OBLIG 1 FYS2160 – second try

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Abstract

In this report assignment we want to use theory about heat transfer and conduction, as well as experimental data to construct a working model for temperature loss in two different mugs/thermoses. We manage to do so, and from coding in python we visualize the accuracy of these models, which we conclude to be very good.

1 Introduction

Studying the rate of cooling of water in different mugs can be a good way of learning about heat transfer. In this report we introduce theory for describing the decreasing temperature of a liquid within a mug. We then visualize a dataset for temperature-reduction, for both a regular *Bodum mug* and a *Temperfect mug* that stores energy from the liquid it holds to first cool it down and then keep it warm. We use the definition of flux of energy through the wall of the mugs to approximate and adapt a model for our real data and make them as accurate as possible. We discuss the models and finally we come to a conclusion as to their accuracy.

2 Theory

We wish to create a model for temperature development for liquid within two different types of mugs – the Bodum mug and the Temperfect. We must first take a look at how the liquid will act, and the process that takes place when a hot drink cools inside the mug. We know that the density of cooler water is higher than that of warmer water, and therefore will float downwards in a mug and create circulation. We simplify the situation by assuming that circulation in the cup results in an even temperature everywhere in the mug.

Figure 1: illustration from the Joeveo homepage linked in the assignment text, explaining the Temperfect technology.

Further, we need to be able to describe both heat conduction between the liquid and mug, as well as heat loss to the environment due to the open lid at the top.

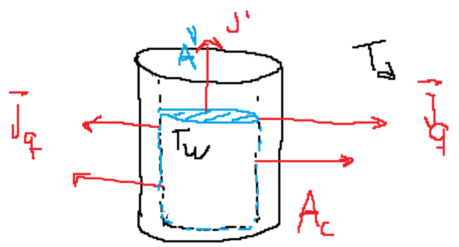

Figure 2: A figure showing how the heat is transferred from the liquid to its surroundings.

We know that if we assume that the heat loss is mostly due to conduction through the walls of the mug, we have the formula from the assignment text;

$$\frac{\Delta T}{\Delta T_0} = e^{-t/\tau} \qquad (1)$$

where T_0 is the start temperature of the drink, ΔT is the difference between water and air temperature, t is the time and τ (or tau) is a time constant. If we wish, we can find an expression for tau in this equation, that takes into account the conduction through the walls as well as the heat to the air at the top of the mug. To do this we use the formula from the assignment text (2), as well as (1);

$$\frac{dQ}{dt} = mc_V \frac{dT}{dt} = A_C J_q + A' J_q' \qquad (4)$$

where $A_C J_q$ where m is mass, c is the heat capacity for the water, Q is the heat, and A are the areas the flux is relevant for, describes the conduction through the walls of the mug, and $A' J_q'$ is the assumed expression for transport equation for the liquid-air at the top of our mug. We know that the heat flux $J_q = -\lambda \nabla T$ where lambda is the thermal conductivity of the material from the assignment text. This gives us an expression $\frac{\Delta T}{\Delta T_0} = e^{-t/\tau}\_$ for our situation where we can express $tau\_ = mc_V / (\frac{A_C \lambda_C}{\Delta x} + \frac{A\prime \lambda\prime}{\Delta x\prime})$. By approximating a value of this $tau\_$ we can then create a model for a mug where the only process that takes place is heat-conduction.

We can also use the formula for heat conduction

$$\frac{\partial T}{\partial t} - \alpha \frac{\partial^2 T}{\partial x^2} = 0 \qquad (2)$$

where T is the temperature and $\alpha = \frac{\lambda}{cp}$ where λ is the thermal conductivity, c is the specific heat capacity and p is the density specific to the block of material. The parameters c and p are both material properties that determine how much heat the material can store, while t and ∂T determine how long and rate of the heat transfer and are therefore all related to the thermal conductivity.

For water we have a thermal conductivity $\lambda = 0.598 W/mK$ at 20 degrees Celsius, a heat capacity c=4.184J/g∘C and a density p=997kg/m^3. We can describe any potential heat loss to the environment by the formula from the assignment text;

$$Q = J_q A \Delta T \qquad (3)$$

where A is the surface area where the heat transport takes place, and J_q is the heat flux, which we set to $0.1 \frac{J}{cm^2 Cmin}$.

3 Results and discussion

We create a python code, presented in Appendix A, that plots the data for temperature on liquid in the two mugs as a function of time, and adapt a few different models to resemble the dataset as accurately as possible. The code uses both formulas (1) and (2) to approximate a model for temperature decay in our Bodum mug, and plot them against the raw data. We get the plot:

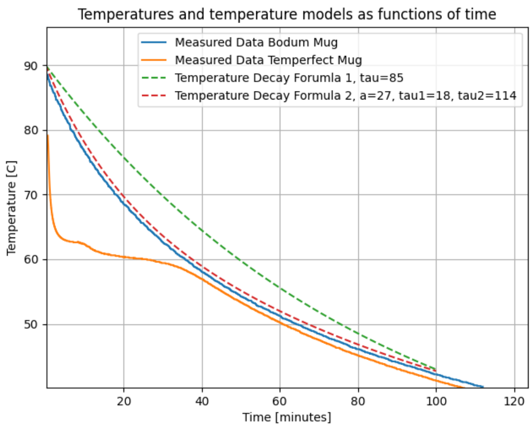

Figure 3: Plots as result of code in Appendix A. This is a comparison of our models for a Bodum mug, and our actual data. We also plotted against the data for our Temperfect mug.

For both the functions our arguments were changed to make the model as accurate as possible. From this plot we can clearly see that the model using formula 2, with an a=27, tau1=18 and tau2=114 is more accurate for our data set than the model based on formula using a tau=85. Therefore, we can conclude that this is the model we should use for this purpose. We can read from this plot that, if we wish to create an accurate model for our Temperfect mug, we have to do this in intervals. The first interval has to model the temperature while heat is stored within the sides of the mug, which we can read takes about 10-12 minutes and brings the temperature of the liquid from about 90 to 63 degrees. The temperature 63degrees is what we call T_m, or the melting temperature of the phase change material in the Temperfect mug. The second interval has to model the timeframe $t\epsilon[12{,}40]$, where the curve is way slacker and the temperature declines from 63 to about 59 degrees. This is the interval where the energy previously stored in the walls of the mug keeps the liquid warm. The third and final interval is one we can see is already modelled well by our function based on formula 2. We code in python to create this model as accurately as possible. We do this by altering the code we've already written.

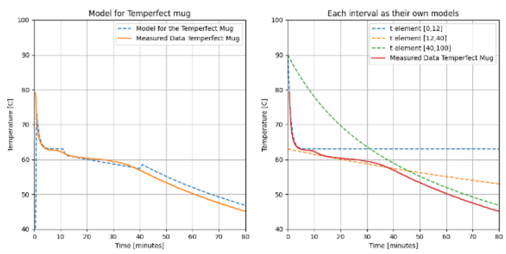

Figure 4: Plots that show our approximation for the model for out Temperfect mug. The code for this plot with explanation in detail can be viewed in Appendix A.

We model the Temperfect mug by dividing the definition into three intervals like we did previously. The model is not entirely accurate overall, but each of the three intervals are pretty good for just those intervals. We have inserted the values tau1=1 and tau2=1 for interval 1, tau1=114 and tau2=500 for interval 2 and tau1=18 and tau2=114 for interval 3, which is what gives us such an accurate model.

Lastly, we use the formula (3) in combination with $mc_V \frac{dT}{dt} = A_C J_q$ and get $Q = mc_V \Delta T$, which are all known to us when m=0.3. $Q = 0.3 * 4184 * (90 - 63) = 33890.4J$.

We were provided with code for a model for heat transfer between two metal blocks in the assignment text, which we can better by using our new understanding of energy-transfer. We compare the model from the code the assignment text references with the experimental data.

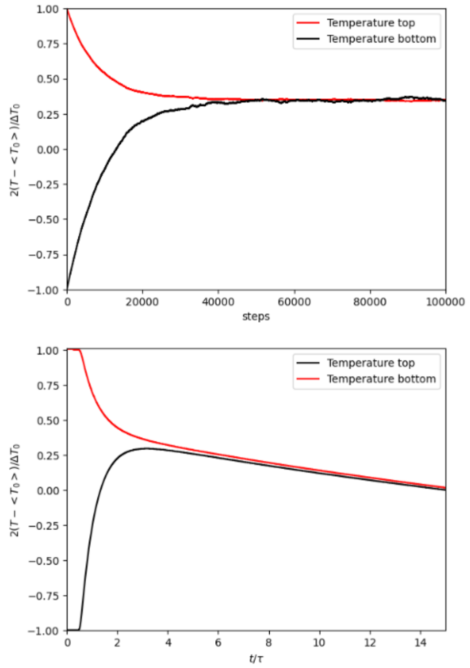

Figure 5: Figure created by code provided in assignment text, adapted with Ctb=10 and tau=200, and additional code representing heat loss to environment. The top plot represents our model that was developed from theory, and the bottom plot represents our real data.

The code was adapted to the heat loss to the environment given by the formula (3).

4 Conclusion

We have, through implementation of theory in code, managed to model both our Temperfect and Bodum mugs to a good extent.

5 References

2023, *Oblig 1 FYS2160*, Fysisk Institutt, UiO.

Shroeder, Daniel V, 2000, *Thermal Physics*, Addison Wesley Longman, The United States.

Appendix A

```python
import numpy as np
import matplotlib.pyplot as plt
import numpy as np


D = np.loadtxt('termokopper.txt', usecols = [0, 1, 2], unpack = True)
tau = 60 #to get time in minutes we set tau=60
t = D[0,:]/tau; T1 = D[1,:]; T2 = D[2,:]

def temp_bodum(t, tau, initial_temp, room_temp):
    temps=np.zeros(len(t))
    for i in range(len(t)):
        temps[i]=room_temp + (initial_temp - room_temp) * np.exp(-t[i] / tau)
    return temps

def temperature(times,a,tau1,tau2):
    initial_temp=90; room_temp=22
    temps=np.zeros(len(times))
    for i in range(len(times)):
        temps[i]=room_temp+(initial_temp-room_temp)*(a*np.exp(-times[i]/tau1)+(1-a)*np.exp(-times[i]/tau2))
    return temps

ts=np.linspace(0,100,100)
plt.title("Temperatures and temperature models as functions of time")
plt.plot(t[70:],T1[70:], label="Measured Data Bodum Mug")
plt.plot(t[70:],T2[70:], label="Measured Data Temperfect Mug")
plt.plot(ts, temp_bodum(ts, 85, 90, 22.0), "--",label="Temperature Decay Forumla 1, tau=85")
plt.plot(ts, temperature(ts,a=0.27,tau1=18,tau2=114), "--", label='Temperature Decay Formula 2, a=27, tau1=18, tau2=114')

plt.xlabel("Time [minutes]")
plt.ylabel(r'Temperature [C]')
plt.legend()
plt.grid()
plt.show()
```

Above is the code that plots Figure3. It imports the .txt-file, and plots the temperature data for both the mugs against the time. Then, it creates both our models for the Bodum mug by defining two functions, that are essentially just representation of our formulas (2) and (3). We remember to include the room temperature in our equation.

```python
def temperfect_model(times,a,tau1,tau2,f2tau1,f2tau2,f3tau1,f3tau2):
    a=0.27; temp1=np.zeros(len(times)); temp2=np.zeros(len(times)); temp3=np.zeros(len(times))
    temps=np.zeros(len(times))
    for i in range(len(times)):
        temp1[i]=63+(90-63)*(a*np.exp(-times[i]/tau1)+(1-a)*np.exp(-times[i]/tau2))
        temp2[i]=22+(63-22)*(a*np.exp(-times[i]/f2tau1)+(1-a)*np.exp(-times[i]/f2tau2))
        temp3[i]=22+(90-22)*(a*np.exp(-times[i]/f3tau1)+(1-a)*np.exp(-times[i]/f3tau2))
    for i in range(1,len(times)):
        if times[i]<12:
            temps[i]=temp1[i]

        elif times[i]>=12 and times[i]<=40:
            temps[i]=temp2[i]
        else:
            temps[i]=temp3[i]
    return temps,temp1,temp2,temp3

t1=np.linspace(0,100,101)
temps,temp1,temp2,temp3=temperfect_model(t1,a=0.27,tau1=1,tau2=1,f2tau1=114,f2tau2=500,f3tau1=18,f3tau2=114)
plt.subplot(1,2,1)
plt.title("Model for Temperfect mug")
plt.plot(t1,temps, "--", label="Model for the Temperfect Mug")
plt.plot(t[70:],T2[70:], label="Measured Data Temperfect Mug")
plt.axis([0,80,40,100])
plt.xlabel("Time [minutes]")
plt.ylabel(r'Temperature [C]')
plt.legend()
plt.grid()

plt.subplot(1,2,2)
plt.title("Each interval as their own models")
plt.plot(t1,temp1, "--", label='t element [0,12]')
plt.plot(t1,temp2, "--", label='t element [12,40]')
plt.plot(t1,temp3, "--", label='t element [40,100]')
plt.plot(t[70:],T2[70:], label="Measured Data Temperfect Mug")
plt.axis([0,80,40,100])
plt.xlabel("Time [minutes]")
plt.ylabel(r'Temperature [C]')
plt.legend()
plt.grid()
plt.show()
```

The code above plots Figure4. We start by defining a function that takes a time-array and two tau -values for each of our three different time-intervals as arguments. Then, the code creates arrays to hold our different temperature-values, and we loop through each of the instances of time to fill these arrays up. To calculate the temperatures the code uses the same form of formula (2) as the most accurate approximation of the Bodum mug did previously. We define $\frac{\Delta T}{\Delta T_0}$ for each interval, and calculate the temperature for each model. Finally, we go through all these temperature-approximations and fills a final temperature-array with the relevant values within each of the intervals. The function returns the final temperature-array as well as the three different models for the temperature. We're interested in seeing how the three different models look in the same plot as well, which is why we bother solving the problem this way. Finally, the program plots the model against the data.

```
In [11]: import numpy as np
         import matplotlib.pyplot as plt

         Ctb=10
         N = 80000
         nstep = 15 * N
         tau = 200
         Tt = np.zeros(nstep, float)
         Tb = np.zeros(nstep, float)
         Tt[0] = 1
         Tb[0] = -1
         Tr = -1

         surface_area=75*38*10**(-2) #cm
         thermal_resistance_env=0.1
         for i in range(1, nstep):
             r = 4 * np.random.rand(1, 1) - 2
             DT = Tt[i - 1] - Tb[i - 1]
             heat_loss_env_top = surface_area * thermal_resistance_env * (Tt[i - 1] - Tr)
             heat_loss_env_bottom = surface_area * thermal_resistance_env * (Tb[i - 1] - Tr)
             if r < DT:
                 Tt[i] = Tt[i - 1] - 1 / N - heat_loss_env_top / N
                 Tb[i] = Tb[i - 1] + Ctb / N + heat_loss_env_bottom / N
             else:
                 Tt[i] = Tt[i - 1] + 1 / N + heat_loss_env_top / N
                 Tb[i] = Tb[i - 1] - Ctb / N - heat_loss_env_bottom / N

         plt.figure(1)
         plt.plot(range(0, nstep), Tt , color = 'r')
         plt.plot(range(0, nstep), Tb, color = 'k')
         plt.xlabel('steps')
         plt.ylabel(r'$2(T-<T_0>)/\Delta T_0$')
         plt.axis([0,100000,-1,1])
         plt.legend(['Temperature top','Temperature bottom'])
         plt.show()

         D = np.loadtxt('metalblocks_lecture.txt', usecols = [0, 1, 2], unpack = True)
         tau = 100
         t = D[0,:] / tau
         T1 = D[1,:]
         T2 = D[2,:]
         T10 = np.mean(T1[70:91])
         T20 = np.mean(T2[70:91])
         DT0 = T20 - T10
         Tmean0 = (T20 + T10) / 2

         plt.figure(2)
         plt.plot(t, 2 * (T1 - Tmean0) / DT0, color = 'k')
         plt.plot(t, 2 * (T2 - Tmean0) / DT0, color = 'r')
         plt.axis([0, 15, -1.0, 1.01])
         plt.xlabel(r'$t/\tau$')
         plt.ylabel(r'$2(T-<T_0>)/\Delta T_0$')
         plt.legend(['Temperature top','Temperature bottom'])
         plt.show()
```

Above is the code from the file *diffusion_model_start.py* attached to the assignment, with minor changes to improve the model. Heat loss was added as a result of formula 3. Plots visualized in *Figure 5*.

Appendix B: Concept questions

1a. The pressure of a set amount of gas in a thermos with a decreasing volume, like we have in this case, the ***pressure of the gas will increase***. We can explain this easily by looking at the definition of pressure – force per unit area. We also know from Boyle's law that a decreasing volume of gas will result in increased pressure value.

1b. ***When the pressure of a gas increases, so will the temperature***. This can be explained by increased friction between the gas molecules moving within the constantly shrinking space. There is explained that there is no heat escaping the space in the thermos, and the increasing friction between the molecules, and energy increase, will therefore result in an increasing temperature. Mathematically, this can be explained by the law for ideal gas, which we can assume is the case here; $PV = nRT$ where *V* is the volume (decreasing), *n* is the gas amount (constant), *R* is the ideal gas constant, *V* is the volume and *T* is the temperature. For the pressure *P* to increase with a decreasing volume, the ***temperature T needs to increase***.

1c. The gas molecules will, when compressed, move within a smaller space. This means they will collide with each other more frequently, which results in an increasing pressure as explained in 1a. We know that the more particles are within the system with decreasing volume, the larger this pressure change will be. With larger amount of collisions between the atoms in the smaller space, comes an increase in kinetic energy from the friction, and therefore also an increase in temperature as explained in 1b.

1d. There are a couple obvious property changes that changes within the gas: The ***density*** of the gas increases when it has less room to exist – the gas becomes denser. The ***energy*** of the gas will also increase, as a result of the decreasing space and increasing temperature.

Naturally there will also be a temperature change for a gas with an increasing pressure, as we discussed earlier.

2a. The bottle in this instance allows for heat to enter or leave the environment inside, and we will therefore assume that there is a constant temperature. We can then understand that there will be ***no increase in the temperature*** of the gas. For this to be possible there must be no increase in the kinetic energy when the particles collide with the walls of the bottle of this system, since this is how energy is transferred to the environment inside of the bottle. For there to be a constant energy during collision there needs to be a constant mean velocity for the particles within the bottle as well – they only change direction during collision, not velocity.

2b. The pressure of the gas will still ***increase*** in this instance because the volume of the gas is still decreasing. Boyle's law is still applicable.

2c. The gas molecules move within a smaller space when compressed, which still results in more frequent collisions between each other, as well as the walls of the bottle. We established an increasing pressure, but when we in this instance have to model our internal energy as constant, we cannot conclude that anything other than the pressure will increase.

2d. ***Density*** will still change as a result of this pressure change.

3a. We know from page 15 formula (1.23) in *"Thermal Physics",* that thermal energy for a system of N molecules with f degrees of freedom is given $U_{thermal} = Nf\frac{1}{2}kT$. We can, since we are dealing with mostly constants, compare degrees of freedom to answer if water or air would have more energy per molecule. I look up degrees of freedom and find a number around 7 for air and 9 for water. ***This means that at low and high temperatures water has higher energy than air.***

3b. We know that water molecules are attracted to each other due to hydrogen bonding – this results in a lower potential energy than for the air molecules – they are only affected by smaller forces. ***The potential energy per molecule of water molecules is more negative than that of air molecules***.

3c. On page 16 in *"Thermal Physics"* it is explained that vibration counts as two degrees of freedom – one for potential and one for kinetic energy. Since $E_{kinetic} = Nf\frac{1}{2}kT$ as well, we can conclude that as long as the temperatures are larger or smaller (not close to room temperature), the kinetic energy will be larger for water than for air due to the different degrees of freedom.

3d. The air and water having the same temperature means that they have a similar average kinetic energy. We know that this energy can be transferred between the system, and that if they reach the same average kinetic energy, the temperature can be the same.

3e. When the water and air are heated slowly to keep the same temperature, the kinetic energy of both will increase, and they will start moving faster. How the pressure changes is dependent on several factors, such as air contents and volume, but I can imagine that since the molecules in the air will move faster the pressure will increase faster than that of the water.

3f. For a glass with ice in an environment with room tempered air, we know that the ice will absorb heat and begin to melt. The ice will keep its temperature until all of it melts, because the energy from the environment is used to break the bonds within the ice instead of increasing the temperature. When the ice is melted the water will heat up till it is the same temperature as the environment.