Geometric Algorithms Fall, 2016

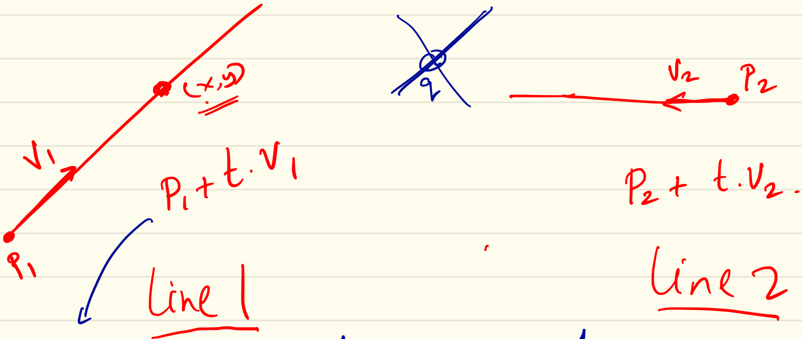
TDR 1 – Basic Geometric Primitives

16 November

Questions

1. Compute the intersection of two lines in $\mathbb{R}^2$ given in parametric form.
2. Give a method to compute the two closest points between two line segments in the plane. Assume you are given the two endpoints of each of the line segments.
3. Compute the intersection of a line and a plane in $\mathbb{R}^3$, where the plane is given in the point-normal form, and the line is given in the parametric form.
4. Give a method to compute the closest distance between a point and a triangle in $\mathbb{R}^3$. Assume you are given the three vertices of the triangle.
5. Give a method to compute the closest distance between two lines in $\mathbb{R}^3$. Both lines are given in parametric form.
6. Give a method to compute the closest distance of a point to a line-segment in $\mathbb{R}^3$. Assume that you are given the two end-points of the line-segment and the plane in the point-normal form.

Compute the intersection of two lines in $\mathbb{R}^2$ given in parametric form.



$$q = p_1 + \boxed{t_1} \cdot \overset{\text{point}}{v_1} = \overset{\text{point}}{p_2} + \boxed{t_2} \cdot v_2$$

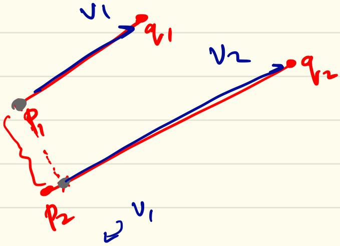
$$\begin{cases} p_1^x + t_1 \cdot v_1^x = p_2^x + t_2 \cdot v_2^x \\ p_1^y + t_1 \cdot v_1^y = p_2^y + t_2 \cdot v_2^y \end{cases}$$

2 variables: t_1, t_2

2 equation

Solve it as we did when we were young.

2. Give a method to compute the two closest points between two line segments in the plane. Assume you are given the two endpoints of each of the line segments.



$\to d(p_1, p_2)$

$\to d(p_1, q_2)$

$\to d(p_2, q_1)$

$\to d(q_1, q_2)$

$\to$ compute intersection

$p_1 + t\cdot(q_1 - p_1)$

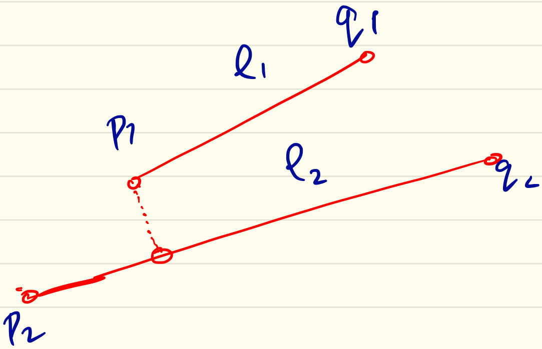
$p_2 + t\,(q_2 - p_2)$

$\to$ 1 Compute intersection of lines passing through the 2 segments.

$\to$ if $\underline{0 \le t_1, t_2 \le 1}$ then the two segments intersect.

2 Compute the distance between p_1 and l_2

q_1	l_2
p_2	l_1
q_2	l_1



Compute the intersection of a line and a plane in $\mathbb{R}^3$, where the plane is given in the point-normal form, and the line is given in the parametric form.

q lies on the line:

$$\Rightarrow \quad q = p_1 + t \cdot v_1$$

q lies on the plane:

$$\Rightarrow \quad (q - p_2) \cdot N = 0$$

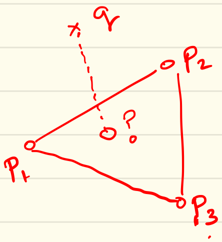
?

$$\big((p_1 + t \cdot v_1) - p_2\big) \cdot N = 0$$

$$(p_1 - p_2) \cdot N + t \cdot v_1 \cdot N = 0$$

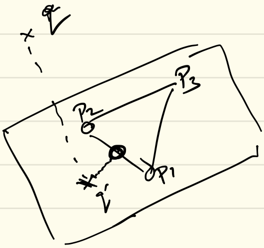
$$t = -\frac{(p_1 - p_2) \cdot N}{v_1 \cdot N} .$$

Give a method to compute the closest distance between a point and a triangle in $\mathbb{R}^3$. Assume you are given the three vertices of the triangle.

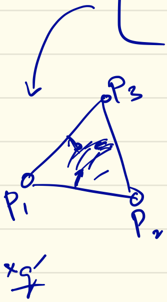


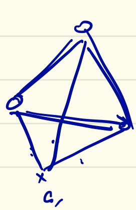
In class:

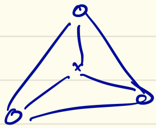
1. compute the closest point of q from the plane defined by P_1, P_2, P_3.



2. If q' lies inside the Δ, we are done.







3. q' lies outside

Closest point from q to segments

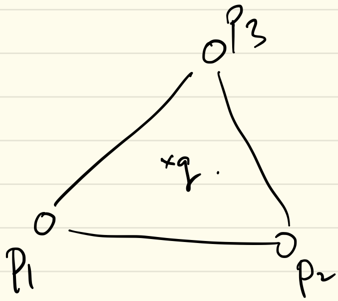
P_1P_2

P_2P_3

P_3P_1

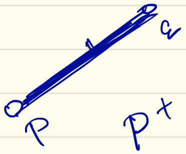
take the smallest distance.

Given a triangle in 2D, and a point q, compute the Barycentric coordinates of q.



Compute α, β, γ for q.

$$\boxed{\begin{aligned} q &= \alpha \cdot p_1 + \beta p_2 + \gamma \cdot p_3 \\ \text{where } & \alpha + \beta + \gamma = 1. \\ & \alpha, \beta, \gamma \geq 0. \end{aligned}}$$



$p + t \cdot (q - p)$

$(1-t)p + t \cdot q$

$\alpha p + \beta \cdot q$ $\quad$ $\alpha + \beta = 1$

We need to find α, β, γ.

But $\alpha + \beta + \gamma = 1$.

We just need to find α, and β.

then $\gamma = 1 - \alpha - \beta$.

$$q = \alpha \cdot p_1 + \beta \cdot p_2 + (1 - \alpha - \beta) \cdot p_3$$

$q^x = \gamma \cdot p_1^x + \beta \cdot p_2^x + (1-\alpha-\beta) \cdot p_3^x$ AND.

$q^y = \gamma \cdot p_1^y + \beta \cdot p_2^y + (1-\alpha-\beta) \cdot p_3^y$.

2 variables: α, β

2 equation:

we can solve! :)

TD

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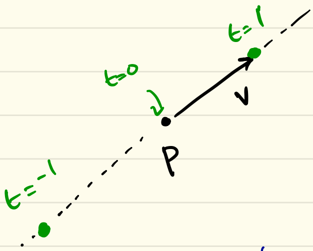
Compute the intersection of two lines in $\mathbb{R}^2$ given in parametric form.



$$\underbrace{p}_{\text{point}} + \underbrace{t}_{\text{number}} \cdot \underbrace{v}_{\text{vector}}$$

$$p_1 + t \cdot v_1$$
$$p_2 + t \cdot v_2$$

- a point on a line fixed by 1 number $\downarrow$ t.

- If a point lies on <u>both</u> the lines, then

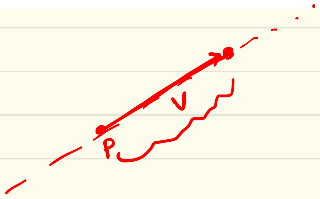
$$\underbrace{p_1}_{\text{point}} + \underbrace{t_1}_{\text{number}} \cdot \underbrace{v_1}_{\text{vector}} = \underbrace{q}_{\text{point}} = \underbrace{p_2}_{\text{point}} + \underbrace{t_2}_{\text{number}} \cdot \underbrace{v_2}_{\text{vector}}$$

$$\begin{cases} p_1^x + t_1 \cdot v_1^x = q^x = p_2^x + t_2 \cdot v_2^x \\ p_1^y + t_1 \cdot v_1^y = q^y = p_2^y + t_2 \cdot v_2^y \end{cases}$$

- need to find t_1 & t_2
- have 2 equations.

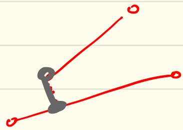
$\Longrightarrow$ solve.

2. Give a method to compute the two closest points between two line segments in the plane. Assume you are given the two endpoints of each of the line segments.



$p + \underbrace{t}\cdot v$

$0 \le t \le 1.$



$p_1 + \underbrace{t_1}\cdot v_1$

line segment : $0 \le t_1 \le 1$

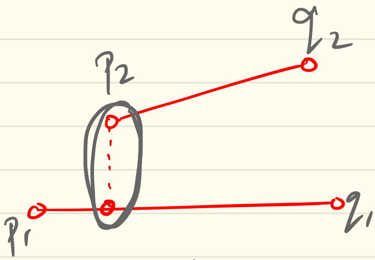
$p_2 + t_2 \cdot v_2$

line segment : $0 \le t_2 \le 1$.

2 main cases :

$p_1 + t_1 \cdot v_1 = p_2 + t_2 v.$

__IF__ : $0 \le t_1 \le 1$, $0 \le t_2 \le 1$, then the two segments intersect.



Compute the nearest point from

p_1 to segment $p_2 q_2$

p_2 to segment $p_1 q_1$

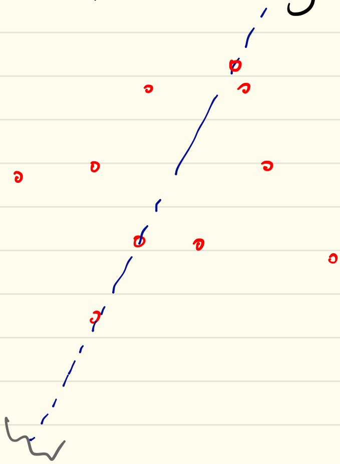
q_1 to segment $p_2 q_2$

q_2 to segment $p_1 q_1$

Take the best answer.

Q: Given a set P of n points in the plane are there 3 points which lie on the same line?

How fast can you check this??



- $O(n^3)$ ← easy
- Is $O(n^2 \log n)$ possible?

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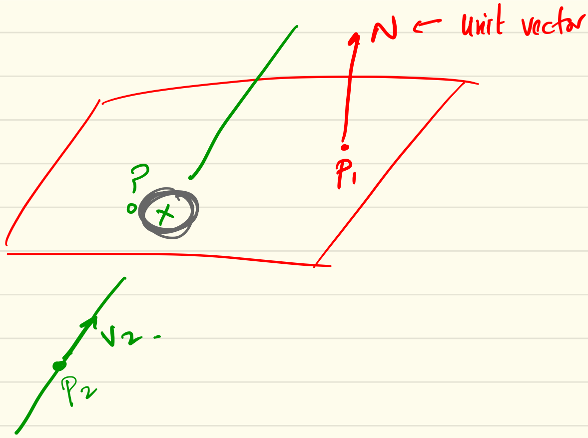

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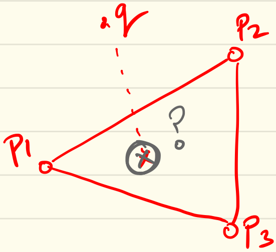

3D

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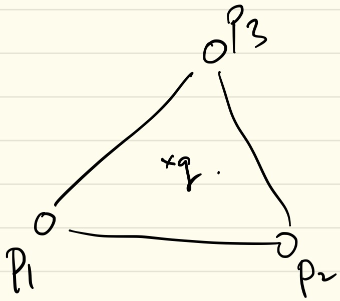

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