IN4112

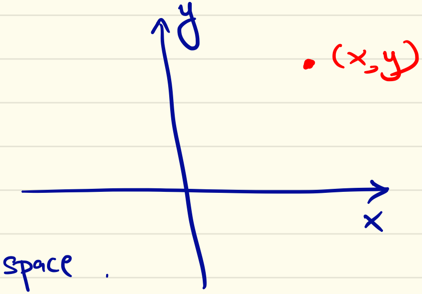
Today:

Today: basic mathematics for meshes.

Point: 2D $\rightarrow$ (x, y)

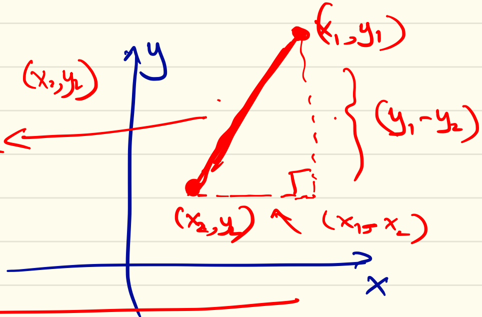
3D $\rightarrow$ (x, y, z)

represents a position in space.



distance between (x_1, y_1) and (x_2, y_2)

$$\sqrt{(y_1-y_2)^2 + (x_1-x_2)^2}$$



In 3D:

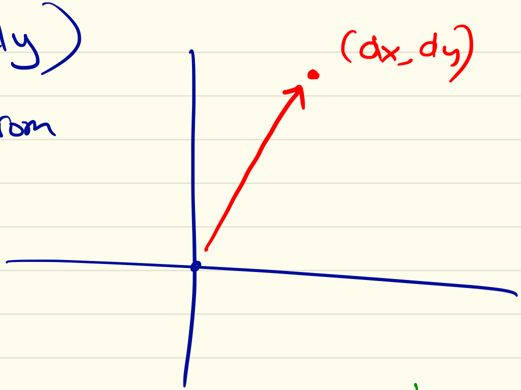
$$\sqrt{(y_1-y_2)^2 + (x_1-x_2)^2 + (z_1-z_2)^2}$$

Vector: 2D → (dx, dy)

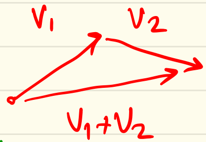
need 2 important quantities from a Vector:

→ which direction ??

→ how much in that direction ??



Vector + vector → vector



point + vector → point

starting position

direction to follow from the starting position.

ending position.

Point + Point →

dot-product of vector.

scalar

$$\underbrace{\text{vector}}\rightarrow V_1 \cdot V_2 \leftarrow \text{vector}$$

$$\|V_1\|\cdot\|V_2\|\cdot\cos\left[\text{angle between } V_1 \text{ and } V_2\right] = (x_1, y_1)\cdot(x_2, y_2) = \overbrace{x_1\cdot x_2 + y_1\cdot y_2}^{\text{number.}}$$

Q: Whats the big deal about dot-products?

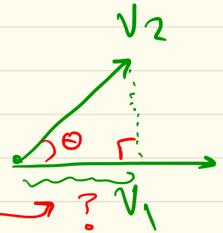
① $V_1 \cdot V_2 = 0$ if and only if $\cos\left[\text{angle between } V_1 \text{ and } V_2\right] = 0$

② Compute angle between V_1 and V_2.

$$V_1, V_2 \rightarrow \text{make unit vector} \rightarrow \cos^{-1}(V_1 \cdot V_2).$$

③

$$\cos\theta = \frac{?}{\|V_2\|}$$



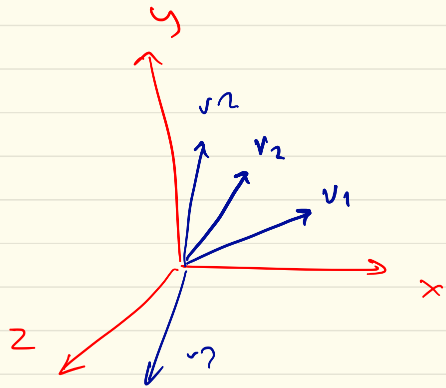
$$? = \|V_2\|\cdot\cos\theta$$

$$\downarrow$$

$V_1 \cdot V_2$ iff V_1 is a unit vector!

Cross-product of vector

$$\underbrace{V_1}_{\text{Vector}} \times \underbrace{V_2}_{\text{Vector}} \longrightarrow \underbrace{V_3}_{\text{vector}}.$$



Vector V : $\underline{(dx, dy, dz)}$

direction, $\underline{\text{length}}$. $\rightarrow \sqrt{dx^2 + dy^2 + dz^2}$

Unit vector: length 1, . .

→ What is the unit vector in the direction of V ?

unit vector: $\left(\frac{dx}{\sqrt{dx^2 + dy^2 + dz^2}}, \frac{dy}{\sqrt{dx^2+dy^2+dz^2}}, \frac{dz}{\sqrt{dx^2+dy^2+dz^2}} \right)$

Checking length:

$$\sqrt{\frac{dx^2}{dx^2+dy^2+dz^2} + \frac{dy^2}{dx^2+dy^2+dz^2} + \frac{dy^2}{dx^2+dy^2+dz^2}}$$

$$= 1.$$

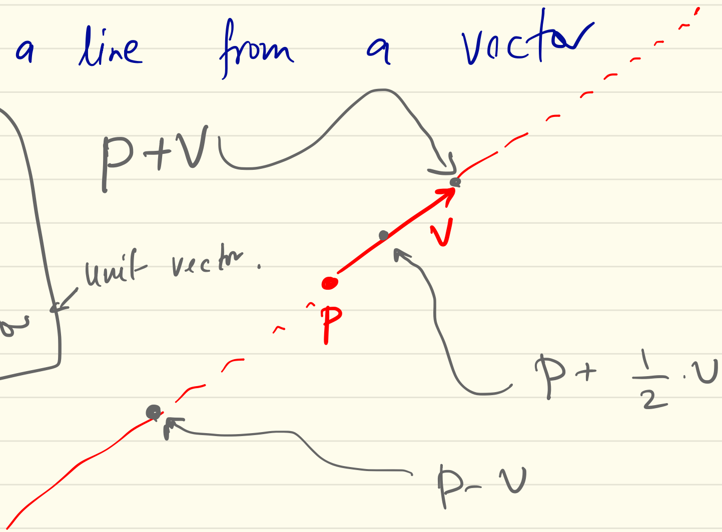
LINES

define a line from a vector

$$\boxed{\underbrace{p}_{\text{point}} + \underbrace{t}_{\text{number}} \cdot \underbrace{v}_{\text{vector}}}$$

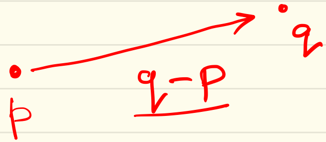
unit vector.

point.

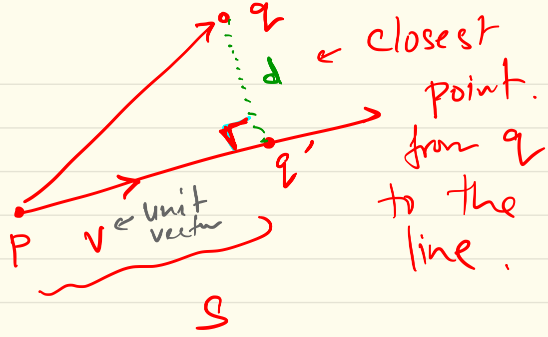


$$\underbrace{p + t \cdot (q-p)}$$

$$\underbrace{p \cdot (1-t) + t \cdot q}$$



Q: what is the distance between q and the line $p + t \cdot v$?

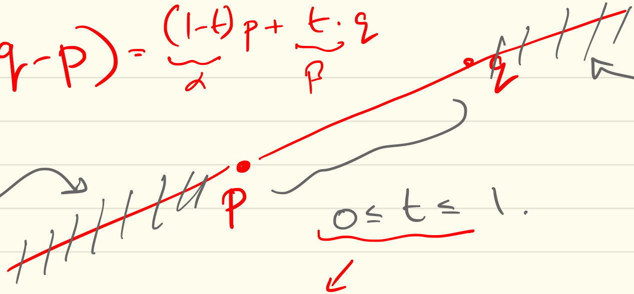


Idea 1: compute s by projection and then use Pythagoras theorem to get 'd'.

$$q' = p + s \cdot v$$

← assumes v is a unit vector.

$$p + t\cdot(q-p) = \underbrace{(1-t)}_{\alpha}\, p + \underbrace{t}_{\beta}\cdot q$$



$t < 0$

$0 \le t \le 1$.

$t > 1$

$$\underbrace{\alpha}_{\text{number}} \cdot \underbrace{p}_{\text{point}} + \underbrace{\beta}_{\text{number}} \cdot \underbrace{q}_{\text{point}},$$ where $\alpha + \beta = 1$.

$\alpha, \beta \ge 0$

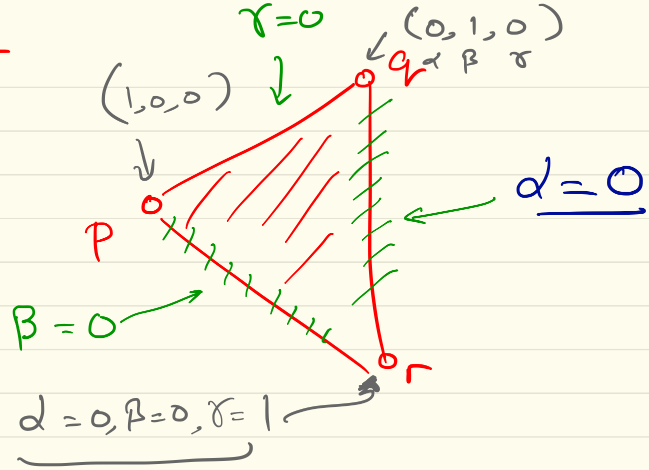
TRIANGLES:

$$\alpha \cdot p + \beta \cdot q + \gamma \cdot r$$

$$\alpha, \beta, \alpha \geq 0$$

$$\underbrace{\alpha + \beta + \alpha} = 1.$$

Barycentric coordinates with respect to the triangle.



Planes in 3D

→ fixed by 3 points } bad!!

→ point + a vector

p   v ← normal vect

lying on the plane.

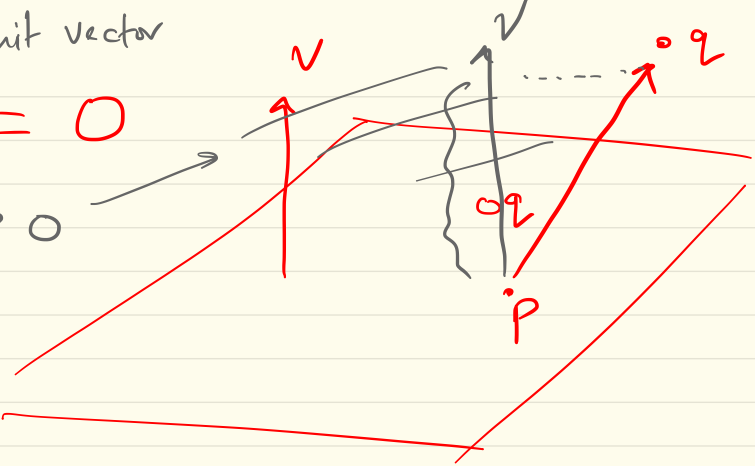
→ Q: When does a point q lie on the plane defined by p and v?

unit vector

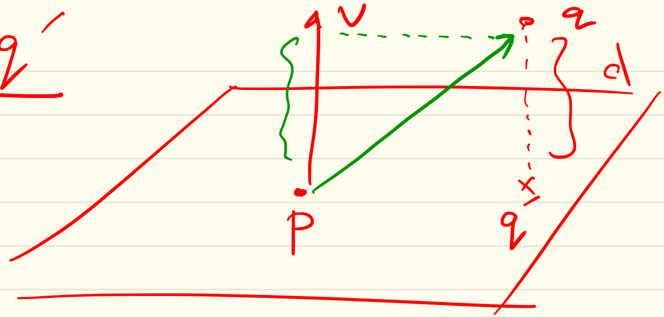
$$(q - p) \cdot \underline{\underline{v}} = 0$$

> 0

< 0



Q: Compute d and q'



$$d = (q-p)\cdot v \;!$$

$$q' = q + (-dv)$$

KEY:

line is stored as a point + vector

plane is stored as a point + $\boxed{\text{vector}}$

unit vector

2D: 2D point + 2D vector

3D: 2D point + 3D vector

Line:

$$p + t \cdot \underline{\underline{V}}$$ vector along the line

<u>Plane:</u>

$$(q - p) \cdot \underset{\sim}{V} = 0$$

vector <u><u>normal</u></u> to the plane.