \geq R_N^*$ if $\overline{m} \leq d^0$ and $\mathsf{P}\{V \geq r(v^0 + s(\overline{m}))\} \geq (r+1)/(2r)$ for some $r > 1$.*

4.3. Contingent Preannounced Pricing

We consider a contingent preannounced pricing setting in which the firm chooses a price in the second period depending on the first-period sales. In this subsection, we compare the two information release policies (i.e., the F setting and the N setting) under contingent preannounced pricing.

4.3.1. Sales Disclosure: The F Setting. At the beginning of the first period in the F setting, the firm charges a price $p_{1,F}$ for the first period and preannounces a pricing scheme $p_{2,F}(d_1)$ as a function of the first-period sales d_1 for the second period. Because $(m \mid \alpha_1, d_1) = d_1/\alpha_1$, $p_{2,F}(d_1)$ can be rewritten as $p_{2,F}(m)$. We write $\boldsymbol{p_{2,F}} = \{p_{2,F}(M)\}$ for simplicity.

Given α_1, d_1, and $\boldsymbol{p_{2,F}}$, the second-period customers identically anticipate that an α_2 fraction of the market size will purchase the good in the second period. The adoption fraction α_2 in an REE, for any given α_1, d_1, and corresponding $p_{2,F}$, must satisfy

$$\alpha_1 + \alpha_2 = \max\{\alpha_1, \bar{G}(p_{2,F}(m) - s((\alpha_1 + \alpha_2)m))\}.$$

We can see that α_2 in an REE is a function of α_1, $p_{2,F}(m)$, and m. Let $\alpha(\alpha_1, p_{2,F}(m), m) = \alpha_1 + \alpha_2(\alpha_1, p_{2,F}(m), m)$ be

the total adoption fraction conditional on α_1, $p_{2,F}(m)$, and m, and let $\tilde{\alpha}(p_{2,F}(m), m)$ be the fixed point of

$$\tilde{\alpha} = \bar{G}(p_{2,F}(m) - \mathsf{E}s(\tilde{\alpha}m)), \quad \forall m. \tag{9}$$

Then the REE condition for the second period can be rewritten as

$$\alpha(\alpha_1, p_{2,F}(m), m) = \max\{\alpha_1, \tilde{\alpha}(p_{2,F}(m), m)\}. \tag{10}$$

In the first period, the analysis of the REE equation is similar to the state-independent pricing scenario. If there are multiple equilibria, by following the same steps as in Section 4.1, we select the equilibrium $(\alpha_1, \tilde{\alpha})$ such that

$$
\begin{aligned}
\alpha_1 = \sup_{x \in [0,1]} \{&x : G^{-1}(1-x) - p_{1,F} \\
&+ \delta \mathsf{E}s(\max\{x, \tilde{\alpha}(p_{2,F}(M), M)\}M) \\
\geq \delta \mathsf{E}[&G^{-1}(1-x) - p_{2,F}(M) \\
&+ \mathsf{E}s(\max\{x, \tilde{\alpha}(p_{2,F}(M), M)\}M)]^+\},
\end{aligned}
\tag{11}
$$

and $\tilde{\alpha}(p_{2,F}(m), m)$ is the largest fixed point of (9).

We use the same notation as in Section 4.1 to denote outcomes in an equilibrium characterized as above. Thus, the firm's problem is

$$
\begin{aligned}
\max_{p_{1,F}, \boldsymbol{p_{2,F}}} R_F\big(p_{1,F}, \boldsymbol{p_{2,F}}\big) \\
= p_{1,F}\alpha_{1,F}\big(p_{1,F}, \boldsymbol{p_{2,F}}\big)\mathsf{E}M \\
+ \mathsf{E}\big[p_{2,F}(M)\alpha_{2,F}\big(p_{1,F}, \boldsymbol{p_{2,F}}, M\big)M\big].
\end{aligned}
$$

With slight abuse of notation, we denote the optimal value of the preceding formulation by R_F^*.

4.3.2. Sales Nondisclosure: The N Setting. In the N setting, it is assumed that because of sales nondisclosure, customers have no knowledge of the exact market size, even in the second period. Hence, in this setting, the firm can only preannounce a price for the second period that is independent of the first-period sales volume. This case was discussed in Section 4.2. Again, with slight abuse of notation, we denote the optimal revenue for this setting by R_N^*.

4.3.3. Sales Disclosure vs. Nondisclosure. In the F setting, if the firm can commit to future prices contingent on the revealed first-period sales, not only can the saturation effect be dampened, because now the firm can commit to a higher price for booming market, but also, the downside of the Matthew effect can be mitigated, because now the firm can commit to a lower price to boost demand if the early sales volume proves to be scant. The following proposition compares the optimal profits in the F and N settings under contingent preannounced pricing.

Proposition 4 (Contingent Preannounced Pricing). $R_F^* \geq R_N^*$.

Proposition 4 confirms our intuition by showing that the firm is better off combining sales disclosure and nimble pricing contingent on the sales. The intuition comes from two aspects. First, with contingent pricing, the firm has the flexibility to moderate the Matthew and saturation effects in the F setting by charging different prices based on the realization of M, whereas in the N setting, the firm just determines two prices depending on the distribution of M. Hence, committing to sales disclosure coupled with pricing flexibility can outperform committing to sales nondisclosure by allowing the firm to better respond to market size uncertainty, especially for the second period.

Second, the other aspect is about commitment and assurance. Preannouncing a contingent pricing policy for the second period helps ensure a relatively high utility for first-period customers who buy immediately. In particular, the firm can preannounce a low second-period price if the first-period sales volume is low. Doing so not only improves the expected network benefit but also reduces the risk that the ultimate network benefit will be disappointing to first-period customers, thereby driving more customers to buy the good in the first period.

We note that the ability to commit to a contingent pricing policy is necessary for sales disclosure to be better off. A firm that cannot make such a commitment to customers might only focus on maximizing the second-period profit when it observes the remaining potential customers at the beginning of that period. As a result, the firm may achieve a higher second-period profit under sales disclosure, but it may not ensure greater attraction in the first period. In this case, some additional conditions are required for sales disclosure to be better off. This case is discussed formally in the next subsection.

4.4. Contingent Pricing Without Commitment
In this subsection, we consider a scenario in which the firm determines the first-period (second-period) price at the beginning of the first (second) period, where the second-period price is contingent on two state variables: the first-period sales d_1 and the first-period adoption fraction α_1. Again, we consider the F and the N settings and compare the firm's profits R_F^* and R_N^* in those two settings.

4.4.1. Sales Disclosure: The *F* Setting.
We study this two-stage game by backward induction. In the second period, given the first-period sales volume d_1 and the first-period adoption fraction α_1, customers can infer the realization m of the random market size M, and when facing the second-period price $p_{2,F}$, they anticipate that an α_2 fraction of customers will buy the

good. The equilibrium adoption fraction α_2 can be characterized by

$$\alpha_1 + \alpha_2 = \max\{\alpha_1, \bar{G}(p_{2,F} - s((\alpha_1 + \alpha_2)m))\}.$$

To optimize its second-period profit, the firm must charge a price such that $\alpha_2 > 0$ for all possible first-period sales volumes as long as $\alpha_1 < 1$. (Otherwise, the firm will gain nothing from the second period.) Consequently, the REE for the second period can be rewritten as

$$\alpha_1 + \alpha_2 = \bar{G}(p_{2,F} - s((\alpha_1 + \alpha_2)m)). \quad (12)$$

Denote by $\alpha_{2,F}(\alpha_1, p_{2,F}, m)$ the largest fixed point in (12). For any α_1 and m, the firm's problem at the second period is $\max_{p_{2,F}} R_{2,F}(\alpha_1, p_{2,F}, m) = p_{2,F}\alpha_{2,F}(\alpha_1, p_{2,F}, m)m$. Clearly, the optimal solution is a function of α_1 and m denoted by $p_{2,F}^*(\alpha_1, m)$. Additionally, we use $\alpha_{2,F}^*(\alpha_1, m)$ to denote the adoption fraction under this optimal price.

In the first period, analogous to the state-independent pricing case, the REE can be characterized by

$$\alpha_1 = \sup_{x \in [0,1]} \{x : G^{-1}(1 - x) - p_{1,F}$$
$$+ \delta Es((x + \alpha_{2,F}^*(x, M))M)$$
$$\geq \delta[G^{-1}(1 - x) - Ep_{2,F}^*(x, M)$$
$$+ Es((x + \alpha_{2,F}^*(x, M))M)]\}$$
$$= \sup_{x \in [0,1]} \{x : G^{-1}(1 - x) - p_{1,F}$$
$$\geq \delta[G^{-1}(1 - x) - Ep_{2,F}^*(x, M)]\}. \quad (13)$$

Note that $G^{-1}(1 - x) - p_{2,F}^*(x, m) + s((x + \alpha_{2,F}^*(x, m))m)$ is always nonnegative for all x and m. It can be seen that α_1 in an REE is a function of $p_{1,F}$ denoted by $\alpha_{1,F}(p_{1,F})$. Then the firm's problem in the first period is $\max_{p_{1,F}} R_F(p_{1,F}) = p_{1,F}\alpha_{1,F}(p_{1,F})EM + E[p_{2,F}^*(\alpha_{1,F}(p_{1,F}), M)\alpha_{2,F}^*(\alpha_{1,F}(p_{1,F}), M)M]$. Its optimal value is denoted by R_F^*.

4.4.2. Sales Nondisclosure: The *N* Setting.
For this setting, we also use backward induction to study the two-period game. In the second period, conditional on the first-period adoption fraction α_1, the second-period adoption fraction α_2 in equilibrium can be characterized by

$$\alpha_1 + \alpha_2 = \bar{G}(p_{2,N} - Es((\alpha_1 + \alpha_2)M)).$$

Denote by $\alpha_{2,N}(\alpha_1, p_{2,N})$ the largest fixed point. For any α_1, the firm's problem in the second period is

$$\max_{p_{2,N}} R_{2,N}(\alpha_1, p_{2,N}) = p_{2,N}\alpha_{2,N}(\alpha_1, p_{2,N})EM.$$

Let $p_{2,N}^*(\alpha_1)$ be the optimal solution to this problem and $\alpha_{2,N}^*(\alpha_1)$ be the corresponding second-period adoption fraction.

In the first period, the REE can be characterized by

$$\alpha_1 = \sup_{x \in [0,1]} \{ x : G^{-1}(1-x) - p_{1,N} + \delta \mathsf{E} s\big((x + \alpha_{2,N}^*(x))M\big)$$

$$\geq \delta \big[G^{-1}(1-x) - p_{2,N}^*(x)$$

$$+ \mathsf{E} s\big((x + \alpha_{2,N}^*(x))M\big) \big] \}$$

$$= \sup_{x \in [0,1]} \{ x : G^{-1}(1-x) - p_{1,N}$$

$$\geq \delta \big[G^{-1}(1-x) - p_{2,N}^*(x) \big] \}.$$

It can be seen that α_1 in an REE is a function of $p_{1,N}$ denoted by $\alpha_{1,N}(p_{1,N})$. Then the firm's problem in the first period is

$$\max_{p_{1,N}} R_N(p_{1,N}) = p_{1,N} \alpha_{1,N}(p_{1,N}) \mathsf{E} M + p_{2,N}^*\big(\alpha_{1,N}(p_{1,N})\big)$$

$$\cdot \alpha_{2,N}^*\big(\alpha_{1,N}(p_{1,N})\big) \mathsf{E} M.$$

Its optimal value is denoted by R_N^*.

4.4.3. Sales Disclosure vs. Nondisclosure.

The second-period price in the F setting will be chosen to maximize the second-period profit given the first-period adoption fraction α_1 and the realized market size m. In contrast, the second-period price in the N setting will be chosen to maximize the second-period profit conditional on α_1. The following proposition compares the second-period profits in the F setting and the N setting under contingent pricing without commitment.

Lemma 7 (Contingent Pricing Without Commitment). *For* $\alpha_1 \in (0,1]$, $\mathsf{E} R_{2,F}^*(\alpha_1, M) \geq R_{2,N}^*(\alpha_1)$.

Lemma 7 shows that for any α_1, the expected second-period profit under sales disclosure always dominates that under sales nondisclosure, unlike in the exogenous pricing case. This result highlights certain benefits of being flexible on pricing in the F setting.

However, we do not always have $\mathsf{E} p_{2,F}^*(\alpha_1, M) \geq p_{2,N}^*(\alpha_1)$. It can be seen from Equation (13) that the first-period demand under sales disclosure does not necessarily dominate that under sales nondisclosure. One special case occurs when the discount factor is sufficiently large. In this case, most customers will delay their purchase to the second period in the F or N setting, and hence, the total profits under sales disclosure and nondisclosure come mainly from the second period. Then it follows from Lemma 7 that committing to sales disclosure is likely to result in a higher profit in this case. This intuition is formalized in the following proposition.

Proposition 5 (Contingent Pricing Without Commitment). *Suppose that* $\sup\{\bar{G}(x)/g(x) : g(x) > 0\}$ *is finite. Then there* exists δ_c *such that* $0 \leq \delta_c < 1$ *and, when* $\delta \in [\delta_c, 1)$, $R_F^* \geq R_N^*$.

5. Concluding Remarks

In this paper, we studied two information disclosure policies, full disclosure (transparency) and no disclosure (secrecy), in selling network goods under market size uncertainty. The problem is nontrivial, because demand is endogenized as an equilibrium outcome depending on customers' knowledge of the market size. We show that the fundamental trade-off is between two countervailing effects, the Matthew effect and the saturation effect, and our results apply to a fairly general class of standalone valuation distributions and market size distributions. The model captures the key trade-off in this class of problems, with insights carrying over to a variety of modeling extensions. Our work shows that the full disclosure policy coupled with a commitment to contingent pricing is always better than the nondisclosure policy. This is consistent with the observation that many crowdfunding platforms reveal full information throughout the pledging process and encourage contingent updates and stimuli. For example, Kickstarter.com encourages a project creator to offer discounts or add new goodies to reenergize the funding process when the funding falls below expectation (see Du et al. 2017).

Our model could in part account for customer risk aversion. Consider the one-period model. In the N setting, if consumers are only risk averse about the uncertain social utility because of the *uncertain* network effects, it is equivalent to replacing $s(\cdot)$ in the base model by $u(s(\cdot))$ for the risk-aversion case, where $u(\cdot)$ is a concave utility function. Nevertheless, the two identified effects remain as long as the demand, as a function of the realized market size, is shaped as in Figure 2. In the F setting, customers face a realized market size without uncertainty; therefore, their behavior stays the same. As $s(\cdot)$ is replaced by $u(s(\cdot))$, the equilibrium adoption fraction in the F setting, $\alpha_F(m)$ is closer to a concave function in the market size, and hence, risk-averse behavior tends to favor sales nondisclosure by dampening the Matthew effect. Moreover, our model assumes that sales information reaches all customers. In reality, this may not be true, and social interaction, such as word of mouth, will lead to a wider spread of the information if the sales volume is higher. Such sales-dependent valence of word of mouth can be approximated and captured by modifying the network benefit functions. The sales-dependent word-of-mouth effect tends to result in a more convex social utility function and is expected to tilt the firm in favor of sales disclosure by boosting the Matthew effect.

Though we study a monopoly model in this paper, many of its insights are ready to be extended to

a competitive setting. First, disclosing early sales information is beneficial to customers. Hence, in a competitive setting, we expect that committing to sales disclosure is more likely to be a firm's information policy in equilibrium as a competitive advantage. Second, in a competitive environment, the market size possessed by each firm can be small. According to Proposition 3(a), when the expected network benefit is relatively weak and the customer valuation distribution has a heavy tail, committing to sales disclosure is likely to be in equilibrium for a firm. Third, Lemma 7 and Proposition 4 have shown that committing to sales disclosure coupled with pricing flexibility may lead to higher profit. Hence, those firms that have more pricing flexibility in reacting to varying market conditions are more likely to implement the full sales disclosure policy.

Finally, although this paper focuses mainly on the comparison between two simple policies (i.e., full and no sales disclosure), there might be other more sophisticated mechanisms by which the seller can disclose sales information in the presence of network externalities. Below we discuss these policies briefly; we leave a thorough study of them to future research.

5.1. Partial Disclosure

It is possible for a firm to consider a partial disclosure policy: for example, to disclose sales information when the sales reach (or fall below) a certain threshold. Such a policy may have the potential to generate more profit for the firm. However, there could be additional challenges in implementation because the firm may have an incentive to deviate from such a policy after learning the sales information. Moreover, there are other forms of partial disclosure that can be more sophisticated than the threshold-type policy.

5.2. Dynamic Disclosure

Although we studied dynamic pricing schemes along with customers' intertemporal purchase behavior, we did not consider dynamic sales information disclosure and the corresponding learning by customers, which can be critical, especially when the market size is evolving over time. One can apply the Bayesian persuasion framework to study such a problem (Ely 2017, Bimpikis et al. 2019). Nevertheless, again, such a study could be quite complicated given the market size as a continuous quantity in our model.

5.3. Randomized Disclosure

This paper considered whether to commit to sales disclosure at a fixed time. One interesting direction for future research would be to consider randomizing the timing of sales disclosure. Price randomization has been shown to be beneficial in the context of markdowns with customers strategically monitoring

the price changes (Moon et al. 2017, Chen et al. 2018). When customers monitor the sales online with heterogeneous monitoring costs, randomizing the timing of sales disclosure might benefit the firm. However, expanding the policy space from a deterministic mapping (from a realized state to an action) to a stochastic one may significantly complicate the problem.

Acknowledgments

Endnotes

[1] In the literature, a belief can be about either the market outcomes or the behavior of other players. However, in a market with many infinitesimal players, a focal customer's belief is usually about market outcomes: for example, the number of adopters in network games (Jackson and Yariv 2007). In other words, when the market is large enough, each customer's private information about his or her own valuation is negligible, and his or her belief about the market outcome is close to his or her belief about other customers in the market.

[2] We refer the readers to Tarski (1955) for a general setting of Tarski's fixed-point theorem.

[3] This assumption is made to simplify the discussion. It does not affect the results in this subsection. In fact, when this assumption is violated (i.e., when no customer buys the good in the first period), the second-period customers are uninformed of the market size even in the F setting. In this case, the two settings, F and N, become effectively identical.

[4] In general, an optimal solution to a profit-maximization problem may not exist. In this case, we focus on the supremum of the objective function, and an optimal price is thought of as a limit of a sequence of prices at which the sequence of objective function values converges to its supremum.

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