Deductive Arguments

Sound: All premises are true $\wedge$ conclusion is true

Valid: All premises are true $\Rightarrow$ conclusion is true

Invalid: All premises are true $\wedge$ conclusion is false

Premises	Conclusions	Validity	Soundness
T	T	Valid (T)	Sound (T)
T	F	Invalid (F)	Not Sound (F)
F	T	Valid (T)	Not Sound (F)
F	F	Valid (T)	Not Sound (F)

Logical Connectives

Symbol	Name	Formal Name	Type	Hierarchy of Connectives
~	Not	Negation	Unary	Low
$\wedge$	And	Conjunction	Binary	Medium
$\vee$	Or	Disjunction	Binary	Medium
$\rightarrow$	If then	Conditional	Binary	High
$\leftrightarrow$	If and only if	Biconditional	Binary	High

Atomic: Statement with no logical connectives (use letters P to Z)

Molecular: Statement with logical connectives

Official Notation

1. () around binary connective ($\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$)
2. No () around unary connective (~) and atomic statement

Informal Notation

1. () around *some* binary connectives ($\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$)
2. No () around unary connective (~) and atomic statement
3. Use hierarchy of connectives to disambiguate

   eg. $(P \vee (Q \rightarrow (P \wedge R))) = P \vee (Q \rightarrow P \wedge R)$

4. Use right-most rule to disambiguate collections of all $\wedge$ and all $\vee$

   eg. $(((P \wedge Q) \wedge R) \wedge S) = P \wedge Q \wedge R \wedge S$

Main Connective: Basically the main/central logical connective of an argument. Scope applies to the entire argument.

Antecedent: Left side of conditional/biconditional (condition)

Consequent: Right side of conditional/biconditional (consequence)

Truth Value: Whether a statement is true or false

Truth Value Assignment (TVA): An assignment of T or V values to all variables in a statement (ie. T and F tables)

Statements	Set of Statements
Tautology: True under every TVA *Contradiction:* False under every TVA *Contingent:* If there're TVAs that can make them true and false	*Consistent:* If there's a TVA that makes them all true *Inconsistent:* If no TVA makes all of them true *Logically Equivalent:* If all of them have the same truth values for all TVAs

A statement/set of statements is...	Find...
Not Tautology	TVA that makes it F
Not Contradiction	TVA that makes it T
Contingent	2 TVAs that makes statement T and F
Not Logically Equivalent	TVA that makes one statement T, the other F
Consistent	TVA that makes it T
Not Inconsistent	TVA that makes it T
Invalid	Case where premises T, conclusion F

Symbolization

Comma = Can help disambiguate, group everything before or after it into a statement

Phrase	Logic
P, provided Q P is necessary for Q Only if/when P, then (is it the case that) Q P if Q P assuming Q P given that Q P on the condition of Q P in case Q	$Q \longrightarrow P$
P, only provided Q Assuming/if/whenever P, then Q For P, Q (is necessary) For P, it is necessary that Q P is enough/sufficient for Q P, only if/when Q P, only on the condition of Q P only if/when Q P means Q	$P \longrightarrow Q$

If P, then Q	
P is necessary & sufficient for Q P exactly on the condition of Q P if and only if Q P just in case Q	$P \leftrightarrow Q$
Neither P nor Q	$\sim P \land \sim Q$ $\sim(P \lor Q)$
Not P and Q	$\sim P \lor \sim Q$ $\sim(P \land Q)$
P exclusive or Q	$(P \lor Q) \land \sim(P \land Q)$ $(P \land \sim Q) \lor (\sim P \land Q)$ $\sim(P \leftrightarrow Q)$
P unless Q P except for/when Q Either P or Q P or Q	$\sim Q \longrightarrow P$ $P \lor Q$ $\sim P \longrightarrow Q$
All of P, Q, R	$P \land Q \land R$
At most two of P, Q, R	None $\lor$ Exactly One $\lor$ Exactly Two $(\sim P \land \sim Q \land \sim R) \lor$ $(P \land \sim Q \land \sim R) \lor (\sim P \land Q \land \sim R) \lor (\sim P \land \sim Q \land R) \lor$ $(P \land Q \land \sim R) \lor (P \land \sim Q \land R) \lor (\sim P \land Q \land R)$ or $\sim(P \land Q \land R)$
At most one of P, Q, R	None $\lor$ Exactly One $(\sim P \land \sim Q \land \sim R) \lor$ $(P \land \sim Q \land \sim R) \lor (\sim P \land Q \land \sim R) \lor (\sim P \land \sim Q \land R)$
At least two of P, Q, R	$(P \land Q) \lor (P \land R) \lor (Q \land Q)$
At least one of P, Q, R	$P \lor Q \lor R$
Exactly two of P, Q, R	$(P \land Q \land \sim R) \lor (P \land \sim Q \land R) \lor (\sim P \land Q \land R)$
Exactly one of P, Q, R	$(P \land \sim Q \land \sim R) \lor (\sim P \land Q \land \sim R) \lor (\sim P \land \sim Q \land R)$
None of P, Q, R	$\sim(P \lor Q \lor R)$ $\sim P \land \sim Q \land \sim R$
X, which is P, Q	$P \land Q$

X, who is P, Q P and Q P but Q P, although Q P, even though Q P; Q	
If P, then Q, and in that case, R	$(P \rightarrow Q) \wedge (Q \rightarrow R)$

Derivation

A__: Assume __
CD: Conditional derivation
ID: Indirect derivation
DD: Direct derivation

Repetition R	Modus Ponens MP	Modus Tollens MT	Double Negation DN
P $\therefore P$	$P \rightarrow R$ P $\therefore R$	$P \rightarrow R$ $\sim R$ $\therefore \sim P$	$\sim\sim P$ $\therefore P$
Adjunction ADJ	**Simplification** S/SL/SR	**Modus Tollendo Ponens** MTP	**Addition** ADD
P R $\therefore P \wedge R$	$P \wedge R$ $\therefore P$ $\quad\therefore R$	$P \vee R$ $\sim P$ $\quad\sim R$ $\therefore R$ $\quad\therefore P$	P $\therefore P \vee R$

Biconditional-Conditional BC	Conditional-Biconditional CB
$P \leftrightarrow R$ $\therefore P \rightarrow R$ $\quad\therefore R \rightarrow P$	$P \rightarrow R$ $R \rightarrow P$ $\therefore P \leftrightarrow R$

Notes:

- Can't simplify $P \leftrightarrow \sim\sim Q$. Double negation only works when ~ is in front.
- Using Modus Tollens on $\sim P \rightarrow \sim R$, you have to write $\sim\sim R$, $\therefore \sim\sim P$.

Negation of Conditional NC		Negation of Biconditional (Exclusive Or) NB	
$\sim(P \rightarrow R)$ $\therefore P \wedge \sim R$	$P \wedge \sim R$ $\therefore \sim(P \rightarrow R)$	$\sim(P \leftrightarrow Q)$ $\therefore P \leftrightarrow \sim Q$	$P \leftrightarrow \sim Q$ $\therefore \sim(P \leftrightarrow Q)$

<table>
<tr><th colspan="2">De Morgan’s Law
DM</th><th colspan="2">Separation of Cases
SC</th></tr>
<tr><td>$\sim(P \lor R)$

$\therefore \sim P \land \sim R$</td><td>$\sim(P \land R)$

$\therefore \sim P \lor \sim R$</td><td>$P \lor Q$
$P \rightarrow R$
$Q \rightarrow R$
$\therefore R$</td><td>$P \rightarrow R$
$\sim P \rightarrow R$

$\therefore R$</td></tr>
<tr><th colspan="4">Conditional as Disjunction
CDJ</th></tr>
<tr><td>$P \rightarrow R$

$\therefore \sim P \lor R$</td><td>$\sim P \lor R$

$\therefore P \rightarrow R$</td><td>$\sim P \rightarrow R$

$\therefore P \lor R$</td><td>$P \lor R$

$\therefore \sim P \rightarrow R$</td></tr>
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Predicate Logic

Atomic Sentence Letters	P-Z
Individual Constants/Names	a-h
Variables	i-z
Operation Letters	a-h, or a^0, a^1, a^2, ...
Predicate Letters	A-O, or F^0, F^1, F^2, ...
Logical Connectives	$\sim$, $\longrightarrow$, $\leftrightarrow$, $\vee$, $\wedge$
Brackets/Parentheses	(), []
Quantifiers	$\exists$, $\forall$
Identity Sign	=
Operator	Logical Connectives + Quantifiers

$\exists x \forall y$ = All y and one specific x

$\forall y \exists x$ = A unique x for all y

$\exists x(Fx \wedge \forall xGx) \equiv \exists x(Fx \wedge \forall yGy)$

“All x isn’t y, except for z” - doesn’t imply z is y

Implicature: Not logical, but implied in everyday language

Implication: Logical

Logical Equivalences

Contrapositive	Biconditional	Exportation	Quantifier Negation	
$P \longrightarrow Q$ $\equiv$ $\sim Q \longrightarrow \sim P$	$P \leftrightarrow Q$ $\equiv$ $(P \longrightarrow Q) \wedge (Q \longrightarrow P)$	$P \wedge Q \longrightarrow R$ $\equiv$ $P \longrightarrow (Q \longrightarrow R)$ $\equiv$ $Q \longrightarrow (P \longrightarrow R)$	$\sim \forall xAx$ $\equiv$ $\exists x \sim Ax$	$\sim \exists xAx$ $\equiv$ $\forall x \sim Ax$

Symbolization

All A are B A are B The A is B Any A is B If you’re A, then you’re B	$\forall x(Ax \longrightarrow Bx)$
Some A are B At least one A is B There exists/is an A that is B	$\exists x(Ax \wedge Bx)$

Only A are B	$\forall x(Bx \rightarrow Ax)$
x is y’s only Z	$x = Zy \land \sim\exists a(a \neq x \land Za)$ *x is y’s only Z, and no other a is* $x = Zy \land \forall a(Za \rightarrow a = x)$ *x is t’s only Z, and every a that is y’s Z is actually just x* $\forall a(a = Zy \leftrightarrow a = x)$ *Every a is y’s Z iff it is x*
X is the best	$\forall a(a \neq x \rightarrow B(xa))$ *x is better than every a that isn’t x* $\sim\exists a(B(ax))$ *There is no a that is better than x*
X did at least 1 thing.	$\exists aD(Xa)$
X did at least 2 (unique) things.	$\exists a(D(Xa) \land \exists b(a \neq b \land D(Xb)))$ *X did a, X did b, and a ≠ b*
X did at least 3 (unique) things.	$\exists a(D(Xa) \land \exists b(a \neq b \land D(Xb) \land \exists c(a \neq c \land b \neq c \land D(Xc))))$ *X did a, X did b, X did C, a ≠ b, a ≠ c, and b ≠ c*
X did at most 1 thing.	$\forall a(D(Xa) \rightarrow \forall n(D(Xn) \rightarrow n = a))$ *If X did any a, then if X did any n, then n = a* $\sim\exists a(D(Xa) \land \exists b(a \neq b \land D(Xb)))$ *X did not do at least 2 things*
X did at most 2 (unique) things.	$\forall a(D(Xa) \rightarrow \forall b(D(Xb) \rightarrow \forall n(D(Xn) \rightarrow n = a \lor n = b \lor a = b)))$ $\forall a \forall b \forall n(D(Xa) \land D(Xb) \land D(Xn) \rightarrow n = a \lor n = b \lor a = b)$ $\sim\exists a(D(Xa) \land \exists b(a \neq b \land D(Xb) \land \exists c(a \neq c \land b \neq c \land D(Xc))))$
X did exactly 1 thing.	$\exists aD(Xa) \land \forall n(D(Xn) \rightarrow n = a)$ *X did a, and if X did any n, then n = a* $\exists aD(Xa) \land \sim\exists n(D(Xn) \land n \neq a)$ *X did a, and there is no n ≠ a that X did* $\forall n \exists a(D(na))$ *n did a if and only if n = X*

Symbolic sentence can only have bound variables (previously introduced by $\exists$ / $\forall$), no free variables

- Eg. $\exists x(Ax \land Bx)$, where x is a bound variable. Good.
- Eg. $\exists x(Ax \land By)$, where y is a free variable. Bad.

Universe of Discourse: Everything in the bounds of a quantifier

Direction Derivation (DD)

Conditional Derivation (CD) - a conditional statement

Indirect Derivation (ID) - assuming the opposite, getting a contradiction

Universal Derivation (UD) - pick an arbitrary member of the universe, prove something for it

Derivation

Universal Instantiation UI	Existential Instantiation EI	Existential Generalization EG	Quantifier Negation QN			
$\forall xAx$ $\therefore Ay$	$\exists xAx$ $\therefore Ay$	Ax $\therefore \exists yAy$	$\sim\forall xAx$ $\therefore \exists x{\sim}Ax$	$\exists x{\sim}Ax$ $\therefore \forall xAx$	$\sim\exists xAx$ $\therefore \forall x{\sim}Ax$	$\forall x{\sim}Ax$ $\therefore \sim\exists xAx$

Alphabetic Variance AV	
$\forall xAx$ $\therefore \forall yAy$	$\exists xAx$ $\therefore \exists yAy$

eg. No one watches every movie

Watches(no one, every movie)

No one = $\sim\exists x(\text{Someone}(x) \land \ldots)$

Every movie = $\forall y(\text{Movie}(y) \rightarrow \ldots)$

$\sim\exists x(\text{Someone}(x) \land \forall y(\text{Movie}(y) \rightarrow \text{Watches}(xy)))$