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The Effect of Multichannel and Omnichannel Retailing on Physical Stores

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Abstract. Most retailers today sell products through an online channel in addition to traditional physical stores. We investigate how such a multichannel or omnichannel retailer should decide the number and size of physical stores. We show that a higher return rate for online purchases can incentivize the retailer to have fewer physical stores that are larger in size. As online shopping becomes more convenient, a retailer may prefer to have more physical stores that are smaller in size. We also study the effect of three popular omnichannel strategies that involve changes of the physical stores' functions: (i) *showrooms* only display products for customers to inspect before they purchase online, removing fulfillment from physical stores; (ii) *return flexibility* expands the functionality of physical stores by allowing customers to return online orders at them; and (iii) *fulfillment flexibility* expands functionality by allowing customers to pick up products purchased online at physical stores. We show that when the physical stores are given fewer (more) functions, as with the showroom (return or fulfillment flexibility) strategy, the omnichannel retailer may find it optimal to increase (reduce) the number and/or size of the physical stores.

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1. Introduction

The long dominance of brick-and-mortar retail has been challenged by the game-changing advent of the internet, bringing about a stunning transformation in the retail industry. Most firms have begun to adopt a multichannel approach by selling products through both online and offline channels. The success of online retail is inextricably linked with the advances in digital technology. Almost all Americans today use the internet, and 77% already own a smartphone, making online shopping widely accessible (Smith 2017). Customers may find online shopping attractive because they can order products in pajamas at home as opposed to having to travel to a store. However, this also comes with its own drawbacks—online shoppers do not have the ability to physically inspect or try on a product being considered for purchase, and also cannot obtain their products immediately (Cegid 2016). Many retailers are investing heavily in digital initiatives to make online shopping more convenient and less of a hassle for customers (JDA and PwC 2017). For example, Target recently invested $7 billion and Kohl's $2 billion in digital initiatives to improve their online shopping experience (Target 2017, Wahba 2017). Accordingly, it is not surprising that many people, such as Marc Andreessen, the creator of Netscape, predict that traditional retail stores will die off and online websites will be the only way for customers to shop in the future (Shontell 2013).

The increasing popularity of online shopping has also led retailers to reevaluate and restructure their physical stores. In 2016, for instance, high fashion retailer Zara replaced four smaller stores spread around the town of La Coruña in Spain with a huge five-story flagship store, which will serve as a model for Zara to roll out globally (Kowsmann 2017). In contrast, around the same time, beauty retailer Sephora and furniture giant IKEA started to add more new and smaller stores across the United States (Petroff 2017, Thomas 2017). Similarly, both Target and Kohl's have increased the number of stores and made them smaller, while simultaneously investing in their online shopping initiatives (Target 2017, Wahba 2017). Given the different strategies observed in practice, an important question in this context is how the presence of an online channel influences a retailer's decisions regarding its physical stores, namely, their number and size. This is the first question we address in this paper.

A major drawback of online retailing is that the customers cannot inspect a product before purchasing it, which leads to a high rate of product return

(Cegid 2016, Saleh 2016). Consequently, the loss incurred by product returns to retailers has been steep (Boyajian 2018). Moreover, what customers want is to be able to assemble their own path of the purchasing process to optimize shopping experience, be it online, in-store, or both. As a result, retailers have begun to adopt different omnichannel strategies, which offer customers flexibility by providing with them an integrated shopping experience online and offline. One popular strategy is to build showrooms, which are physical stores that do not stock merchandise, but instead display products for customers to try on before they purchase online. Given the potential of this strategy to reduce the returns associated with online purchases, showroom stores have become popular with many retailers such as Warby Parker and Bonobos (Bell et al. 2014).

Compared with showrooms, which remove the fulfillment function from physical stores, other popular omnichannel strategies incorporate new functions in them. For example, some retailers like IKEA, J.Crew, and Sephora allow customers to return their online purchases to the physical stores. Such "return flexibility" can reduce the return handling costs given that it is typically cheaper for retailers to collect and return products from stores instead of directly from customers (AlixPartners 2017). More recently, other retailers like Best Buy, Gap, and Home Depot have started to offer customers the option to pick up an online order from a physical store. Such "fulfillment flexibility" enables customers to receive a product purchased online in merely a couple of hours (Chen and Rosmarin 2019).

Given the rise in popularity of omnichannel strategies, a natural question is how they influence a retailer's decisions regarding physical stores; that is, should the retailer have more or fewer physical stores under these omnichannel strategies and carry more or fewer products in them. This is the second question we investigate in this paper. The effect of omnichannel strategies on physical stores has important and significant managerial implications, given that by definition, physical stores form an integral part of omnichannel retailing. However, despite the growing body of research on omnichannel retailing, the existing literature does not offer any insights for how a retailer should modify its physical stores.

In order to address our research questions, we develop a stylized model that captures essential elements of the multichannel and omnichannel retailing context. We consider a retailer who sells products through both an online channel and physical stores. We utilize a circular location model for customers, where a fraction of them face valuation uncertainty. When these "uninformed" customers purchase a product online, there is a chance that they will dislike the product and return it back to the retailer. Purchasing the product online also incurs customers an online hassle cost, which represents the disutility from having to wait, delivery costs, or other inconveniences involved in online purchases. In contrast, if uninformed customers visit a physical store, they can evaluate whether they like a product before purchasing it, avoiding any costs to return the product. However, visiting a physical store requires incurring an offline hassle cost for traveling to the nearest store and finding the product. Customers choose where to purchase the product by maximizing their individual net utility. The rest of the customers are "informed" customers who do not face valuation uncertainty. The retailer chooses the number of physical stores and products carried in them to maximize its expected profit, by taking into account facility costs for physical stores. The main trade-off for the retailer in having more and/or bigger stores is that they help reduce return handling costs, as more uninformed customers will visit physical stores to inspect products before purchasing them, but lead to higher facility costs.

We first consider a multichannel retailer with conventional stores, which do not interact directly with the online channel. This allows us to highlight the effect of the online channel on the retailer's physical store decisions. Conventional wisdom would suggest that retailers facing a higher return rate should have more stores to make it more convenient for customers to come and physically try out the product, so as to reduce online product returns and the associated costs. However, we show that for a higher product return rate, a retailer may prefer to have fewer physical stores that are larger in size. One would also expect that as online shopping becomes more convenient as the result of technology and delivery innovations, a retailer should reduce the number of physical stores. However, we show that when the online hassle cost is lowered due to such advances, as long as it is optimal to have stores, the retailer may instead have an incentive to open more physical stores that are smaller in size.

We subsequently study the effect of three popular omnichannel strategies, namely, showroom stores, return flexibility, and fulfillment flexibility, on the retailer's physical store decisions. Specifically, we consider an omnichannel retailer that has both physical stores and an online channel. We find that our insights from the base model with conventional stores regarding the effect of return rate and online hassle cost continue to qualitatively hold in the presence of these strategies. More interestingly, we show that when physical stores are given more (fewer) functions due to the adoption of omnichannel strategies, the retailer may find it optimal to reduce (increase) their number. The retailer may need to increase the store size when implementing showrooms or fulfillment flexibility, but decrease it when implementing return flexibility. That is, with the growth of omnichannel retailing, it is increasingly important for retailers to carefully evaluate their physical store presence.

2. Literature Review

Our work is related to and builds on the literature on multichannel retailers that sell products through both online and offline channels, and the recent literature on omnichannel retailing.

A large body of literature in multichannel retailing studies the benefit of adding an online channel to a firm that traditionally sells product through a brick-and-mortar store, which could be operated by a third-party retailer (Chiang et al. 2003, Chen et al. 2008) or by the firm itself (Lal and Sarvary 1999, Bernstein et al. 2008). Our paper belongs to the second category; that is, we study a single retailer who manages both online and store channels, as commonly observed in practice today. Ofek et al. (2011) study the impact of product returns on a multichannel retailer and examine how pricing strategies and store assistance levels change as a result of the additional online outlet. However, none of these papers consider a retailer's decisions regarding the number and size of physical stores. We contribute to this literature by investigating how a multichannel retailer should make decisions regarding the physical stores given the presence of the online channel.

The growth of omnichannel retailing in practice has spurred increasing attention in the academic literature. A series of papers study how a retailer should manage online and offline channels in an integrated manner, so as to provide customers with a seamless shopping experience across channels (Rigby 2011, Brynjolfsson et al. 2013, Bell et al. 2014, Gallino and Moreno 2014, Glaeser et al. 2019, Aflaki and Swinney 2021). A crucial feature of this context is the role of customers' ex ante uncertainty regarding their valuation for the product due to nondigital product attributes that cannot be fully evaluated online (Lal and Sarvary 1999). Physical stores allow customers to inspect and evaluate a product, helping customers resolve their product value uncertainty before purchasing it. In contrast, customers who purchase the product online may end up not liking it, and therefore will have to deal with returning it. In a single-channel setting, various mechanisms have been studied to induce customer purchase in the presence of product value uncertainty, for example, refund (Che 1996, Su 2009), money-back guarantees (Davis et al. 1995), and advance selling (Xie and Shugan 2001).

In the omnichannel context, in order to mitigate costs associated with customers' valuation uncertainty, retailers adopt different strategies that change the functionality of the physical stores (Gao and Su 2019). A recent stream of literature studies the use of showroom stores, which do not hold any inventory. Instead, they only display samples for customers to inspect a product before purchasing it online. Using quasi-experimental data on showroom openings by WarbyParker.com, Bell et al. (2018) empirically find that showroom stores help reduce online returns. However, Gao and Su (2017b) show that physical showrooms may increase online product returns, as a result of changes in the retailer's inventory decisions. Another stream of literature studies omnichannel strategies that introduce new functions to physical stores. Nageswaran et al. (2020) investigate the pricing and return policy decisions of an omnichannel retailer utilizing a flexible return policy, where customers are allowed to return their online purchases to a physical store. Such return flexibility can reduce the potential costs incurred by a customer when they do not like their online purchase. Gallino and Moreno (2014) empirically test the impact of allowing customers to pick up their online purchases at a physical store on a retailer's sales in both online and offline channels. Gao and Su (2017a) theoretically study the implementation of such a flexible fulfillment policy on channel coordination. Many retailers use ship-from-store fulfillment, where they fulfill online orders from nearby stores rather than from distribution centers. Acimovic and Graves (2014) and Jasin and Sinha (2015) develop efficient algorithms to help retailers determine where to pull the inventory from under such ship-from-store programs. However, all of the above papers either do not consider decisions related to the number and size of physical stores or implicitly assume them to be exogenous and fixed. We contribute to this literature by investigating how the adoption of three omnichannel retailing strategies, that is, showrooms, return flexibility, and fulfillment flexibility, influence a retailer's decisions regarding the number and size of physical stores.

3. The Base Model: Conventional Stores

We begin by developing a model of a multichannel retailer who sells products both online and in physical stores. In this section, we consider conventional brick-and-mortar stores, which do not directly interact with the online sales channel. This analysis will help us offer important insights into how the online channel influences the retailer's physical store decisions. Subsequently, in Section 4, we will consider the effects of several different omnichannel retailing strategies.

3.1. Customer and Product Characteristics

We assume that the retailer can offer an entire set of distinct products, labeled by I, in the online channel. The retailer needs to determine the number of physical stores, denoted by n, and what products to offer in these stores, denoted by I_s, which is a subset of I. If a product $i \in I$ is offered both online and in stores, a customer can decide to purchase it from either channel. We assume that the price p_i for product i is the same across the two channels. This is in line with practice, where price consistency across different channels has been found to be an important attribute for customers (Taylor 2019). A customer is interested in only one specific product, and we label the customers interested in product $i \in I$ as class i customers.[1] Therefore, there are in total $|I|$ products and $|I|$ customer classes, with $|\cdot|$ denoting the cardinality of a set.

The size of customer class i is assumed to be deterministic and denoted by $D_i > 0$. Customers of each class $i \in I$ are distributed uniformly on a circle of unit circumference (Salop 1979). The circular location model has been used extensively in the existing literature to model interactions between customers and a retailer (see Bakos 1997, Balasubramanian 1998, Dewan et al. 2003, Shulman et al. 2009). For analytical tractability and ease of exposition, we consider a symmetric case where the n physical stores are evenly located on the circle, and each store carries the same set of products $I_s \subseteq I$. Accordingly, $|I_s|$ can be interpreted as the breadth of the products carried in physical stores and is hereafter referred to simply as the *product breadth*.

We build on recent research of omnichannel retail operations (see, e.g., Gao and Su 2017b) to model customers in this multichannel setting. We assume that customers within each class $i \in I$ are heterogeneous in terms of whether they experience valuation uncertainty for the product or not. In particular, a fraction θ of the customers in each class are uninformed in that, with probability $\gamma \in (0,1)$, they will dislike the product and their valuation for the product will be zero. With probability $1-\gamma$, the customers will like the product and derive a valuation $v_i > 0$. An uninformed customer can ascertain their valuation for the product only by examining it in person. Therefore, she can resolve the valuation uncertainty before purchasing the product by visiting a physical store. If the customer instead purchases the product through the online channel, she will learn her valuation only after receiving and examining the product. The remaining fraction $1-\theta$ of customers in each class $i \in I$ are informed in that they do not face valuation uncertainty and derive a valuation v_i for product i. This may be because these customers may have seen the product before.

We next outline utilities of both types of customers from each of the purchasing options.

3.1.1. Buying a Product Online. Consider a class i customer who purchases product $i \in I$ online. She will incur a hassle cost associated with online shopping, denoted by $h_o \geq 0$, which includes any inconvenience of using the website or app, delivery costs, and the wait for the product to be delivered. Upon receiving the online purchase, an uninformed customer will like the product with probability $1-\gamma$, in which case she will keep the product. If this uninformed customer dislikes the product, which will happen with probability γ, she will return the product to the retailer through the online process for a full refund. The return process will lead the customer to incur a return hassle cost, denoted by $h_r > 0$, which includes the inconvenience and expense associated with shipping the product back. In our base model with conventional stores, the two channels are separate. We consider the possibilities of customers returning and picking up online purchases at a physical store in Sections 4.2 and 4.3, respectively.

For the rest of this paper, we simply refer to γ as the return rate. Note that we make an implicit assumption that return rates are the same for all the products. This assumption is reasonable for retailers that specialize in selling products in the same or similar product categories, for example, Best Buy for electronics, Tiffany & Co. for jewelry, and IKEA for furniture.[2]

The expected utility of an uninformed class i customer from buying product i through the online channel is given by

$$u_{i,o} = (1-\gamma)(v_i - p_i) - \gamma h_r - h_o \text{ for } i \in I, \tag{1}$$

where $(1-\gamma)(v_i - p_i)$ represents the expected utility from buying the product, γh_r represents the expected return hassle cost, and h_o represents the online hassle cost.

The expected utility of an informed class i customer from buying product i through the online channel is given by

$$\tilde{u}_{i,o} = v_i - p_i - h_o \text{ for } i \in I, \tag{2}$$

where the superscript $\sim$ is used to denote informed customers throughout this paper.

3.1.2. Buying a Product in a Physical Store. For a product carried in physical stores (i.e., $i \in I_s$), a class i customer can purchase the product online or from a store. Visiting a physical store allows an uninformed customer to examine and test the product *before* purchasing it. Following Gao and Su (2017b), we assume that a customer can completely resolve her valuation uncertainty by physically inspecting the product in a store. Note that this is in contrast to when the customer purchases the product through the online channel, where she can resolve it only after purchasing and receiving the product. Accordingly, when an uninformed customer visits a physical store, she will purchase the product with probability $1-\gamma$. If she does not like the product, with probability γ, she will simply not purchase it. However, visiting a physical store and finding the product there requires customers to incur an offline hassle cost given by $h_s(x) = tx$, where $t > 0$ is a cost factor and x is the distance between the customer's location on the circular city and her nearest store. Therefore, the expected utility of an uninformed class i customer from buying product i at a physical store is given by

$$u_{i,s}(x) = (1-\gamma)(v_i - p_i) - h_s(x) \text{ for } i \in I_s. \quad (3)$$

Finally, the expected utility for an informed class i customer from buying product i at a physical store is given by

$$\tilde{u}_{i,s}(x) = v_i - p_i - h_s(x) \text{ for } i \in I_s. \quad (4)$$

Comparing Equations (2) and (4), it can be seen that for products carried both online and in physical stores, an informed customer will choose between them by comparing their offline hassle cost for visiting a store with the online shopping hassle cost. For uninformed customers, by comparing Equations (1) and (3), it can be seen that they will compare their offline hassle cost for visiting a store with the expected hassle costs of online shopping *and* return.[3] Given our focus on multichannel and omnichannel retail operations, we assume that customer valuation v_i is sufficiently large such that the expected utility of customers from purchasing through either channel is nonnegative, that is, $u_{i,o} \geq 0$ and $u_{i,s}(0) \geq 0$ (for a similar approach, see Gao and Su 2017b, Mehra et al. 2017). This implies full market coverage in that all customers decide between purchasing the product online and purchasing at a physical store.[4]

3.2. Demand and Retailer Profit

For products not offered in physical stores ($i \notin I_s$), all class i customers will purchase the product online. The total demand for product i for the online channel is then simply given by D_i. If we denote the demand faced by the online channel and the physical store locations, from uninformed and informed customers, by $d_{i,o}$, $\tilde{d}_{i,o}$, $d_{i,s}$, and $\tilde{d}_{i,s}$, respectively, then we have that for products available only online,

$$d_{i,o} = \theta D_i, \;\; \tilde{d}_{i,o} = (1-\theta)D_i, \text{ and } d_{i,s} = \tilde{d}_{i,s} = 0 \text{ for } i \notin I_s. \quad (5)$$

For products offered in physical stores ($i \in I_s$), uninformed customers will compare the expected utilities from each channel, given by (1) and (3), to choose where to purchase the product. In particular, an uninformed class i customer will visit a store as opposed to buying online if and only if $u_{i,s}(x) \geq u_{i,o}$ or, equivalently, $x \leq (\gamma h_r + h_o)/t$. Similarly, informed customers will compare the expected utilities from (2) and (4) to choose where to purchase a product $i \in I_s$. It follows that an informed class i customer will visit a store as opposed to buying online if and only if $\tilde{u}_{i,s}(x) \geq \tilde{u}_{i,o}$ or, equivalently, $x \leq h_o/t$. Therefore, only the customers who live close enough to a physical store location (within a distance of $(\gamma h_r + h_o)/t$ for an uninformed customer and h_o/t for an informed customer) will choose to purchase the product in a store.[5] For a given number n of uniformly located stores, the distance between two nearby stores on the circular city is $1/n$, and the maximum distance from a customer's location to her nearest store is $1/(2n)$. Therefore, it follows that if $(\gamma h_r + h_o)/t \geq 1/(2n)(h_o/t \geq 1/(2n))$, all uninformed (informed) class i customers will visit a physical store. The demand faced by the online channel and physical stores, of uninformed and informed customers, respectively, will then be given by

$$\begin{aligned}
d_{i,o} &= \left[1 - 2\min\left\{\frac{\gamma h_r + h_o}{t}, \frac{1}{2n}\right\} n\right]\theta D_i,\\
d_{i,s} &= \left[2\min\left\{\frac{\gamma h_r + h_o}{t}, \frac{1}{2n}\right\} n\right]\theta D_i \text{ for } i \in I_s,\\
\tilde{d}_{i,o} &= \left[1 - 2\min\left\{\frac{h_o}{t}, \frac{1}{2n}\right\} n\right](1-\theta)D_i, \text{ and}\\
\tilde{d}_{i,s} &= \left[2\min\left\{\frac{h_o}{t}, \frac{1}{2n}\right\} n\right](1-\theta)D_i \text{ for } i \in I_s.
\end{aligned} \quad (6)$$

Among the class i uninformed customers who visit the physical stores (i.e., $d_{i,s}$), on average, a total of $(1-\gamma)d_{i,s}$ units of product i will be sold and not returned, because a customer purchases the product only if she likes it, which occurs with probability $1-\gamma$. In contrast, for products purchased by class i uninformed customers online (i.e., $d_{i,o}$), on average, $(1-\gamma)d_{i,o}$ units will be eventually kept by the customers and $\gamma d_{i,o}$ units will be returned to the retailer. The total sales of product i by uninformed customers across the two channels is then given by $(1-\gamma)d_{i,s}$, + $(1-\gamma)d_{i,o} = (1-\gamma)\theta D_i$, and the total number of product i returned is $\gamma d_{i,o}$. On the other hand, the total sales of product i by informed customers across the two channels is $\tilde{d}_{i,s} + \tilde{d}_{i,o} = (1-\theta)D_i$, and the total number of product i returned is zero. If we use m_i to denote the retailer's per-unit profit of selling product i,[6] and k_i to denote the handling cost for the retailer for each returned product i due to repackaging and restocking, then the total expected profit from product i earned by the retailer is given by

$$\pi_i = m_i[(1-\gamma)\theta D_i + (1-\theta)D_i] - k_i\gamma d_{i,o}. \quad (7)$$

Based on (5) and (6), we can rewrite the profit in (7) for a product $i \notin I_s$ and $i \in I_s$ as follows:

$$\pi_i = \begin{cases} m_i\big[(1-\gamma)\theta D_i + (1-\theta)D_i\big] - k_i\gamma\theta D_i & \text{for } i \notin I_s; \\ m_i\big[(1-\gamma)\theta D_i + (1-\theta)D_i\big] & \\ \quad -k_i\gamma\Big[1 - 2\min\Big\{\frac{\gamma h_r + h_o}{t}, \frac{1}{2n}\Big\}n\Big]\theta D_i & \text{for } i \in I_s. \end{cases}$$

The retailer has to invest capital in building or renting retail space for retail stores. Typically, a physical store requires space for displaying and storing inventory for each product carried in the store. Let this space required for product $i \in I_s$ be denoted by Ω_i. In addition, a physical store requires some space for common areas such as entry, access, fitting rooms, or restrooms, which we denote by $\Omega_0 > 0$. Accordingly, the total size of a physical store, Ω, can be written as

$$\Omega = \sum_{i \in I_s} \Omega_i + \Omega_0, \text{ where } \Omega_i = \alpha_i + \beta_i \frac{(1-\gamma)d_{i,s} + \tilde{d}_{i,s}}{n}$$
$$\text{for some } \alpha_i, \beta_i > 0, \text{ for all } i \in I_s, \tag{8}$$

where the first term in Ω_i, α_i, represents the space required to display product $i \in I_s$. The second term in Ω_i, $\beta_i \frac{(1-\gamma)d_{i,s}+\tilde{d}_{i,s}}{n}$, represents the space required to store units of product $i \in I_s$, where $\frac{(1-\gamma)d_{i,s}+\tilde{d}_{i,s}}{n}$ is the total sales of this product faced by a physical store, and $\beta_i > 0$ is the space required to store a unit of this product.[7]

For the rest of this paper, we simply refer to Ω as the store size. The cost of building or renting space typically increases linearly in the store size. For example, commercial building costs or rent are typically quoted on a per-square-foot basis (Gharib 2015). Therefore, we model the facility cost for each store as $\xi\Omega$, where $\xi > 0$ is the unit cost for facility space. The retailer's total expected profit is then given by summing the profit earned from all the products less the total facility cost for n stores, that is,

$$\pi = \sum_{i \in I} \pi_i - n\xi\Omega$$
$$= \sum_{i \in I}\big\{m_i\big[(1-\gamma)\theta D_i + (1-\theta)D_i\big] - k_i\gamma d_{i,o}\big\} - n\xi\Omega. \tag{9}$$

The retailer maximizes its total expected profit π given in (9) by choosing the number of physical stores (n) and the subset of products to carry in them (I_s). In order to do so, we assume that prices and margins are exogenous, which is a common assumption in the omnichannel retailing literature (see, e.g., Gao and Su 2017b, Aflaki and Swinney 2021). This is also consistent with practice, where many retailers benchmark their pricing decisions using keystone pricing, which is essentially doubling the wholesale price or production cost of the product they have paid for the product, and does not depend on the number of stores and product breadth carried in them (Carroll 2012). Finally, as can be seen from (9), the retailer's trade-off in terms of whether it prefers customers to shop online or in physical stores is between the product return handling costs associated with online purchases and the facility costs from having physical stores (i.e., $\sum_{i \in I} k_i\gamma d_{i,o}$ versus $n\xi\Omega$).[8]

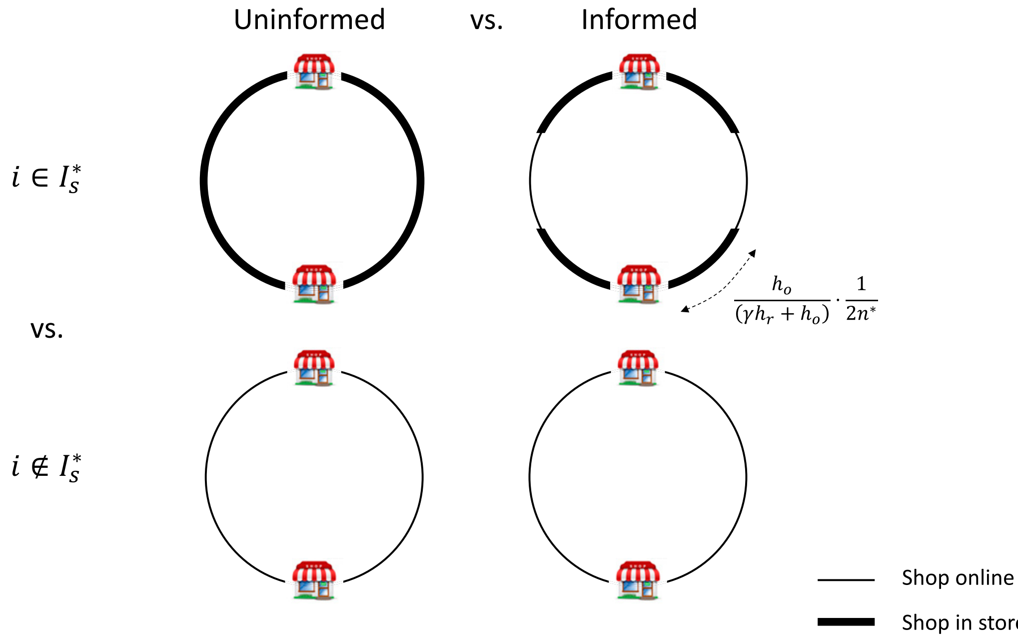
3.3. Analysis and Results

We begin by characterizing the retailer's optimal decisions n^* and I_s^*. In order to do so, we relax the integrality of number of stores to allow it to be a continuous variable hereafter, which is a common approach used in the literature (see Cachon 2014, Park et al. 2015).[9] Note that the retailer will open physical stores only if it is optimal to carry at least one product in them, that is, $n^* > 0$ if and only if $I_s^* \neq \varnothing$. For brevity and ease of exposition, let $\boldsymbol{D} \doteq (D_1, D_2, \ldots, D_{|I|})$, $\boldsymbol{m} \doteq (m_1, m_2, \ldots, m_{|I|})$, $\boldsymbol{k} \doteq (k_1, k_2, \ldots, k_{|I|})$, $\boldsymbol{\alpha} \doteq (\alpha_1, \alpha_2, \ldots, \alpha_{|I|})$, and $\boldsymbol{\beta} \doteq (\beta_1, \beta_2, \ldots, \beta_{|I|})$.

Lemma 1. *There exists a threshold $G(\theta, \boldsymbol{D}, h_o, h_r, \boldsymbol{k}, t, \boldsymbol{\alpha}, \boldsymbol{\beta}, \Omega_0, \xi)$ such that the retailer prefers to have physical stores ($n^* > 0$ and $I_s^* \neq \varnothing$) if and only if $\gamma > G(\cdot)$. Under this condition, the optimal physical store decisions for the retailer are given by $n^* = \frac{t}{2(\gamma h_r + h_o)}$ and $I_s^* = \{i \in I : k_i\gamma\frac{2(\gamma h_r + h_o)}{t}\theta D_i > \xi[\alpha_i + \beta_i(\frac{2h_o}{t}(1-\theta)D_i + (1-\gamma)\frac{2(\gamma h_r + h_o)}{t}\theta D_i)]\}$.*

When the product return rate is high (i.e., $\gamma > G(\cdot)$), the retailer benefits from having physical stores in order to allow uninformed customers to visit a store and inspect a product prior to purchasing it. This avoids returns from an uninformed customer purchasing online and not liking the product. Therefore, the retailer sets the density of physical stores such that an uninformed customer in the middle of two adjacent stores is indifferent between shopping in the store or the online channel. Accordingly, in equilibrium, all uninformed class i customers will visit the nearest physical store as long as product i is carried in the store. However, physical stores meet the demand for product i from only a fraction of informed customers. In particular, a fraction $h_o/(\gamma h_r + h_o)$ of the informed class i customers will choose to purchase product i in-store, and the rest of them will buy it online (see (6) and Figure 1). The demand of a product $i \in I$ from uninformed and informed customers received by each physical store are given by $\frac{2(\gamma h_r + h_o)}{t}\theta D_i$ and $\frac{2h_o}{t}(1-\theta)D_i$, respectively. Therefore, the criterion of I_s^* in Lemma 1 implies that the retailer should carry product i in the physical store when the cost associated with the expected product returns from the uninformed customers (i.e., $k_i\gamma\frac{2(\gamma h_r + h_o)}{t}\theta D_i$) exceeds the marginal

Figure 1. (Color online) Channel Choice of Class i Customers for $n^* = 2$



Notes. We consider only two stores for simplicity in illustration. When there are n^* stores, a fraction $1/n^*$ of uninformed class i customers and a fraction $(h_o/(\gamma h_r + h_o))/n^*$ of informed class i customers will visit any particular store, for each $i \in I_s^*$.

facility cost from including the product in the physical store (i.e., $\xi[\alpha_i + \beta_i(\frac{2h_o}{t}(1-\theta)D_i + (1-\gamma)\frac{2(\gamma h_r + h_o)}{t}\theta D_i)]$).

When the product return rate is sufficiently small (i.e., $\gamma \le G(\cdot)$), uninformed customers face lower valuation uncertainty, and thus prefer purchasing products online instead of incurring offline hassle costs to visit a physical store. Therefore, the retailer finds it optimal to operate only the online channel under this setting (i.e., $n^* = 0$ and $I_s^* = \varnothing$) to avoid the facility costs for physical stores. Although this result is expected, our next result will show that the effect of product return rate γ on the retailer's optimal decisions is not straightforward.

For the rest of this section, we will restrict our attention to the multichannel setting where $n^* > 0$ (or, equivalently, $\gamma > G(\cdot)$). Let Ω^* denote the size of a physical store under the optimal store decisions (n^*, I^*) for the retailer.

Proposition 1. *Let $n^* > 0$. The retailer should have fewer stores n^* and larger product breadth $|I_s^*|$ for a higher product return rate γ. Moreover, the store size Ω^* is larger for a higher product return rate γ if $h_r(1-2\gamma) > h_o$ (equivalent to $\gamma < \frac{h_r - h_o}{2h_r}$), which holds only if $\gamma < 1/2$ and $h_o < h_r$.*

Conventional wisdom would suggest that a higher return rate incentivizes the retailer to have more physical stores, so that it can reduce the returns associated with online purchases. However, the above result shows that the retailer should have fewer physical stores when faced with a higher return rate. To understand this result, note that a higher return rate implies a higher probability of an uninformed customer incurring the return hassle cost associated with an online purchase. Therefore, uninformed customers will be willing to incur a greater offline hassle cost to visit a physical store so that they can inspect the product before purchasing it. Given this stronger incentive for the customers to visit physical stores, the retailer can reduce the number of stores in order to save on the facility costs. Accordingly, the retailer also prefers to increase the product breadth $|I_s^*|$ to attract more customers and reduce the return handling costs associated with online purchases.

Given that a higher return rate incentivizes the retailer to carry more products in stores, it can also lead to a larger store size if the return hassle cost is sufficiently large, that is, $h_r(1-2\gamma) > h_o$. Under this condition, the total demand faced by a store for a product, and thus the storage space required, increases in the return rate γ. The reason for this is as follows: A higher return rate γ has two contrasting effects on the demand for a product. On the one hand, a higher γ results in fewer stores, and thus, a larger number of uninformed customers visit a store, that is, $\frac{d_{i,s}^*}{n^*} = \frac{2(\gamma h_r + h_o)}{t}\theta D_i$ increases in γ. On the other hand, the probability that when an uninformed customer visits the store, she will purchase the product, $1-\gamma$, is decreasing in γ. When the return hassle cost is sufficiently large, it is more attractive for uninformed customers to visit the store, leading the former effect to dominate. As a result, a higher return rate leads to greater demand for a product at each store, resulting in a larger store size. Note that this occurs if $\gamma < 1/2$ and $h_o < h_r$, which usually hold in practice because product return rates are typically no higher than 30% (Briggs 2013) and the online hassle cost is typically lower than the return hassle cost.

Proposition 1 offers one potential explanation for the contrasting strategies used by the furniture retailer IKEA and the fashion retailer Zara in recent years. In particular, IKEA started to add smaller outlets around the globe since 2015, and there are now 44 such stores, which are roughly one-tenth the size of a typical big-box IKEA store (Petroff 2017). In contrast, Zara decided to replace four smaller stores spread around the town of La Coruña in Spain with a huge five-story flagship store, and will use this as a template to roll out globally (Kowsmann 2017). This is in line with our results, which suggest that as furniture has a lower product return rate than fashion apparel (Toktay 2003, Briggs 2013), IKEA should have more stores that are smaller in size, but Zara should consolidate its several smaller stores into fewer large ones.

We next investigate the impact of the online hassle cost h_o on the retailer's decisions.

Proposition 2. *Let $n^* > 0$. The retailer should have more stores n^* for a lower online hassle cost h_o. Moreover, both the product breadth $|I_s^*|$ and store size Ω^* are smaller for a lower online hassle cost h_o if $h_r < \min_{i\in I} \frac{t\alpha_i}{2\beta_i\gamma(1-\theta)D_i}$.*

Conventional wisdom would suggest that as advances in mobile technology and delivery innovation further lower the online hassle cost for customers, retailers may not need as many stores and could reduce their number. However, the above result shows that, as long as it is optimal to have physical stores (i.e., $n^* > 0$), the retailer should open more stores when customers' online hassle cost is lowered. To understand this result, recall that when the online hassle cost decreases, it becomes more attractive for customers to purchase products online. This in turn leads to more returns from the uninformed customers, and thus, higher return handling costs for the retailer. Accordingly, the retailer has an incentive to open more physical stores to induce some of the uninformed customers to visit them and ascertain their valuation before purchasing the product. However, note that although opening more physical stores can reduce the return handling costs, it leads to higher facility costs, especially if the customers' return hassle cost is sufficiently low (i.e., $h_r < \min_{i\in I} \frac{t\alpha_i}{2\beta_i\gamma(1-\theta)D_i}$) such that the retailer needs to open a large enough number of physical stores to attract customers to visit them. Under this setting, to mitigate the additional facility costs, the retailer finds it optimal to reduce the breadth of the products carried in stores, which in turn leads to smaller stores, for a lower online hassle cost. Note that the condition $h_r < \min_{i\in I} \frac{t\alpha_i}{2\beta_i\gamma(1-\theta)D_i}$ is likely to hold for retailers such as those selling fashion clothes and accessories because they typically require less storage space (low β) and have a higher proportion of uninformed customers (high θ).

Proposition 2 also offers an important managerial insight for a multichannel retailer who sells products both online and in physical stores. Typically, the implicit assumption is that the online channel and physical stores are substitutes, in that there should be fewer physical stores as the online channel becomes more attractive. However, we show that these channels can be complementary; that is, the online channel becoming more attractive because of a decrease in online shopping hassle cost should be accompanied by an increase in physical stores. Our result also offers one potential explanation for some retailers' strategies observed in practice. For example, Target is investing \$7 billion in digital initiatives to improve its online shopping experience, while also adding 130 small-format stores by 2019 (Target 2017). Similarly, Kohl's chief executive officer thinks that "having more stores is better," and Kohl's is making hundreds of its stores smaller while also investing heavily to make online shopping more convenient for customers (Wahba 2017).

We next investigate how the online hassle cost h_o and return rate γ influence the retailer's profit. Let π^* denote the retailer's profit under the optimal store decisions (n^*, I_s^*). The expressions for the thresholds in the following result are relegated to the online appendix for brevity.

Proposition 3. *Let $n^* > 0$. A lower online hassle cost h_o leads to lower retailer profit π^* if $h_r < H_r(\theta, \gamma, \boldsymbol{D}, t, \boldsymbol{\alpha}, \boldsymbol{\beta})$. Moreover, if the return handling costs $\boldsymbol{k}$ satisfy $\min_{i\in I} k_i > K(\theta, \gamma, \boldsymbol{D}, h_o, h_r, \boldsymbol{m}, t, \boldsymbol{\alpha}, \boldsymbol{\beta}, \Omega_0)$ and the unit facility cost ξ satisfies $\Xi_1(\theta, \gamma, \boldsymbol{D}, h_o, h_r, \boldsymbol{k}, \boldsymbol{m}, t, \boldsymbol{\alpha}, \boldsymbol{\beta}, \Omega_0) < \xi < \Xi_2(\theta, \gamma, \boldsymbol{D}, h_o, h_r, \boldsymbol{k}, \boldsymbol{m}, t, \boldsymbol{\alpha}, \boldsymbol{\beta}, \Omega_0)$, then a higher product return rate γ leads to higher retailer profit π^*.*

Although one would expect that a lower online hassle cost is beneficial for the retailer, the first part of Proposition 3 shows that it can be detrimental. The reason for this is as follows: When the return hassle cost is sufficiently low (i.e., $h_r < H_r(\cdot)$), a lower online hassle cost makes it more attractive for uninformed customers to purchase online as opposed to through a physical store. This increases the returns associated with online purchases, hurting the retailer's profit.

One would also expect that a higher product return rate is detrimental for the retailer. However, the second part of the above result shows that, under a limited set of conditions, a higher product return rate can lead to a higher profit for the retailer. This occurs when the unit facility cost is sufficiently low and the return handling costs k are sufficiently large, that is, $\xi < \Xi_2(\cdot)$ and $\min_{i\in I} k_i > K(\cdot)$,[10] which provides the retailer with an incentive to carry all of the products in physical stores, that is, $I_s^* = I$ (see discussion following Lemma 1), and all uninformed customers visit a store. Recall that as the product return rate γ increases, the retailer prefers to have fewer stores, resulting in facility cost savings (see Proposition 1). However, as γ increases, the demand from uninformed customers who purchase in-store is also lower, resulting in lower revenues for the retailer. Under this setting, when the unit facility cost is not too small, that is, $\xi > \Xi_1(\cdot)$, the savings due to the lower facility costs arising from having fewer stores outweigh the loss in revenues due to lower demand, resulting in higher profit for the retailer.

4. The Impact of Omnichannel Retail Strategies

We now extend our base model with conventional stores to study the effect of omnichannel strategies on a retailer's decisions regarding the number, product breadth, and size of physical stores. We focus on three strategies prevalent in practice: showroom stores, allowing flexibility in return of online purchases, and allowing flexibility in fulfillment of online purchases. Some of these omnichannel retailing strategies can benefit the retailer by potentially reducing the return handling costs arising due to customers' valuation uncertainty. They do so through different mechanisms: For example, showroom stores allow customers to evaluate whether they like the product or not before purchasing it online; that is, they prevent returns from occurring in the first place. In contrast, flexibility in returning online purchases aims to reduce return costs once the need for returning the product has already occurred.

Furthermore, conventional stores (discussed in the previous section) mainly serve two functions for the retailer, namely, fulfilling in-store demand and allowing customers to evaluate the product. However, under different omnichannel strategies, physical stores play a different role. In particular, showroom stores offer fewer functions, as the in-store demand is fulfilled by the online channel. Under flexible return or fulfillment strategies, physical stores have more functions, namely, they also handle online returns and fulfill online orders, respectively. We next analyze how an omnichannel retailer should change its physical store decisions when the stores are given more or fewer functions due to the adoption of different omnichannel strategies.[11]

4.1. Showroom Stores: See In-Store and Buy Online

We begin by considering a retailer who sells products through an online channel and also operates physical showrooms, that is, physical stores that only carry samples. Note that under such a strategy, order fulfillment is carried out only through the online channel; that is, customers can only inspect the products in the showrooms but have to purchase them through the online channel. For ease of exposition, we will continue to use I to denote the entire set of distinct products offered in the online channel, and denote the number of showrooms by n and the subset of products displayed in them by I_s. We use the superscript s to denote the decisions and thresholds in this setting with showroom stores.

The expected utility of a class i customer, uninformed or informed, from buying product i through the online channel will be the same as that in the base model, that is,

$$u_{i,o}^s = (1-\gamma)(v_i - p_i) - \gamma h_r - h_o \text{ and}$$
$$\tilde{u}_{i,o}^s = v_i - p_i - h_o \text{ for } i \in I, \tag{10}$$

which includes the online hassle cost h_o and the expected return hassle cost γh_r for uninformed customers if they end up not liking the product.

For a product i displayed in the showrooms (i.e., $i \in I_s$), customers can choose to visit them to inspect the product before purchasing it online, or they can directly purchase it online without inspecting it first. If an uninformed customer visits a showroom, with probability $1-\gamma$, she will like the product and make a purchase online by incurring the online hassle cost h_o. With probability γ, the uninformed customer will not like the product and decide not to purchase it. In either case, the uninformed customer incurs the offline hassle cost $h_s(x)$ to visit the showroom but can avoid the return hassle cost. Accordingly, the expected utility of an uninformed customer choosing to go to a store will be different from the base model in Section 3 and will be given by

$$u_{i,s}^s(x) = (1-\gamma)(v_i - p_i - h_o) - h_s(x) \text{ for } i \in I_s. \tag{11}$$

An informed customer, who does not face valuation uncertainty, will always prefer to directly purchase the product online instead of visiting a showroom to inspect it. This is because her utility from visiting a showroom before buying the product online is given by

$$\tilde{u}_{i,s}^s(x) = v_i - p_i - h_o - h_s(x) \text{ for } i \in I_s, \tag{12}$$

which is strictly lower than her utility from buying online, that is, $\tilde{u}_{i,o}^s$ in (10).

Therefore, if product i is displayed in the showroom, that is, $i \in I_s$, customers will decide whether to visit the showroom or not by comparing their expected utilities in (11) and (12) with those in (10). For brevity, we relegate the detailed expressions for the resulting demand of each channel to the proof of Lemma B.1 in the online appendix, Section D. For products not available in the showroom stores ($i \notin I_s$), customers can purchase them only through the online channel. The demand for them is then given by $d^s_{i,o} = \theta D_i$ and $\tilde{d}^s_{i,o} = (1-\theta)D_i$ for $i \notin I_s$.

Given that showrooms only carry samples for products, they require less space, and thus will cost less than conventional stores (see *Economist* 2016). In particular, they do not require space for storing inventory, but still require space for displaying each product and common area. Therefore, we can adapt the specification for the total size of a store given by (8) to represent showroom stores as follows:

$$\Omega^s = \sum_{i\in I_s} \Omega^s_i + \Omega_0 = \sum_{i\in I_s} \alpha_i + \Omega_0,$$

where the term associated with space for storing inventory for products $i \in I_s$ is no longer present (i.e., $\beta_i = 0$).

The retailer's total expected profit from all products, taking into account the facility costs, is then given by

$$\pi^s = \sum_{i\in I} \pi^s_i - n\xi\Omega^s = \sum_{i\in I} \pi^s_i - n\xi\left[\sum_{i\in I_s} \alpha_i + \Omega_0\right],$$

where $\pi^s_i = m_i[(1-\gamma)\theta D_i + (1-\theta)D_i] - k_i\gamma d^s_{i,o}$.

The retailer maximizes its expected profit by choosing the number of showroom stores (n) and the subset of products to display in them (I_s). With a slight abuse of notation, we use the superscript s here to denote the optimal decisions under the setting with showrooms, that is, n^s and I^s_s. For brevity and ease of exposition, the complete analytical characterization of n^s and I^s_s is relegated to Lemma B.1 in the online appendix, Section B.[12] We also show that our results in Propositions 1 and 2 related to the effect of return rate γ and online hassle cost h_o for conventional stores continue to qualitatively hold for showroom stores (see Propositions B.1 and B.2 in the online appendix, Section B).

In what follows, we investigate an omnichannel retailer's decisions for showrooms as compared with conventional stores by focusing on the setting where both $n^* > 0$ and $n^s > 0$. In order to understand these results, we first outline two different effects of showrooms: First, the expected utility from visiting the store is lower for customers. This is because they only carry samples and customers still have to purchase the product online, incurring the online hassle cost. This, along with the observation that the expected utility from directly purchasing the product from the online channel remains the same, implies that visiting stores becomes relatively less attractive when they are showrooms. We refer to this as the *effect of showrooms on demand* for brevity hereafter. Second, given that showrooms do not require space for storing inventory, they cost less than conventional stores. We refer to this as the *effect of showrooms on facility cost* for brevity hereafter.

Proposition 4. *The retailer should have more stores when they are showrooms as compared with conventional stores (i.e., $n^s > n^*$). It should also carry more products in them (i.e., $|I^s_s| \geq |I^*_s|$) if $h_o < H^s_o(\theta, \gamma, \boldsymbol{D}, h_r, \boldsymbol{k}, t, \boldsymbol{\alpha}, \boldsymbol{\beta}, \xi)$. Moreover, showrooms are larger than conventional stores (i.e., $\Omega^s \geq \Omega^*$), if $|I^s_s| > |I^*_s|$, $\alpha_i = \alpha$, $\beta_i = \beta$ for all $i \in I$, and $\alpha > A^s(\theta, \gamma, \boldsymbol{D}, h_o, h_r, \boldsymbol{k}, t, \beta, \xi)$.*

Recall that, compared with conventional stores, showrooms no longer fulfill in-store orders, which are instead fulfilled through the online channel. Interestingly, the above result shows that despite the stores performing fewer functions, the retailer prefers to open more stores (i.e., $n^s > n^*$). The reason for this is as follows: Compared with conventional stores, the effect of showrooms on demand implies a potential demand shift from stores to the online channel, as showrooms are a less convenient way for customers to shop. That is, more uninformed customers will directly purchase the product online without visiting a store, resulting in higher return handling costs faced by the retailer. This provides the retailer with a greater incentive to have more stores (and thus make them closer to customers) in order to induce more people to visit a showroom and evaluate the product before purchasing it online. Moreover, the effect of showrooms on facility cost also makes it cheaper to have stores. Accordingly, both the demand and the facility cost effect of showrooms make it more attractive for the retailer to have more stores.

The above result also shows that the retailer prefers to carry more products in showrooms if the online hassle cost is sufficiently low (i.e., $h_o < H^s_o(\cdot)$). This result is driven by the balance (or lack thereof) between the contrasting influences of the demand and cost effects of showrooms on the product breadth $|I_s|$ carried in the store. The effect of showrooms on facility cost implies that it is cheaper for the retailer to carry more products in the store. However, the effect of showrooms on demand makes it less attractive for the retailer to carry more products in the store. This is because visiting stores becomes relatively less attractive when they are showrooms, as customers still need to purchase the product online, incurring the online hassle cost. Accordingly, when the online hassle cost is sufficiently low (i.e., $h_o < H^s_o(\cdot)$), the influence of the cost effect of showrooms dominates, resulting in more products being carried in showrooms ($|I^s_s| \geq |I^*_s|$).

Finally, we also find that even though showrooms do not require space to store inventory, they can be larger than conventional stores, that is, $\Omega^s \geq \Omega^*$. Proposition 4 shows that this can occur in the special case where every product requires the same display and storage space (i.e., $\alpha_i = \alpha$ and $\beta_i = \beta$ for all $i \in I$), if each product requires a significant amount of space for display (i.e., $\alpha > A^s(\cdot)$) and the retailer prefers to carry more products in a showroom (i.e., $|I_s^s| > |I_s^*|$). Under this setting, the significant amount of display space required due to the larger product breadth in showrooms outweighs the elimination of space required for storing inventory.

4.2. Return Flexibility: Buy Online and Return In-Store

Another omnichannel strategy adopted by many retailers is to provide customers with flexibility in returning their online purchases. Under such return flexibility, if a customer does not like the product they purchased online, she can choose to return it to a physical store. In order to consider this, we extend our base model with conventional stores and use the superscript r to denote the decisions and thresholds in this setting under return flexibility.

When an uninformed customer purchases a product online, she will dislike it with probability γ, and the customer will now have two options to return the product back to the retailer: (i) ship it back directly to the retailer through the online return process, incurring a return hassle cost h_r as before, or (ii) return it to a nearby physical store by incurring an offline return hassle cost, $h_r^r(x) = t^r x$, which can be different from her offline hassle cost to purchase a product in-store (i.e., $h_s(x)$). The customer will choose between these two return options based on which one minimizes her cost of return, which is given by $\min\{h_r, h_r^r(x)\}$. Therefore, the expected utility of an uninformed class i customer who purchases the product through the online channel is

$$u_{i,o}^r = (1-\gamma)(v_i - p_i) - \gamma \min\{h_r, h_r^r(x)\} - h_o \text{ for } i \in I. \tag{13}$$

If an uninformed customer visits a physical store to purchase product $i \in I_s$, her expected utility is the same as in the base model, that is,

$$u_{i,s}^r(x) = (1-\gamma)(v_i - p_i) - h_s(x) \text{ for } i \in I_s.$$

An informed customer does not face valuation uncertainty, and thus does not face any need to return the product. Therefore, the expected utility of a class i informed customer, from purchasing the product through the online channel or by visiting a physical store, will be the same as that in the base model, that is,

$$\tilde{u}_{i,o}^r(x) = v_i - p_i - h_o \text{ for } i \in I, \text{ and}$$
$$\tilde{u}_{i,s}^r(x) = v_i - p_i - h_s(x) \text{ for } i \in I_s.$$

For products available in the stores, $i \in I_s$, customers will decide whether to visit the store or purchase the product online by comparing their expected utilities from each option. For brevity, we relegate the detailed expressions for the resulting demand for each channel to the proof of Lemma B.2 in the online appendix, Section D. For products not available in the stores, customers can purchase them only through the online channel, and thus $d_{i,o}^r = \theta D_i$ and $\tilde{d}_{i,o}^r = (1-\theta)D_i$ for $i \notin I_s$. An uninformed customer who purchases the product online will dislike it with probability γ and need to return it. Then, according to (13), such a customer will return the product by visiting a physical store only if her distance from the nearest store is sufficiently small, that is, $x \le h_r/t^r$. Let $d_{i,o,s}^r$ and $d_{i,o,o}^r$ denote the demand from such customers who prefer to return product i in-store and online, respectively, and their detailed expressions are relegated to the proof of Lemma B.2 in the online appendix, Section D, for brevity.

When online purchases are returned to a physical store, they are rarely put back on the floor; instead, they are sent back to the distribution center to be either repackaged for sales or get sold to a liquidator (Northrup 2014). Because of first-reverse-mile effects, it is typically cheaper for a retailer to collect returns from physical stores instead of them being directly returned by customers; that is, products returned to physical stores are typically sent back together to distribution centers using backhauling, whereas direct returns by customers are more expensive given their smaller size (Inbound Logistics 2017). To capture the lower cost of handling returns at physical stores, we denote by k_i^r the retailer's per-unit handling cost for product $i \in I$ returned to a physical store, which is assumed to be lower than the handling cost for a product returned online (i.e., k_i). Given that stores only temporarily keep the returned online purchases and send them back to the distribution center, the specification for the size of a store continues to be the same as in our base model.

The retailer's total expected profit from all products, taking into account the facility costs, is then given by

$$\pi^r = \sum_{i \in I} \pi_i^r - n\xi\Omega^r$$
$$= \sum_{i \in I} \pi_i^r - n\xi\left[\sum_{i \in I_s}\left(\alpha_i + \beta_i \frac{(1-\gamma)d_{i,s}^r + \tilde{d}_{i,s}^r}{n}\right) + \Omega_0\right].$$

where $\pi_i^r = m_i[(1-\gamma)\theta D_i + (1-\theta)D_i] - k_i\gamma d_{i,o,o}^r - k_i^r\gamma d_{i,o,s}^r$.

The retailer maximizes its expected profit by choosing the number of stores (n) and the subset of products to carry in them (I_s). With a slight abuse of notation, we use the superscript r here to denote the optimal decisions under return flexibility, that is, n^r and $|I_s^r|$. For brevity and ease of exposition, the complete analytical characterization of n^r and $|I_s^r|$ is relegated to

Lemma B.2 in the online appendix, Section B. We also show that our results in Propositions 1 and 2 related to the effect of return rate γ and online hassle cost h_o for conventional stores continue to qualitatively hold under return flexibility (see Propositions B.1 and B.2 in the online appendix, Section B).

In what follows, we investigate an omnichannel retailer's decision under return flexibility as compared with conventional stores by focusing on the setting where both $n^* > 0$ and $n^r > 0$. In order to understand these results, we first outline two different effects of return flexibility. First, for uninformed customers, expected utility from purchasing a product online is higher because of return flexibility. This, along with the observation that the expected utility from visiting a physical store to purchase a product remains the same, implies that purchasing online becomes relatively more attractive under return flexibility. We refer to this as the *effect of return flexibility on demand* for brevity hereafter. Second, given that it is typically cheaper to collect returns from stores instead of being directly returned by customers, return flexibility lowers return handling costs for the retailer. We refer to this as the *effect of return flexibility on return handling cost* for brevity hereafter.

Proposition 5. *The retailer should have fewer stores under return flexibility (i.e., $n^r < n^*$) if and only if $h_o < H_o^r(\theta, \gamma, \boldsymbol{D}, h_r, \boldsymbol{k}, \boldsymbol{k}^r, t, t^r, \boldsymbol{\alpha}, \boldsymbol{\beta}, \xi)$, which holds only if $\gamma t^r < t$. Moreover, under return flexibility, the stores should carry fewer products (i.e., $|I_s^r| \le |I_s^*|$) and be smaller in size (i.e., $\Omega^r \le \Omega^*$).*

Recall that under return flexibility, stores perform more functions as they also handle returns of online purchases. However, the above result shows that the retailer prefers to have fewer stores under return flexibility if the online hassle cost is sufficiently low (i.e., $h_o < H_o^r(\cdot)$). Note that this occurs only if $\gamma t^r < t$; otherwise, uninformed customers will never use the in-store return option for products carried in stores.[13] This result is driven by the contrasting influence of the effect of return flexibility on the demand versus on the return handling costs. Recall that the effect of return flexibility on return handling cost makes returns associated with online purchases less costly for the retailer, which makes having more stores to reduce online returns less attractive. However, the effect of return flexibility on demand implies that purchasing online becomes relatively more attractive. This could lead to more product returns from uninformed customers, resulting in higher return handling costs for the retailer. It can attempt to moderate such costs by having more stores; that is, the effect of return flexibility on demand provides the retailer with an incentive to have more stores. When the online hassle cost is sufficiently low (i.e., $h_o < H_o^r(\cdot)$), customers have a strong preference to purchase online, making it difficult for the retailer to induce more uninformed customers to purchase in stores by building more stores; that is, under this setting, the effect of return flexibility on return handling cost dominates, which implies that making stores accessible for uninformed consumers to check the product before purchase is less of a priority for the retailer, resulting in fewer stores (i.e., $n^r < n^*$).[14] If the online hassle cost is higher (i.e., $h_o \ge H_o^r(\cdot)$), then the effect of return flexibility on the demand dominates, leading to more stores (i.e., $n^r \ge n^*$).

The above result also shows that under return flexibility, the retailer prefers to carry fewer products in stores, which are smaller in size. Recall that the effect of return flexibility on demand implies purchasing online is relatively more attractive for uninformed customers. This provides the retailer with a lower incentive to carry a product in the store. In a similar vein, return flexibility makes it cheaper for the retailer to handle returns associated with online purchases made by uninformed customers. Accordingly, the effect of return flexibility on demand and the return handling cost both incentivize the retailer to carry fewer products in-store (i.e., $|I_s^r| \le |I_s^*|$). Consequently, less space is required in stores, resulting in smaller stores (i.e., $\Omega^r \le \Omega^*$).

4.3. Fulfillment Flexibility: Buy Online and Pick Up In-Store

Another omnichannel strategy adopted by many retailers is to provide customers with flexibility in how their online purchases are fulfilled. In particular, for products available in physical stores, customers have the option to pick up an online order from a physical store.[15] Typically, the product will be ready for pickup within one to two hours after the order is placed (Chen and Rosmarin 2019). In order to consider a setting with such fulfillment flexibility, we extend our base model with conventional stores, and use the subscript f to denote the decisions and thresholds in this setting.

If a customer chooses to purchase a product online and have it shipped directly to home, her expected utility remains the same as in the conventional case, that is,

$$u_{i,o}^f = (1-\gamma)(v_i - p_i) - \gamma h_r - h_o$$
$$\text{for and } \tilde{u}_{i,o}^f = v_i - p_i - h_o \text{ for } i \in I.$$

For a product that is carried in physical stores (i.e., $i \in I_s$), the customer can choose to buy the product by visiting a physical store or buy the product online but visit a physical store to receive the product right away. Following Gao and Su (2017a), we assume that a customer incurs a portion of both the online and offline hassle costs when she buys a product online and picks it up in-store. In particular, we denote the pickup hassle cost as $h_s^f(x) = \delta_s h_s(x) + \delta_o h_o$, where $\delta_s, \delta_o \in (0,1)$ are scale factors, and x denotes the distance from the customer's location to her nearest store. This specification of the pickup hassle cost captures that the customer still needs to go to the store but can pick up their order at their convenience and no longer needs to search for the product in the store; at the same time, she incurs the hassle cost for placing the order online but can avoid the shipping and waiting time for the product to arrive. Finally, when a customer shows up at a store to pick up her online order, if she does not like the product after physically evaluating it, she need not complete the transaction.[16] Let $u_{i,s}^f$ ($\tilde{u}_{i,s}^f$) denote the expected utility of an uninformed (informed) customer from purchasing a product $i \in I_s$ by visiting a store. These utilities are similar to those in the base model except that the offline hassle cost, $h_s(x)$, is replaced by $\min\{h_s(x), h_s^f(x)\}$ to reflect the additional option of buying online but picking up the product in the store, that is,

$$\begin{aligned} u_{i,s}^f(x) &= (1-\gamma)(v_i - p_i) - \min\{h_s(x),\ h_s^f(x)\} \text{for } i \in I_s, \\ \tilde{u}_{i,s}^f(x) &= v_i - p_i - \min\{h_s(x),\ h_s^f(x)\} \text{ for } i \in I_s. \end{aligned} \tag{14}$$

With a slight abuse of notation, let $d_{i,s}^f$ and $d_{i,o}^f$ ($\tilde{d}_{i,s}^f$ and $\tilde{d}_{i,o}^f$) denote the demand from uninformed (informed) customers who obtain product i from the store and online, respectively. In particular, $d_{i,s}^f$ and $\tilde{d}_{i,s}^f$ include both the demand from customers who visit a store directly to purchase the product and the demand from customers who purchase the product online but pick it up in the store. For a product i that is carried in store, customers will decide by comparing their expected utilities from the two options. The detailed expressions of the resulting demands, $d_{i,s}^f, d_{i,o}^f, \tilde{d}_{i,s}^f$, and $\tilde{d}_{i,o}^f$ for $i \in I_s$, are relegated to the proof of Lemma B.3 in the online appendix, Section D, for brevity. For a product i not carried in-store, customers can purchase it only online and wait for it to be delivered, and thus $d_{i,o}^f = \theta D_i$, $\tilde{d}_{i,o}^f = (1-\theta)D_i$, and $d_{i,s}^f = \tilde{d}_{i,s}^f = 0$ for $i \notin I_s$.

Among the class i uninformed customers who choose to obtain the product in physical stores (i.e., $d_{i,s}^f$), on average, a total of $(1-\gamma)d_{i,s}^f$ will like the product and make a purchase. In contrast, for those that buy online and choose to have the product directly shipped to them (i.e., $d_{i,o}^f$), on average, a total of $(1-\gamma)d_{i,o}^f$ will like and keep the product, whereas $\gamma d_{i,o}^f$ will dislike and return it to the retailer. Accordingly, the total sales of product i to uninformed customers across the two channels is given by $(1-\gamma)d_{i,s}^f + (1-\gamma)d_{i,o}^f = (1-\gamma)\theta D_i$, and the total returns of product i is given by $\gamma d_{i,o}^f$. The total sales of product i to informed customers across the two channels is given by $\tilde{d}_{i,s}^f + \tilde{d}_{i,o}^f = (1-\theta)D_i$ with no product returns. The expected profit earned by the retailer from product i, from both uninformed and informed customers, is then given by

$$\pi_i^f = m_i[(1-\gamma)\theta D_i + (1-\theta)D_i] - k_i\gamma d_{i,o}^f.$$

The retailer's total expected profit from all products, taking into account the total facility costs, is then given by

$$\begin{aligned} \pi^f &= \sum_{i\in I}\pi_i^f - n\xi\Omega^f \\ &= \sum_{i\in I}\pi_i^f - n\xi\left[\sum_{i\in I_s}\left(\alpha_i + \beta_i\frac{(1-\gamma)d_{i,s}^f + \tilde{d}_{i,s}^f}{n}\right) + \Omega_0\right]. \end{aligned}$$

The retailer maximizes its expected profit by choosing the number of stores (n) and the subset of products to display in showrooms (I_s). With a slight abuse of notation, we use the superscript f here to denote the optimal store decisions under fulfillment flexility, that is, n^f and $|I_s^f|$. For brevity and ease of exposition, the complete analytical characterization of n^f and $|I_s^f|$ is relegated to Lemma B.3 in the online appendix, Section B. We also show that our results in Propositions 1 and 2 related to the effect of return rate γ and online hassle cost h_o for conventional stores continue to qualitatively hold under fulfillment flexibility (see Propositions B.1 and B.2 in the online appendix, Section B).

In what follows, we focus on the case when both $n^* > 0$ and $n^f > 0$ to investigate the impact of fulfillment flexibility on an omnichannel retailer's physical store decisions.

Proposition 6. *The retailer should have fewer stores under fulfillment flexibility (i.e., $n^f \le n^*$), which carry more products (i.e., $|I_s^f| \ge |I_s^*|$) and are larger in size (i.e., $\Omega^f \ge \Omega^*$).*

Recall that under fulfillment flexibility, stores can also fulfill online purchases. Interestingly, the above result shows that despite them performing more functions, the retailer prefers to have fewer stores. The reason for this is as follows: The effect of fulfillment flexibility on demand makes it less attractive for the retailer to have more stores. This is because the expected utility from visiting a store is higher for both informed and uninformed customers given that the additional option of buying the product online and picking it up in the store lowers their hassle cost. This makes visiting a store relatively more attractive for customers, which implies that the retailer does not need as many stores. Moreover, uninformed customers who purchase online but pick up in store, do not complete their transaction if they do not like the product. That is, fulfillment flexibility also reduces returns associated with online purchases. Accordingly, given that fulfillment flexibility makes visiting stores relatively more attractive, and reduces returns, the retailer prefers to have fewer stores to lower the associated facility costs (i.e., $n^f \leq n^*$).

The above result also shows that the retailer carries more products in stores under fulfillment flexibility ($|I_s^f| \geq |I_s^*|$). There are several reasons for this: First, as mentioned above, fulfillment flexibility makes visiting a store relatively more attractive for customers, providing the retailer with a greater incentive to carry a product in the store. Second, fulfillment flexibility reduces returns associated with online purchases of products that are also available in stores; that is, fulfillment flexibility makes it more attractive to carry a product in the stores. Therefore, fulfillment flexibility incentivizes the retailer to carry more products in stores ($|I_s^f| \geq |I_s^*|$), requiring more space and resulting in larger stores ($\Omega^f \geq \Omega^*$).

5. Extensions

We next examine several extensions that relax some of the assumptions used in our main model and capture additional considerations.

5.1. Different Margins Across Channels

Our main model assumes that margins are the same in the online channel and physical stores. We now generalize our model and analysis to consider a setting with different margins. Let $m_{i,o}$ and $m_{i,s}$ denote the margin for product $i \in I$ in the online channel and physical stores, respectively. For brevity, the details of this extension are relegated to the online appendix, Sections C.1 and C.4.2. In what follows, we summarize the results and insights from this extension: First, we can analytically show that all our structural results and key insights continue to hold if $m_{i,s} - \xi\beta_i \leq m_{i,o}$ for all $i \in I$; that is, the margin in the online channel is larger than that in physical stores (see the online appendix, Section C.1). This condition typically holds in practice given that selling a product through physical stores is commonly discussed to be less profitable than selling through the online channel (see, e.g., Helmore 2017, Kapner and Safdar 2018). Second, when this condition does not hold, solving for the equilibrium decisions is analytically intractable. Nevertheless, we can numerically verify that our key results continue to hold under this setting as well (see the online appendix, Section C.4.2).

We can also analyze which types of products the retailer should carry in stores when the margins are different across channels. In order to do so, let $\mu_i = m_{i,s} - m_{i,o}$ denote the margin for a product $i \in I$ in-store relative to the online channel. For analytical tractability, we focus on a special case where $\alpha_i = \alpha$, $\beta_i = \beta$, $k_i = k$, $k_i^r = k^r$, and $D_i = D$ for all $i \in I$. We can analytically show that it is more attractive for the retailer to carry a product with a larger μ_i in stores[17] (see Propositions C.1.4, C.1.7, and C.1.9 in the online appendix, Section C.1). That is, the retailer prefers to carry a product in-store that has a relatively higher margin in-store than online.

5.2. Impact of Physical Stores on Demand

In our main model, we assume that all customers choose between purchasing the product through a store or the online channel. We now discuss two different extensions that modify this assumption.

First, we consider a setting where, despite the presence of physical stores, some customers purchase a product only through the online channel. In particular, we modify our model to assume that a fraction ψ_i of class i customers ($i \in I$) do not visit a store and only use the online channel, whereas the remaining fraction consider both channels as in our main model. It can be shown that the analysis for this extension is equivalent to our main model with the demand D_i replaced with $(1 - \psi_i)D_i$ for all $i \in I$ (see the online appendix, Section C.2, for details). Accordingly, our structural results and key insights continue to hold when some customers purchase only through the online channel.

Second, we consider a setting where some customers may consider purchasing only through the physical stores, that is, physical stores may be able to attract additional customers that would otherwise not purchase a product (see Bell et al. 2018). We can modify our model to assume that the demand faced by physical stores increases by a fraction of λ for each store to capture this setting. In particular, we assume that the demand for product i in each store is given by $(1 + \lambda)\frac{(1-\gamma)d_{i,s}+\tilde{d}_{i,s}}{n}$, where $\frac{(1-\gamma)d_{i,s}+\tilde{d}_{i,s}}{n}$ is the demand from our main model.[18] That is, when $\lambda = 0$, this extension is the same as our main model. In the online appendix, Section C.3, we can analytically show that this extension is equivalent to the extension in Section 5.1 with $m_{i,o}$, $m_{i,s}$, and β_i being replaced by m_i, $(1 + \lambda)m_i$, and $(1 + \lambda)\beta_i$, respectively; that is, it can be analytically shown the structural results and key insights continue to hold if $(1+\lambda)m_i - \xi(1+\lambda)\beta_i \leq m_i$ for all $i \in I$ (see the online appendix, Section C.1). If this condition does not hold, solving for the equilibrium decisions is analytically intractable. Nevertheless, we can numerically verify that our key results continue to hold under this setting as well (see the online appendix, Section C.4.3).

5.3. Different Return Rates for Products

We make an implicit assumption in our main model that return rates are the same for all products. In practice, different products may be returned at different rates. In order to consider such a setting, we modify our main model to assume that the return rate is different for different products and is denoted by γ_i

for product i. For brevity, we relegate the details of this extension to the online appendix, Section C.4. Given that solving for the retailer's decisions is analytically intractable, we resort to numerical analysis to investigate the effect of different return rates for products. We can numerically verify that our key results continue to hold under this setting as well (see the online appendix, Section C.4).

We can also analyze which types of products a firm should carry in stores if they differ in their return rates. We do so by considering a special case where $\alpha_i = \alpha, \beta_i = \beta, k_i = k, k_i^r = k^r, D_i = D$, and $m_i = m$ for $i \in I$. We can analytically show that the retailer prefers to carry products with a higher return rate in the stores (see Proposition C.4.1 in the online appendix, Section C.4). This is because carrying such products in the store reduces their returns by inducing more uninformed customers to visit a store and resolve their valuation uncertainty.

6. Concluding Remarks

Omnichannel retailing has increasingly become the norm, providing customers with an integrated shopping experience across both offline and online channels. However, it is not clear a priori how this affects the decisions by a multichannel or omnichannel retailer, who has both online and store channels, regarding the number, product breadth, and size of physical stores. This is the main research question we address in this paper. We find that as the online channel becomes more convenient or for a lower product return rate, the retailer may prefer to have more and smaller physical stores. We examine three prevalent omnichannel strategies, namely, showroom stores, return flexibility, and fulfillment flexibility. Compared with conventional stores, showrooms offer fewer functions, whereas stores that enable return or fulfillment flexibility offer more functions. However, we show that the retailer may find it optimal to reduce the physical store presence under the showroom strategy and increase it under return or fulfillment flexibility. This result demonstrates that as stores are given fewer (more) responsibilities due to a retailer's adoption of omnichannel strategies, the retailer should have more (fewer) stores. This highlights the importance of carefully modifying their physical presence when retailers adopt omnichannel strategies.

Our findings can also help explain some real-word observations. For example, our results explain how the return rate of products carried by a retailer influences the decisions regarding physical stores. This can explain why some retailers operate no physical stores, whereas others prefer to have a few big stores or many small ones. There has also been a heated argument in the media in recent years whether the rise of interest in online shopping would be the death knell for brick-and-mortar stores. Our findings suggest that this may be far from the case. As online purchases become more convenient for customers, it can increase the return costs incurred by retailers. This incentivizes retailers to maintain or increase the number of physical stores to offer customers an opportunity to evaluate the products before purchasing them. Therefore, as technology advances make online shopping more and more convenient, not only will physical stores not die off, they will have an increasingly important role with the growth of omnichannel retailing.

Finally, there has been a recent stream of empirical research that studies the effect of omnichannel strategies on metrics such as inventory and demand (for some examples, see Gallino and Moreno 2014, Gallino et al. 2016, Bell et al. 2018). Our results can be utilized to formulate hypotheses for empirical research addressing how omnichannel strategies affect physical stores. In particular, our results offer the following hypotheses: (i) As online shopping becomes more convenient, retailers should prefer to have more stores, which carry fewer products and are smaller in size when returning online purchases is not too difficult. (ii) A shift to showrooms is associated with more stores, which carry more products in them. (iii) Allowing return flexibility is associated with stores that are smaller and carry fewer products. Moreover, if online shopping is convenient, allowing return flexibility is associated with fewer stores. (iv) Allowing fulfillment flexibility is associated with fewer stores, which carry more products and are larger in size.

We conclude with discussion of the limitations of our paper and directions for future research. First, we focused on a retailer's strategic decisions for physical stores, that is, their number and size. We also used a long-run perspective, where we did not consider any fluctuations in the product costs, prices, or inventory, over time. Therefore, a promising direction for future research is to consider operational and tactical decisions such as pricing and/or inventory (see Hu et al. 2018), especially in a dynamic context. Second, our research highlights how a retailer should make physical store decisions to moderate the handling cost of returns from online purchases. A fruitful direction for future research is to study other decisions, such as pricing or inventory, that a retailer can use to achieve the same goal. We also focused on different omnichannel strategies separately to clearly identify their impact on the physical store decisions. Accordingly, an interesting direction for future research is to consider the effect of joint implementation of different omnichannel strategies, which will be more complex to analyze. However, we expect that the effect of their joint presence will be a combination of their separate effects identified by our results.

In addition, in order to focus on our main research question and for analytical tractability, we assumed that each customer is interested in only one product. Therefore, our results and key insights are applicable only for products where the consideration set is one or few products. For settings, where a customer's consideration set includes many products, a customer may search for a suitable product from an assortment, and may switch its preference among available products in a store. Moreover, a set of products may be frequently ordered together, incentivizing the retailer to carry and package them together. Accordingly, studying assortment decisions for products carried in stores is a promising direction for future research, especially focusing on settings where customers consider among different products, and there are substitution or complementarities between different products. However, given the additional complexity, investigating this will require simplifying other aspects of our model or resorting to a simulation or numerical approach. We also assume that customers have perfect information about whether a product is available in physical store or not, which is a common assumption in the literature (see Gu and Tayi 2017, Mehra et al. 2017). When a certain fraction of customers are naive and mistakenly assume that a product is available in store, we expect that our key results and insights will continue to hold as the retailer can help them at the store to purchase the product through the online channel. Nevertheless, an interesting direction for future research is to study the setting where customers may have imperfect information regarding store assortments. Given that our main research question is to study a multichannel or omnichannel retailer's store decisions, throughout this paper, we focused on the case where the retailer finds it optimal to have physical stores. A promising direction for future research is to consider the perspective of an e-tailer who is considering adding physical stores.

For parsimony and analytical tractability, we used a one-dimensional circular location model to capture the distribution of customers. A promising direction for future research is to extend our analysis to consider a two-dimensional location model such as in Cachon (2014), Belavina et al. (2016), and Belavina (2021); also see Glaeser et al. (2019) for an empirical application. This will significantly complicate the analysis and require additional simplifying assumptions. However, we conjecture that our basic insights and results will continue to hold given the fundamental trade-offs considered in our paper. Finally, another fruitful direction for future research is to examine other new roles of stores in an omnichannel environment. For example, stores can take on more functionalities and serve as online fulfillment locations through programs such as ship-to-store (Gallino et al. 2016) or ship-from-store (Karimi et al. 2018). Although we did not study such strategies in this paper, we can offer some initial insights for how they may influence a retailer's store decisions. For example, ship-from-store fulfillment strategy can potentially reduce a retailer's shipping cost, providing a retailer with an incentive to have more stores. However, more space will be required to store these products in stores, increasing the facility cost, and providing the retailer with an incentive to have fewer stores. Therefore, the overall effect of a ship-from-store strategy on a retailer's physical store decisions is not straightforward. We conjecture that if the facility cost is sufficiently high, a ship-from-store strategy will lead to fewer stores; that is, as in our main results, the expansion of physical stores' functions through ship-from-store may lead the retailer to reduce the number of physical stores. Otherwise, if the facility cost is low, we conjecture that a ship-from-store strategy will lead to more stores. Accordingly, the expansion of physical stores' functions through ship-from-store option may instead lead the retailer to increase the number and/or size of physical stores. In sum, we hope that this paper spurs more research into the effect of other omnichannel strategies on physical stores.

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Endnotes

[1] Note that we assume each customer is interested in only one product for analytical tractability. In practice, a customer's consideration set may include more than one product. Nevertheless, our model still captures, albeit in a simple manner, one of the crucial roles of physical stores in that they allow customers to inspect a product before purchasing it. Please see the discussion in Section 6 regarding the implications of this assumption.

[2] Nevertheless, we consider an extension where products have different effective return rates in Section 5.3 and show that our key insights continue to hold.

[3] Note that a customer's decision of whether to purchase the product online or by visiting a physical store does not depend on the price of the product p_i because it is equal across the two channels.

[4] In practice, some customers may only shop online or in physical stores. We consider such settings in Section 5.2 and find that all our structural results and qualitative insights continue to hold.

[5] Note that uninformed customers are willing to travel more to a physical store (i.e., $\frac{\gamma h_r + h_o}{t} > \frac{h_o}{t}$) because doing so allows them to evaluate the product before purchasing it, thus avoiding the expected return hassle cost γh_r.

[6] Note that we make an implicit assumption that margins are equal across online and store channels. See Section 5.1 for an extension

where we generalize our model to consider different margins across the channels.

[7] Given our assumption that demand is deterministic, the retailer can stock its physical stores exactly equal to the demand faced by them.

[8] Given our assumption that the product prices p_i and margins m_i are equal across the two channels, it can be noted that the retailer's decisions will not depend on p_i or m_i.

[9] We numerically find that our key insights continue to hold when the number of stores is restricted to be an integer (details available on request).

[10] Note that if return handling costs are all low, that is, $k_i \to 0$ for $i \in I$, it is optimal for the retailer to not have any physical stores, that is, $n^* = 0$. Under this setting, a higher product return rate γ leads to lower retailer profit (see the online appendix, Section A, for the proof of this result). Similarly, if we consider a special case with $\alpha_i = \alpha$, $\beta_i = \beta$, $k_i = k$, $D_i = D$, and $m_i = m$ for all $i \in I$, a higher product return rate leads to lower retail profit if and only if $\xi \geq \tilde{\Xi}_2(\theta, \gamma, D, h_o, h_r, k, m, t, \alpha, \beta, \Omega_0)$ or $\xi < \tilde{\Xi}_1(\theta, \gamma, D, h_o, h_r, k, m, t, \alpha, \beta, \Omega_0)$ (see the online appendix, Section A, for the proof of this result).

[11] We provide a discussion regarding the results when the retailer has only an online channel before or after adopting an omnichannel strategy. If the retailer has only an online channel in the base model but prefers to open physical stores after adopting an omnichannel strategy, then it is straightforward that the number and size of stores is then larger under the omnichannel strategy. If instead the retailer closes physical stores after adopting an omnichannel strategy, then it is straightforward that the number and size of stores is smaller because of the omnichannel strategy.

[12] Showrooms have become increasingly popular among traditionally pure online retailers (e-tailers), for example, Bonobos and Warby Parker. We show in Lemma B.1 that it is optimal for an e-tailer to open showrooms (i.e., $n^s > 0$) only if the return rate γ is sufficiently high (see the online appendix, Section B).

[13] For $i \in I_s$, uninformed class i customer's expected utility of purchasing the product online and returning it in a store (if necessary) is $(1-\gamma)(v_i - p_i) - \gamma t^r x - h_o$, which is smaller than the expected utility of visiting a store to purchase the product (i.e., $(1-\gamma)(v_i - p_i) - tx$) for any $x \geq 0$ if $\gamma t^r \geq t$.

[14] When $n^r < n^*$, with return flexibility, we find that the store channel does not cover the entire market of class i uninformed customers in equilibrium (if $i \in I_s^r$). Please see proof of Proposition 5 for more details.

[15] In practice, a retailer may also provide the in-store pickup option for a product available only online, in which case the product will be shipped to the store from the retailer's online distribution center. This strategy is thus called "ship-to-store" (STS); Gallino et al. 2016), for which customers typically have to wait for a week or even longer before the pickup is ready. Given that one of the main reasons for customers to pick up online orders in-store is to avoid the wait for shipping (Forrester Consulting 2014), we will focus on the case when the store pickup option is available only to products that are carried in-store and do not consider STS in this paper. This is consistent with other papers in the literature (see, e.g., Gallino and Moreno 2014, Gao and Su 2017a). However, it is a promising direction for future research to study the impact of STS on a retailer's store decisions.

[16] See Gap for an example of the practice: https://www.digitalcommerce360.com/2013/06/19/gap-launches-store-pickup-items-reserved-online/ (accessed February 24, 2021). Note that a canceled pick up is not counted as a return given that the product was already at the physical store (Shopify 2017).

[17] Note that this result is not valid when stores are showrooms. This is because showrooms do not carry any inventory and all purchases are fulfilled through the online channel. As a result, the relative margin between the channels is not relevant for the retailer's decisions.

[18] This extension implicitly assumes that the demand in each store depends on store's attractiveness (as a result of the retailer's choice of number and size of stores). However, in practice, some customers will choose to purchase only through a physical store no matter how unattractive it is. Given that such customers are not affected by the retailer's store decisions, we conjecture that our key insights will continue to hold as long as such customers are not a significant portion of the population.

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