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A TABLE OF CONNECTED GRAPHS ON SIX VERTICES

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This paper contains a table of 112 connected graphs on six vertices. The graphs are ordered lexicographically by their spectral moments in non-increasing order. The pictures of graphs are given to show as much symmetry as possible. Several data such as the spectrum, and its main part, coefficients of the characteristic and of the matching polynomial, numbers of circuits, etc., are given for each graph in the table. Several observations implied by this table are noted.

The purpose of this paper is to present a table of connected graphs on six vertices together with several useful data about them.

Tables of 6-vertex graphs already appeared in the literature (see, for example, [9]). In fact, the table from [9] was the basis for producing this table. New topics in this table are the way of graph presentation, the ordering of graphs and some data on these graphs.

We tried to produce 'nice' pictures of graphs. Automorphisms of graphs are identified with geometric symmetries of the picture as much as possible. Planar representations are preferred unless they contradict the first principle. The 112 connected graphs on six vertices are given in Table 1 and each graph is provided by an identification number. In planar graphs with a non-planar representation the identification numbers are provided by an asterisk.

Let $\lambda_1, \ldots, \lambda_n$ be the eigenvalues of a graph G. Then the quantity

$$S_k = \sum_{i=1}^{n} \lambda_i^k \qquad (k = 0, 1, \ldots)$$

is called the kth spectral moment of G. We have $S_0 = n$, $S_1 = l$, $S_2 = 2m$, $S_3 = 6t$, where n, l, m, t denote the number of vertices, the number of loops, the number of edges and the number of triangles of G, respectively.

Of course, two graphs are cospectral if and only if they have the same spectral moments.

The graphs are ordered lexicographically by their spectral moments in non-increasing order. Each group of graphs with a constant number of edges is separated in the tables from the neighbouring groups.

Table 1. Connected graphs on six vertices

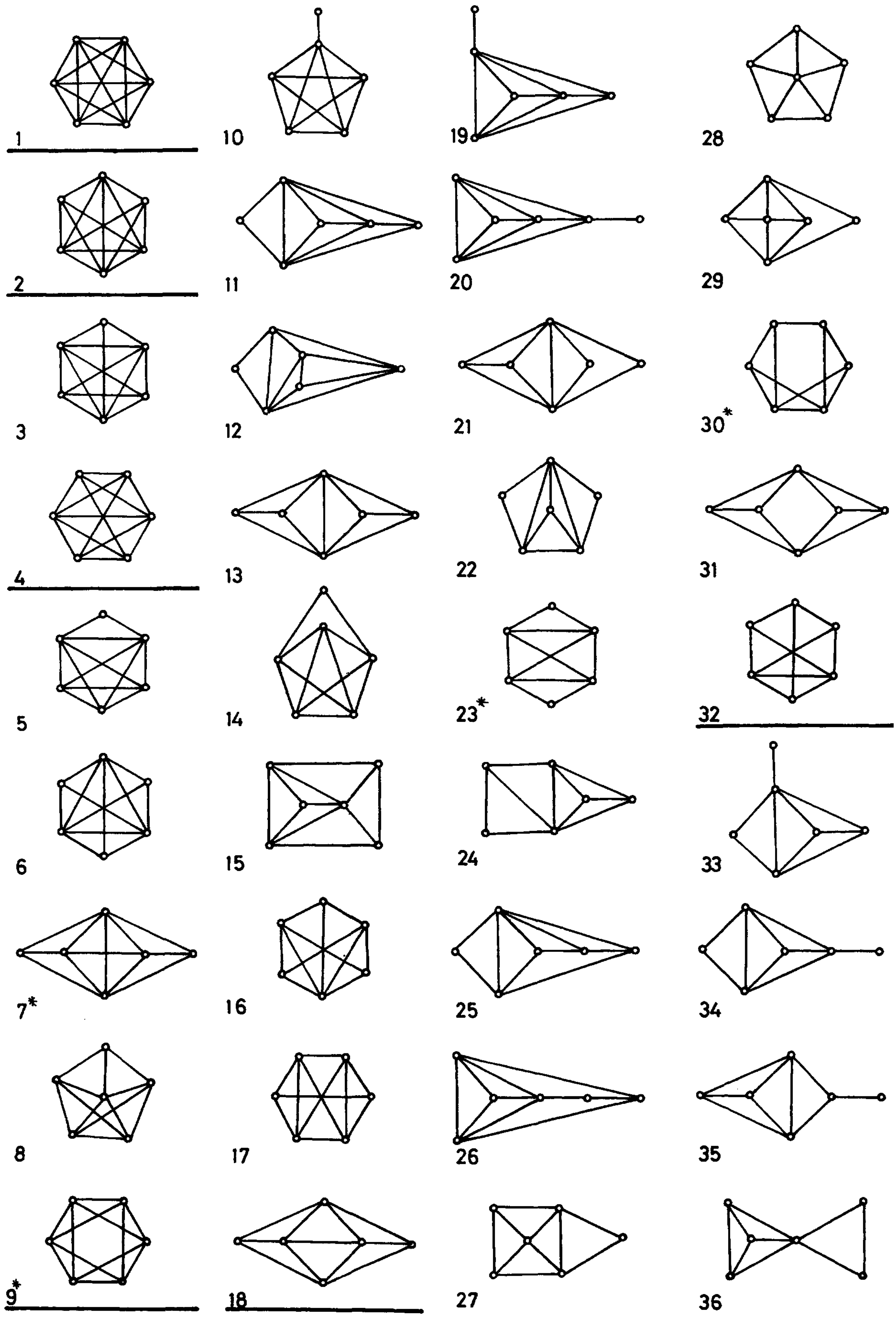
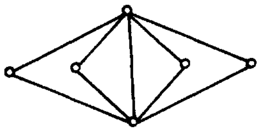


Table 1. (*contd.*)



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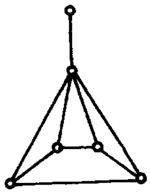

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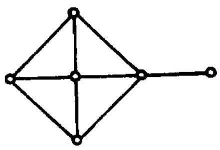

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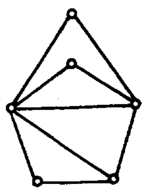

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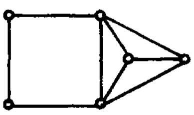

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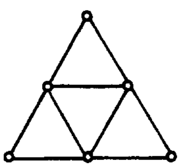

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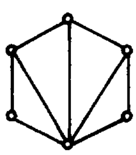

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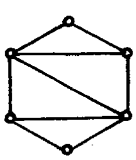

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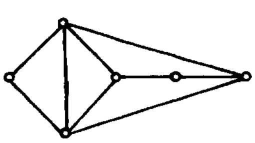

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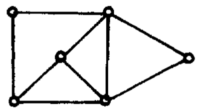

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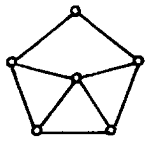

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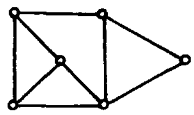

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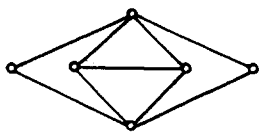

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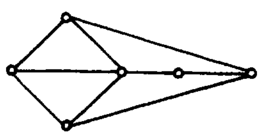

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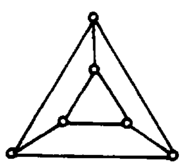

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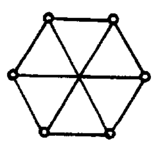

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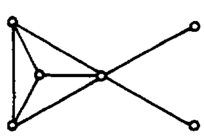

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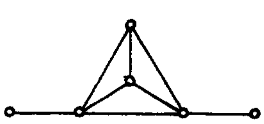

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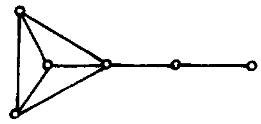

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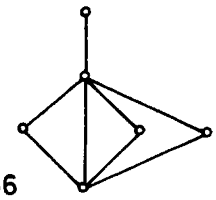

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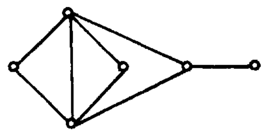

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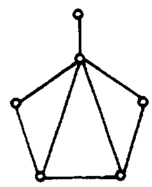

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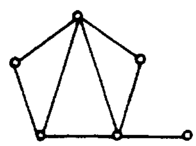

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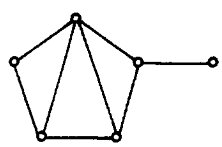

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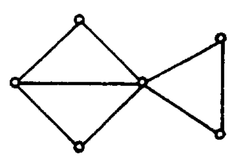

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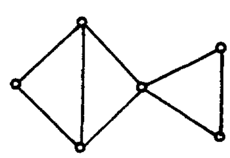

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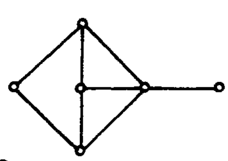

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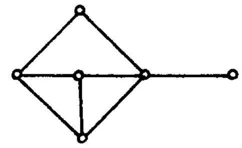

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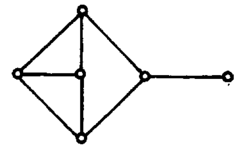

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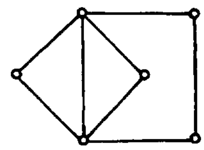

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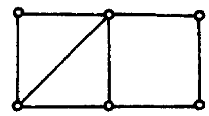

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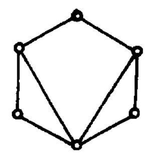

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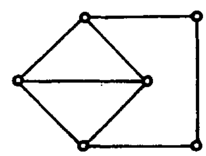

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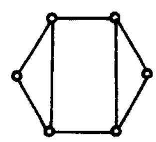

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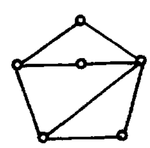

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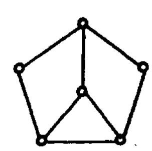

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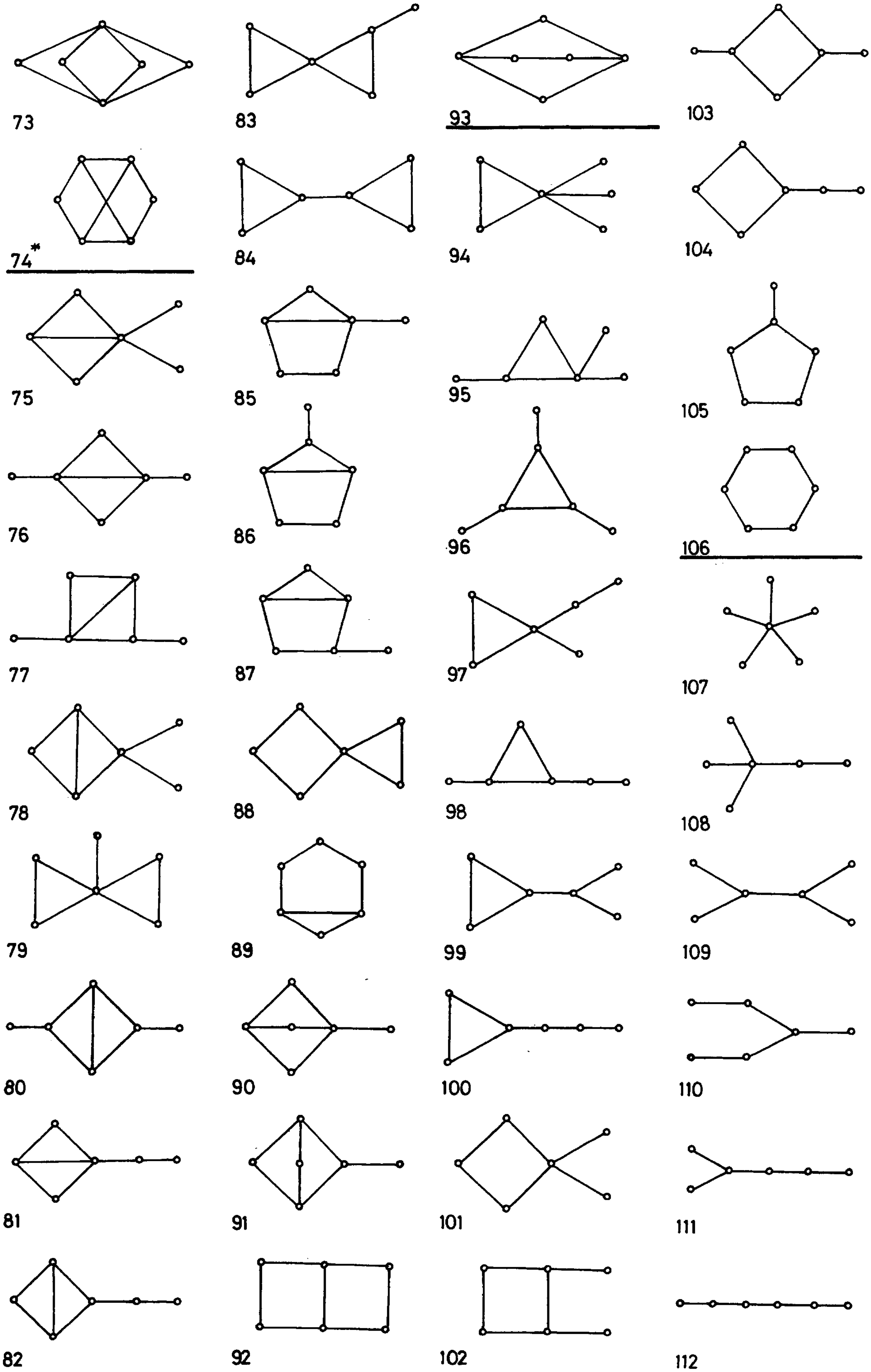
Table 1. (*contd.*)



Ordering graphs by their spectral moments is a slight modification of ordering by eigenvalues, first used in [1] and further discussed in [3]. The present table again amounts to the naturality of such an ordering. The effects are specially visible in the set of trees, unicyclic and bicyclic graphs.

To order the only two cospectral graphs in the table (no. 79 and no. 80) we used the corresponding eigenvectors, as suggested in [3].

To give an ordering to graphs 79 and 80 it was sufficient to look at the eigenvector corresponding to the largest eigenvalue. If this (positive) vector is normalized, the sum of its coordinates (which is a graph invariant) is equal to 2.3434 for graph no. 79 and equal to 2.2686 for no. 80.

The problem of ordering cospectral graphs is of minor interest here but in sets of graphs with a larger number of vertices cospectral graphs occur very often and one should know the way of ordering them. The above criterion does not solve the problem in general since there are non-isomorphic graphs with the same sum of coordinates of the eigenvector of the largest eigenvalue. For example, one cannot distinguish between regular graphs in this way.

Table 2 contains information (coded in two lines) for each of the 112 graphs from Table 1.

The numbers of the first line represent:

- graph identification number k (referring to Table 1);
- spectral moments $S_2, \ldots, S_6$ ($S_0=6$ and $S_1=0$ for all graphs);
- eigenvalues $\lambda_1, \ldots, \lambda_6$ in non-increasing order with three exact digits behind point for non-integer values, main eigenvalues[1] being given in bold face.

Line 2 contains:

- coefficients $a_2, a_3, \ldots, a_6$ of the characteristic polynomial $\sum_{i=0}^{6} a_i\lambda^{6-i}$ (of course, $a_0=1$, $a_1=0$ and $-a_2$ equals the number of edges for all graphs);
- coefficients p_2 and p_3 of the matching polynomial (p_k denotes the number of sets of k mutually non-adjacent edges);
- the numbers c_3, c_4, c_5, c_6 of circuits of length 3, 4, 5, 6, respectively,
- degree sequence $d_1, \ldots, d_6$ (in non-increasing order);
- radius r;
- diameter d;
- chromatic number γ;
- (vertex-) connectivity c;
- l, where $l=1$, if the graph is a line graph, and $l=0$ otherwise;
- $\bar{k}$, where $\bar{k}$ is the identification number of the complement of the graph (referring again to Table 1), if the complement is connected, and $\bar{k}=0$ otherwise.

Some of these data require the use of a computer to be determined (e.g. eigenvalues) but majority of them can be simply determined ad hoc. Nonetheless such data are included into the table so that one can confront several properties of the graphs when going through the table.

[1] An eigenvalue is called *main* if its eigenspace contains an eigenvector the sum of whose coordinates is different from zero.

Table 2. Data of graphs[a]

k	S_2	S_3	S_4	S_5	S_6	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
	a_2	a_3	a_4	a_5	a_6/p_2	p_3/c_3	c_4 c_5	c_6/d_1 $\cdots$	$\cdots$	$d_6/r/d/\gamma/c/l/$	$\bar{k}$
1	30	120	630	3120	15630	**5**	−1	−1	−1	−1	−1
	−15	−40	−45	−24	−5/45	15/20	45 72	60/5 5 5	5 5	5/1/1/6/5/1/	0
2	28	96	500	2280	10828	**4.701**	0	−1	−1	−1	**−1.701**
	−14	−32	−27	−8	0/39	12/16	33 48	36/5 5 5	5 4	4/1/2/5/4/0/	0
3	26	78	398	1680	7574	**4.427**	**0.375**	−1	−1	−1	**−1.803**
	−13	−26	−15	2	3/33	9/13	24 30	17/5 5 5	4 4	3/1/2/5/3/0/	0
4	26	72	386	1560	7058	**4.372**	0	0	−1	**−1.372**	−2
	−13	−24	−12	0	0/34	10/12	23 32	25/5 5 4	4 4	4/1/2/4/4/0/	0
5	24	66	324	1290	5532	**4.201**	**0.545**	−1	−1	−1	**−1.746**
	−12	−22	−9	6	4/27	6/11	18 18	5/5 5 4	4 4	2/1/2/5/2/1/	0
6	24	60	324	1200	5304	**4.162**	0	0	−1	−1	**−2.162**
	−12	−20	−9	0	0/27	6/10	18 18	6/5 5 5	3 3	3/1/2/4/3/0/	0
7	24	60	304	1160	4926	**4.119**	0.618	**−0.431**	−1	−1.618	**−1.687**
	−12	−20	−4	8	3/28	7/10	16 18	9/5 5 4	4 3	3/1/2/4/3/0/	0
8	24	54	300	1060	4644	**4.067**	**0.361**	0	−1	**−1.244**	**−2.184**
	−12	−18	−3	4	0/29	8/ 9	16 20	14/5 4 4	4 4	3/1/2/4/3/0/	0
9	24	48	288	960	4224	**4**	0	0	0	−2	−2
	−12	−16	0	0	0/30	8/ 8	15 24	17/4 4 4	4 4	4/2/2/3/4/1/	0
10	22	60	278	1080	4438	**4.051**	**0.482**	−1	−1	−1	**−1.534**
	−11	−20	−9	4	3/21	3/10	15 12	0/5 4 4	4 4	1/1/2/5/1/1/	0
11	22	48	250	860	3562	**3.895**	**0.397**	0	−1	**−1.292**	−2
	−11	−16	−2	4	0/22	4/ 8	12 10	2/5 5 4	3 3	2/1/2/4/2/0/	0
12	22	48	238	830	3346	**3.858**	**0.779**	**−0.379**	−1	**−1.475**	**−1.783**
	−11	−16	1	10	3/23	5/ 8	11 10	3/5 4 4	4 3	2/1/2/4/2/0/0	
13	22	48	230	800	3190	**3.828**	1	−1	−1	−1	**−1.828**
	−11	−16	3	16	7/23	5/ 8	10 8	5/5 5 3	3 3	3/1/2/4/2/0/	0
14	22	42	242	750	3250	**3.820**	**0.459**	0	−1	−1	**−2.279**
	−11	−14	0	4	0/24	6/ 7	12 12	6/4 4 4	4 4	2/2/2/4/2/0/	0
15	22	42	226	730	2986	**3.778**	**0.710**	0	−1	**−1.489**	−2
	−11	−14	4	8	0/24	5/ 7	10 11	8/5 4 4	3 3	3/1/2/4/3/1/	0
16	22	36	242	660	3094	**3.766**	0	0	0	**−1.282**	**−2.483**
	−11	−12	0	0	0/24	6/ 6	12 12	6/5 4 4	3 3	3/1/2/3/3/0/	0
17	22	36	230	640	2902	**3.732**	0.414	**0.267**	−1	−1	−2.414
	−11	−12	3	4	−1/25	7/ 6	11 12	11/4 4 4	4 3	3/2/2/4/3/0/	0
18	22	36	222	640	2764	**3.713**	0.618	0	**−0.482**	−1.618	**−2.230**
	−11	−12	5	4	0/25	6/ 6	10 14	8/4 4 4	4 3	3/2/2/3/3/0/	0
19	20	42	204	680	2648	**3.710**	**0.440**	0	−1	**−1.384**	**−1.766**
	−10	−14	−1	4	0/17	2/ 7	9 6	0/5 4 4	3 3	1/1/2/4/1/0/	0
20	20	42	200	660	2570	**3.690**	**0.753**	**−0.578**	−1	−1	**−1.865**
	−10	−14	0	8	3/18	3/ 7	9 6	0/4 4 4	4 3	1/2/3/4/1/0/108	

Table 2. (*contd.*)

k	S_2	S_3	S_4	S_5	S_6	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
	a_2	a_3	a_4	a_5	a_6/p_2	p_3/c_3	c_4 c_5	c_6/d_1	$\cdots\cdots d_6$	$/r/d/\gamma/c/l/$	$\bar{k}$
21	20	36	196	580	2372	**3.626**	**0.515**	0	−1	−1	**−2.141**
	−10	−12	1	4	0/17	2/ 6	8 4	0/5	5 3 3 2	2/1/2/4/2/0/	0
22	20	36	184	570	2198	**3.592**	0.618	**0.158**	−1	−1.618	**−1.751**
	−10	−12	4	6	−1/18	3/ 6	7 4	1/5	4 4 3 2	2/1/2/4/2/0/	0
23	20	36	180	540	2108	**3.561**	1	**−0.561**	−1	−1	−2
	−10	−12	5	12	4/19	4/ 6	7 4	2/4	4 4 4 2	2/2/3/4/2/1/	109
24	20	36	172	530	1988	**3.534**	**1.082**	**−0.407**	−1	**−1.511**	**−1.698**
	−10	−12	7	14	4/19	4/ 6	6 4	2/5	4 3 3 3	2/1/2/4/2/1/	0
25	20	30	180	480	2000	**3.514**	**0.669**	0	**−0.528**	**−1.478**	**−2.176**
	−10	−10	5	4	0/19	4/ 5	7 6	2/5	4 3 3 3	2/1/2/3/2/0/	0
26	20	30	176	470	1946	**3.497**	**0.729**	**0.150**	−1	**−1.187**	**−2.190**
	−10	−10	6	6	−1/20	5/ 5	7 6	4/4	4 4 3 3	2/2/2/4/2/1/	111
27	20	30	168	460	1820	**3.467**	**0.912**	0	**−0.798**	**−1.581**	−2
	−10	−10	8	8	0/20	4/ 5	6 6	2/4	4 4 3 3	2/2/2/3/2/1/	110
28	20	30	160	460	1730	**3.449**	0.618	0.618	−**1.449**	−1.618	−1.618
	−10	−10	10	8	−5/20	5/ 5	5 6	5/5	3 3 3 3	3/1/2/4/3/0/	0
29	20	24	184	400	1952	**3.460**	**0.349**	0	0	**−1.338**	**−2.471**
	−10	−8	4	0	0/20	4/ 4	8 8	2/4	4 4 3 3	2/2/2/3/2/0/	0
30	20	24	164	380	1658	**3.388**	0.801	**0.187**	−0.554	**−1.575**	−2.246
	−10	−8	9	4	−1/21	5/ 4	6 8	4/4	4 3 3 3	3/2/2/3/3/0/	112
31	20	24	164	360	1652	**3.372**	1	0	−1	−1	**−2.372**
	−10	−8	9	8	0/21	4/ 4	6 8	4/4	4 3 3 3	3/2/2/3/2/0/	0
32	20	18	188	300	1928	**3.392**	**0.325**	0	0	−1	**−2.717**
	−10	−6	3	0	0/21	6/ 3	9 6	6/4	4 3 3 3	3/2/2/3/3/0/	0
33	18	30	150	430	1602	**3.403**	**0.489**	**0.251**	−1	**−1.282**	**−1.861**
	−9	−10	3	4	−1/13	1/ 5	5 2	0/5	4 3 3 2	1/1/2/4/1/0/	0
34	18	30	146	420	1542	**3.383**	**0.742**	**0**	−1	**−1.327**	**−1.798**
	−9	−10	4	6	0/14	2/ 5	5 2	0/4	4 4 3 2	1/2/3/4/1/1/	95
35	18	30	142	400	1470	**3.353**	**1**	**−0.476**	−1	−1	**−1.877**
	−9	−10	5	10	3/15	3/ 5	5 2	0/4	4 3 3 3	1/2/3/4/1/0/	101
36	18	30	126	360	1230	**3.261**	**1.339**	−1	−1	−1	**−1.601**
	−9	−10	9	18	7/15	3/ 5	3 0	0/5	3 3 3 2	2/1/2/4/1/1/	0
37	18	24	162	360	1650	**3.372**	0	0	0	−1	**−2.372**
	−9	−8	0	0	0/12	0/ 4	6 0	0/5	5 2 2 2	2/1/2/3/2/0/	0
38	18	24	146	360	1434	**3.323**	**0.357**	0	0	**−1.681**	−2
	−9	−8	4	0	0/14	2/ 4	5 4	0/5	3 3 3 3	1/1/2/3/1/0/	0
39	18	24	142	340	1380	**3.294**	**0.734**	0	**−0.597**	**−1.292**	**−2.139**
	−9	−8	5	4	0/15	2/ 4	5 4	0/4	4 3 3 3	1/2/3/3/1/0/	97
40	18	24	138	340	1326	**3.281**	**0.771**	0	**−0.512**	**−1.540**	**−2**
	−9	−8	6	4	0/14	2/ 4	4 2	0/5	4 3 2 2	2/1/2/3/2/0/	0

Table 2. (*contd.*)

k	S_2	S_3	S_4	S_5	S_6	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
	a_2	a_3	a_4	a_5	a_6/p_2	p_3/c_3	c_4 c_5	$c_6/d_1 \cdots\cdots$		d_6 $/r/d/\gamma/c/l/$	$\bar{k}$
41	18	24	130	320	1218	**3.236**	**1**	0	−1	**−1.236**	−2
	−9	−8	8	8	0/16	4/ 4	4 2	2/4 4 3 3 2		2/2/2/4/2/1/103	
42	18	24	126	330	1188	**3.236**	0.618	0.618	**−1.236**	−1.618	−1.618
	−9	−8	9	6	−4/15	2/ 4	3 3	1/4 4 4 2 2		2/2/2/3/2/1/ 96	
43	18	24	126	320	1170	**3.222**	1	**0.112**	−1	**−1.526**	**−1.808**
	−9	−8	9	8	−1/15	3/ 4	3 2	1/5 3 3 3 2		2/1/2/3/2/0/ 0	
44	18	24	122	300	1092	**3.181**	1.246	−0.445	**−0.593**	**−1.588**	−1.801
	−9	−8	10	12	3/16	3/ 4	3 2	1/4 4 3 3 2		2/2/3/3/2/0/102	
45	18	18	138	250	1242	**3.188**	**0.834**	0	**−0.627**	−1	**−2.396**
	−9	−6	6	4	0/16	4/ 3	5 2	2/4 4 3 3 2		2/2/2/3/2/0/ 99	
46	18	18	130	260	1140	**3.169**	**0.728**	**0.279**	**−0.466**	**−1.505**	**−2.205**
	−9	−6	8	2	−1/16	3/ 3	4 4	1/4 4 3 3 2		2/2/2/3/2/0/ 98	
47	18	18	118	250	996	**3.114**	**0.745**	0.618	**−0.860**	−1.618	−2
	−9	−6	11	4	−4/17	4/ 3	3 4	3/4 3 3 3 3		2/2/2/3/2/1/105	
48	18	18	118	230	978	**3.086**	**1.155**	**0.109**	−1	**−1.173**	**−2.178**
	−9	−6	11	8	−1/17	3/ 3	3 4	2/4 3 3 3 3		2/2/2/3/2/0/104	
49	18	12	146	180	1290	**3.141**	**0.484**	0	0	−1	**−2.626**
	−9	−4	4	0	0/16	2/ 2	6 4	0/4 4 3 3 2		2/2/2/3/2/0/ 0	
50	18	12	134	180	1128	**3.092**	**0.701**	0	0	**−1.285**	**−2.508**
	−9	−4	7	0	0/17	4/ 2	5 4	2/4 3 3 3 3		2/2/2/3/2/0/100	
51	18	12	114	180	858	**3**	1	0	0	−2	−2
	−9	−4	12	0	0/18	4/ 2	3 6	4/3 3 3 3 3		3/2/2/3/3/1/106	
52	18	0	162	0	1458	**3**	0	0	0	0	−3
	−9	0	0	0	0/18	6/ 0	9 0	6/3 3 3 3 3		3/2/2/2/3/0/ 0	
53	16	24	116	300	1072	**3.177**	**0.678**	0	−1	−1	**−1.855**
	−8	−8	3	4	0/ 9	0/ 4	3 0	0/5 3 3 3 1		1/1/2/4/1/0/ 0	
54	16	24	112	300	1030	**3.164**	0.618	**0.227**	−1	**−1.391**	−1.618
	−8	−8	4	4	−1/10	1/ 4	3 0	0/4 4 3 3 1		1/2/3/4/1/1/ 76	
55	16	24	104	270	910	**3.096**	**1.116**	**−0.508**	−1	−1	**−1.704**
	−8	−8	6	10	3/12	3/ 4	3 0	0/4 3 3 3 2		1/2/3/4/1/1/ 90	
56	16	18	116	240	988	**3.102**	**0.344**	0	0	**−1.322**	**−2.123**
	−8	−6	3	0	0/ 9	0/ 3	3 0	0/5 4 2 2 2		1/1/2/3/1/0/ 0	
57	16	18	108	220	892	**3.047**	**0.821**	0	**−0.756**	−1	**−2.112**
	−8	−6	5	4	0/11	2/ 3	3 0	0/4 4 3 2 2		1/2/3/3/1/0/ 78	
58	16	18	104	230	850	**3.043**	0.618	**0.328**	**−0.548**	−1.618	**−1.824**
	−8	−6	6	2	−1/10	1/ 3	2 1	0/5 3 3 2 2		1/1/2/3/1/0/ 0	
59	16	18	100	220	802	**3.014**	**0.848**	**0.196**	**−0.724**	**−1.477**	**−1.856**
	−8	−6	7	4	−1/11	1/ 3	2 1	0/4 4 3 2 2		1/2/3/3/1/0/ 77	
60	16	18	96	210	748	**2.980**	**1.041**	**0**	**−0.706**	**−1.537**	**−1.779**
	−8	−6	8	6	0/12	2/ 3	2 1	0/4 3 3 3 2		1/2/3/3/1/1/ 85	

Table 2. (*contd.*)

k	S_2 / a_2	S_3 / a_3	S_4 / a_4	S_5 / a_5	S_6 / a_6/p_2	λ_1 / p_3/c_3	λ_2 / c_4 c_5	λ_3 / $c_6/d_1\cdots\cdots$	λ_4	λ_5 / $d_6/r/d/\gamma/c/l/$	λ_6 / $\bar{k}$
61	16	18	92	200	700	**2.947**	**1.159**	0	−1	**−1.285**	**−1.820**
	−8	−6	9	8	0/11	2/ 3	1 0	0/5 3 2 2 2		2/1/2/3/1/0/	0
62	16	18	84	170	580	**2.842**	**1.506**	**−0.506**	−1	−1	**−1.842**
	−8	−6	11	14	4/13	2/ 3	1 0	0/4 3 3 2 2		2/2/3/3/1/1/	91
63	16	12	104	160	784	**2.943**	**0.664**	**0**	0	**−1.368**	**−2.240**
	−8	−4	6	0	0/12	2/ 2	3 2	0/4 3 3 3 2		1/2/3/3/1/0/	83
64	16	12	104	150	790	**2.932**	**0.727**	**0.308**	**−0.656**	−1	**−2.311**
	−8	−4	6	2	−1/12	1/ 2	3 2	0/4 3 3 3 2		1/2/3/3/1/0/	81
65	16	12	100	140	736	**2.895**	**1**	0	**−0.602**	−1	**−2.292**
	−8	−4	7	4	0/13	2/ 2	3 2	0/3 3 3 3 3		1/2/3/3/1/0/	88
66	16	12	96	160	688	**2.903**	**0.806**	0	0	**−1.709**	−2
	−8	−4	8	0	0/12	2/ 2	2 2	0/4 4 2 2 2		2/2/2/3/2/0/	80
67	16	12	92	140	646	**2.852**	**1.055**	**0.183**	**−0.661**	**−1.271**	**−2.158**
	−8	−4	9	4	−1/13	3/ 2	2 1	1/4 3 3 2 2		2 2/3/3/2/0/	87
68	16	12	84	140	568	**2.813**	1	**0.529**	−1	**−1.342**	−2
	−8	−4	11	4	−4/13	2/ 2	1 2	1/4 3 3 2 2		2/2/2/3/2/1/	86
69	16	12	80	140	526	**2.791**	**1**	0.618	−1	−1.618	**−1.791**
	−8	−4	12	4	−5/14	3/ 2	1 2	2/ 3 3 3 3 2		2/2/2/3/2/0/	93
70	16	12	80	120	496	**2.732**	1.414	0	**−0.732**	−1.414	−2
	−8	−4	12	8	0/14	2/ 2	1 2	1/3 3 3 3 2		2/2/3/3/2/1/	92
71	16	6	100	80	700	**2.796**	**0.853**	0	0	**−1.195**	**−2.454**
	−8	−2	7	0	0/13	2/ 1	3 2	0/4 3 3 2 2		2/2/2/3/2/0/	82
72	16	6	88	90	562	**2.741**	**0.710**	0.618	**−0.231**	−1.618	**−2.220**
	−8	−2	10	−2	−1/14	3/ 1	2 3	1/3 3 3 3 2		2/2/2/3/2/0/	89
73	16	0	128	0	1024	**2.828**	0	0	0	0	**−2.828**
	−8	0	0	0	0/12	0/ 0	6 0	0/4 4 2 2 2		2/2/2/2/2/0/	0
74	16	0	112	0	832	**2.732**	0.732	0	0	**−0.732**	−2.732
	−8	0	4	0	0/14	4/ 0	5 0	2/3 3 3 3 2		2/2/3/2/2/0/	84
75	14	12	82	140	566	**2.813**	**0.529**	0	0	**−1.342**	**−2**
	−7	−4	4	0	0/ 6	0/ 2	1 0	0/5 3 2 2 1		1/1/2/3/1/0/	0
76	14	12	78	140	524	**2.791**	0.618	**0**	0	−1.618	**−1.791**
	−7	−4	5	0	0/ 7	0/ 2	1 0	0/4 4 2 2 1		1/2/3/3/1/0/	54
77	14	12	74	130	488	**2.753**	**0.772**	**0.306**	**−0.609**	**−1.329**	**−1.894**
	−7	−4	6	2	−1/ 8	1/ 2	1 0	0/4 3 3 2 1		1/2/3/3/1/0/	59
78	14	12	74	120	482	**2.732**	**1**	0	**−0.732**	−1	**−2**
	−7	−4	6	4	0/ 8	0/ 2	1 0	0/4 3 3 2 1		1/2/3/3/1/0/	57
79	14	12	70	120	446	**2.709**	1	**0.193**	−1	−1	**−1.903**
	−7	−4	7	4	−1/ 7	1/ 2	0 0	0/5 2 2 2 2		1/1/2/3/1/0/	0
80	14	12	70	120	446	**2.709**	1	**0.193**	−1	−1	**−1.903**
	−7	−4	7	4	−1/ 9	1/ 2	1 0	0/3 3 3 3 1		1/2/4/3/1/0/	66

Table 2. (*contd.*)

k	S_2	S_3	S_4	S_5	S_6	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
	a_2	a_3	a_4	a_5	a_6/p_2	p_3/c_3	c_4 c_5	$c_6/d_1 \cdots\cdots d_6$		$/r/d/\gamma/c/l/$	$\bar{k}$
81	14	12	70	120	440	**2.705**	**1.056**	0	**−0.559**	**−1.350**	**−1.851**
	−7	−4	7	4	0/ 9	2/ 2	1 0	0/4 3 2 2 2		1/2/3/3/1/0/	64
82	14	12	66	110	398	**2.655**	**1.210**	**0**	−1	−1	**−1.866**
	−7	−4	8	6	0/10	2/ 2	1 0	0/3 3 3 2 2		1/2/4/3/1/1/	71
83	14	12	62	110	362	**2.628**	**1.229**	**0.139**	−1	**−1.319**	**−1.678**
	−7	−4	9	6	−1/ 9	1/ 2	0 0	0/4 3 2 2 2		1/2/3/3/1/1/	63
84	14	12	54	80	254	**2.414**	1.732	**−0.414**	−1	−1	−1.732
	−7	−4	11	12	3/11	1/ 2	0 0	0/3 3 2 2 2		2/2/3/3/1/1/	74
85	14	6	70	70	410	**2.599**	**0.766**	**0.466**	**−0.384**	**−1.305**	**−2.141**
	−7	−2	7	0	−1/ 9	1/ 1	1 1	0/4 3 2 2 2		1/2/3/3/1/0/	60
86	14	6	66	70	362	**2.561**	1	**0**	0	**−1.561**	−2
	−7	−2	8	0	0/10	2/ 1	1 1	0/3 3 3 2 2		1/2/3/3/1/1/	68
87	14	6	66	60	368	**2.539**	**1.082**	**0.261**	**−0.540**	**−1.206**	**−2.136**
	−7	−2	8	2	−1/10	1/ 1	1 1	0/3 3 3 2 2		1/2/3/3/1/0/	67
88	14	6	66	50	362	**2.503**	**1.264**	0	**−0.576**	−1	**−2.191**
	−7	−2	8	4	0/10	2/ 1	1 0	0/4 2 2 2 2		2/2/3/3/1/0/	65
89	14	6	54	60	260	**2.438**	**1.138**	0.618	**−0.820**	−1.618	**−1.756**
	−7	−2	11	2	−4/11	2/ 1	0 1	1/3 3 2 2 2		2/2/3/3/2/1/	72
90	14	0	86	0	560	**2.557**	**0.677**	0	0	**−0.677**	**−2.557**
	−7	0	3	0	0/ 9	0/ 0	3 0	0/4 3 2 2 2		1/2/3/2/1/0/	55
91	14	0	82	0	518	**2.524**	**0.792**	0	0	**−0.792**	**−2.524**
	−7	0	4	0	0/10	2/ 0	3 0	0/3 3 3 2 2		1/2/3/2/1/0/	62
92	14	0	70	0	398	**2.414**	1	0.414	**−0.414**	−1	−2.414
	−7	0	7	0	−1/11	3/ 0	2 0	1/3 3 2 2 2		2/2/3/2/2/0/	70
93	14	0	62	20	308	**2.391**	**0.772**	0.618	0	−1.618	**−2.164**
	−7	0	9	−4	0/11	2/ 0	1 2	0/3 3 2 2 2		2/2/2/3/2/0/	69
94	12	6	60	60	336	**2.514**	**0.571**	0	0	−1	**−2.086**
	−6	−2	3	0	0/ 3	0/ 1	0 0	0/5 2 2 1 1		1/1/2/3/1/0/	0
95	12	6	52	60	264	**2.445**	**0.796**	**0**	0	**−1.370**	**−1.872**
	−6	−2	5	0	0/ 5	0/ 1	0 0	0/4 3 2 1 1		1/2/3/3/1/0/	34
96	12	6	48	60	234	**2.414**	0.618	0.618	**−0.414**	−1.618	−1.618
	−6	−2	6	0	−1/ 6	1/ 1	0 0	0/3 3 3 1 1		1/2/3/3/1/1/	42
97	12	6	48	50	234	**2.379**	**1**	**0.291**	**−0.751**	−1	**−1.920**
	−6	−2	6	2	−1/ 6	1/ 1	0 0	0/4 2 2 2 1		1/2/3/3/1/0/	39
98	12	6	44	50	198	**2.334**	**1.099**	**0.274**	**−0.594**	**−1.373**	**−1.739**
	−6	−2	7	2	−1/ 7	1/ 1	0 0	0/3 3 2 2 1		1/2/4/3/1/1/	46
99	12	6	44	40	192	**2.278**	**1.317**	0	**−0.704**	−1	**−1.891**
	−6	−2	7	4	0/ 7	0/ 1	0 0	0/3 3 2 2 1		1/2/3/3/1/0/	45
100	12	6	40	40	162	**2.228**	**1.360**	**0.185**	−1	−1	**−1.774**
	−6	−2	8	4	−1/ 8	1/ 1	0 0	0/3 2 2 2 2		1/2/4/3/1/1/	50

Table 2. (*contd.*)

k	S_2	S_3	S_4	S_5	S_6	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
	a_2	a_3	a_4	a_5	a_6/p_2	p_3/c_3	c_4 c_5	$c_6/d_1 \cdots\cdots d_6/r/d/\gamma/c/l/\bar{k}$			
101	12	0	56	0	288	**2.288**	**0.874**	0	0	**−0.874**	**−2.288**
	−6	0	4	0	0/ 6	0/ 0	1 0	0/4 2 2 2 1 1/2/3/2/1/0/ 35			
102	12	0	52	0	258	**2.246**	0.801	**0.554**	−0.554	**−0.801**	−2.246
	−6	0	5	0	−1/ 7	1/ 0	1 0	0/3 3 2 2 1 1/2/3/2/1/0/ 44			
103	12	0	52	0	252	**2.236**	1	**0**	0	−1	**−2.236**
	−6	0	5	0	0/ 7	0/ 0	1 0	0/3 3 2 2 1 1/2/4/2/1/0/ 41			
104	12	0	48	0	216	**2.175**	**1.126**	**0**	0	**−1.126**	**−2.175**
	−6	0	6	0	0/ 8	2/ 0	1 0	0/3 2 2 2 2 1/2/4/2/1/0/ 48			
105	12	0	40	10	150	**2.144**	1	0.618	**−0.254**	−1.618	**−1.860**
	−6	0	8	−2	−1/ 8	1/ 0	0 1	0/3 2 2 2 2 1/2/3/3/1/0/ 47			
106	12	0	36	0	132	**2**	1	1	−1	−1	−2
	−6	0	9	0	−4/ 9	2/ 0	0 0	1/2 2 2 2 2 2/3/3/2/2/1/ 51			
107	10	0	50	0	250	**2.236**	0	0	0	0	**−2.236**
	−5	0	0	0	0/ 0	0/ 0	0 0	0/5 1 1 1 1 1/1/2/2/1/0/ 0			
108	10	0	38	0	160	**2.074**	**0.835**	0	0	**−0.835**	**−2.074**
	−5	0	3	0	0/ 3	0/ 0	0 0	0/4 2 1 1 1 1/2/3/2/1/0/ 20			
109	10	0	34	0	130	**2**	1	0	0	**−1**	−2
	−5	0	4	0	0/ 4	0/ 0	0 0	0/3 3 1 1 1 1/2/3/2/1/0/ 23			
110	10	0	30	0	106	**1.931**	1	**0.517**	**−0.517**	−1	**−1.931**
	−5	0	5	0	−1/ 5	1/ 0	0 0	0/3 2 2 1 1 1/2/3/2/1/0/ 27			
111	10	0	30	0	100	**1.902**	**1.175**	**0**	0	**−1.175**	**−1.902**
	−5	0	5	0	0/ 5	0/ 0	0 0	0/3 2 2 1 1 1/2/4/2/1/0/ 26			
112	10	0	26	0	76	**1.801**	1.246	**0.445**	−0.445	**−1.246**	−1.801
	−5	0	6	0	−1/ 6	1/ 0	0 0	0/2 2 2 2 1 1/3/5/2/1/1 30			

[a] Main eigenvalues indicated in bold face.

When compiling this table we used the data existing in the literature (e.g., [9, 10]), we used the interactive programming system ‘Graph’ [7] to compute or to check some data and, finally, a lot of things was determined immediately from the picture of the graph. M. Doob and I. Gutman helped in completing this table.

There are several interesting graphs in Table 1. Graphs nos. 13, 28, 35, 44, 69, 80 are Beineke’s forbidden subgraphs for line graphs (there are also two forbidden subgraphs on 5 vertices and one on 4). If we add graphs nos. 3, 7, 12, 20, 22, 33, 43, 58, 59, 77, 79, 97, 105, 110, we get the twenty graphs coming from the root system E_6 (c.f. [8]). They all have the least eigenvalue >-2. Further addition of graphs nos. 6, 21, 37, 56, 57, 94, 107, 108, completes the list of 28 graphs, 6-vertex forbidden subgraphs for generalized line graphs [6] (there are 3 more forbidden subgraphs on 5 vertices). The only graphs with at most 6 vertices with $\lambda_2>1$ are the 23 graphs from this table. There are 18 graphs in this table with exactly two main eigenvalues.

The following unsolved problems are related to these facts:

(1) Characterize graphs with $\lambda_2 \leq 1$ [4];

(2) Characterize graphs with exactly two main eigenvalues [2].

Finally, Table 2 gives rise to the following proposition.

Proposition. *Let $M(\lambda) = (\lambda - \mu_1) \cdots (\lambda - \mu_k)$, where $\mu_1, \ldots, \mu_k$ are the main eigenvalues of a graph G. Then coefficients of $M(\lambda)$ are rational.*

Proof. By [5, p. 55], the function

$$P_{\bar{G}}(\lambda - 1)/P_G(\lambda),$$

where $P_G(\lambda)$ is the characteristic polynomial of graph G, and $\bar{G}$ is the complement of G, has simple poles $\mu_1, \ldots, \mu_k$ and no other poles. Since the greatest common divisor of polynomials in numerator and denominator (obtained by Euclid's algorithm) has rational coefficients, so does $M(\lambda)$. This completes the proof. □

Tables of 6-vertex graphs have been used in several investigations. For example, this table was the starting point for finding forbidden subgraphs for generalized line graphs, mentioned above.

Note that spectra and characteristic polynomials of graphs up to five vertices are given, for example, in [5].

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References

[1] F.C. Bussemaker, S. Čobeljič, D.M. Cvetković and J.J. Seidel, Computer investigation of cubic graphs, Technological University Eindhoven, T.H. Report 76-WSK-01; Cubic graphs on ≤14 vertices, J. Combin. Theory (B) 23 (1977) 234–235.

[2] D. Cvetković, The main part of the spectrum, divisors and switching of graphs, Publ. Inst. Math. (Beograd) 23 (37) (1978) 31–38.

[3] D.M. Cvetković, Some possible directions in further investigations of graph spectra, in: L. Lovász and V.T. Sos, eds., Algebraic Methods in Graph Theory I (North-Holland, Amsterdam, 1981) 47–67.

[4] D. Cvetković, On graphs whose second largest eigenvalue does not exceed 1, Publ. Inst. Math. (Beograd) 31 (45) (1982) 15–20.

[5] D. Cvetković, M. Doob and H. Sachs, Spectra of Graphs – Theory and Application (Deutscher Verlag der Wissenschaften – Academic Press, Berlin–New York, 1980; second ed., 1982).

[6] D. Cvetković, M. Doob and S. Simić, Generalized line graphs, J. Graph Theory 5 (1981) 385–399.

[7] D. Cvetković, L. Kraus and S. Simić, Discussing graph theory with a computer I, Implementation of graph theoretic algorithms, Univ. Beograd, Publ. Elektrotehn. Fak., Ser. Mat. Fiz. No. 716-No. 734 (1981) 100–104.
[8] M. Doob and D.M. Cvetković, On spectral characterizations and embeddings of graphs, Linear Algebra Appl. 27 (1979) 17–26.
[9] F. Harary, Graph Theory (Addison-Wesley, Reading, MA, 1969).
[10] C. King, Characteristic polynomials of 7-node graphs, Sci. Rept., Univ. of West Indies CC6 (AFORS project 1026–66), Kingston, 1967.