Properties of Network Functions

14.1 INTRODUCTION

The basic definition of one port and two port network being discussed earlier, here we will discuss about the *transform of excitation and response* alongwith their relations. A *network function* exhibits the relationship between the transform of the source or excitation to the transform of the response for a electrical network. Further to this, we will discuss the stability of the network function mathematically formulating the network function through "pole-zero" concept.

14.2 DRIVING POINT IMPEDANCE AND ADMITTANCE

The *driving point impedance* of a one port network is defined as

$$Z(s) = \frac{V(s)}{I(s)} \quad \text{...(14.1)}$$

while the driving point admittance is given as

$$Y(s) = \frac{I(s)}{V(s)} \quad \text{...(14.2)}$$

for the one port network.

Similarly, for the two port network, the *driving point impedance* and *admittance at port* 1 is defined as

$$Z_{11}(s) = \frac{V_1(s)}{I_1(s)} \quad \text{...(14.3)}$$

and

$$Y_{11}(s) = \frac{I_1(s)}{V_1(s)} \quad \text{...(14.4)}$$

While the *driving point impedance and admittance at the port* 2 are designated as

$$Z_{22}(s) = \frac{V_2(s)}{I_2(s)} \quad \text{...[14.4(a)]}$$

and

$$Y_{22}(s) = \frac{I_2(s)}{V_2(s)} \quad \text{...(14.5)}$$

14.3 TRANSFER IMPEDANCE AND ADMITTANCE

Transfer impedance is defined as the ratio of transformed voltage at output port to the transformed current at the input port of a two port network.

This gives,

$$Z_{12}(s) = \frac{V_2(s)}{I_1(s)} \quad \text{...(14.6)}$$

In a similar way, the *transfer admittance* is defined as the ratio of current transform at output port to the voltage tranform at the input port. It is given as

$$Y_{12}(s) = \frac{I_2(s)}{V_1(s)} \quad ...(14.7)$$

14.4 VOLTAGE AND CURRENT TRANSFER RATIO

Voltage and current transfer ratios are basically transfer functions. *Voltage transfer ratio* is the ratio of voltage transform at output port to the voltage transform at the input port. It is usually denoted by $G(s)$. Thus,

$$G_{12}(s) = \frac{V_2(s)}{V_1(s)} \quad ...(14.8)$$

In a similar way, the ratio of transform of current at output port to that at the input port of a two port network gives the *current transfer ratio*. It is represented by $\alpha(s)$. Thus,

$$\alpha_{12}(s) = \frac{I_2(s)}{I_1(s)} \quad ...(14.9)$$

EXAMPLE 14.1 *A series R-L circuit is in parallel with a capacitance. Find the input impedance in Laplace domain.*

SOLUTION. Figure E14.1 represents the parallel combination of one series *R-L* circuit with a capacitance in Laplace domain. The input impedance is $Z(s)$ which is obviously

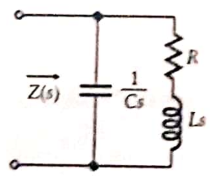


Fig. E14.1

$$Z(s) = \frac{(1/Cs)(R+Ls)}{\frac{1}{Cs}+(R+Ls)}$$

$$= \frac{R+Ls}{1+RCs+LCs^2}$$

$$= \frac{s+\frac{R}{L}}{C\left(s^2+\frac{R}{L}s+\frac{1}{LC}\right)}$$

∴ Input impedance

$$Z(s) = \frac{s+(R/L)}{C\left(s^2+\frac{R}{L}s+\frac{1}{LC}\right)}$$

EXAMPLE 14.2 *Find $Z_{11}(s)$, $Z_{21}(s)$ in the following circuit. (Fig. E14.2)*

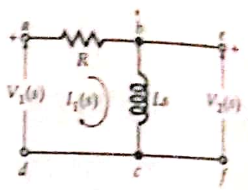


Fig. E14.2

SOLUTION. KVL gives for *loop abcd*,

$$RI_1(s) + I_1(s)Ls = V_1(s) \quad ...(i)$$

or,

$$I_1(s)(R+Ls) = V_1(s)$$

$$\therefore \quad Z_{11}(s) = \frac{V_1(s)}{I_1(s)} = (R+Ls) \quad ...(ii)$$

Again in *loop bcfe*, KVL yields

$$V_2(s) = I_1(s)Ls$$

which gives,

$$Z_{12}(s) = \frac{V_2(s)}{I_1(s)} = Ls. \quad ...(iii)$$

Equations (*ii*) and (*iii*) give the required result.

EXAMPLE 14.3 *Obtain the transfer function $V_2(s)/V_1(s)$ in Fig. E14.3.*

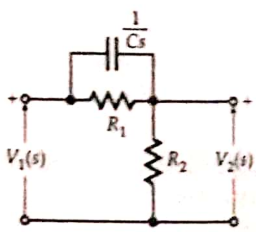


Fig. E14.3

SOLUTION. The equivalent impedance of $\frac{1}{Cs}$ and R_1 is given by $Z(s)$.

Obviously,

$$Z(s) = \frac{R_1 \times \frac{1}{Cs}}{R_1 + \frac{1}{Cs}} = \frac{R_1}{R_1 Cs + 1}.$$

Applying KVL in *left loop* of Fig. E14.3, we get

$$V_1(s) = I_1(s)[Z(s) + R_2]$$

$$V_2(s) = R_2\, I_1(s).$$

$$\therefore \quad V_2(s)/V_1(s) = \frac{R_2}{R_2 + Z(s)}$$

$$= \frac{R_2}{R_2 + \dfrac{R_1}{R_1 Cs + 1}}$$

$$= \frac{R_2(1 + R_1 Cs)}{R_2 + R_1 R_2 Cs + R_1}$$

$$= \frac{R_2 + R_1 R_2 Cs}{R_1 + R_2 + R_1 R_2 Cs}$$

$$= \frac{R_2 R_1 C\left(s + \dfrac{1}{R_1 C}\right)}{R_2 R_1 C\left(s + \dfrac{R_1 + R_2}{R_2 R_1 C}\right)}$$

i.e., Voltage transfer ratio

$$= \frac{\mathbf{R_2 R_1 C\left(s + \dfrac{1}{R_1 C}\right)}}{\mathbf{R_2 R_1 C\left(s + \dfrac{R_1 + R_2}{R_1 R_2 C}\right)}}.$$

EXAMPLE 14.4 *Obtain $Z_{12}(s)$ for a parallel R-C network where $R = 1\Omega$; $C = 1$ F.*

SOLUTION. In Fig. E14.4,

$$V_2(s) = I_1(s)\left[\frac{R \times \dfrac{1}{Cs}}{R + \dfrac{1}{Cs}}\right]$$

or, $$\frac{V_2(s)}{I_1(s)} = \frac{R}{RCs + 1} = \frac{1}{\left(s + \dfrac{1}{RC}\right)C}$$

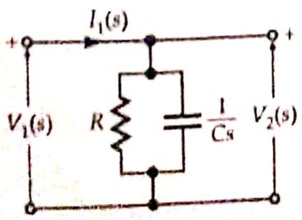


Fig. E14.4

$$\therefore \quad Z_{12}(s) = \frac{V_2(s)}{I_1(s)}$$

$$= \frac{1}{C\left(s + \dfrac{1}{RC}\right)} = \frac{1}{1(s+1)}$$

$$\therefore \quad Z_{12}(s) = \frac{1}{s+1}.$$

EXAMPLE 14.5 *Find (V_2/V_1) and (V_2/I_1) in Fig. E14.5.*

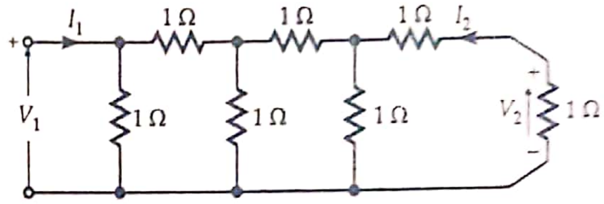


Fig. E14.5

SOLUTION. The figure is redrawn (Fig. E14.6) with marking of nodes and voltages at the node.

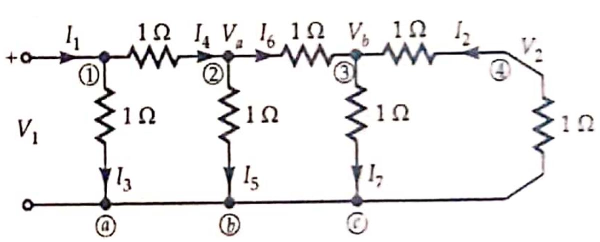


Fig. E14.6

At *node (1)*, application of KCL yields

$$I_1 = I_3 + I_4 = \frac{V_1}{1} + \frac{V_1 - V_a}{1} = 2V_1 - V_a \qquad \ldots(1)$$

At *node (2)*,

$$I_4 = I_5 + I_6$$

or, $$\frac{V_1 - V_a}{1} = \frac{V_a}{1} + \frac{V_a - V_b}{1}$$

or, $$V_1 - V_a = V_a + V_a - V_b$$

or, $$V_1 - 3V_a + V_b = 0 \qquad \ldots(2)$$

At *node (3)*,

$$I_6 = I_7 - I_2$$

or $$\frac{V_a - V_b}{1} = \frac{V_b}{1} - \frac{V_2 - V_b}{1}$$

or, $$V_a - V_b = V_b - V_2 + V_b$$

or $$V_2 + V_a - 3V_b = 0 \qquad \ldots(3)$$

C H A P T E R 14

At node (4) KCL yields,

Current through load of 1Ω = Current flows between nodes (3) and (4)

$$\frac{V_2}{1} = \frac{V_b - V_2}{1} \quad \ldots(4)$$

i.e.,

$$2V_2 - V_b = 0$$

or

From (4), $V_b = 2V_2$ and from (3) we then get $V_a = 5V_2$

From (2), we get

$$V_1 - 3(5V_2) + (2V_2) = 0 \quad \text{or,} \quad V_1 = 13\,V_2$$

and from (1), we get

$$I_1 = 2\,(13\,V_2) - (5V_2) = 21\,V_2$$

Then finally, we get

$$V_2 / V_1 = \frac{1}{13} \text{ and } V_2 / I_1 = \frac{1}{21}.$$

Example 14.6 *Find Z(s) for the following network (Fig. E14.7).*

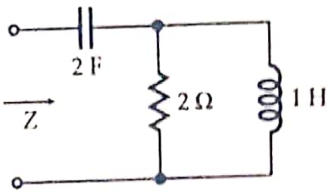


Fig. E14.7

Solution. The transform network is shown in Fig. E14.7(*a*).

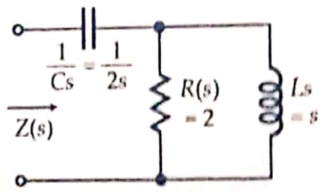


Fig. E14.7 (*a*)

$$Z(s) = [R(s) \,||\, Ls] + \frac{1}{Cs} = \frac{2 \times s}{2 + s} + \frac{1}{2s},$$

$$= \frac{4s^2 + s + 2}{2s(s+2)} = \frac{4s^2 + s + 2}{2s^2 + 4s}$$

$$\therefore \quad Z(s) = \frac{4s^2 + s + 2}{2s^2 + 4s}.$$

Example 14.7 *Find the driving point impedance for the network shown in Fig. E14.8.*

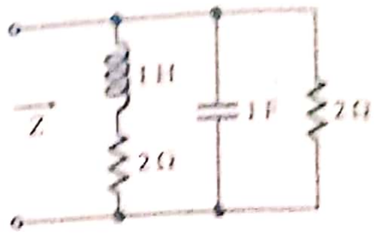


Fig. E14.8

Solution. The transfer network is shown in Fig. E14.8(*a*).

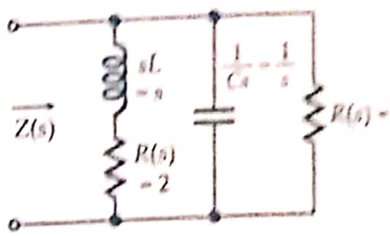


Fig. E14.8 (*a*)

$$\text{Here } Z(s) = \left[R_2(s) \,||\, \frac{1}{Cs}\right] \,||\, [SL + R_1(s)]$$

$$= \frac{2 \times \frac{1}{s}}{2 + \frac{1}{s}} \,||\, (2 + s) = \frac{\frac{2}{s}}{\frac{2s+1}{s}} \,||\, (2 + s)$$

$$= \frac{2}{(2s+1)} \,||\, (2 + s) = \frac{\frac{2}{(2s+1)} \times (2 + s)}{\frac{2}{2s+1} + 2 + s}$$

$$= \frac{2(s+2)}{2 + 4s + 2 + 2s^2 + s}$$

$$= \frac{2(s+2)}{2s^2 + 5s + 4}.$$

i.e., $Z(s)$, the driving point impedance $= \dfrac{2(s+2)}{2s^2 + 5s + 4}$.

Example 14.8 *Find the driving point admittance of the network shown in Fig. E14.9.*

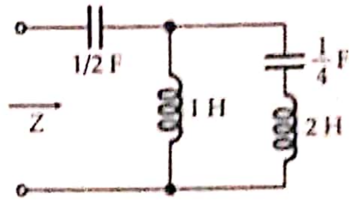


Fig. E14.9

Solution. The transform network is shown in Fig. E14.9(*a*)

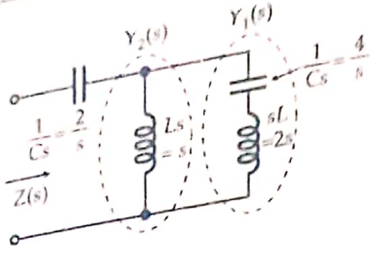


Fig. E14.9 (a)

$$\text{Here } Y_1(s) = \frac{1}{2s + \frac{4}{s}} = \frac{1}{\frac{2s^2+4}{s}} = \frac{s}{2s^2+4}$$

$$Y_2(s) = \frac{1}{s}$$

$$Y_E(s) = Y_1(s) + Y_2(s) = \frac{s}{2s^2+4} + \frac{1}{s}$$

$$= \frac{3s^2+4}{2s^3+4s}$$

$$\therefore \quad Z_E(s) = \frac{1}{Y(s)} = \frac{2s^3+4s}{3s^2+4}$$

$$\text{Thus } \quad Z(s) = Z_E(s) + \frac{1}{Cs}$$

$$= \frac{2s^3+4s}{3s^2+4} + \frac{2}{s}$$

$$= \frac{2s^4+10s^2+8}{3s^3+4s}$$

$$\because \quad Y(s) = \frac{1}{Z(s)},$$

here $$Y(s) = \frac{3s^3+4s}{2s^4+10s^2+8}.$$

EXAMPLE 14.9 *Find the transfer function of the network shown in Fig. E14.10.*

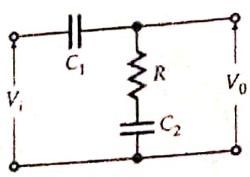


Fig. E14.10

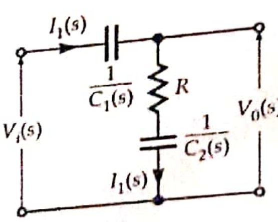


Fig. E14.10 (a)

SOLUTION. Transforming the given network in the Laplace domain [Fig. E14.10(a)].

$$V_i(s) = I_1(s)\left[R + \frac{1}{C_1 s} + \frac{1}{C_2 s}\right] \qquad \ldots(1)$$

and
$$V_0(s) = I_1(s)\left[R + \frac{1}{C_2 s}\right]$$

$$\therefore \quad \frac{V_0(s)}{V_i(s)} = \frac{R + \frac{1}{C_2 s}}{R + \frac{1}{C_1 s} + \frac{1}{C_2 s}}$$

$$= \frac{(RC_2 s + 1) C_1 s}{(RC_1 C_2 s^2 + C_1 s + C_2 s)}$$

$$= \frac{C_1 (1 + RC_2 s)}{C_1 + C_2 + RC_1 C_2 s}$$

i.e., the transfer function (voltage transfer ratio) is given by $\left[\dfrac{C_1 (1 + RC_2 s)}{C_1 + C_2 + RC_1 C_2 s}\right]$.

EXAMPLE 14.10 *Find the expression of voltage transfer ratio for the network shown in Fig. E14.11.*

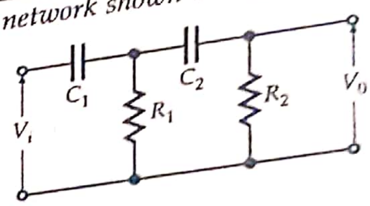


Fig. E14.11

SOLUTION. The equivalent circuit of Fig. E14.11 in Laplace domain is shown in Fig. E14.11(*a*).

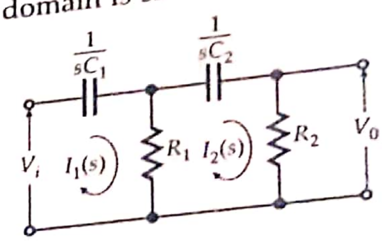


Fig. E14.11 (*a*)

Here, in the *leftmost loop*,

$$V_i(s) = I_1(s)\left[R_1 + \frac{1}{C_1 s}\right] - R_1 I_2(s) \qquad \ldots(a)$$

in the *second loop*,

$$-R_1 I_1(s) + \left(R_1 + R_2 + \frac{1}{C_2 s}\right) I_2(s) = 0 \qquad \ldots(b)$$

and in the *rightmost loop*,

$$R_2 I_2(s) = V_0(s) \qquad \ldots(c)$$

CHAPTER 14

Solving for $I_2(s)$,

$$I_2(s) = \frac{\begin{vmatrix} R_1 + \frac{1}{C_1 s} & V_i(s) \\ -R_1 & 0 \end{vmatrix}}{\begin{vmatrix} R_1 + \frac{1}{C_1 s} & -R_1 \\ -R_1 & R_1 + R_2 + \frac{1}{C_2 s} \end{vmatrix}}$$

$$\text{or, } I_2(s) = \frac{R_1 V_i(s) C_1 s C_2 s}{(R_1 + R_2) C_2 s + 1 + R_1 C_1 s (R_2 C_2 s + 1)}$$

$\therefore$ From (c),

$$V_0(s) = I_2(s) R_2$$

$$= \frac{R_1 R_2 V_i(s) C_1 s C_2 s}{(R_1 + R_2) C_2 s + 1 + R_1 C_1 S (R_2 C_2 s + 1)}$$

i.e.,

$$\frac{V_0(s)}{V_i(s)} = \frac{R_1 R_2 C_1 s C_2 s}{(R_1 + R_2) C_2 s + 1 + R_1 C_1 s (R_2 C_2 s + 1)}$$

i.e., Voltage transfer ratio

$$= \frac{(R_1 C_1)(R_2 C_2) s^2}{(R_1 + R_2) C_2 s + 1 + s (R_1 C_1)(R_2 C_2 s + 1)}.$$

14.5 CONCEPT OF POLES AND ZEROS IN A NETWORK FUNCTION

A network function $H(s)$ may be written as

$$H(s) = \frac{A(s)}{B(s)} = \frac{a_0 s^n + a_1 s^{n-1} + \ldots + a_{n-1} s + a_n}{b_0 s^m + b_1 s^{m-1} + \ldots + b_{m-1} s + b_m} \quad \ldots(14.10)$$

where, $a_0, a_1, a_2, \ldots\ldots, a_n$ and $b_0, b_1, \ldots\ldots, b_m$ are the coefficients of the polynomials $A(s)$ and $B(s)$. They are real and +ve for a passive network.

Factorising the numerator and denominator, the network function can be rewritten as

$$H(s) = \frac{A(s)}{B(s)} = \frac{a_0 (s - z_1)(s - z_2)\ldots\ldots(s - z_n)}{b_0 (s - p_1)(s - p_2)\ldots\ldots(s - p_m)} \quad \ldots(14.11)$$

where, $z_1, z_2, \ldots\ldots, z_n$ are the n roots for $A(s) = 0$;

$p_1, p_2, \ldots\ldots, p_m$ are the m roots for $B(s) = 0$;

(a_0 / b_0) is a constant (say K) and is called *scale factor*.

Here, $z_1, z_2, \ldots\ldots, z_n$ are called "*zeros*" and denoted by a small circle in the complex frequency diagram while $p_1, p_2, \ldots\ldots, p_m$ are called "*poles*" and denoted by a "*cross*" on the plot. The network function $H(s)$ becomes zero if "s" is equal to any of the zeros and becomes infinity when s is equal to any of the poles.

The network function is said to be having simple poles or simple zeros if the poles and zeros are not repeated. On the other hand, if the poles and zeros are repetitive, the function is said to have multiple poles or zeros.

EXAMPLE 14.11 *A function is given by* $Z(s) = \frac{s+4}{s}$

Find the pole-zero plot.

SOLUTION. In the function $Z(s) = \frac{s+4}{s}$, the zero is at $s = -4$ and pole is at $s = 0$. This gives the pole-zero plot as shown in Fig. E14.12.

Fig. E14.12

EXAMPLE 14.12 *A function is given by* $Z(s) = \frac{2s}{s^2 + 16}$

Draw its pole zero plot.

SOLUTION. For the given function factorisation of denominator gives

$$(s^2 + 16) = (s + j4)(s - j4)$$

$$\therefore \quad Z(s) = \frac{2s}{(s + j4)(s - j4)}$$

The scale factor is 2, poles are at $s = +j4$ and $-j4$, zero is at $s = 0$. The pole-zero plot is shown in Fig. E14.13.

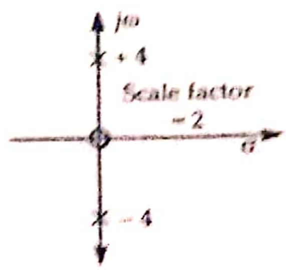


Fig. E14.13

EXAMPLE 14.13 *An admittance function is given by*

$$Y(s) = \frac{1}{s+2}.$$

Find the pole-zero plot.

SOLUTION. For the given function, the pole is at $s = -2$ and the zero is at infinity. The plot is shown in Fig. E14.14.

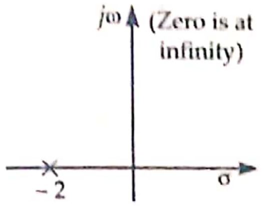


Fig. E14.14

EXAMPLE 14.14 *A network function is given by*

$$P(s) = \frac{2s}{(s+2)(s^2+2s+2)}$$

Obtain the pole-zero diagram.

SOLUTION. In the given function, on factorisation, the denominator is obtained as

$$(s+2)(s+1-j\,1)(s+1+j\,1).$$

Hence, the function becomes

$$P(s) = \frac{2s}{(s+2)(s+1-j\,1)(s+1+j\,1)}.$$

Then the poles are located at $s = -2, (-1+j\,1), (-1-j\,1)$ and the zero is at $S = 0$.

The scale factor is 2. [Fig. E14.15]

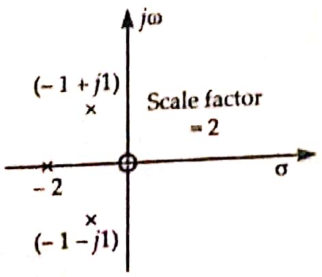


Fig. E14.15

14.6 RESTRICTION ON LOCATION OF POLES AND ZEROS IN DRIVING POINT FUNCTIONS

The following list provides the listing of restrictions of locations of poles and zeros in driving point function :

(*a*) The co-efficients of the polynomials $A(s)$ and $B(s)$ of the network function $H(s)$ must be real and positive.

(*b*) Poles and zeros, if complex or imaginary, must occur in conjugate pairs.

(*c*) The real parts of all poles and zeros must be zero or negative.

[If the real part is zero, then the pole and zero must be simple]

(*d*) The polynomial $A(s)$ or $B(s)$ cannot have any missing term between those of highest and lowest order values unless all even order or all odd order terms are missing.

(*e*) The degree of $A(s)$ and $B(s)$ may differ by zero or one only.

(*f*) The lowest degree in $A(s)$ and $B(s)$ may differ in degree by at the most one.

EXAMPLE 14.15 *Check, if the driving point impedance* $Z(s)$, *given by* $Z(s) = \dfrac{s^4+s^2+1}{s^3+2s^2-2s+10}$ *can represent a passive one port network.*

SOLUTION. The given function is not suitable to represent the impedance of a one port network due to following reasons :

(*i*) In the numerator, one coefficient is missing.

(*ii*) In the denominator, one coefficient is negative.

EXAMPLE 14.16 *Explain, with reasons, why the following expression for the driving point impedance Z(s) is not suitable for representing a passive network.*

$$Z(s) = \frac{s^4 - s^3 + 2s^2}{s+5}$$

SOLUTION. The given function is not suitable for representating the driving point impedance due to following reasons :

(*i*) In the numerator, one coefficient is –ve

(*ii*) The degree of numerator is 4 while that of denominator is 1. Thus a difference of 3 exists between the degree of numerator and denominator and is not permitted.

(*iii*) The numerator gives $s^2(s^2 - s + 2)$ *i.e.*, double zero, at $s = 0$, $s = -0.5 + j\sqrt{\frac{7}{4}}$ and $s = -0.5 - j\sqrt{\frac{7}{4}}$. This is not permitted.

(*iv*) The term of the lowest degree in numerator is of 2 while that in the denominator is zero.

CHAPTER 14

14.7 NECESSARY CONDITIONS FOR TRANSFER FUNCTIONS

The limitations on pole and zero location in transfer function are as follows :

(i) The coefficients of polynomials $A(s)$ and $B(s)$ are to be real; coefficients of $B(s)$ must be +ve though few coefficients of $A(s)$ may be −ve.

(ii) The poles and zeros; if complex or imaginary, must be in conjugate.

(iii) The real part of poles must be negative or zero. If the real part is zero, the pole must be simple.

(iv) The polynomial $A(s)$ may have missing terms between the lowest and highest degree. However, polynomial $B(s)$ should not have any missing term between the highest and lowest degree unless all even or all odd terms are missing.

(v) Degree of $A(s)$ may be zero, independent of the degree of $B(s)$.

(vi) For the voltage or current transfer ratio the maximum degree of $A(s)$ must equal the degree of $B(s)$.

(vii) For the transfer impedance and admittance, the maximum degree of $A(s)$ must equal the degree of $B(s)$ plus one.

14.8 NATURAL RESPONSE OF A NETWORK

The *natural response* of a network is the response that occurs when the source *excitation is reduced to zero value* and the system responds to its *initial conditions* (*i.e.*, initial stored energy). It is then also called *source free response* or *zero state behaviour*.

Basically, the source free response is the *complementary* solution of the homogeneous differential equation. It is called the *transient response* too. It depends only on the values of the elements and their interconnection but independent of the excitation. The natural response of the system may also be described as the response to *unit impulse excitation* and is characterised by the natural frequencies of oscillation of the system (*natural frequencies being defined as those to which a network*

On the other hand, if a source excitation is applied with frequency ω_0, the network is forced to react at that frequency. However, the source "being unit impulse function, it does not force any specific frequency in the network and simple introduces some energy instantaneously and then leaves the network to react naturally".

Thus, a network, being exited by an impulse function, reacts *naturally* and the resulting response singularities are defined as the *natural frequencies*.

14.8.1 Open Circuit Natural Frequency (OCNF)

Let the drivng point impedance be given as

$$Z_{11}(s) = K\frac{(s-z_1)(s-z_2)\ldots(s-z_n)}{(s-p_1)(s-p_2)\ldots(s-p_m)} \quad \ldots(14.12)$$

Also, $V_1(s) = Z_{11}(s)\, I_1(s)$.

With $I_1(s) = 0$, $V_1(s)$ becomes zero unless $Z_{11}(s)$ be infinite *i.e.*, a voltage to exist at the open circuit input with no excitation, it is necessary that $Z_{11}(s)$, the driving point impedance is infinite. The poles of $Z_{11}(s)$ *i.e.*, $p_1, p_2, \ldots, p_n$ are then the *natural frequencies* of the network with input open. These frequencies are called *open circuit natural frequencies* of that network.

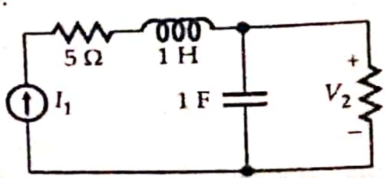
14.8.2 Short Circuit Natural Frequencies (SCNF)

Also, for a network,

$$I_1(s) = Y_{11}(s)\, V_1(s) = \frac{1}{Z_{11}(s)} V_1(s).$$

If $V_1(s)$ is zero, *i.e.*, the input ports are shorted, $I_1(s)$ would be zero unless $Y_{11}(s)$ is infinite *i.e.*, $Z_{11}(s)$ is zero. Thus, for a current to exist with short circuited input, it is necessary that the driving point impedance is zero. Then the zeros of $Z_{11}(s)$ *i.e.*, $z_1, z_2, \ldots$ represent the short circuit natural frequencies of the network.

EXAMPLE 14.17 *Find the pole-zero plots of the driving point and transfer impedances of the network shown in Fig. E14.16.*



SOLUTION. Let us first transform the given circuit to s-domain as shown in Fig. E14.17 ; where Z_L (s) is the impedance of circuit across terminals x-y.

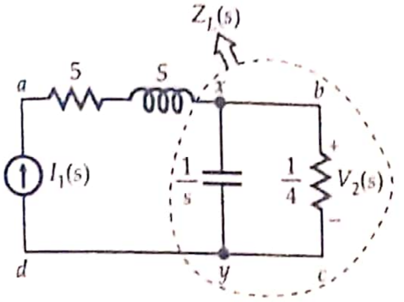


Fig. E14.17

Obviously, $Z_L(s) = \dfrac{1}{Y_L(s)} = \dfrac{1}{s+4}$

$\because$ $V_2(s) = I_1(s)\, Z_L(s)$

$$\frac{V_2(s)}{I_1(s)} = Z_L(s) = \frac{1}{s+4}$$

Also, if V_1 (s) be the voltage across the current source, application of KVL in the *loop abcd* (Fig. E14.17) yields,

$$(5+s)\, I_1(s) + V_2(s) = V_1(s)$$

or,
$$V_1(s) = I_1(s)\left[s + 5 + \frac{1}{s+4}\right]$$

$$= I_1(s)\left[\frac{s^2 + 9s + 20 + 1}{s+4}\right]$$

$$\therefore \quad \frac{V_1(s)}{I_1(s)} = \frac{s^2 + 9s + 21}{s+4}$$

i.e., we ultimately obtained

$$Z_{11}(s) = \frac{s^2 + 9s + 21}{s+4}$$

$$Z_{12}(s) = \frac{1}{s+4}$$

Pole-zero plot of Z_{11} (s)

The numerator of Z_{11} (s) has roots

$$s_{1,2} = \frac{-9 \pm \sqrt{81 - 84}}{2} = -4.5 \pm j\frac{\sqrt{3}}{2}$$

$\therefore$ Z_{11} (s) has zeros at $\left(-4.5 \pm j\dfrac{\sqrt{3}}{2}\right)$

while pole at -4 on s plane.

Pole zero plot of Z_{21} (s)

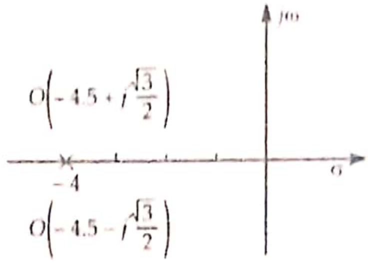


Fig. E14.18

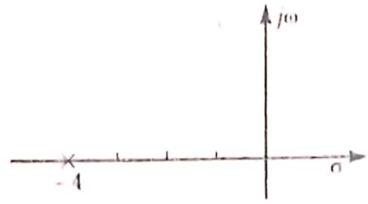


Fig. E14.19

As there is no "s" term in the numerator of Z_{21} (s), it has **no define** *zero* while the pole is at -4 on s plane.

EXAMPLE 14.18 *In the circuit of Fig. E14.20, find Z_{in} (s). Also find the voltage transfer function.*

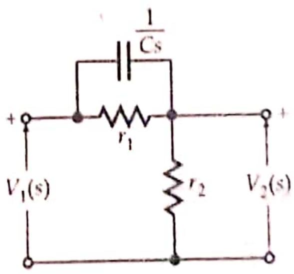


Fig. E14.20

SOLUTION. $Z_{in}(s) = \left(r_1 \,||\, \dfrac{1}{Cs}\right) + r_2$

$$= \frac{\frac{r_1}{Cs}}{r_1 + \frac{1}{Cs}} + r_2 = \frac{r_1}{r_1 Cs + 1} + r_2$$

i.e.,
$$Z_{in}(s) = \frac{r_1 + r_1 r_2 Cs + r_2}{(r_1 Cs + 1)}.$$

$$\therefore \quad I(s) = \frac{V_1(s)}{Z_{in}(s)} = \frac{V_1(s)}{\frac{r_1 + r_1 r_2 Cs + r_2}{r_1 Cs + 1}}$$

$$= V_1(s)\, \frac{r_1 Cs + 1}{r_1 + r_2 r_1 Cs + r_2}$$

$V_2(s) = r_2 I(s) = V_1(s)\dfrac{r_1 r_2 Cs + r_2}{r_1 + r_2 + r_1 r_2 Cs}$

i.e. Voltage transfer function

$$\frac{V_2(s)}{V_1(s)} = \frac{r_1 r_2 Cs + r_2}{r_1 + r_2 + r_1 r_2 Cs}.$$

EXAMPLE 14.19 *Find $Z_{21}(s)$ in the circuit of Fig. E14.21.*

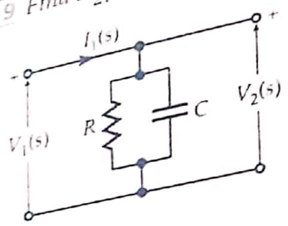


Fig. E14.21

SOLUTION. $Y_{(input)}(s) = \dfrac{1}{R} + Cs$

However,

$$I_1(s) = V_1(s)\, Y_{in}(s)$$

or,

$$I_1(s) = V_1(s)\left[\frac{1}{R} + Cs\right]$$

But $V_2(s) = V_1(s)$ [by observation] ;

$$\therefore \quad \frac{V_2(s)}{I_1(s)} = \frac{1}{\frac{1}{R} + Cs} = \frac{1}{C\left[s + \frac{1}{RC}\right]}$$

$$Z_{12}(s) = \frac{1}{C\left[s + \frac{1}{RC}\right]}.$$

EXAMPLE 14.20 *Find $G_{21}(s)$, Voltage transfer for ratio, for the network shown in Fig. E14.22.*

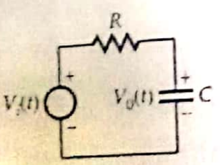


Fig. E14.22 .

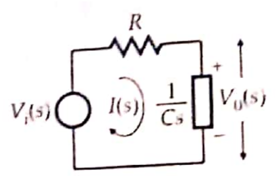


Fig. E14.22 (*a*)

SOLUTION. Transforming the given circuit of Fig. E14.22(*a*), into the *s*-domain circuit, we get,

$$V_i(s) = I(s)\left[R + \frac{1}{Cs}\right] \qquad \ldots(1)$$

and

$$V_0(s) = I(s)\cdot\frac{1}{Cs} \qquad \ldots(2)$$

$$\therefore \quad \frac{V_0(s)}{V_i(s)} = \frac{1/Cs}{R + \frac{1}{Cs}} = \frac{1}{1 + sCR} = \frac{1}{CR}\left[\frac{1}{s + \frac{1}{RC}}\right]$$

Then,

$$G_{12}(s) = \frac{1}{RC\left(s + \frac{1}{RC}\right)}.$$

It may be noted that this function *does not have any definite zero* as there is no *s* term in the numerator. The pole is at $-\dfrac{1}{RC}$, *i.e.*, on the –ve real axis.

EXAMPLE 14.21 *In the network of Fig. E14.23, find $\dfrac{V_C}{V}$. Also find the pole zero locations.*

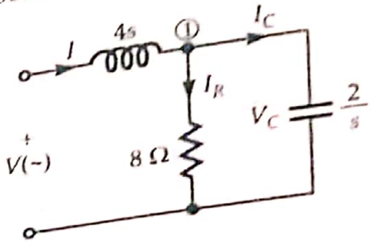


Fig. E14.23

SOLUTION. Using KCL at *node (1)*,

$$\frac{V_C - V}{4s} + \frac{V_C}{8} + \frac{V_C}{2/s} = 0$$

or,

$$\frac{V_C}{4s} - \frac{V}{4s} + \frac{V_C}{8} + \frac{V_C \cdot s}{2} = 0$$

or,

$$V_C\left[\frac{1}{4s} + \frac{1}{8} + \frac{s}{2}\right] = \frac{V}{4s}$$

or,

$$V_C\left[\frac{2 + s + 4s^2}{8s}\right] = \frac{V}{4s}$$

$$\therefore \quad V_C = V\frac{2}{2 + s + 4s^2}$$

giving

$$\frac{V_C}{V} = \frac{2}{4s^2 + s + 2}$$

Again,

$$\frac{V_C}{V} = \frac{2}{4s^2 + s + 2} = H(s), \text{ Say.}$$

i.e.,

$$G_{12}(s) = \frac{2}{\left(s + \frac{1}{8} - j\frac{\sqrt{31}}{8}\right)\left(s + \frac{1}{8} + j\frac{\sqrt{31}}{8}\right)}$$

This function $H(s)$ **does not have any zero** as the s term is absent in the numerator. The poles are at $\left(-\frac{1}{8}+j\frac{\sqrt{31}}{8}\right)$ and at $\left(-\frac{1}{8}-j\frac{\sqrt{31}}{8}\right)$ in the s-plane (Fig. E14.24).

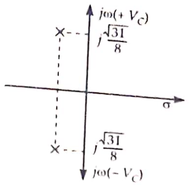


Fig. E14.24

EXAMPLE 14.22 *Find the pole zero locations of the current transfer ratio I_2/I_1 in s-domain for circuit shown in Fig. E14.25.*

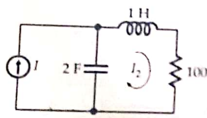


Fig. E14.25

SOLUTION. By the current division formula, the current $I_2(s)$ in Fig. E14.25(*a*) (the *s*-domain circuit of the given network) is given by

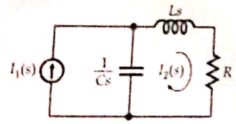


Fig. E14.25 (*a*)

$$I_2(s) = I_1(s)\frac{\frac{1}{Cs}}{R+Ls+\frac{1}{Cs}}$$

or,
$$\frac{I_2(s)}{I_1(s)} = \frac{\frac{1}{Cs}}{R+Ls+\frac{1}{Cs}}$$

$$= \frac{1}{1+sCR+s^2 LC}$$

$$= \frac{1}{1+200\,s+2s^2} = \frac{1}{D}$$

where $D = 1 + 200\,s + 2s^2$

$$\therefore \quad s_1, s_2 = \frac{-200 \pm \sqrt{200^2-8}}{2}$$

$$= -100 \pm \sqrt{10{,}000-2}$$

$$= -100 \pm 99.99 = \text{either } 0 \text{ or } -200$$

i.e., **"zero" being not existing** (as numerator of $I_2(s)/I_1(s)$ does not contain s term), the poles are **either at zero or at −200.**

14.9 MAGNITUDE AND PHASE ANGLE OF THE COEFFICIENTS IN THE NETWORK FUNCTION

Let the network function be rewritten as shown in eqn. (14.11).

$$H(s) = \frac{A(s)}{B(s)} = K\frac{(s-z_1)(s-z_2)\ldots\ldots(s-z_n)}{(s-p_1)(s-p_2)\ldots\ldots(s-p_m)}$$

$$= \frac{K_a}{s-p_1}+\frac{K_b}{s-p_2}+\ldots+\frac{K_r}{(s-p_r)}+\ldots+\frac{K_m}{(s-p_m)} \quad \ldots(14.13)$$

Here,

$$K_r = K\frac{(s-z_1)(s-z_2)\ldots(s-z_n)}{(s-p_1)(s-p_2)\ldots(s-p_r)\ldots(s-p_m)}(s-p_r)\Bigg|_{s=p_r}$$

or, $$K_r = K\frac{(p_r-z_1)(p_r-z_2)\ldots(p_r-z_n)}{(p_r-p_1)(p_r-p_2)\ldots(p_r-p_m)} \quad \ldots(14.14)$$

Expression (14.14) consists of factors of the general form $(p_r - p_n)$, where p_r and p_n are known *complex numbers*. The difference of two complex numbers is also another complex number which may be written in polar form as

$$(p_r - p_n) = M_{nr}\, e^{j\phi_{nr}} \quad \ldots(14.15)$$

M_{nr} being the magnitude of the phasor $(p_r - p_n)$ and ϕ_{nr} the phase angle of the same phasor.

Thus,

$$K_r = K\frac{M_{1r}\,.\,M_{2r}\,.\,M_{3r}\,\ldots\ldots\,M_{nr}}{M_{ar}\,.\,M_{br}\,.\,M_{cr}\,\ldots\,M_{mr}}\, e^{j(\phi_{1r}+\phi_{2r}+\ldots-\phi_{ar}-\phi_{br}-\ldots)} \quad \ldots(14.16)$$

The above equation (14.16) gives the magnitude and phase angle of the coefficient K_r. Thus all the coefficients from K_a to K_m can be evaluated.

The magnitude of $M_{1r}, M_{2r}, \ldots\ldots$ and $M_{ar}, M_{br}, \ldots\ldots$ can be evaluated from pole and zero plot on the s-plane. In a similar way, the phase angles $\phi_{1r}, \phi_{2r}, \ldots$ and $\phi_{ar}, \phi_{br}, \ldots$ can also be measured from the same pole-zero plot.

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zeros of the given network function to scale.

Step 2 Calculate (or measure) the distances $M_1, M_2, \ldots$ of a given pole from each of the zeros. Also calculate (or measure) the distances $M_a, M_b, \ldots$ of the same pole from each of the other finite poles.

Step 3 Calculate or measure the angle from each of the other finite poles (i.e. $\phi_a, \phi_b, \ldots$) as well as from each of the other zeros ($\phi_1, \phi_2, \ldots$).

Step 4 Substitute these values in equation (14.16) to get the value of K_1.

Step 5 Repeat the above procedure for all the coefficients $K_1, K_2, \ldots$

14.10 ROUTH-HURWITZ CRITERION OF STABILITY OF NETWORK FUNCTION

The stability of a network function can be observed utilising following steps –

(i) The array is to be constructed first.

(ii) Two rows of coefficients are formed, first row containing even numbered coefficients and the second row odd number coefficients.

(iii) The array is to be completed.

Illustration

Let the polynomial be

$$B(s) = b_0 s^m + b_1 s^{m-1} + b_2 s^{m-2} + \ldots + b_m.$$

First row and second row coefficients give

$$\begin{array}{lll} b_0 & b_2 & b_4 \ldots\ldots \\ b_1 & b_3 & b_5 \ldots\ldots \end{array}$$

Let $m = 5$, the array will contain $(m+1)$ i.e. 6 rows.

Array

s^5	b_0	b_2	b_4
s^4	b_1	b_3	b_5
s^3	c_1	c_2	
s^2	d_1	d_2	
s^1	e_1		
s^0	f_1		

$$c_1 = \frac{\begin{vmatrix} b_0 & b_2 \\ b_1 & b_3 \end{vmatrix}}{b_1} = \frac{b_1 b_2 - b_0 b_3}{b_1}$$

$$c_2 = \frac{\begin{vmatrix} b_0 & b_4 \\ b_1 & b_5 \end{vmatrix}}{b_1} = \frac{b_1 b_4 - b_0 b_5}{b_1}$$

$$d_1 = \frac{\begin{vmatrix} b_1 & b_3 \\ c_1 & c_2 \end{vmatrix}}{c_1} = \frac{c_1 b_3 - b_1 c_2}{c_1}$$

$$d_2 = \frac{\begin{vmatrix} b_1 & b_5 \\ c_1 & 0 \end{vmatrix}}{c_1} = \frac{b_5 c_1}{c_1} = b_5$$

$$e_1 = \frac{\begin{vmatrix} c_1 & c_2 \\ d_1 & d_2 \end{vmatrix}}{d_1} = \frac{c_2 d_1 - c_1 d_2}{d_1}$$

$$f_1 = \frac{\begin{vmatrix} d_1 & d_2 \\ e_1 & 0 \end{vmatrix}}{e_1} = \frac{d_2 e_1}{e_1} = d_2$$

According to *Routh Hurwitz criterion*, the system is said to be stable, *if and only if, there are no changes in signs of the first column of the array.* This gives the roots with negative real parts and hence gives the condition of stability.

EXAMPLE 14.23 *Show the pole-zero plot of the given network function $V(s)$ and obtain $v(t)$.*

$$V(s) = \frac{10\,s}{(s+3)(s+2)}$$

SOLUTION. Using the techniques of partial fraction,

$$V(s) = \frac{10\,s}{(s+3)(s+2)} = \frac{K_1}{(s+3)} + \frac{K_2}{(s+2)}$$

$$v(t) = K_1 e^{-3t} + K_2 e^{-2t} \quad \ldots (i)$$

The pole zero plot is shown in Fig. E14.26.

First we consider the pole at –2.

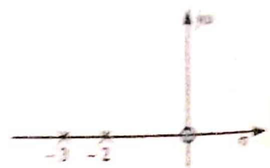

Fig. E14.26

$\therefore M_{02}$ (distance between "zero" to "pole" at $-2) = 2$

and $\phi_{02} = 180°$

Also, $M_{32} = 1$ and $\phi_{32} = 0°$

[$\because$ the distance between pole at -3 and pole at -2 is 1 unit and ϕ_{32} is directed in opposite sense to ϕ_{02}]

$$\therefore \quad K_2 = H \frac{M_{02}\, e^{j\phi_{02}}}{M_{32}\, e^{j\phi_{32}}} = 10 \frac{2e^{j180°}}{e^{j0°}}$$

$$= 20\, e^{j180°} = -20.$$

Next we consider the pole at -3.

$\therefore \quad M_{03} = 3\,;\ \phi_{03} = 180°$

Also, $M_{23} = 1\,;\ \phi_{23} = 180°$

$$\therefore \quad K_1 = H \frac{M_{03}\, e^{j\phi_{03}}}{M_{23}\, e^{j\phi_{23}}}$$

$$= 10 \frac{3\, e^{j180°}}{e^{j180°}} = 30$$

This gives $v(t) = 30\, e^{-3t} - 20\, e^{-2t}$.

EXAMPLE 14.24 *Obtain the pole zero diagram of the given function and obtain the time domain response.*

$$I(s) = \frac{2s}{(s+1)(s^2+2s+4)}$$

SOLUTION. Using partial fraction,

$$I(s) = \frac{2s}{(s+1)(s^2+2s+4)}$$

$$= \frac{K_1}{(s+1)} + \frac{K_2}{(s+1-j\sqrt{3})} + \frac{K_3}{(s+1+j\sqrt{3})}$$

$$\therefore i(t) = K_1\, e^{-t} + K_2\, e^{-(1-j\sqrt{3})t} + K_3\, e^{-(1+j\sqrt{3})t}$$

The pole zero plot has been exhibited in the adjacent figure (Fig. E14.27).

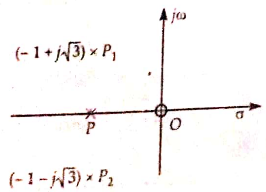

Fig. E14.27

$$K_1 = H \frac{M_{OP}\, e^{j\phi_{OP}}}{M_{P_1-P}\, M_{P_2-P}\, e^{j(\phi_{P_1-P} + \phi_{P_2-P})}}$$

$$= 2 \frac{1 \cdot e^{j180}}{(\sqrt{3} \times \sqrt{3})\, e^{j(-90°+90°)}}$$

[$\because M_{OP} = 1\,;\ M_{P_1-P} = M_{P_2-P} = \sqrt{3}$
$\phi_{OP} = 180°\,;\ \phi_{P_1-P} = -90°\,;\ \phi_{P_2-P} = 90°$]

$\therefore \quad K_1 = 0.67\, e^{j180°} = -0.67$

and
$$K_2 = H \frac{M_{OP_1}\, e^{j\phi_{OP_1}}}{M_{P-P_1}\, M_{P_2-P_1}\, e^{j(\phi_{P-P_1} + \phi_{P_2-P_1})}}$$

(Please check with Fig. E14.28)

$$= 2 \frac{2e^{j\left[\tan^{-1}\frac{OP}{PP_1} + 90°\right]}}{(\sqrt{3} \times 2\sqrt{3})\, e^{j(90°+90°)}}$$

$$= 0.67\, e^{j\left[\tan^{-1}\frac{OP}{PP_1} + 90° - 180°\right]}$$

$$= 0.67\, e^{-j(60°)}$$

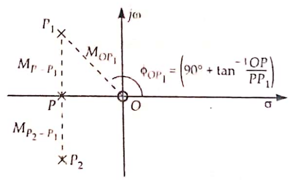

Fig. E14.28

$\therefore K_3$, being the conjugate of K_2 will be given by

$$K_3 = 0.67\, e^{j(60°)}\ [K_3 = K_2^*]$$

$$\therefore \quad i(t) = -0.67\, e^{-t} + 0.67\, e^{+j(60°)}\, e^{-(1+j\sqrt{3})t} + 0.67\, e^{-j(60°)}\, e^{-(1-j\sqrt{3})t}.$$

EXAMPLE 14.25(a) *Check the stability of the following polynomial by applying Routh-Hurwitz criterion :*

$$P(s) = s^4 + 2s^3 + 4s^2 + 12s + 10$$

SOLUTION. Routh array of the polynomial can be obtained from the following coefficients.

$$b_0 = 1\,;\ b_2 = 4\,;\ b_4 = 10\,;\ b_1 = 2\,;\ b_3 = 12$$

$$c_1 = \frac{b_1\, b_2 - b_0\, b_3}{b_1} = \frac{8-12}{2} = -2$$

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$$= \frac{20 - 1\times 0}{2} = 10$$

$$= \frac{2\times 12 - 2\times 10}{-2} = 22$$

$$d_1 = \ldots$$

$$d_2 = b_3 = 0$$

$$e_1 = \frac{c_2 d_1 - c_1 d_2}{d_1} = \frac{10\times 16 - (-2)\times 0}{16} = 10$$

$$f_1 = d_2 = 0$$

This gives

s^4	1	4	10
s^3	2	12	
s^2	−2	10	
s^1	22	0	
s^0	10		

There is a sign change in the 1st column of the array. Hence the system is **not stable.**

EXAMPLE 13.25(b) *For what value of 'K' the polynomial $s^4 + 10s^3 + 35s^2 + Ks + 24 = 0$ is said to be stable ? Solve by using Routh Hurwitz Array.*

SOLUTION. Given Polynomial is

$$s^4 + 10s^3 + 35s^2 + Ks + 24 = 0.$$

Routh Hurwitz Array is shown below

s^4	1	35	24
s^3	10	K	
s^2	$\frac{35\times 10 - K\times 1}{10} = a_1$	$\frac{10\times 24 - 0\times 1}{10} = 24$	
s^1	$\frac{a_1\times K - 24\times 10}{a_1}$		
s^0	24		

For stability of the network, $a_1 > 0$

i.e., $\frac{350 - K}{10} > 0$ or $K < 350$.

Hence the range K is $(0 < K < 350)$.

EXAMPLE 14.26(a) *A polynomial is given by*

$$P(s) = s^3 + 2s^2 + 4s + M.$$

"M" is adjustable. Apply Routh Hurwitz criterion.

SOLUTION. Routh array gives

s^3	1	4
s^2	2	M
s^1	$\frac{8-M}{2}$	
s^0	M	

We see that if M is less than 8, the system is stable. There will be no change in the sign of 1st column. If $M = 8$, then also there is no change in sign in the first column and hence the system will also be stable.

EXAMPLE 13.26(b) *Check whether the following polynomial $P(s) = s^4 + s^3 + 2s^2 + 2s + 3$ is stable or not Comment on your findings.*

SOLUTION. Given polynomial is

$$P(s) = s^4 + s^3 + 2s^2 + 2s + 3$$

where total number of rows are $(4 + 1) = 5$

Routh Hurwitz Array is shown below :

s^4	1	2	3
s^3	1	2	
s^2	$\frac{2-2=0}{1} = 0$	3	
s^1	∞		
s^0			

In above array, we found third row of first column has a zero value. Hence the subsequent value is infinite. Let us now assume a small value ε in place of the zero value. The modified array is given below :

s^4	1	2	3
s^3	1	2	
s^2	ε	3	
s^1	$\frac{2\varepsilon - 3}{\varepsilon} > 0$		
s^0	3		

For stability point of view, $\varepsilon > 0$ *i.e.*, $\frac{2\varepsilon - 3}{\varepsilon} > 0$.

This gives $\varepsilon > 3/2$

Hence to have the stability of the function, we should have any value of ε more than $(3/2)$.

EXAMPLE 14.27 *What should be the value of L such that $Z(s) = 1$ in the network of Fig. E14.29 ?*

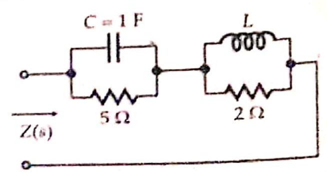

Fig. E14.29

Solution. Let us first convert the network of Fig. E14.29 to s-domain [Fig. E14.29(a)]

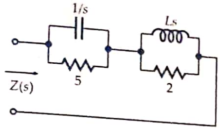

Fig. E14.29 (a)

$$Z(s) = \frac{5 \times \frac{1}{Cs}}{5 + \frac{1}{Cs}} + \frac{2 \times Ls}{2 + Ls} = \frac{5}{5s+1} + \frac{2\ Ls}{2 + Ls}$$

$$= \frac{10 + 5\ Ls + 10\ Ls^2 + 2\ Ls}{(5s+1)(2 + Ls)}$$

$$= \frac{10 + 7Ls + 10\ Ls^2}{(5s+1)(2 + IS)}$$

However, $Z(s) = 1$

$$\therefore \quad \frac{10 + 7\ Ls + 10Ls^2}{10\ s + 5Ls^2 + 2 + Ls} = 1$$

or, $\quad 5Ls^2 + 6Ls - 10\ s + 8 = 0$

$$\therefore \quad L = \frac{10\ s + 8}{s\ (5s + 6)}.$$

14.11 ADDITIONAL EXAMPLES

Example 14.28 *Find Z_{in} in Laplace domain in the* [...]*uit of Fig. E14.30 and find the voltage transfer* [...]*ction of the same network if a voltage V_1 (s) is applied at 1 - 1′ terminal that produces a drop V_2 (s) across r_2.*

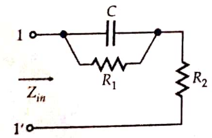

Fig. E14.30

Solution. Let us transform the given network to s-domain first (Fig. E14.31)

$$Z_{in}\ (s) = \left[\frac{1}{Cs} \,||\, R_1\right] + R_2 = \frac{R_1}{1 + R_1\ Cs} + R_2$$

$$= \frac{R_1 + R_2 + R_1\ R_2\ Cs}{1 + R_1\ Cs}$$

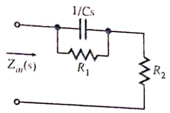

Fig. E14.31

or, $$Z_{in}\ (s) = \frac{s + \frac{1}{R_2\ C} + \frac{1}{R_1\ C}}{s + \frac{1}{R_1\ C}} \cdot \frac{R_1\ R_2\ C}{R_1\ C}$$

i.e., $$Z_{in}\ (s) = R_2 \cdot \frac{s + \frac{1}{R_2\ C} + \frac{1}{R_1 C}}{s + \frac{1}{R_1\ C}}$$

$$\text{Voltage transfer ratio} = \frac{V_2\ (s)}{V_1\ (s)} = \frac{I\ (s) . R_2}{I\ (s) . Z_{in}\ (s)}$$

$$= \frac{R_2}{Z_{in}\ (s)} = \frac{R_2}{R_2 \dfrac{s + \frac{1}{R_2\ C} + \frac{1}{R_1\ C}}{s + \frac{1}{R_1\ C}}}$$

$$= \frac{s + \frac{1}{R_1\ C}}{s + \frac{1}{R_2\ C} + \frac{1}{R_1\ C}}$$

$$\therefore \text{ Voltage transfer ratio} = \frac{s + \frac{1}{R_1\ C}}{s + \frac{1}{R_2\ C} + \frac{1}{R_1\ C}}.$$

Example 14.29 *Obtain the transfer function $\frac{V_2\ (s)}{V_1\ (s)}$ of the network shown in Fig. E14.32. Find V_2 (t) when $V_1\ (t) = 10\ e^{-2t}$ V.*

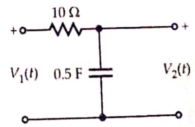

Fig. E14.32

Solution. Let the network be transformed to s-domain (Fig. E14.33).

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E
R
14

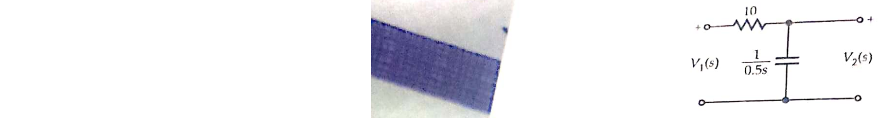


Fig. E14.33

Here, $V_1(s) = \left(10 + \frac{1}{0.5\,s}\right) I_1(s) = \left(10 + \frac{2}{s}\right) I_1(s)$

Also, $V_2(s) = I_1(s) \times \frac{1}{0.5\,s} = \frac{2}{s} I_1(s)$

$\therefore$ Thus, the required transfer function

$$= \frac{V_2(s)}{V_1(s)} = \frac{\frac{2}{s}}{10 + \frac{2}{s}}$$

$\therefore$ Voltage transfer function $= \dfrac{2}{s\left(10 + \frac{2}{s}\right)}$

However,

$$V_1 = 10\,e^{-2t} \quad i.e., \; V_1(s) = \frac{10}{(s+2)}$$

$\because \quad V_2(s) = V_1(s) \cdot \dfrac{2}{s\left(10 + \frac{2}{s}\right)},$

hence $\quad V_2(s) = \dfrac{10}{(s+2)} \dfrac{2}{s(10 + 2/s)}$

$$= \frac{20s}{s(s+2)(10s+2)}$$

$$= \frac{K_1}{s+2} + \frac{K_2}{10s+2}$$

$$K_1 = \left.\frac{20}{(10s+2)}\right|_{s=-2} = -1.11$$

$$K_2 = \left.\frac{20}{(s+2)}\right|_{s=-\frac{2}{10}} = 11.11.$$

$\therefore \quad V_2(s) = -\dfrac{1.11}{s+2} + \dfrac{11.11}{10s+2}$

$\therefore \quad \mathbf{V_2(t) = -1.11\,e^{-2t} + 1.11\,e^{-0.2t}}.$

EXAMPLE 14.30 *V_1 is applied at the input terminals $1-1'$. Find the voltage transfer ratio G_{21} in the network of Fig. E14.34.*

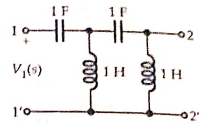


Fig. E14.34

SOLUTION. The network of Fig. E14.34 is redrawn in s-domain in Fig. E14.35 alongwith assumed voltages and currents.

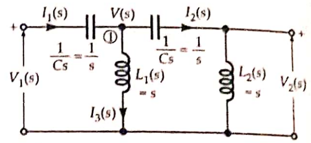


Fig. E14.35

In the leftmost loop, applying KCL at node (1),

$$I_1(s) = I_2(s) + I_3(s)$$

or, $\quad \dfrac{V_1(s) - V(s)}{\frac{1}{s}} = \dfrac{V(s)}{s} + \dfrac{V(s) - V_2(s)}{\frac{1}{s}}$

or, $\quad sV_1(s) = V(s)\left[2s + \frac{1}{s}\right] - sV_2(s)$ ...(1)

By observation, it is evident that $I_2(s)$ flows through $L_2(s)$ too producing output voltage $V_2(s)$ across 2-2′ terminal.

$\therefore \quad \dfrac{V(s) - V_2(s)}{\frac{1}{s}} = \dfrac{V_2(s)}{s}$

or, $\quad 0 = -sV(s) + V_2(s)\left[s + \frac{1}{s}\right]$

or, $\quad V(s) = \dfrac{V_2(s)}{s}\left[s + \frac{1}{s}\right]$ ...(2)

Substituting the value of $V(s)$ from (2) in (1),

$$sV_1(s) = \left[\frac{V_2(s)}{s}\left(s + \frac{1}{s}\right)\right]\left[2s + \frac{1}{s}\right] - sV_2(s)$$

$$= V_2(s)\left[\left\{\frac{1}{s}\left(s + \frac{1}{s}\right)\right\}\left\{2s + \frac{1}{s}\right\} - s\right]$$

$$= V_2(s)\left[\left(1 + \frac{1}{s^2}\right)\left(2s + \frac{1}{s}\right) - s\right]$$

$$= V_2(s)\left[2s + \frac{1}{s} + \frac{2}{s} + \frac{1}{s^3} - s\right]$$

$$= V_2(s)\left[\frac{2s^4 + s^2 + 2s^2 - s^4 + 1}{s^3}\right]$$

$$= V_2(s)\left[\frac{s^4 + 3s^2 + 1}{s^3}\right]$$

$$\therefore \quad \frac{V_2(s)}{V_1(s)} = \frac{s^4}{s^4 + 3s^2 + 1} = G_{21}(s)$$

$$G_{12}(s) = \frac{s^4}{s^4 + 3s^2 + 1}.$$

Example 14.31 *Find $G_{21}(s)$ for the network shown in Fig. E14.36 when $V_1(s)$ is the applied voltage at the input terminals.*

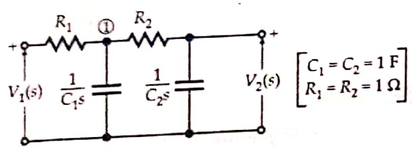


Fig. E14.36

Solution. In the s-domain network of Fig. E14.36, let $V_3(s)$ be the potential at the junction of R_1, R_2, C_1 *i.e.*, at *node (1)*.

Applying KCL at node (1),

$$\frac{V_1(s) - V_3(s)}{R_1} + \frac{V_2(s) - V_3(s)}{R_2} = \frac{V_3(s)}{\frac{1}{C_1 s}}$$

$$= V_3(s) C_1 s$$

or, $$\frac{V_1(s)}{R_1} = V_3(s)\left[sC_1 + \frac{1}{R_1} + \frac{1}{R_2}\right] - \frac{V_2(s)}{R_2} \quad \text{...(1)}$$

It may also be evident that the current passing through R_2 also passes through C_2

i.e., $$\frac{V_3(s) - V_2(s)}{R_2} = \frac{V_2(s)}{\frac{1}{sC_2}}$$

or, $$0 = \left[\frac{-V_3(s)}{R_2}\right] + V_2(s)\left[sC_2 + \frac{1}{R_2}\right]$$

i.e., $$V_3(s) = V_2(s)[sC_2 R_2 + 1] \quad \text{...(2)}$$

Utilising (2) in (1),

$$\frac{V_1(s)}{R_1} = V_2(s)[1 + sC_2 R_2]\left[sC_1 + \frac{1}{R_1} + \frac{1}{R_2}\right] - \frac{V_2(s)}{R_2}$$

or, $$V_1(s) = V_2(s)\left[\{R_1 + sC_2 R_2 R_1\}\left\{sC_1 + \frac{1}{R_1} + \frac{1}{R_2}\right\} - \frac{R_1}{R_2}\right]$$

or, $$\frac{V_2(s)}{V_1(s)} = G_{12}(s)$$

$$= \frac{1}{\left[\{R_1 + sC_2 R_1 R_2\}\left\{sC_1 + \frac{1}{R_1} + \frac{1}{R_2}\right\} - \frac{R_1}{R_2}\right]}$$

$$= \frac{1}{[\{1 + s\}\{s + 1 + 1\} - 1]} = \frac{1}{(s+1)[(s+2) - 1]}$$

i.e., $$G_{12(s)} = \frac{1}{(s+1)^2}.$$

Example 14.32 *Obtain the voltage transfer function of the network shown in Fig. E14.37.*

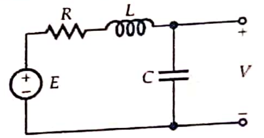


Fig. E14.37

Solution. Let us first transform the network to frequency domain (Fig. E14.38).

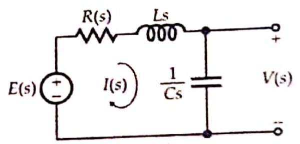


Fig. E14.38

Here, $$I(s) = \frac{E(s)}{R + Ls + \frac{1}{Cs}}$$

$$\therefore \quad V(s) = I(s) \times \frac{1}{Cs} = \frac{E(s)}{R + Ls + \frac{1}{Cs}} \cdot \frac{1}{Cs}$$

$$\therefore \quad \frac{V(s)}{E(s)} = \frac{\frac{1}{Cs}}{R + Ls + \frac{1}{Cs}} = \frac{1}{LCs^2 + RCs + 1}$$

Obviously, $\frac{V(s)}{E(s)}$ gives the voltage transfer ratio and

$$\frac{V(s)}{E(s)} = \frac{1}{LCs^2 + RCs + 1}.$$

CHAPTER 14

network shown in Fig. E14.39. Find the poles and zeros of the network and locate them in the S-plane.

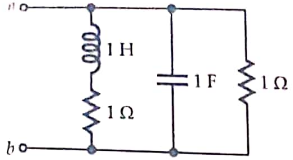


Fig. E14.39

Solution. Let us first transform the impedance to s-domain (Fig. E14.40).

$$Z_1(s) = \frac{1 \times \frac{1}{s}}{1 + \frac{1}{s}} = \frac{1}{s+1}$$

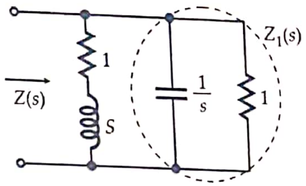


Fig. E14.40

$$\therefore \quad Z(s) = (s+1) \,||\, \frac{1}{(s+1)} = \frac{(s+1)\frac{1}{s+1}}{s+1+\frac{1}{s+1}}$$

$$= \frac{s+1}{s^2+2s+1+1}.$$

$$\therefore \quad Z(s) = \frac{s+1}{s^2+2s+2}.$$

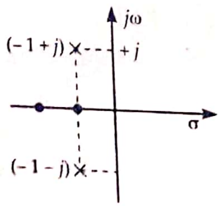


Fig. E14.41

$Z(s)$ being the driving point impedance.

or, $$Z(s) = \frac{s+1}{(s+1+j)(s+1-j)}.$$

Thus the poles are at $-1-j$ and $-1+j$ and there is one zero at -1.

... Find the voltage transfer ratio of the ideal Op-Amp model shown in Fig. E14.42.

[Assume $\Delta V = V_0 / A$, A being the gain].

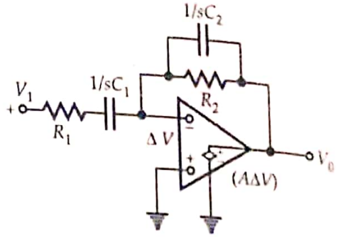


Fig. E14.42

Solution. Application of KCL at the negative terminal of the input of the Op-amp in Fig. E14.42, gives

$$\frac{(-\Delta V)-V_1}{R_1+\frac{1}{C_1 s}} + \frac{(-\Delta V)-V_0}{R_2 \,||\, \frac{1}{C_2 s}} = 0 \qquad ...(1)$$

For an ideal op-amp., the gain A is infinity.

$$\therefore \quad \Delta V = V_0 / A = 0$$

Using $\Delta V = 0$ in (1),

$$-\frac{V_1}{R_1+\frac{1}{C_1 s}} - \frac{V_0}{\frac{R_2 \times \frac{1}{C_2 s}}{R_2+\frac{1}{C_2 s}}} = 0$$

or, $$\frac{V_1(C_1 s)}{(R_1 C_1 s+1)} + \frac{V_0\left(R_2+\frac{1}{C_2 s}\right)}{\frac{R_2}{C_2 s}} = 0$$

or, $$\frac{V_1(C_1 s)}{1+R_1 C_1 s} + \frac{V_0(R_2 C_2 s+1)}{R_2} = 0$$

$$\therefore \quad \frac{V_0}{V_1} = -\left(\frac{C_1 s}{1+R_1 C_1 s}\right)\left(\frac{R_2}{1+R_2 C_2 s}\right)$$

The above expression gives the voltage transfer ratio.

Example 14.35 *What is the value of Z_{21} in the circuit of Fig. E14.43 ?*

Solution. Let the voltage at node (1) be V_1. Application of KCL at node (1) gives

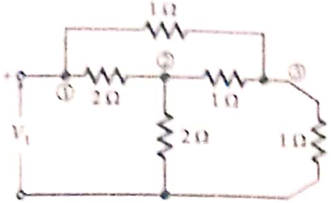


Fig. E14.43

$$I_1 = \frac{V_1 - V_0}{2} + \frac{V_1 - V_2}{1}$$

$$= 0.5\,V_1 - 0.5\,V_0 + V_1 - V_2$$

[node voltage at (2) being V_0]

$$I_1 = 1.5\,V_1 - 0.5\,V_0 - V_2 \qquad ...(1)$$

At *node* (2), application of KCL at node (2) gives

$$\frac{V_1 - V_0}{2} - \frac{V_0 - V_2}{1} = \frac{V_0}{2}$$

or, $\quad 0.5\,V_1 - 0.5\,V_0 - V_0 + V_2 = 0.5\,V_0$

or, $\quad 0.5\,V_1 - 2V_0 + V_2 = 0 \qquad ...(2)$

At *node* (3), application of KCL yields

$$\frac{V_1 - V_2}{1} = \frac{V_2 - V_0}{1} + \frac{V_2}{1}$$

or, $\quad V_1 - V_2 = V_2 - V_0 + V_2$

or, $\quad 0 = -V_1 - V_0 + 3V_2$

or, $\quad V_1 + V_0 - 3V_2 = 0 \qquad ...(3)$

Multiplying equation (2) by 3, we get

$$1.5\,V_1 - 6V_0 + 3V_2 = 0 \qquad ...(4)$$

Adding (3) and (4),

$$-5V_0 + 2.5\,V_1 = 0 \quad \text{or} \quad V_0 = 0.5\,V_1$$

Also, from (2),

$\therefore \quad V_0 = V_2$

$\therefore$ Utilising the values of V_0 and V_2 in (1), we get

$$I_1 = 1.5\,V_1 - 0.5 \times 0.5\,V_1 - 0.5\,V_1$$

$$= 0.75\,V_1$$

$\therefore \quad \frac{V_1}{I_1} = \frac{1}{0.75}$ *i.e.*, $\mathbf{Z_{11} = 1.33\,\Omega}$.

Also, from (1), putting $V_1 = 2V_2$ [from (2)] and $V_0 = 0.5\,V_1$, we get

$$I_1 = 1.5\,V_1 - 0.5\,V_0 - V_2$$

$$= 1.5 \times 2V_2 - 0.5 \times V_2 - V_2 = 1.5\,V_2$$

$$\frac{V_2}{I_1} = Z_{21} = \frac{1}{1.5} = 0.667\,\Omega$$

i.e., $\quad Z_{21} = 0.667\,\Omega$.

EXAMPLE 14.36 *Obtain the current transfer ratio of the network shown in Fig. E14.44.*

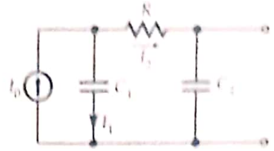


Fig. E14.44

SOLUTION. In Fig. E14.45, the s-domain network of Fig. E14.44.

$$I_0(s) = I_1(s) + I_2(s) = V_1(s)[Y_1(s) + Y_2(s)]$$

where $Y_1(s)$ is the admittance of branch *ab* and $Y_2(s)$ is that of branch *acd*.

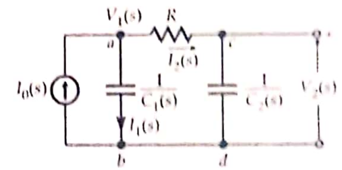


Fig. E14.45

However,

$$I_2(s) = Y_2(s)\,V_1(s)$$

$\therefore \quad I_2(s) = Y_2(s)\dfrac{I_0(s)}{Y_1(s) + Y_2(s)}$

$$= I_0(s)\frac{Y_2(s)}{Y_1(s) + Y_2(s)}$$

$\therefore \quad \dfrac{I_2(s)}{I_0(s)} =$ current transfer ratio

$$= \frac{Y_2(s)}{Y_1(s) + Y_2(s)}$$

However,

$$Y_1(s) = C_1\,s; \quad Y_2(s) = \frac{1}{R + \frac{1}{C_2\,s}} = \frac{sC_2}{RC_2\,s + 1}$$

This gives

$$I_2(s) = I_0(s)\,\frac{\dfrac{C_2 s}{1+RC_2 s}}{C_1 s + \dfrac{C_2 s}{1+RC_2 s}}$$

$$= I_0(s)\,\frac{C_2 s}{C_1 s(1+RC_2 s) + C_2 s};$$

$$\frac{I_2(s)}{I_0(s)} = \text{Current transfer ratio}$$

$$= \frac{sC_2}{sC_2 + sC_1(1+RC_2 s)}$$

Thus, current transfer ratio

$$= \frac{C_2}{C_2 + C_1(1+RC_2 s)}.$$

<u>EXAMPLE 14.37</u> *Obtain the driving point admittance of the network shown in Fig. E14.46.*

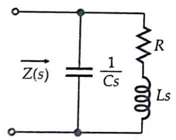


Fig. E14.46

SOLUTION.

$$Z(s) = \frac{(R+Ls)\dfrac{1}{Cs}}{R+Ls+\dfrac{1}{Cs}} = \frac{\dfrac{1}{Cs}(R+Ls)}{\dfrac{1+RCs+LCs^2}{Cs}}$$

$$= \frac{R+Ls}{1+RCs+LCs^2}$$

$\therefore$ $Y(s)$, the driving point admittance

$$= \frac{1}{Z(s)} = \frac{LCs^2+RCs+1}{R+Ls},$$

i.e., $$\mathbf{Y(s) = \frac{1+RCs+LCs^2}{R+Ls}}.$$

<u>EXAMPLE 14.38</u> *Find the driving point impedance of the network shown in Fig. E14.47.*

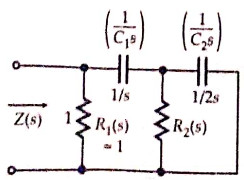


Fig. E14.47

SOLUTION. R_2 and C_2 are parallel

$$\therefore \quad Y_2(s) = Y_{R_2}(s) + Y_{C_2}(s)$$

$$= 1 + \frac{1}{Z_{C_2}(s)} = 1 + \frac{1}{\frac{1}{2s}} = 1+2s$$

i.e., $$Z_2(s) = \frac{1}{1+2s}$$

However, $Z_2(s)$ is in series with C_1

$$\therefore \quad Z_1(s) = Z_2(s) + \frac{1}{C_1 s} = \frac{1}{1+2s} + \frac{1}{s} = \frac{3s+1}{s(2s+1)}$$

$$\therefore \quad Y_1(s) = \frac{s(2s+1)}{3s+1}$$

Again $Z_1(s)$ and $R_1(s)$ are in parallel.

$$\therefore \quad Y(s) = \frac{1}{Z_1(s)} + \frac{1}{R_1(s)}$$

$$= \frac{s(2s+1)}{3s+1} + 1 = \frac{2s^2+4s+1}{3s+1}$$

$$\therefore \quad Z(s) = \frac{1}{Y(s)} = \frac{3s+1}{2s^2+4s+1}$$

i.e., Driving point impedance $= \mathbf{\dfrac{3s+1}{2s^2+4s+1}}$.

<u>EXAMPLE 14.39</u> *In the network of Fig. E14.48, find pole-zero plot.*

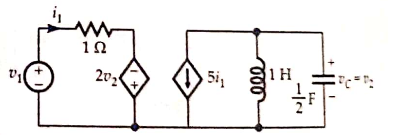


Fig. E14.48

SOLUTION. Let us first convert Fig. E14.48 to s-domain circuit (Fig. E14.49).

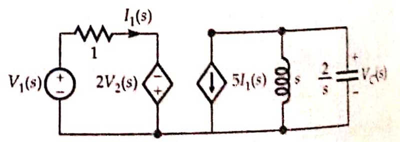


Fig. E14.49

Using KCL at *right hand loop*,

$$5I_1(s) + \frac{V_C(s)}{s} + \frac{V_C(s)}{2/s} = 0 \qquad \text{...(1)}$$

While at *left loop*,

$$I_1(s) = \frac{V_1(s) + 2V_C(s)}{1} \qquad \text{...(2)}$$

Using (2) in (1),

$$5 \cdot \frac{V_1(s) + 2V_c(s)}{1} + \frac{V_C(s)}{s} + \frac{sV_C(s)}{2} = 0$$

or, $$10V_C(s) + \frac{V_C(s)}{s} + \frac{sV_C(s)}{2} + 5V_1(s) = 0$$

or, $$V_C(s)\frac{20s + 2 + s^2}{2s} = -5V_1(s)$$

$$\frac{V_C(s)}{V_1(s)} = G_{12}(s) = -\frac{10s}{20s + 2 + s^2}$$

$\therefore$ Zero is at $s = 0$.

Poles are at $\left(-\frac{1}{20} + j\frac{\sqrt{76}}{40}\right)$ and $\left(-\frac{1}{20} - j\frac{\sqrt{76}}{40}\right)$

The pole-zero plot is shown in Fig. E14.50.

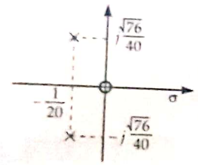


Fig. E14.50

EXAMPLE 14.40 *Find the driving point admittance function and the respective pole-zero plot for Fig. E14.51.*

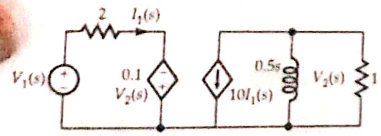


Fig. E14.51

SOLUTION. In the left loop,

$$I_1(s) = \frac{V_1(s) + 0.1V_2(s)}{2}, \qquad \text{...(1)}$$

and in the right hand loop, applying KCL,

$$\frac{V_2(s)}{0.5s} + \frac{V_2(s)}{1} + 10I_1(s) = 0$$

or, $$\frac{2}{s}V_2(s) + V_2(s) + 10I_1(s) = 0.$$

or, $$V_2(s)\left[\frac{2}{s} + 1\right] = -10I_1(s)$$

or, $$V_2(s)\left[\frac{2+s}{s}\right] = -10I_1(s)$$

or, $$V_2(s) = -\frac{10}{\frac{2+s}{s}}I_1(s) = -\frac{10s}{s+2}I_1(s) \qquad \text{...(2)}$$

Using (2) in (1),

$$I_1(s) = \frac{V_1(s) + 0.1\left[-\frac{10s}{s+2}\right]I_1(s)}{2}$$

$$= \frac{1}{2}V_1(s) + 0.05I_1(s)\left[-\frac{10s}{s+2}\right]$$

or, $$I_1(s)\left[1 + \frac{0.5s}{s+2}\right] = 0.5V_1(s)$$

$\therefore$ Driving point admittance

$$= \frac{I_1(s)}{V_1(s)} = \frac{0.5}{1 + \frac{0.5s}{s+2}} = \frac{0.5(s+2)}{s+2+0.5s}$$

or, $$Y_{11}(s) = \frac{0.5(s+2)}{2+1.5s} = \frac{0.5s+1}{1.5s+2}$$

i.e., **pole at $-\frac{4}{3}$; Zero at -2**

Pole zero Plot is shown in Fig. E14.52.

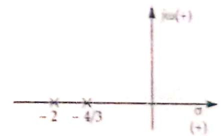


Fig. E14.52

EXAMPLE 14.41 *In the network of Fig. E14.53, "S" is switched on at t = 0. Find the driving point impedance and the source current in s-domain.*

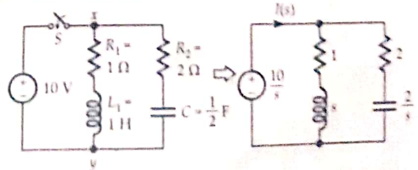


Fig. E14.53

C H A P T E R 14

SOLUTION. Let $Z_1(s)$ be the impedance of R-L branch and $Z_2(s)$ be that of RC branch.

$$Z_1(s) = R_1 + sL_1 = (1+s)\ i.e., Y_1(s) = \frac{1}{1+s}$$

$$Z_2(s) = \left(R_2 + \frac{1}{Cs}\right) = \left(2 + \frac{1}{\frac{1}{2}s}\right)$$

$$= \left(2 + \frac{2}{s}\right) = \frac{2s+2}{s}$$

i.e., $$Y_2(s) = \frac{s}{2s+2}$$

$Y(s)$ = admittance of the network across terminals x-y

$$= Y_1(s) + Y_2(s) = \frac{1}{1+s} + \frac{s}{2s+2}$$

$$= \frac{1}{s+1} + \frac{s}{2(s+1)}$$

$$= \frac{2+s}{2(s+1)} = \frac{(s+2)}{2(s+1)}$$

$$\therefore \quad Z(s) = \frac{2(s+1)}{(s+2)}$$

[$Z(s)$ is the driving point impedance]

However,

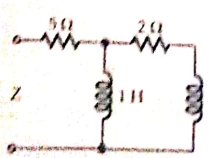
$$Z(s) = \frac{V(s)}{I(s)}$$

$$\therefore \quad I(s) = \frac{V(s)}{Z(s)} = \frac{(10/s)}{\frac{2(s+1)}{(s+2)}} = \frac{10}{s} \times \frac{s+2}{2(s+1)}$$

$$= \frac{10(s+2)}{2s(s+1)} = \frac{5(s+2)}{s(s+1)}$$

$$\therefore \text{ Source current } I(s) = \frac{5(s+2)}{s(s+1)}.$$

EXAMPLE 14.42 *Obtain the pole zero plot in the s-plane of the driving point impedance function for the network shown in Fig. E14.54.*



SOLUTION.

$$Z(s) = 5 + \frac{s(2+2s)}{s+2+2s} = 5 + \frac{2s+2s^2}{3s+2}$$

$$= \frac{15s + 10 + 2s + 2s^2}{3s+2} = \frac{2s^2 + 17s + 10}{3s+2}$$

$$= \frac{(s+4.25+3.61)(s+4.25-3.61)}{(3s+2)}$$

This gives, zeros

at $-4.25 + 3.61$ and $-4.25 - 3.61$

i.e., **at -0.64 and -7.86 and pole at $-\frac{2}{3}$.**

The pole zero plot is shown in Fig. E14.55.

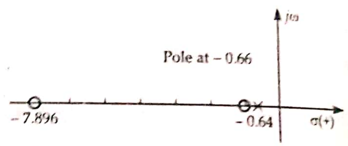


Fig. E14.55

EXAMPLE 14.43 *What is the impedance and admittance functions in the circuit of Fig. E14.56 in complex frequency domain ?*

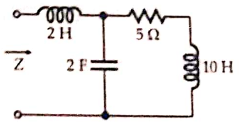


Fig. E14.56

SOLUTION. Let us first transform the given network to frequency domain (Fig. E14.57).

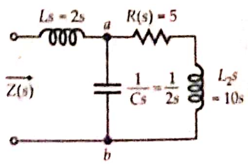


Fig. E14.57

Here, $Z_e(s)$, the equivalent impedance across a-b points is given by

$$Z_e(s) = \left(\frac{1}{Cs}\right) || (R + L_2 s) = \frac{1}{2s} || (5 + 10s)$$

$$= \frac{\frac{1}{2s} \times (5+10s)}{} = \frac{(5+10s)}{}$$

$$\therefore \quad Z(s) = Z_e(s) + L_1 s = \frac{5+10s}{1+10s+20s^2} + 2s$$

$$= \frac{5 + 10s + 2s + 20s^2 + 40s^3}{1 + 10s + 20s^2}$$

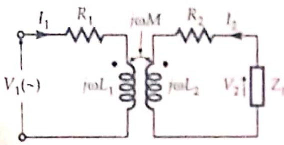
$\therefore$ $Z(s)$ = impedance function

$$= \frac{40s^3 + 20s^2 + 12s + 5}{s^2 + 10s + 1}$$

and $Y(s)$ = admittance function

$$= \frac{1}{Z(s)} = \frac{20s^2 + 10s + 1}{40s^3 + 20s^2 + 12s + 5}.$$

EXAMPLE 14.44 *In the coupled circuit of Fig. 14.58, find input impedance forward voltage and current transfer functions.*



14.58

SOLUTION. The matrix equation of this coupled circuit (Fig. E14.58) is given by [Refer to Chapter *Coupled Circuits*]

$$\begin{bmatrix} V_1 \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} + Z_L \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

where, $Z_{11} = R_1 + j\omega L_1$, $\quad Z_{12} = Z_{21} = j\omega M$,

$Z_{22} = R_2 + j\omega L_2$

The determinant of the impedance matrix is given by $\quad \Delta_z = Z_{11}(Z_{22} + Z_L) - Z_{12} Z_{21}$

Using Cramer's Rule,

$$I_1 = \frac{\begin{vmatrix} V_1 & Z_{12} \\ 0 & Z_{22} + Z_L \end{vmatrix}}{\Delta_Z} = \frac{(Z_{22} + Z_L)}{\Delta_Z} V_1$$

and $$I_2 = \frac{\begin{vmatrix} Z_{11} & V_1 \\ Z_{21} & 0 \end{vmatrix}}{\Delta_Z} = \frac{-Z_{21}}{\Delta_Z} . V_1$$

The input impedance is thus given by

$$Z_1 = \frac{V_1}{I_1} = \frac{V_1}{\frac{(Z_{22} + Z_L)}{\Delta_Z} V_1} = \frac{\Delta_Z}{Z_{22} + Z_L}$$

$$= \frac{Z_{11}(Z_{22} + Z_L) - Z_{12} Z_{21}}{Z_{22} + Z_L}$$

i.e., Z_1 (input impedance)

$$= \frac{Z_{11}(Z_{22} + Z_L) - Z_{12} Z_{21}}{Z_{22} + Z_L}$$

Again, $V_2 = -I_2 Z_L = +\frac{Z_{21}}{\Delta_Z} V_1 Z_L$

$\therefore$ $\frac{V_2}{V_1}$ = forward voltage transfer function

$$= \frac{Z_{21} Z_L}{\Delta_Z}$$

and $$\frac{I_2}{I_1} = \frac{-\frac{Z_{21}}{\Delta_Z} . V_1}{\frac{(Z_{22} + Z_L)}{\Delta_Z} . V_1} = -\frac{Z_{21}}{Z_{22} + Z_L}$$

where $\frac{I_2}{I_1}$ is the forward current transfer function

EXAMPLE 14.45 *What is the driving point and transfer impedance of the network shown in Fig. E14.59 ?*

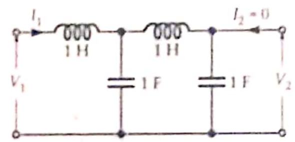


Fig. E14.59

SOLUTION. Let us first transform the given circuit to *s*-domain (Refer to Fig. E14.60).

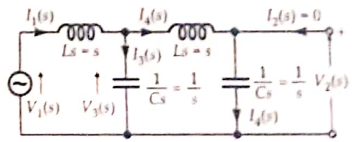


Fig. E14.60

Here, $I_4(s) = \frac{V_2(s)}{\frac{1}{Cs}} = sV_2(s)$

Also $V_3(s) = V_2(s) + I_4(s) . Ls$

$= V_2(s) + sV_2(s) . Ls$

$= V_2(s)[s^2 L + 1] = V_2(s)[1 + s^2]$

$I_1(s) = I_3(s) + I_4(s) = \frac{V_3(s)}{1/Cs} + sV_2(s)$

$= CsV_2(s)(1 + s^2) + sV_2(s)$

$= V_2(s)[s(1 + s^2) + s]$

$= V_2(s)[s^3 + 2s]$

CHAPTER 14

Now, $V_1(s) = V_3(s) + I_1(s)\,Ls$

$$= V_2(s)[1+s^2] + V_2(s)[s^3+2s]\,s$$
$$= V_2(s)[1+s^2+s^4+2s^2]$$
$$= V_2(s)[s^4+3s^2+1]$$

$\therefore$ Driving point impedance

$$= \frac{V_1(s)}{I_1(s)} = \frac{V_2(s)[s^4+3s^2+1]}{V_2(s)[s^3+2s]},$$

or, driving point impedance $= \dfrac{\mathbf{1+3s^2+s^4}}{\mathbf{2s+s^3}}$.

Transfer impedance

$$= \frac{V_2(s)}{I_1(s)} = \frac{V_2(s)}{V_2(s)[s^3+2s]} = \frac{1}{s^3+2s}$$

i.e., transfer impedance $= \dfrac{\mathbf{1}}{\mathbf{s^3+2s}}$.

EXAMPLE 14.46 *The pole zero plot of the driving point impedance of a circuit is shown in Fig. E14.61 in s-plane. Find the time domain response.*

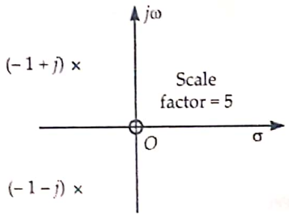


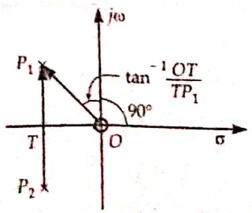
Fig. E14.61

SOLUTION. It is evident from Fig. E14.61, that the zero is at 0 while poles are at $(-1+j)$ and $(-1-j)$, $H = 5$

$$\therefore \quad Z(s) = \frac{5s}{(s+1+j)(s+1-j)}$$

i.e., $\quad Z(t) = K_1\, e^{-(1+j)t} + K_2\, e^{-(1-j)t}$

To find K_1 and K_2, Fig. E14.62 and E14.63 may be referred.



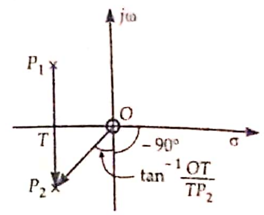


Fig. E14.63

With reference to the text,

$$K_1 = H\,\frac{M_{O-P_1}\ \angle 90^\circ + \tan^{-1}\dfrac{OT}{TP_1}}{M_{P_2-P_1}\ \angle +90^\circ}$$

[Refer to Fig. E14.62]

$$= 5\,\frac{\sqrt{OT^2+TP_1^2}\ \angle 90^\circ + (+45^\circ)}{2\times 1\angle +90^\circ}$$

$$= 5\,\frac{\sqrt{2}\ \angle 135^\circ}{2\ \angle +90^\circ} = 3.54\ \angle 45^\circ$$

$$K_2 = H\,\frac{M_{O-P_2}\ \angle -90^\circ - \tan^{-1}\dfrac{OT}{TP_2}}{M_{P_1-P_2}\ \angle -90^\circ}$$

[Refer to Fig. E14.63]

$$= 5\,\frac{\sqrt{2}\ \angle -90^\circ - 45^\circ}{2\times 1\angle -90^\circ} = 3.54\ \angle -45^\circ$$

$$\therefore \quad \mathbf{Z(t) = 3.54\ \angle 45^\circ\, e^{-(1+j)t} + 3.54\ \angle -45^\circ\, e^{-(1-j)t}\ \Omega.}$$

EXAMPLE 14.47 *A current transfer function is given by*

$$I(s) = \frac{5s}{(s+2)(s^2+2s+2)}.$$

Obtain its time domain response.

SOLUTION. The given transfer function is

$$I(s) = \frac{5s}{(s+2)(s^2+2s+2)}$$
$$= \frac{5s}{(s+2)(s+1-j)(s+1+j1)}$$

$\therefore$ The zero is at origin and the poles are at $(-2), -(1 \pm j1)$

$$\therefore \quad I(s) = \frac{5s}{\{s-(-2)\}\{(s-(-1+j)\}\{s-(-1-j)\}}$$

$$= \frac{K_1}{\ldots} + \frac{K_2}{\ldots} + \frac{K_3}{\{s-(-1-j)\}}$$

Now, $V_1(s) = V_3(s) + I_1(s)\,Ls$

$$= V_2(s)[1+s^2] + V_2(s)[s^3+2s]\,s$$

$$= V_2(s)[1+s^2+s^4+2s^2]$$

$$= V_2(s)[s^4+3s^2+1]$$

$\therefore$ Driving point impedance

$$= \frac{V_1(s)}{I_1(s)} = \frac{V_2(s)[s^4+3s^2+1]}{V_2(s)[s^3+2s]},$$

or, driving point impedance $= \dfrac{1+3s^2+s^4}{2s+s^3}$.

Transfer impedance

$$= \frac{V_2(s)}{I_1(s)} = \frac{V_2(s)}{V_2(s)[s^3+2s]} = \frac{1}{s^3+2s}$$

i.e., transfer impedance $= \dfrac{1}{s^3+2s}$.

EXAMPLE 14.46 *The pole zero plot of the driving point impedance of a circuit is shown in Fig. E14.61 in s-plane. Find the time domain response.*

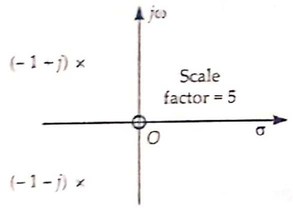


Fig. E14.61

SOLUTION. It is evident from Fig. E14.61, that the zero is at 0 while poles are at $(-1+j)$ and $(-1-j)$, $H = 5$

$$\therefore \quad Z(s) = \frac{5s}{(s+1+j)(s+1-j)}$$

i.e., $\quad Z(t) = K_1\, e^{-(1+j)t} + K_2\, e^{-(1-j)t}$

To find K_1 and K_2, Fig. E14.62 and E14.63 may be referred.

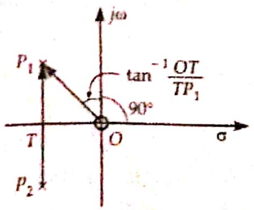


Fig. E14.62

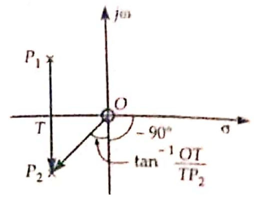


Fig. E14.63

With reference to the text,

$$K_1 = H\,\frac{M_{O-P_1}\ \angle 90^\circ + \tan^{-1}\dfrac{OT}{TP_1}}{M_{P_2-P_1}\ \angle +90^\circ}$$

[Refer to Fig. E14.62]

$$= 5\,\frac{\sqrt{OT^2+TP_1^2}\ \angle 90^\circ + (+45^\circ)}{2\times 1\ \angle +90^\circ}$$

$$= 5\,\frac{\sqrt{2}\ \angle 135^\circ}{2\ \angle +90^\circ} = 3.54\ \angle 45^\circ$$

$$K_2 = H\,\frac{M_{O-P_2}\ \angle -90^\circ - \tan^{-1}\dfrac{OT}{TP_2}}{M_{P_1-P_2}\ \angle -90^\circ}$$

[Refer to Fig. E14.63]

$$= 5\,\frac{\sqrt{2}\ \angle -90^\circ - 45^\circ}{2\times 1\ \angle -90^\circ} = 3.54\ \angle -45^\circ$$

$$\therefore \quad \mathbf{Z(t) = 3.54\ \angle 45^\circ e^{-(1+j)t} + 3.54\ \angle -45^\circ e^{-(1-j)t}\ \Omega.}$$

EXAMPLE 14.47 *A current transfer function is given by*

$$I(s) = \frac{5s}{(s+2)(s^2+2s+2)}.$$

Obtain its time domain response.

SOLUTION. The given transfer function is

$$I(s) = \frac{5s}{(s+2)(s^2+2s+2)}$$

$$= \frac{5s}{(s+2)(s+1-j)(s+1+j1)}$$

$\therefore$ The zero is at origin and the poles are at $(-2), -(1 \pm j\,1)$

$$\therefore \quad I(s) = \frac{5s}{[s-(-2)]\,[(s-(-1+j)]\,[s-(-1-j)]}$$

$$= \frac{K_1}{[s-(-2)]} + \frac{K_2}{[s-(-1+j)]} + \frac{K_3}{[s-(-1-j)]}.$$

Thus the function response in time domain is given by

$$I(t) = K_1 e^{-2t} + K_2 e^{(-1+j)t} + K_3 e^{(-1-j)t}$$

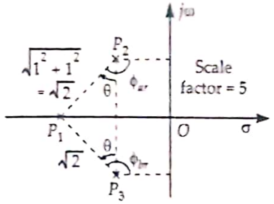


Fig. E14.64

Here

$$K_1 = 5\,\frac{M_{OP_1}\ \angle \text{ of } P_1 \text{ with respect to zero} (\phi_{1r})}{M_{P_2P_1}\ \angle\phi_{ar} \times M_{P_3P_1}\ \angle\phi_{br}}$$

[Refer to Fig. E14.64]

where ϕ_{ar} = angle of $P_2\ P_1$ and ϕ_{br} = angle of $P_3\ P_1$

$$= 5\,\frac{2\ \angle 180^\circ}{\sqrt{2}\ \angle -(90^\circ+\theta)\times\sqrt{2}\ \angle(90^\circ+\theta)}$$

$$\left[\text{where } \theta = \tan^{-1}\frac{1}{1} = 45^\circ\right]$$

$$= 5\,\frac{2\ \angle 180^\circ}{\sqrt{2}\ \angle -135^\circ \times \sqrt{2}\ \angle 135^\circ} = 5\ \angle 180^\circ$$

ind K_2, let us refer to Fig. E14.65.

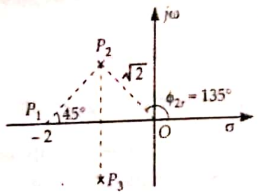


Fig. E14.65

$$K_2 = 5\,\frac{M_{OP_2}\ \angle \text{ of } OP_2\ (\phi_{2r})}{M_{P_1P_2}\ \angle \text{of } P_1P_2(\phi_{P_2r}) \times M_{P_3P_2}\ \angle \text{of } P_3P_2(\phi_{P_3r})}$$

$$= 5\,\frac{\sqrt{2}\ \angle 135^\circ}{\sqrt{2}\ \angle 45^\circ \times 2\ \angle 90^\circ} = 2.5\ \angle 0^\circ = 2.5 + j\,0 = 2.5$$

and $K_3 = K_2^* = 2.5 - j\,0 = 2.5$

$$\therefore\quad I(t) = 5\ \angle 180^\circ e^{-2t} + 2.5\,e^{(-1+j)t} + 2.5\,e^{(-1-j)t}$$

$$= [-5e^{-2t} + 2.5\,e^{(-1+j)t} + 2.5\,e^{(-1-j)t}]\text{ A.}$$

EXAMPLE 14.48 *A transfer admittance is given by*

$$Y(s) = \frac{s(s^2+2)}{s^2+1}.$$

Find the pole-zero plot.

SOLUTION. $Y(s) = \dfrac{s(s+j\sqrt{2})(s-j\sqrt{2})}{(s+j\,1)(s-j\,1)}$

$\therefore$ The zeros would occur at $s = \pm j\sqrt{2}$ and at origin $(s=0)$ while the poles would occur at $S = \pm j\,1$.

The pole zero plot is shown in Fig. E14.66.

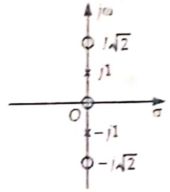


Fig. E14.66

EXAMPLE 14.49 *A driving point impedance is given by* $Z(s) = \dfrac{s^2-7s+10}{s^2+s+50}$. *Find pole zero plot.*

SOLUTION. The numerator of the given function can be factorised as follows :

$$s^2 - 7s + 10 = s^2 - 5s - 2s + 10$$
$$= s(s-5) - 2(s-5) = (s-2)(s-5).$$

$\therefore$ Zeros would appear at $s = 2$, $s = 5$.

However, the denominator being $s^2 + s + 50$, the roots are

$$\frac{-1 \pm \sqrt{1-200}}{2} = -0.5 \pm j\,\frac{\sqrt{199}}{2}$$

$\therefore$ The poles would appear at

$$s = -0.5 + j\,\frac{\sqrt{199}}{2}$$

and $$s = -0.5 - j\,\frac{\sqrt{199}}{2}.$$

The pole zero plot is shown in Fig. E14.67.

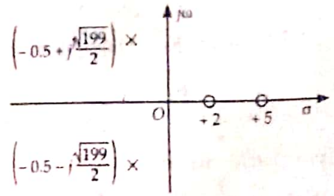


Fig. E14.67

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Example 14.50 *A transfer function is given by* $Y(s) = \frac{50}{s^2 + 2s + 2}$. *Find the pole zero plot.*

Solution. As there is no s term in the numerator of the function, hence there will be no finite zero in the function.

The denominator can be factored as

$$(s^2 + 2s + 2) = (s + 1 + j)(s + 1 - j)$$

$\therefore$ Poles will appear at $s = -1 - j, -1 + j$

The pole zero plot is shown in Fig. E14.68.

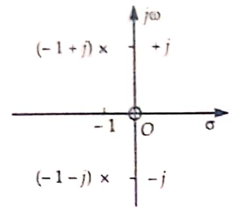

Fig. E14.68

Example 14.51 *The transfer function of a network is given by* $T(s) = \frac{1}{1 + sCR}$. *Find its impulse response.*

Solution. Impulse response is given by

$$I(t) = \pounds^{-1}\, T(s) = \pounds^{-1}\left[\frac{1}{1 + sCR}\right]$$

$$= \pounds^{-1}\left[\frac{1/RC}{s + 1/RC}\right] = \frac{1}{RC} e^{-t/RC},$$

i.e., impulse response $= \frac{1}{RC} e^{-t/RC}$.

Example 14.52 *A transfer function is given by*

$$Y(s) = \frac{10\, s}{[(s + 5 + j\, 15)(s + 5 - j\, 15)]}.$$

Find the time domain response.

Solution. The given function can be written as

$$Y(s) = \frac{10\,(s - 0)}{[s - (-5 + j\, 15)][s - (-s - j\, 15)]}$$

$$= \frac{K_1}{s - (-5 + j\, 15)} + \frac{K_2}{s - (-5 - j\, 15)}$$

$\therefore$ Time domain response is given by

$$Y(t) = K_1\, e^{-(5 + j\, 15)t} + K_2\, e^{-(5 - j\, 15)t}.$$

In order to determine K_1 and K_2, the pole-zero plot is shown in Fig. E14.69(*a*).

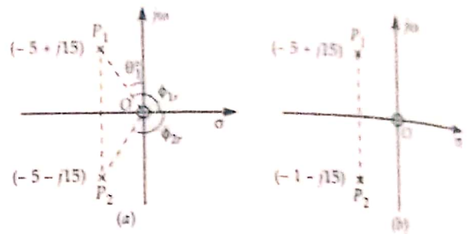

Fig. E14.69

Here, $M_{P_{2r}}$ [distance of the pole P_2 from the zero] is given by,

$$M_{P_{2r}} = OP_2 = \sqrt{5^2 + 15^2} = 15.8 = M_{2r}$$

Similarly, $M_{P_{1r}} = 15.8 = M_{1r}$

Again ϕ_{1r} (angle of P_1 with zero) $= 90° + \theta_1$

where $\theta_1 = \tan^{-1}\frac{5}{15} = 18.4°$

$\therefore \quad \phi_{1r} = 90 + 18.4 = 108.4°$

Similarly, $\phi_{2r} = -108.4°$.

Next, redrawing the pole zero plot, following Fig. E16.69(*b*).

M_{ar} [the distance between two poles P_1 and P_2] is given by,

$$M_{ar} = 15 + 15 = 30$$

while $\phi_{ar(P_1)} = 90°$ and $\phi_{ar(P_2)} = -90°$

[where $\phi_{ar(P_1)}$ is the angle of P_1 with left x-axis and $\phi_{ar(P_2)}$ is the angle of P_2 with left x-axis]

$$\therefore \quad K_1 = 10\frac{M_{1r}\,\angle\phi_{1r}}{M_{ar}\,\angle\phi_{ar(P_1)}} = 10\frac{15.8\,\angle 108.4°}{30\,\angle 90°}$$

$$= 5.26\,\angle 18.4°$$

and
$$K_2 = 10\frac{M_{2r}\,\angle\phi_{2r}}{M_{ar(P_2)}\,\angle\phi_{ar}} = 10\frac{15.8\,\angle -108.4°}{30\,\angle -90°}$$

$$= 5.26\,\angle -18.4°$$

Thus,

$$Y(t) = 5.26\,\angle 18.4°\, e^{-(5 + j\,15)t} + 5.26\,\angle -18.4°\, e^{-(5 - j\,15)t}$$

EXAMPLE 14.53 *Obtain the transfer impedance for the circuit shown in Fig. E14.70.*

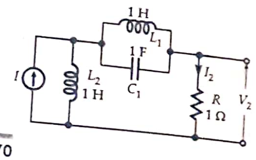

Fig. E14.70

SOLUTION. Let us first transform the given network to s-domain [Fig. E14.70(a)].

Here, $\quad V_2(s) = I_2(s) \, . \, R \quad$ ...(1)

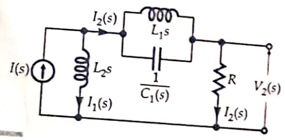

Fig. E14.70 (a)

Voltage across the inductor is given by using KVL

$$V_L(s) = V_2(s) + I_2(s)\left[\frac{1}{\left(sC_1 + \frac{1}{sL_1}\right)}\right]$$

$$= V_2(s) + \frac{V_2(s)}{R} \cdot \frac{sL_1}{1 + s^2 L_1 C_1} \quad \text{...(2)}$$

However,

$$I(s) = I_1(s) + I_2(s) = \frac{V_L(s)}{sL_2} + \frac{V_2(s)}{R}$$

$$= \frac{1}{sL_2}\left[V_2(s)\left\{1 + \frac{1}{R} \cdot \frac{sL_1}{1+s^2L_1C_1}\right\}\right] + \frac{V_2(s)}{R}$$

$$= V_2(s)\left[\frac{1}{sL_2}\left\{1 + \left(\frac{1}{R}\right)\left(\frac{sL_1}{1+s^2L_1C_1}\right)\right\} + \frac{1}{R}\right]$$

$$= V_2(s)\left[\frac{R(1+s^2L_1C_1) + sL_1 + sL_2(1+s^2L_1C_1)}{sL_2 \, R(1+s^2 L_1 C_1)}\right]$$

$$= V_2(s)\left[\frac{s^3L_1L_2C_1 + s(L_1+L_2) + R + RL_1C_1s^2}{sL_2 \, R(1+s^2 L_1 C_1)}\right]$$

$$\therefore \quad Z_{12}(s) = \frac{V_2(s)}{I(s)}$$

$$= \frac{sL_2 R(1+s^2L_1C_1)}{s^3L_1L_2C_1 + s(L_1+L_2) + R + RL_1C_1s^2}$$

Substituting the numerical values

$$Z_{12}(s) = \frac{s(1+s^2)}{s^3+s^2+2s+1}.$$

EXAMPLE 14.54 *In a linear two port network $v_1(t)$ and $v_2(t)$ be the input voltage and its response at the output. $v_2(t) = te^{-2t}\,u(t)$. If the response to an impulse voltage be given by $v(t) = (e^{-t} + e^{-2t})\,u(t)$, find $v_1(t)$ as well as the transfer function of the network.*

SOLUTION. The transfer function $T(s)$ of the network is given by

$$T(s) = \pounds\,[(e^{-t} + e^{-2t})\,u(t)]$$

$$= \frac{1}{s+1} + \frac{1}{s+2} = \frac{(s+2)+(s+1)}{(s+1)(s+2)}$$

or, $\quad T(s) = (2s+3)/(s+1)(s+2)$

Again,

$$v_2(t) = te^{-2t}\,u(t) \text{ [given]}$$

$$\therefore \quad V_2(s) = \frac{1}{(s+2)^2}$$

$$[\because \pounds\, v_2(t) = V_2(s) = \pounds\,[te^{-2t}\,u(t)]]$$

But $\quad V_2(s) = T(s)\,V_1(s)$

$$\therefore \quad V_1(s) = \frac{V_2(s)}{T(s)} = \frac{1}{(s+2)^2} \times \frac{(s+1)(s+2)}{(2s+3)}$$

$$= \frac{(s+1)}{(s+2)(2s+3)} = \frac{K_1}{s+2} + \frac{K_2}{2s+3}$$

Using partial fraction method,

$$K_1 = 1\,;\; K_2 = -1$$

$$\therefore \quad V_1(s) = \frac{1}{s+2} + \frac{(-1)}{2s+3} = \frac{1}{s+2} - \frac{1/2}{s+1.5}$$

i.e., $\quad v_1(t) = [e^{-2t} - 0.5\,e^{-1.5t}]\,u(t)$

i.e., $\quad T(s) = \text{transfer function} = \dfrac{2s+3}{(s+1)(s+2)}$

and $\quad v_1(t) = [e^{-2t} - 0.5\,e^{-1.5t}]\,u(t)$.

EXAMPLE 14.55 *In the coupled circuit of Fig. E14.71, $L_1 = 4H$, $L_2 = 6H$, $C = \frac{1}{18}$ F, $R_1 = 9\,\Omega$, $M = 2H$. Find : (i) the voltage transfer function (ii) input impedance of the network seen from the source size.*

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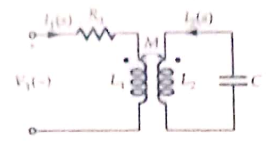

Fig. E14.71

SOLUTION. From the concept of coupled theory,

$$V_1(s) = (R_1 + L_1 s) I_1(s) + Ms I_2$$

or, $$V_1(s) = (9 + 4s) I_1(s) + 2s I_2(s) \quad \text{...(1)}$$

Utilising KVL in the output loop,

$$0 = Ms I_1(s) + sL_2 I_2(s) + I_2(s) \times \frac{1}{Cs}$$

$$= 2s I_1(s) + 6sI_2(s) + \frac{18}{s} I_2(s)$$

or, $$I_1(s) = -\left(3 + \frac{9}{s^2}\right) I_2(s) = -3\left(\frac{s^2+3}{s^2}\right) I_2(s) \quad \text{...(2)}$$

Substitution of the value of $I_1(s)$ from (2) to (1),

$$V_1(s) = (9+4s)\left[-3\left(\frac{s^2+3}{s^2}\right)\right] I_2(s) + 2s\, I_2(s)$$

$$= I_2(s)\left[-(9+4s)3\left(\frac{s^2+3}{s^2}\right) + 2s\right]$$

$$= I_2(s)\left[-27 - \frac{81}{s^2} - 12s - \frac{36}{s} + 2s\right]$$

$$= I_2(s)\left[\frac{-27s^2 - 81 - 12s^3 - 36s + 2s^3}{s^2}\right]$$

$$= I_2(s)\left[\frac{-10s^3 - 27s^2 - 36s - 81}{s^2}\right] \quad \text{...(3)}$$

$$\therefore \quad I_2(s) = \frac{-s^2 V_1(s)}{10s^3 + 27s^2 + 36s + 81}$$

However, the output voltage $V_2(s)$ is given by

$$V_2(s) = -I_2(s) \times \frac{1}{Cs} = \frac{18}{s}[-I_2(s)]$$

$$= \frac{18}{s}\left[\frac{s^2 V_1(s)}{10s^3 + 27s^2 + 36s + 81}\right]$$

$$\therefore \quad \frac{V_2(s)}{V_1(s)} = \frac{18s}{10s^3 + 27s^2 + 36s + 81}$$

= **voltage transfer function.**

We can rewrite equation (2) as

$$I_2(s) = \frac{-s^2}{3(s^2+3)} I_1(s) \quad \text{...(4)}$$

Substitution of the value of $I_2(s)$ from (4) in (3) yields

$$V_1(s) = \frac{-s^2}{3(s^2+3)} I_1(s)\left[\frac{-10s^3 - 27s^2 - 36s - 81}{s^2}\right]$$

$$\therefore \quad \frac{V_1(s)}{I_1(s)} = \frac{10s^3 + 27s^2 + 36s + 81}{3(s^2+3)}$$

= **input impedance.**

EXAMPLE 14.56 *For the network of Fig. E14.72 find the current ratio transfer function given by* $\alpha = (I_2/I_1)$.

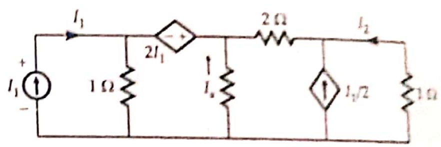

Fig. E14.72

SOLUTION. First, we convert current source into equivalent voltage source as shown in Fig. E14.72(*a*).

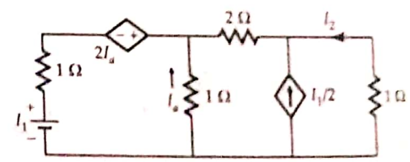

Fig. E14.72 (*a*)

Let I be the current in First (leftmost) loop. Applying KVL we get

$$I_1 = I - 2I_a - I_a \quad \text{...(i)}$$

$$I = -\left(I_a + \frac{I_1}{2} + I_2\right) \quad \text{...(ii)}$$

From equations (*i*) and (*ii*), eliminating I, we get

$$I_1 = -\left(I_a + \frac{I_1}{2} + I_2\right) - 2I_a - I_a$$

$$I_a = -\frac{1}{4}\left(\frac{3I_1}{2} + I_2\right) \quad \text{...(iii)}$$

Next applying KVL in the external loop consisting resistances 1 Ω, 2 Ω and 1 Ω we have from Fig. E14.72(*a*)

$$I_2 + 2\left(I_2 + \frac{I_1}{2}\right) - I_a = 0$$

or
$$3I_2 + I_1 - I_a = 0 \qquad \text{...}(iv)$$

From equations (*iii*) and (*iv*), eliminating I_a, we obtain

$$3I_2 + I_1 + \frac{1}{4}\left(\frac{3I_1}{2} + I_2\right) = 0$$

$$3I_2 + \frac{1}{4}I_2 + I_1\frac{3}{8}I_1 = 0$$

$$\frac{13}{4}I_2 + \frac{11}{8}I_1 = 0$$

$$\alpha = \frac{I_2}{I_1} = -\frac{11}{26}.$$

EXAMPLE 14.57. *Find the driving point admittance function and respective pole-zero location plot of the network shown in Fig. E14.73.*

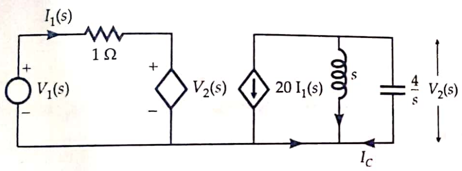

Fig. E14.73

SOLUTION. Considering left hand side loop (Fig. E14.73), for KVL we can write,

$$V_1(s) - I_1(s) + V_2(s) = 0$$

i.e.,
$$I_1(s) = V_1(s) + V_2(s) \qquad \text{...}(i)$$

Similarly, for right hand loop, applying KCL we get

$$20I_1(s) - I_L + I_C = 0 \qquad \text{...}(ii)$$

or
$$20I_1(s) + \frac{V_2(s)}{s} + \frac{V_2(s)}{\frac{4}{s}} = 0$$

or
$$20\,I_1(s) = -V_2(s)\left[\frac{1}{s} + \frac{s}{4}\right] = -V_2(s)\left[\frac{4+s^2}{4s}\right]$$

$$\therefore \quad V_2(s) = \frac{-20 \times 4s}{(4+4s^2)}I_1(s) = \frac{-80\,s}{s^2+4} \times I_1(s).$$

Substituting the value of $V_2(s)$ from equation (*i*) in equation (*ii*) we get

$$I_1(s) = V_1(s) + \left(\frac{-80\,s}{s^2+4}I_1(s)\right)$$

or
$$I_1(s)^0\left[1 + \frac{80s}{s^2+4}\right] = V_1(s)$$

∴ Driving point admittance

$$= \frac{I_1(s)}{V_1(s)} = 1 + \frac{80s}{s^2+4} = \frac{s^2+4+80s}{s^2+4}.$$

CHAPTER 14

EXERCISES

1. Find $Y_{11}(s)$ of the circuit shown in Fig. P14.1.

[**Hint.** $V_1(s) = I_1(s)\left[R + \frac{1}{Cs}\right]$

$$\therefore \quad Y_{11}(s) = \frac{I_1(s)}{V_1(s)} = \frac{1}{\left(R + \frac{1}{Cs}\right)} = \frac{1}{R}\left[\frac{s}{s + \frac{1}{R}}\right]\ ]$$

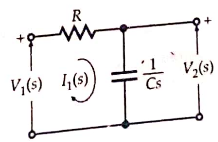

Fig. P14.1