

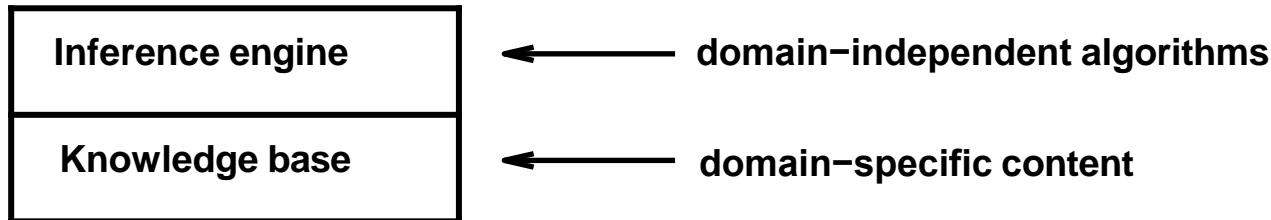
Logical Agents

Chapter 7

Outline

- ◆ Knowledge-based agents
- ◆ Wumpus world
- ◆ Logic in general—models and entailment
- ◆ Propositional (Boolean) logic
- ◆ Equivalence, validity, satisfiability
- ◆ Inference rules and theorem proving
 - forward chaining
 - backward chaining
 - resolution

Knowledge bases



Knowledge base = set of sentences in a **formal** language

Declarative approach to building an agent (or other system):

Tell it what it needs to know

Then it can Ask itself what to do—answers should follow from the KB

Agents can be viewed at the **knowledge level**

i.e., what they know, regardless of how implemented

Or at the **implementation level**

i.e., data structures in KB and algorithms that manipulate them

A simple knowledge-based agent

```
function KB-Agent(percept) returns an action
    static: KB, a knowledge base
        t, a counter, initially 0, indicating time
    Tell(KB, Make-Percept-Sentence(percept, t))
    action  $\leftarrow$  Ask(KB, Make-Action-Query(t))
    Tell(KB, Make-Action-Sentence(action, t))
    t  $\leftarrow$  t + 1
    return action
```

The agent must be able to:

- Represent states, actions, etc.
- Incorporate new percepts
- Update internal representations of the world
- Deduce hidden properties of the world
- Deduce appropriate actions

Wumpus World P E A S description

Performance measure

gold +1000, death -1000

-1 per step, -10 for using the arrow

Environment

Squares adjacent to wumpus are smelly

Squares adjacent to pit are breezy

Glitter iff gold is in the same square

Shooting kills wumpus if you are facing it

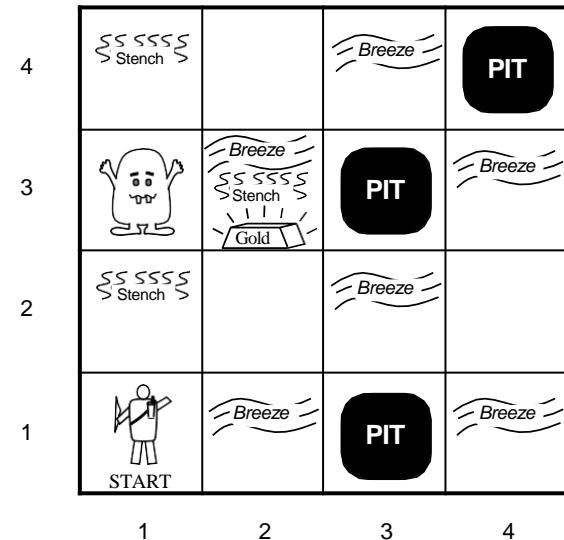
Shooting uses up the only arrow

Grabbing picks up gold if in same square

Releasing drops the gold in same square

Actuators Left turn, Right turn,
Forward, Grab, Release, Shoot

Sensors Breeze, Glitter, Smell



Wumpus world characterization

Observable?? No—only local perception

Deterministic?? Yes—outcomes exactly specified

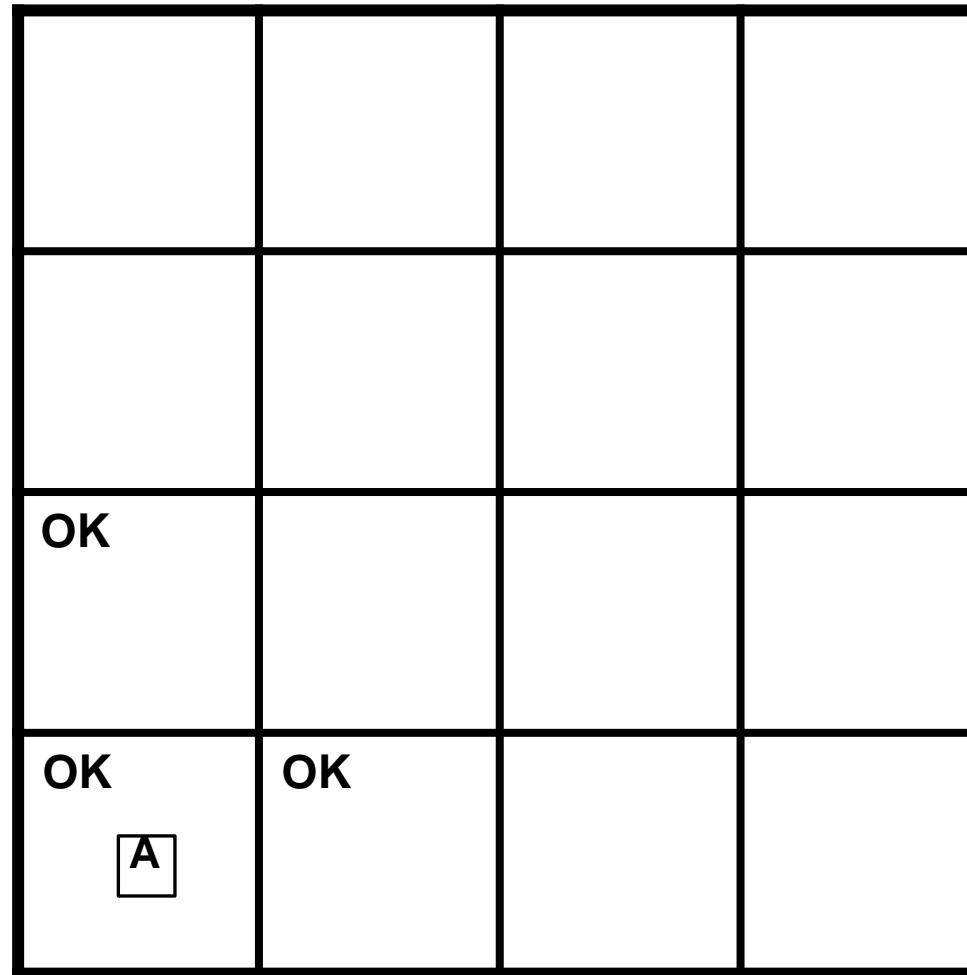
Episodic?? No—sequential at the level of actions

Static?? Yes—Wumpus and Pits do not move

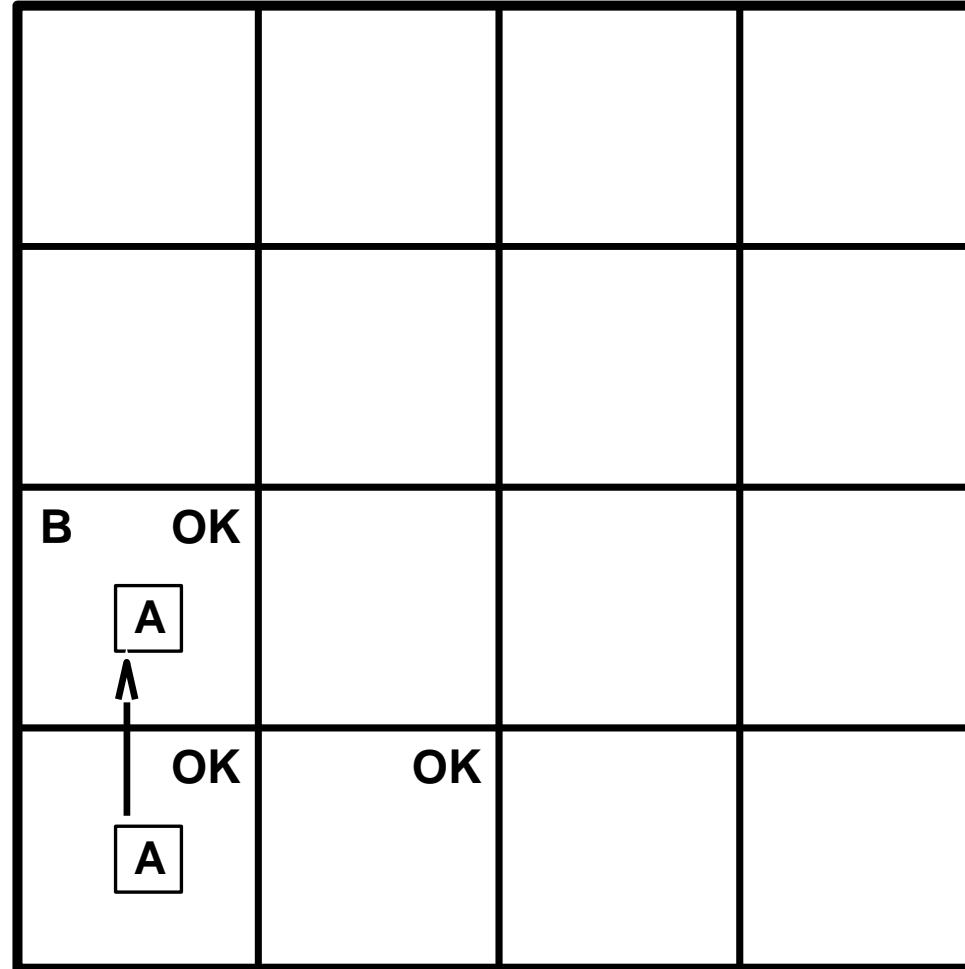
Discrete?? Yes

Single-agent?? Yes—Wumpus is essentially a natural feature

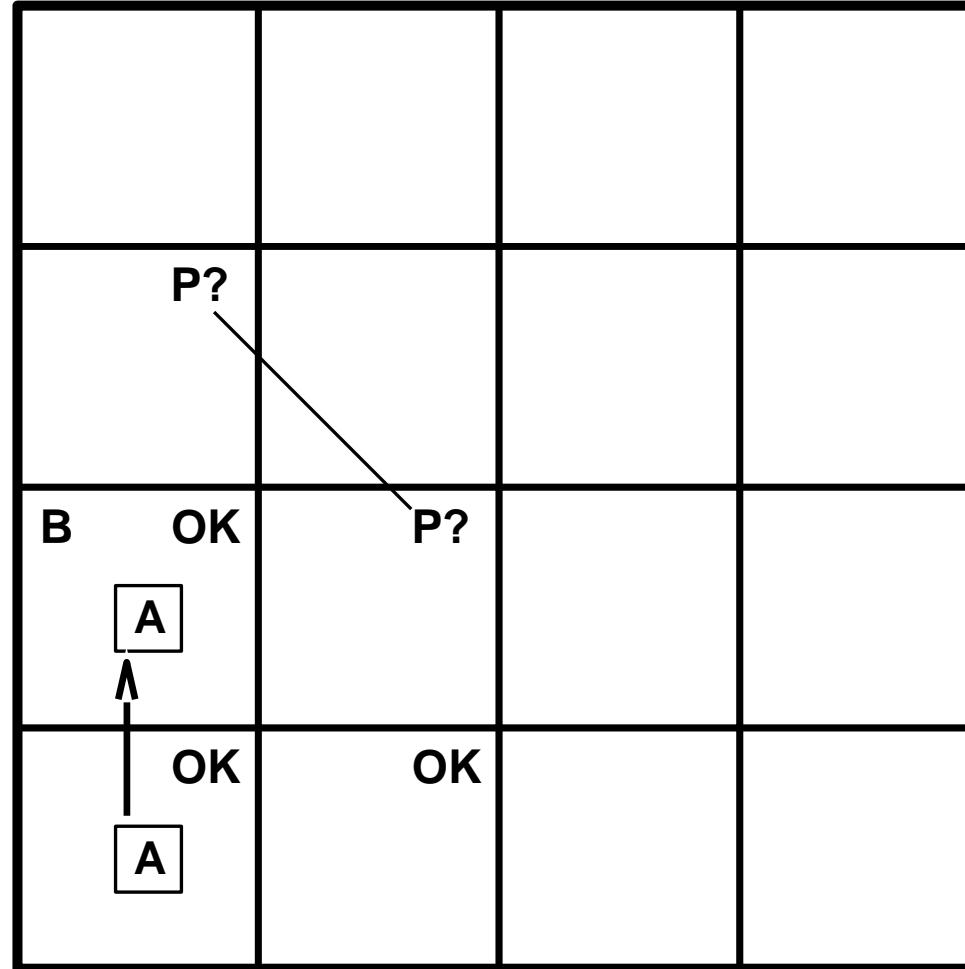
Exploring a wumpus world



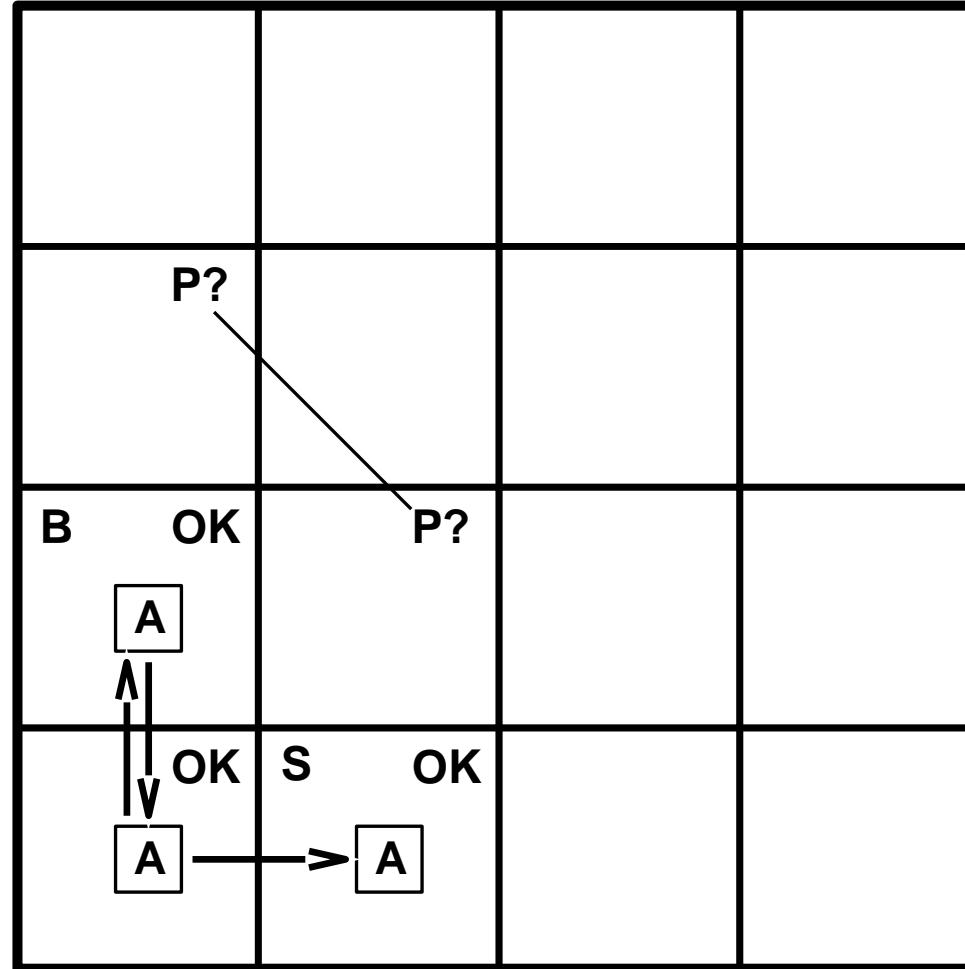
Exploring a wumpus world



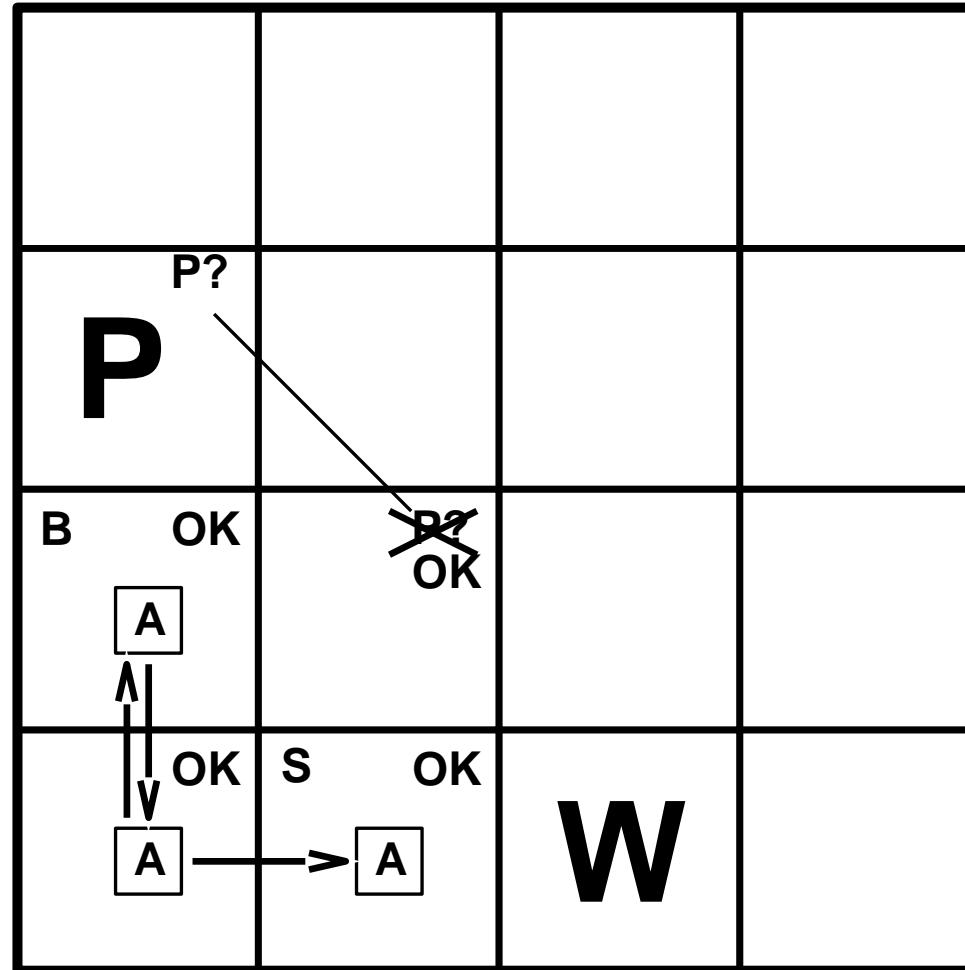
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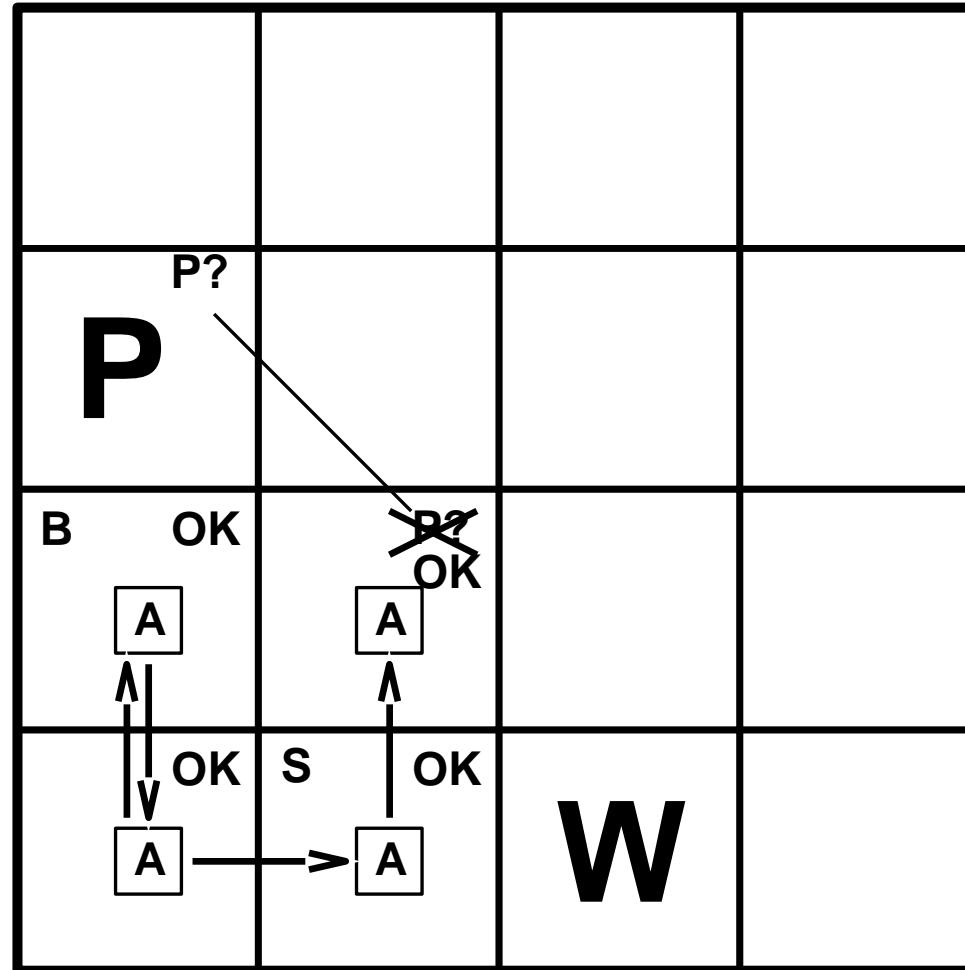
Exploring a wumpus world



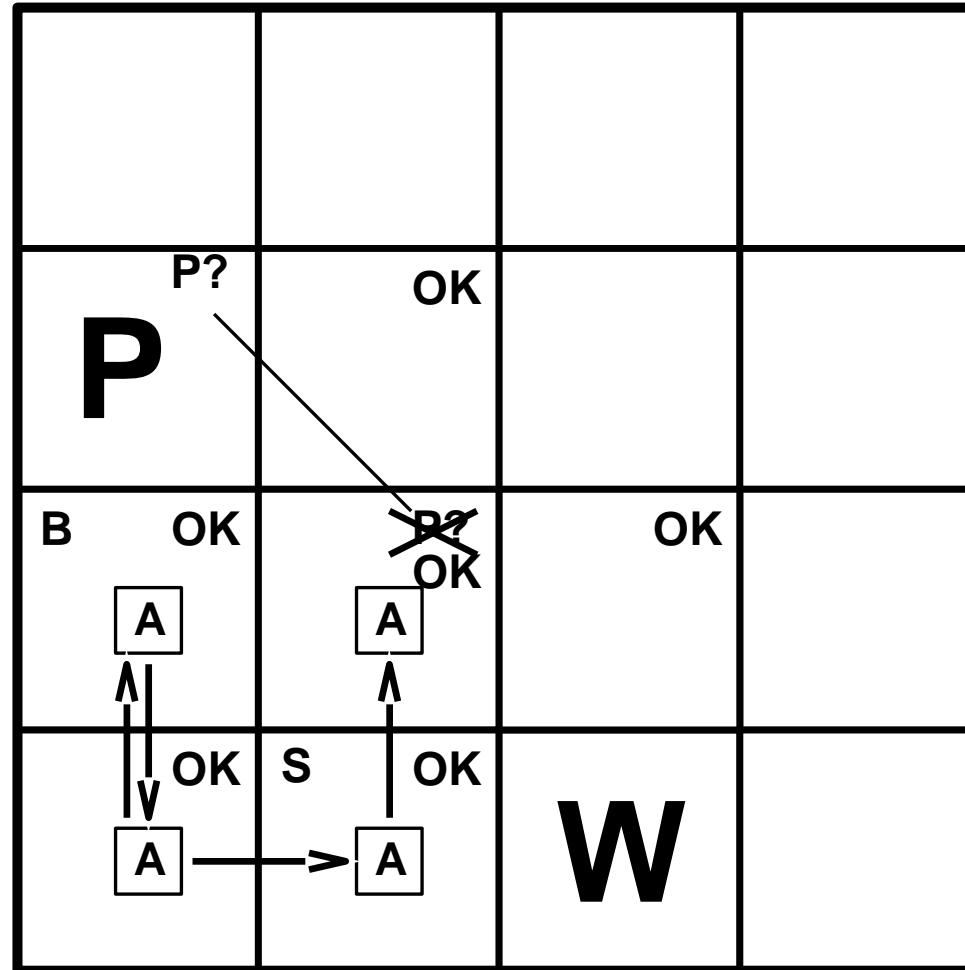
Exploring a wumpus world



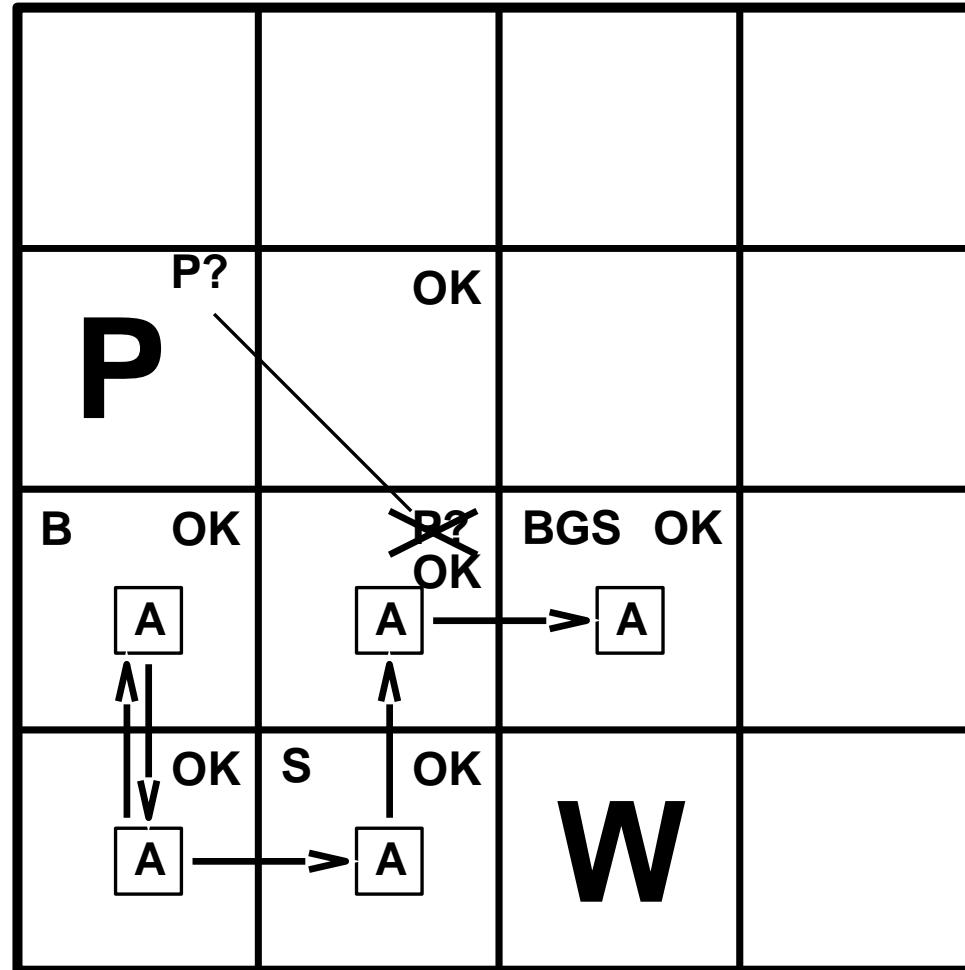
Exploring a wumpus world



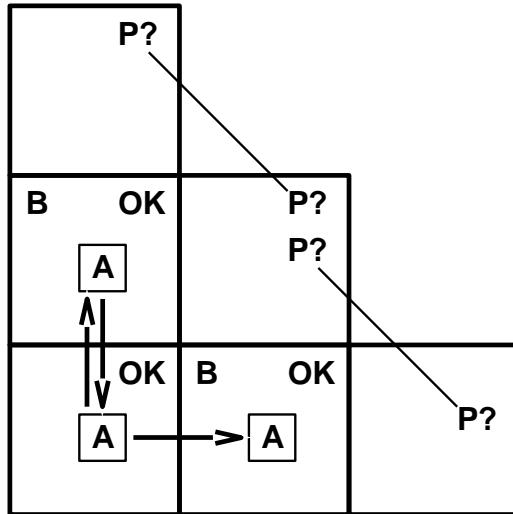
Exploring a wumpus world



Exploring a wumpus world



Other tight spots

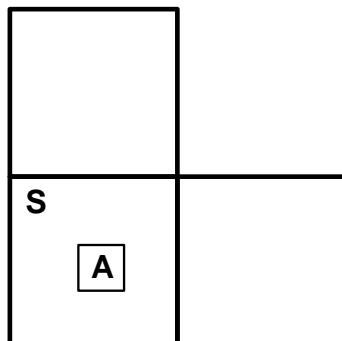


Breeze in (1,2) and (2,1)
⇒ no safe actions

Assuming pits uniformly distributed,
(2,2) has pit w/ prob 0.86, vs. 0.31

Smell in (1,1)
⇒ cannot move

Can use a strategy of **coercion**:
shoot straight ahead
wumpus was there ⇒ dead ⇒ safe
wumpus wasn't there ⇒ safe



Logic in general

Logics are formal languages for representing information such that conclusions can be drawn

Syntax defines the sentences in the language

Semantics define the “meaning” of sentences;
i.e., define truth of a sentence in a world

E.g., the language of arithmetic

$x + 2 \geq y$ is a sentence; $x2 + y >$ is not a sentence

$x + 2 \geq y$ is true iff the number $x + 2$ is no less than the number y

$x + 2 \geq y$ is true in a world where $x = 7, y = 1$

$x + 2 \geq y$ is false in a world where $x = 0, y = 6$

Entailment

Entailment means that one thing follows from another:

$$KB \models \alpha$$

Knowledge base KB entails sentence α

if and only if

α is true in all worlds where KB is true

E.g., the KB containing “the Giants won” and “the Reds won”
entails “Either the Giants won or the Reds won”

E.g., $x + y = 4$ entails $4 = x + y$

Entailment is a relationship between sentences (i.e., syntax)
that is based on semantics

Note: brains process syntax (of some sort)

Models

Logicians typically think in terms of **models**, which are formally structured worlds with respect to which truth can be evaluated

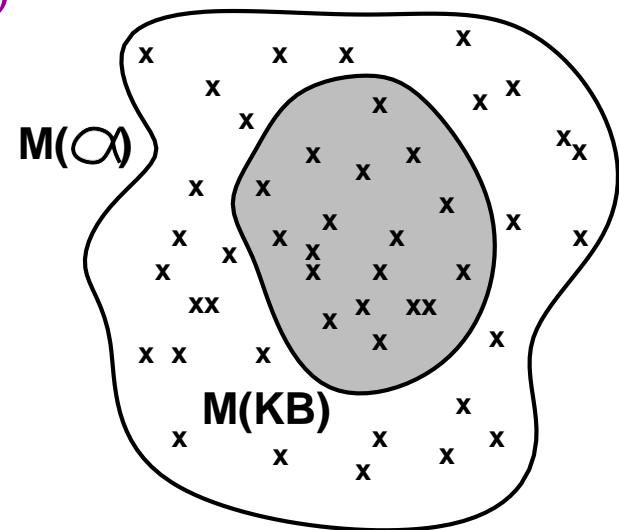
We say m is a model of a sentence α if α is true in m

$M(\alpha)$ is the set of all models of α

Then $KB \models \alpha$ if and only if $M(KB) \subseteq M(\alpha)$

E.g. KB = Giants won and Reds won

α = Giants won

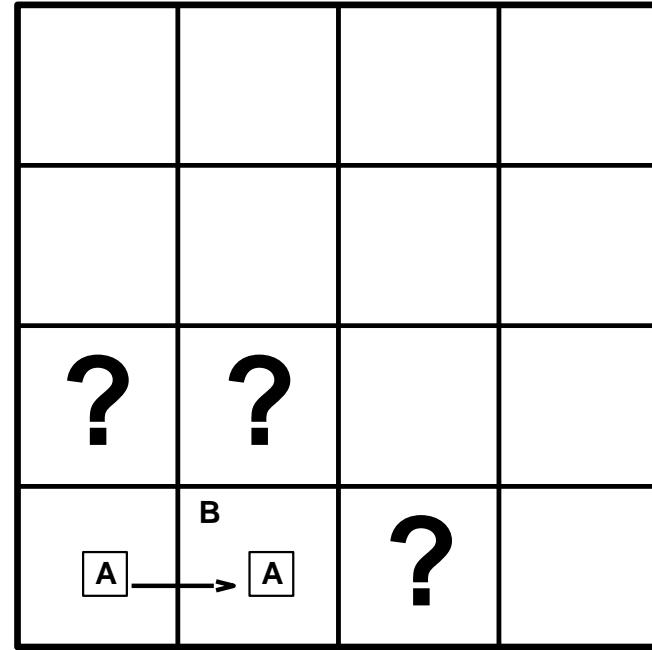


Entailment in the wumpus world

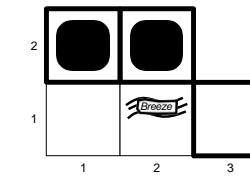
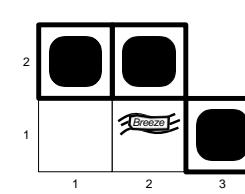
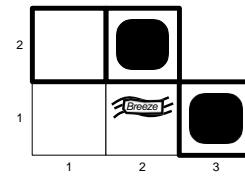
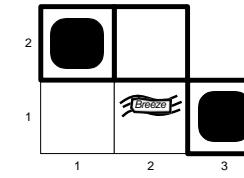
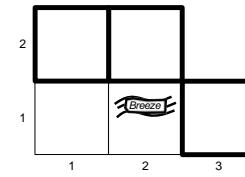
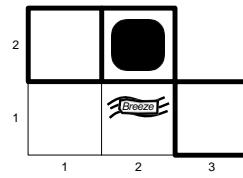
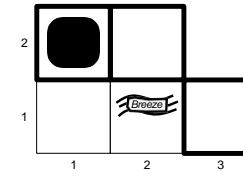
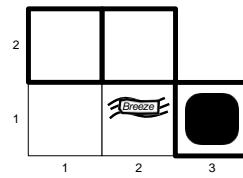
Situation after detecting nothing in [1,1],
moving right, breeze in [2,1]

Consider possible models for ?s
assuming only pits

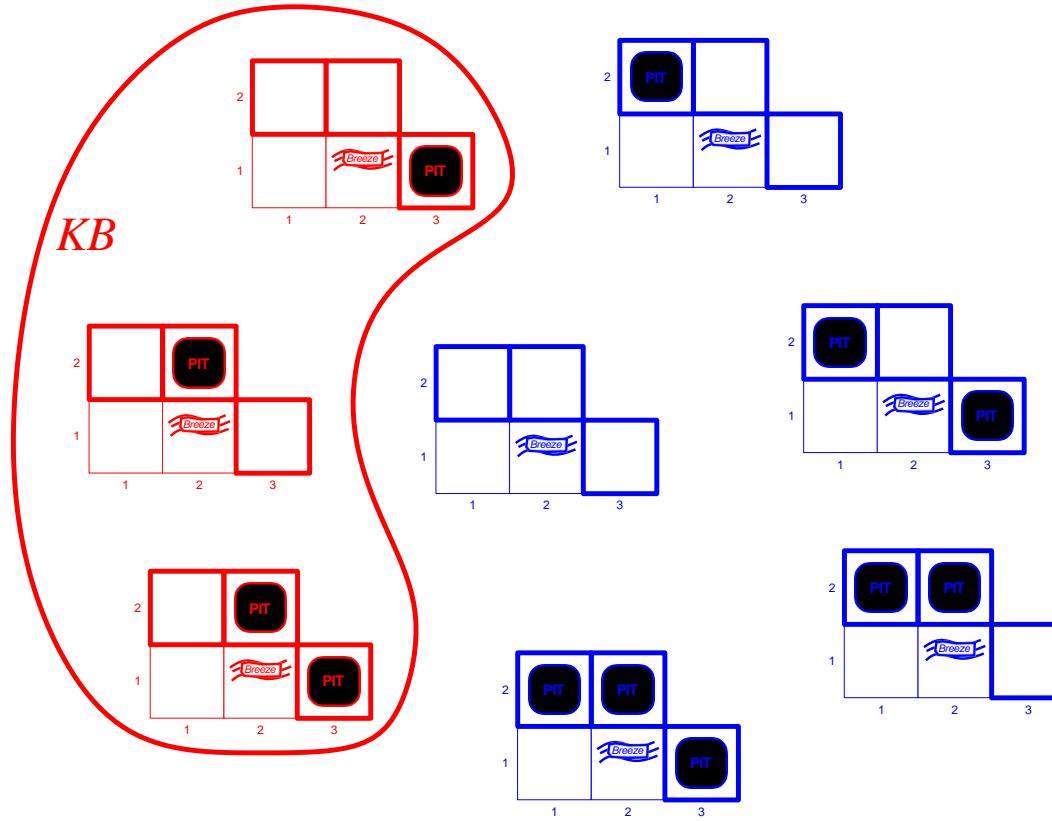
3 Boolean choices \Rightarrow 8 possible models



Wumpus models

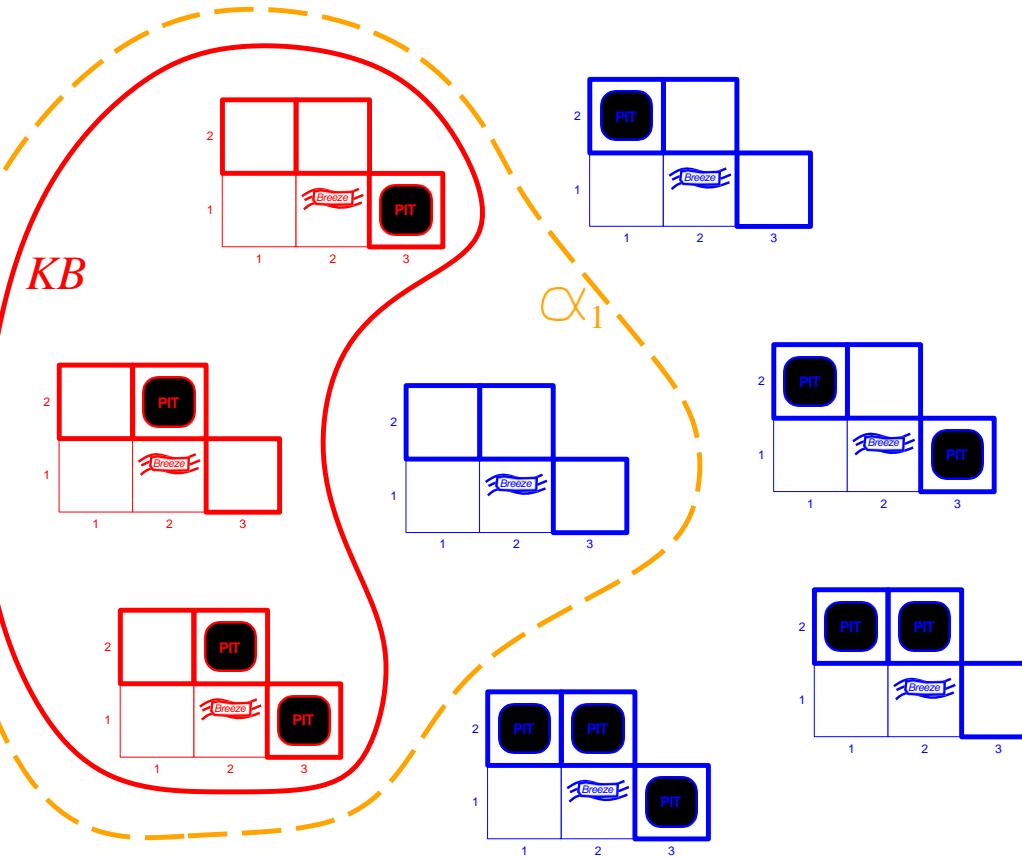


Wumpus models



KB = wumpus-world rules + observations

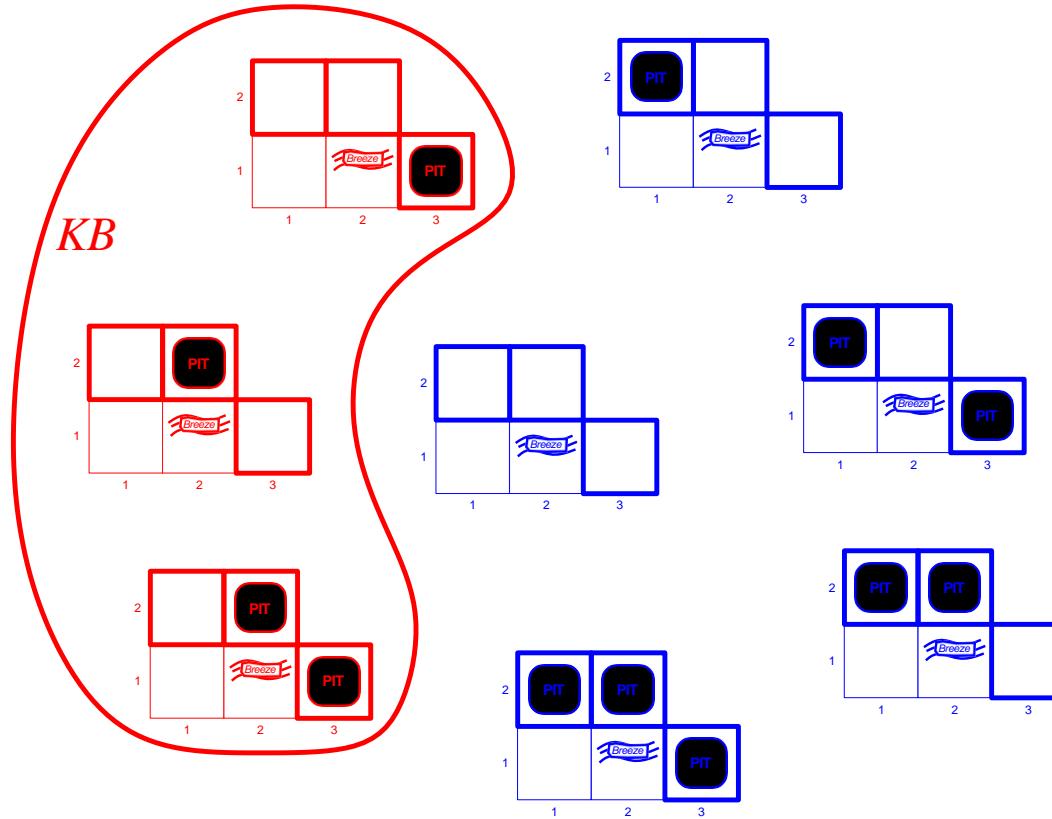
Wumpus models



KB = wumpus-world rules + observations

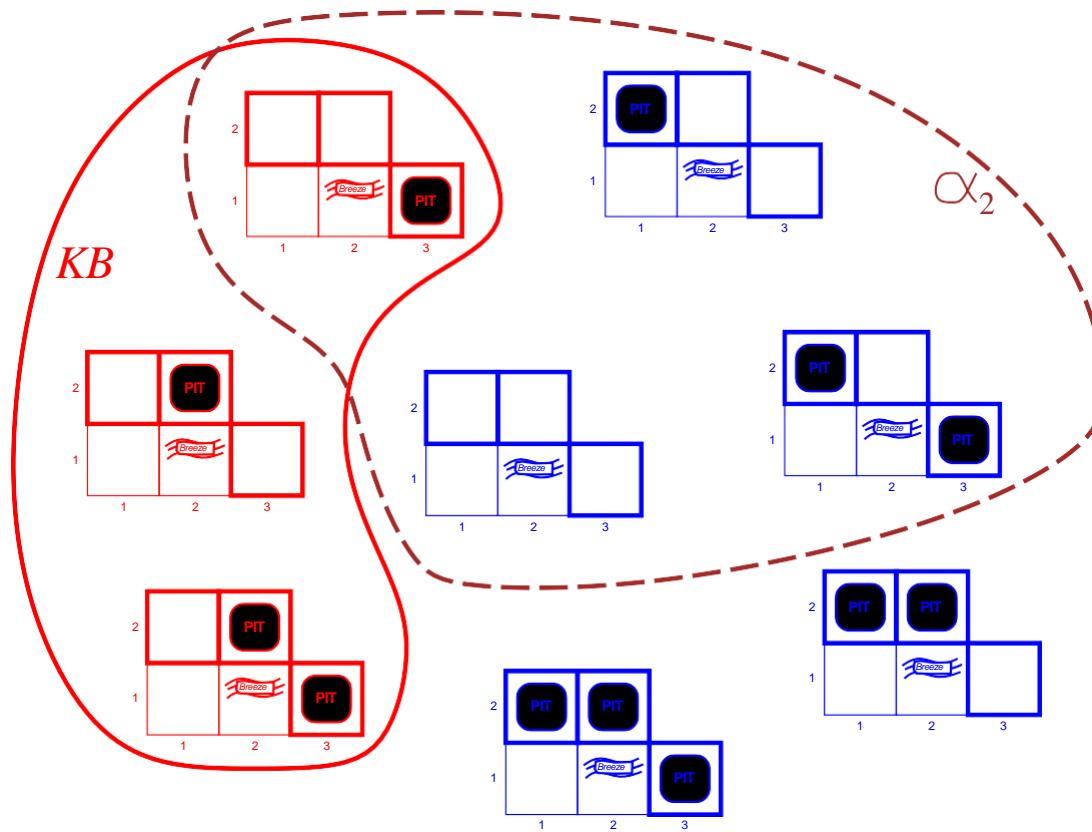
α_1 = “[1,2] is safe”, $KB \models \alpha_1$, proved by model checking

Wumpus models



KB = wumpus-world rules + observations

Wumpus models



KB = wumpus-world rules + observations

α_2 = “[2,2] is safe”, *KB* |= α_2

Inference

$KB \vdash_i \alpha$ = sentence α can be derived from KB by procedure i

Consequences of KB are a haystack; α is a needle.

Entailment = needle in haystack; inference = finding it

Soundness: i is sound if

whenever $KB \vdash_i \alpha$, it is also true that $KB \models \alpha$

Completeness: i is complete if

whenever $KB \models \alpha$, it is also true that $KB \vdash_i \alpha$

Preview: we will define a logic (first-order logic) which is expressive enough to say almost anything of interest, and for which there exists a sound and complete inference procedure.

That is, the procedure will answer any question whose answer follows from what is known by the KB .

Propositional logic: Syntax

Propositional logic is the simplest logic—illustrates basic ideas

The proposition symbols P_1, P_2 etc are sentences

If S is a sentence, $\neg S$ is a sentence (negation)

If S_1 and S_2 are sentences, $S_1 \wedge S_2$ is a sentence (conjunction)

If S_1 and S_2 are sentences, $S_1 \vee S_2$ is a sentence (disjunction)

If S_1 and S_2 are sentences, $S_1 \Rightarrow S_2$ is a sentence (implication)

If S_1 and S_2 are sentences, $S_1 \Leftrightarrow S_2$ is a sentence (biconditional)

Propositional logic: Semantics

Each model specifies true/false for each proposition symbol

E.g. $P_{1,2}$ $P_{2,2}$ $P_{3,1}$
true true false

(With these symbols, 8 possible models, can be enumerated automatically.)

Rules for evaluating truth with respect to a model m :

$\neg S$	is true iff	S	is false	
$S_1 \wedge S_2$	is true iff	S_1	is true and	S_2 is true
$S_1 \vee S_2$	is true iff	S_1	is true or	S_2 is true
$S_1 \Rightarrow S_2$	is true iff	S_1	is false or	S_2 is true
i.e.,	is false iff	S_1	is true and	S_2 is false
$S_1 \Leftrightarrow S_2$	is true iff	$S_1 \Rightarrow S_2$	is true and	$S_2 \Rightarrow S_1$ is true

Simple recursive process evaluates an arbitrary sentence, e.g.,

$$\neg P_{1,2} \wedge (P_{2,2} \vee P_{3,1}) = \text{true} \wedge (\text{false} \vee \text{true}) = \text{true} \wedge \text{true} = \text{true}$$

Truth tables for connectives

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
<i>false</i>	<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>true</i>
<i>false</i>	<i>true</i>	<i>true</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>false</i>
<i>true</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>
<i>true</i>	<i>true</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>

Wumpus world sentences

Let $P_{i,j}$ be true if there is a pit in $[i,j]$.

Let $B_{i,j}$ be true if there is a breeze in $[i,j]$.

$\neg P_{1,1}$

$\neg B_{1,1}$

$B_{2,1}$

“Pits cause breezes in adjacent squares”

Wumpus world sentences

Let $P_{i,j}$ be true if there is a pit in $[i,j]$.

Let $B_{i,j}$ be true if there is a breeze in $[i,j]$.

$$\neg P_{1,1}$$

$$\neg B_{1,1}$$

$$B_{2,1}$$

“Pits cause breezes in adjacent squares”

$$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$$

$$B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$$

“A square is breezy if and only if there is an adjacent pit”

Truth tables for inference

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	true	true	true	true	false	false						
false	false	false	false	false	false	true	true	true	false	true	false	false
.
false	true	false	false	false	false	false	true	true	false	true	true	false
false	✓	false	false	false	false	✓	✓	✓	✓	✓	✓	✓
false	✓	false	false	false	✓	✓	✓	✓	✓	✓	✓	✓
false	✓	false	false	✓	✓	✓	✓	✓	✓	✓	✓	✓
false	true	false	false	true	false	false	true	false	false	true	true	false
.
true	false	true	true	false	true	false						

Enumerate rows (different assignments to symbols),
if KB is true in row, check that α is too

Inference by enumeration

Depth-first enumeration of all models is sound and complete

```
function TT-Entails?(KB, a) returns true or false
    inputs: KB, the knowledge base, a sentence in propositional logic
            a, the query, a sentence in propositional logic
    symbols  $\leftarrow$  a list of the proposition symbols in KB and a
    return TT-Check-All(KB, a, symbols, [ ])



---


function TT-Check-All(KB, a, symbols, model) returns true or false
    if Empty?(symbols) then
        if PL-True?(KB, model) then return PL-True?(a, model)
        else return true
    else do
        P  $\leftarrow$  First(symbols); rest  $\leftarrow$  Rest(symbols)
        return TT-Check-All(KB, a, rest, Extend(P, true, model)) and
               TT-Check-All(KB, a, rest, Extend(P, false, model))
```

$O(2^n)$ for n symbols; problem is co-NP-complete

Logical equivalence

Two sentences are **logically equivalent** iff true in same models:

$\alpha \equiv \beta$ if and only if $\alpha \models \beta$ and $\beta \models \alpha$

- | | |
|--|--|
| $(\alpha \wedge \beta) \equiv (\beta \wedge \alpha)$ | commutativity of \wedge |
| $(\alpha \vee \beta) \equiv (\beta \vee \alpha)$ | commutativity of \vee |
| $((\alpha \wedge \beta) \wedge \gamma) \equiv (\alpha \wedge (\beta \wedge \gamma))$ | associativity of \wedge |
| $((\alpha \vee \beta) \vee \gamma) \equiv (\alpha \vee (\beta \vee \gamma))$ | associativity of \vee |
| $\neg(\neg \alpha) \equiv \alpha$ | double-negation elimination |
| $(\alpha \Rightarrow \beta) \equiv (\neg \beta \Rightarrow \neg \alpha)$ | contraposition |
| $(\alpha \Rightarrow \beta) \equiv (\neg \alpha \vee \beta)$ | implication elimination |
| $(\alpha \Leftrightarrow \beta) \equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha))$ | biconditional elimination |
| $\neg(\alpha \wedge \beta) \equiv (\neg \alpha \vee \neg \beta)$ | De Morgan |
| $\neg(\alpha \vee \beta) \equiv (\neg \alpha \wedge \neg \beta)$ | De Morgan |
| $(\alpha \wedge (\beta \vee \gamma)) \equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma))$ | distributivity of \wedge over \vee |
| $(\alpha \vee (\beta \wedge \gamma)) \equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma))$ | distributivity of \vee over \wedge |

Validity and satisfiability

A sentence is **valid** if it is true in **all** models,

e.g., True , $A \vee \neg A$, $A \Rightarrow A$, $(A \wedge (A \Rightarrow B)) \Rightarrow B$

Validity is connected to inference via the **Deduction Theorem**:

$KB \models a$ if and only if $(KB \Rightarrow a)$ is valid

A sentence is **satisfiable** if it is true in **some** model

e.g., $A \vee B$, C

A sentence is **unsatisfiable** if it is true in **no** models

e.g., $A \wedge \neg A$

Satisfiability is connected to inference via the following:

$KB \models a$ if and only if $(KB \wedge \neg a)$ is unsatisfiable

i.e., prove a by *reductio ad absurdum*

Proof methods

Proof methods divide into (roughly) two kinds:

Application of inference rules

- Legitimate (sound) generation of new sentences from old
- **Proof** = a sequence of inference rule applications
 - Can use inference rules as operators in a standard search alg.
- Typically require translation of sentences into a **normal form**

Model checking

truth table enumeration (always exponential in n)

improved backtracking, e.g., Davis–Putnam–Logemann–Loveland

heuristic search in model space (sound but incomplete)

e.g., min-conflicts-like hill-climbing algorithms

Forward and backward chaining

Horn Form (restricted)

KB = conjunction of Horn clauses

Horn clause =

- ◆ proposition symbol; or
- ◆ (conjunction of symbols) \Rightarrow symbol

E.g., $C \wedge (B \Rightarrow A) \wedge (C \wedge D \Rightarrow B)$

Modus Ponens (for Horn Form): complete for Horn KBs

$$\frac{a_1, \dots, a_n, \quad a_1 \wedge \dots \wedge a_n \Rightarrow \beta}{\beta}$$

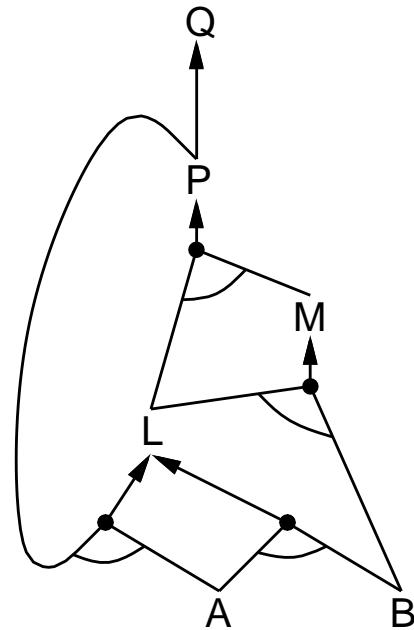
Can be used with forward chaining or backward chaining.

These algorithms are very natural and run in linear time

Forward chaining

Idea: fire any rule whose premises are satisfied in the *KB*,
add its conclusion to the *KB*, until query is found

$$\begin{aligned} P &\Rightarrow Q \\ L \wedge M &\Rightarrow P \\ B \wedge L &\Rightarrow M \\ A \wedge P &\Rightarrow L \\ A \wedge B &\Rightarrow L \\ A \\ B \end{aligned}$$



Forward chaining algorithm

```
function PL-FC-Entails?(KB, q) returns true or false
```

inputs: KB , the knowledge base, a set of propositional Horn clauses

q , the query, a proposition symbol

local variables: $count$, a table, indexed by clause, initially the number of premises

$inferred$, a table, indexed by symbol, each entry initially $false$

$agenda$, a list of symbols, initially the symbols known in KB

while $agenda$ is not empty do

$p \leftarrow \text{Pop}(agenda)$

 unless $inferred[p]$ do

$inferred[p] \leftarrow \text{true}$

 for each Horn clause c in whose premise p appears do

 decrement $count[c]$

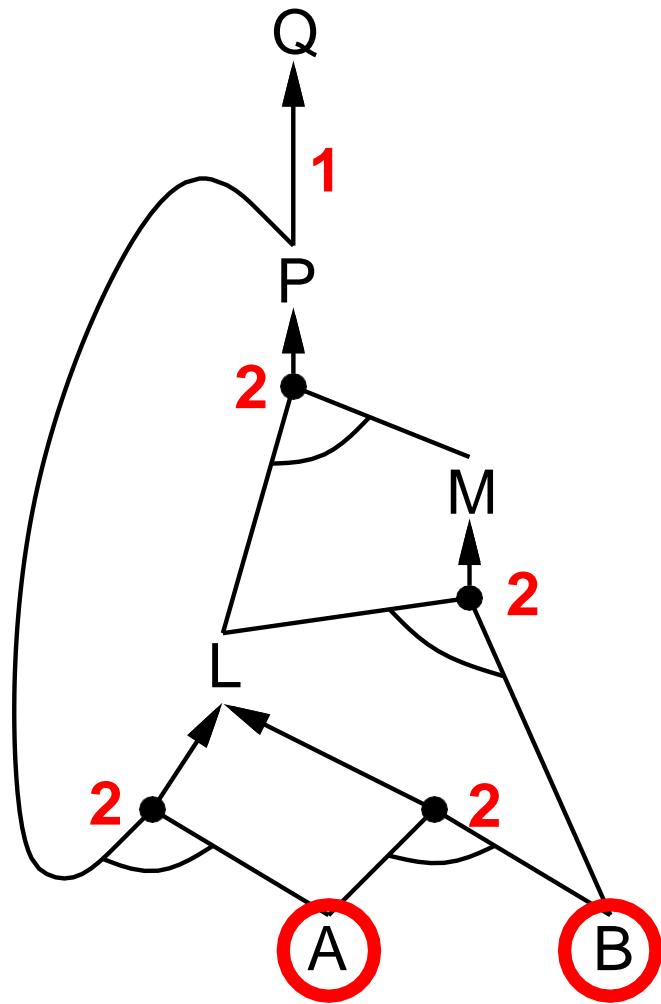
 if $count[c] = 0$ then do

 if $\text{Head}[c] = q$ then return $true$

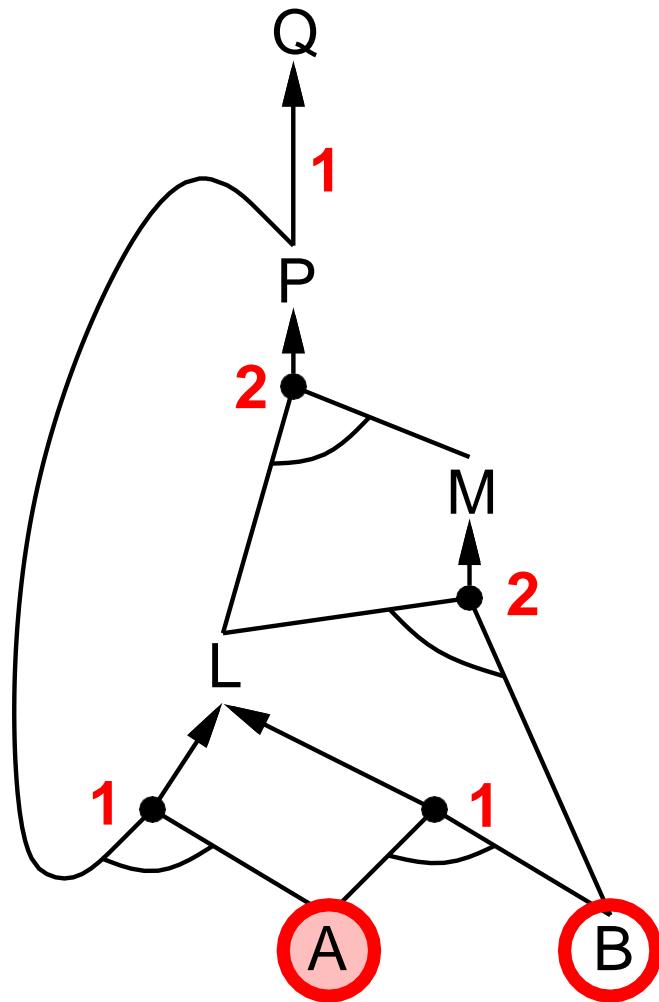
 Push($\text{Head}[c]$, $agenda$)

return $false$

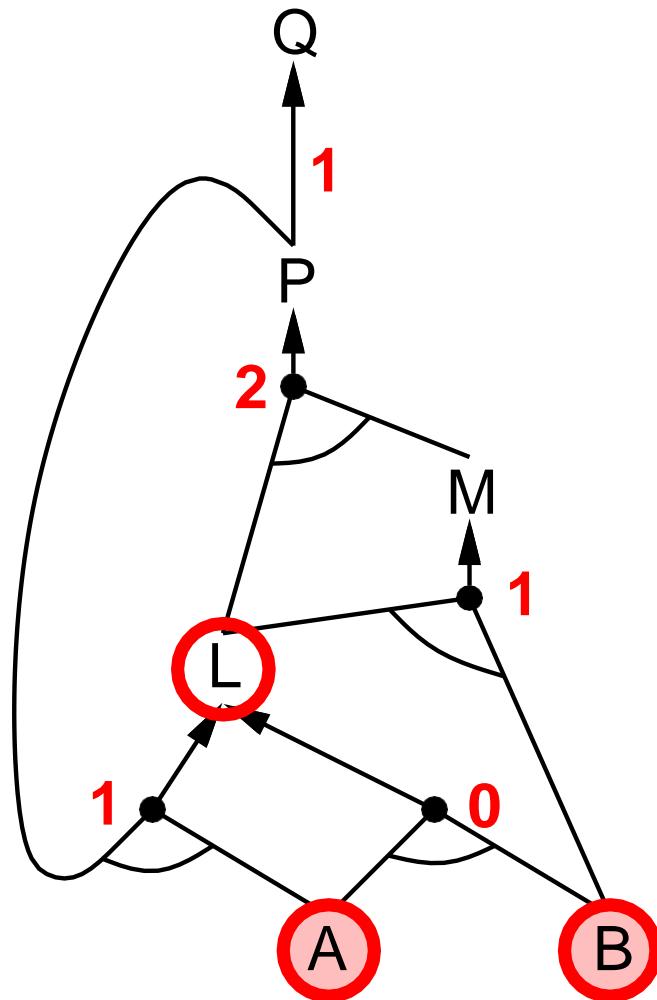
Forward chaining example



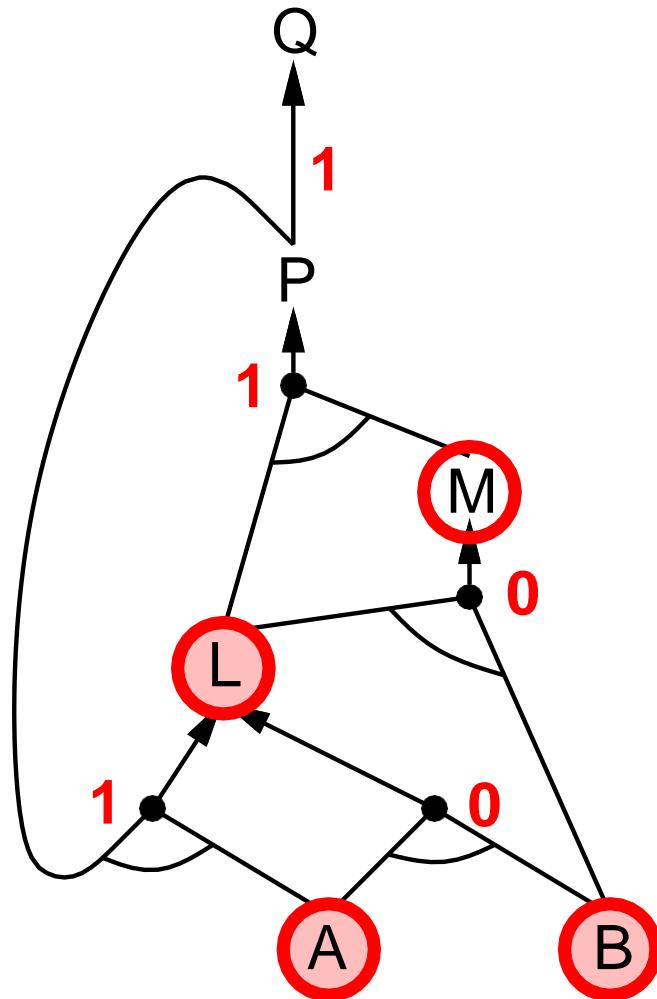
Forward chaining example



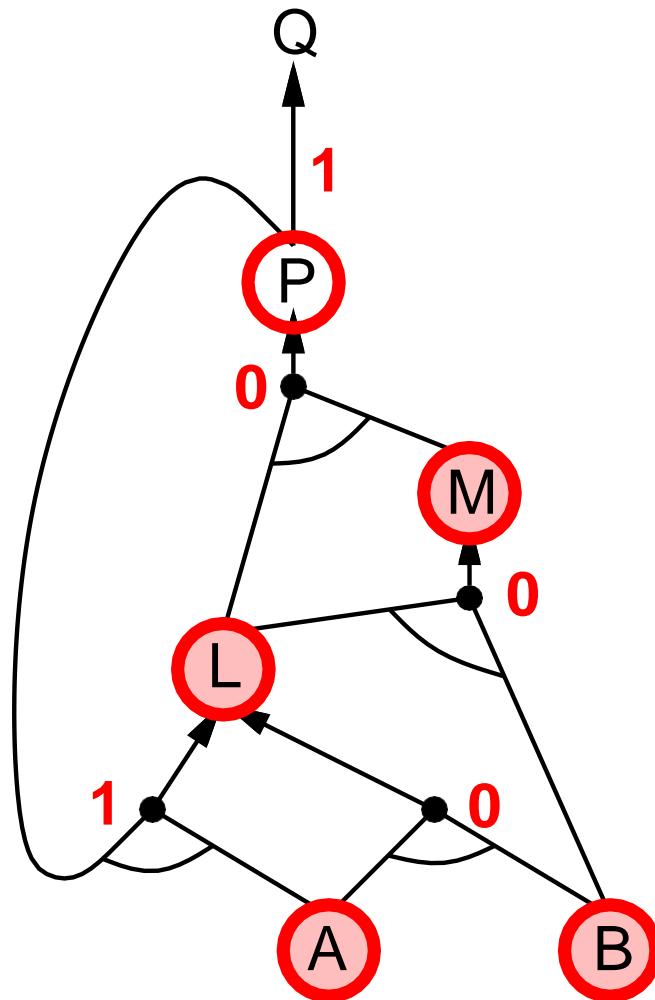
Forward chaining example



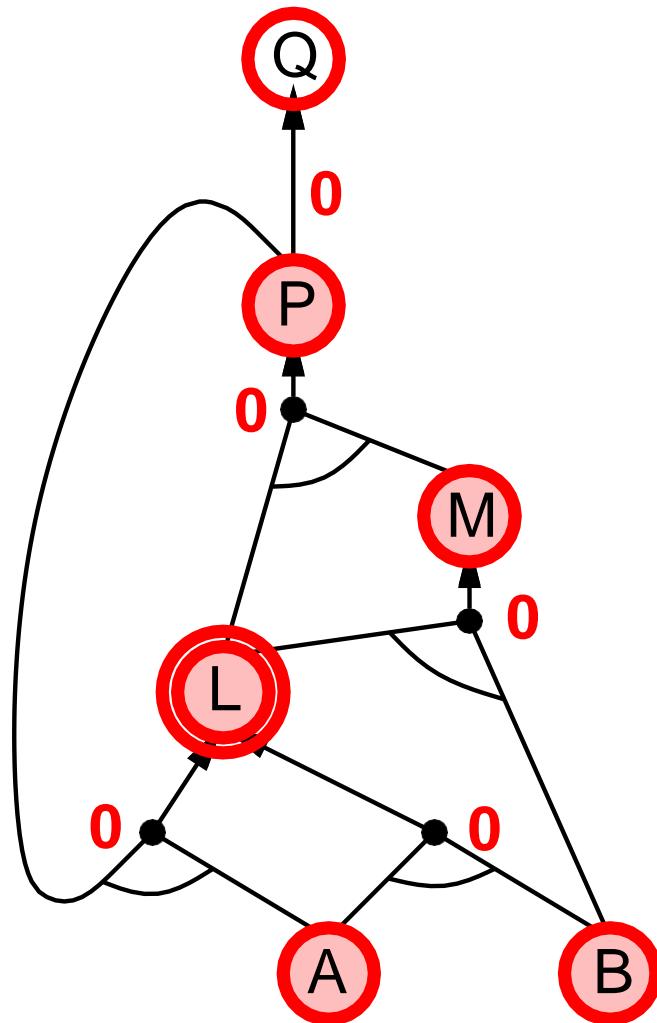
Forward chaining example



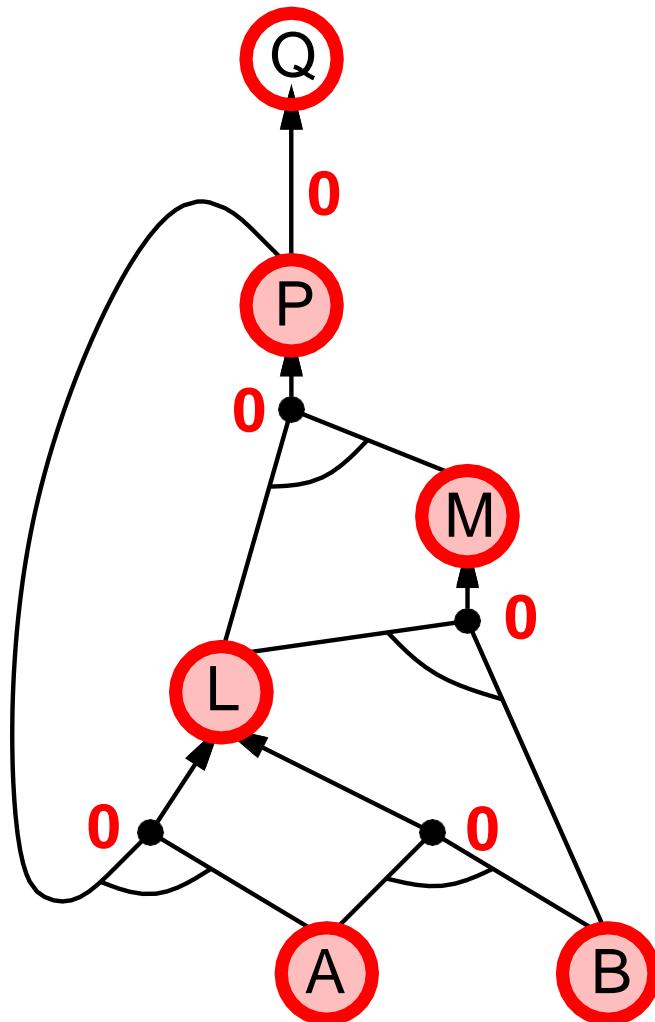
Forward chaining example



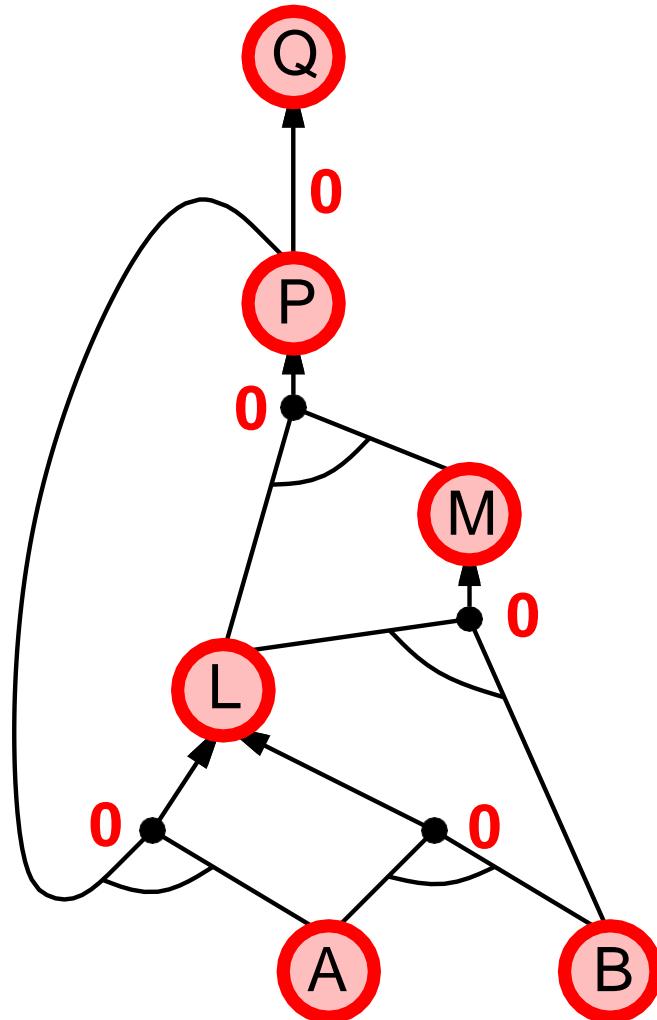
Forward chaining example



Forward chaining example



Forward chaining example



Proof of completeness

FC derives every atomic sentence that is entailed by KB

1. FC reaches a **fixed point** where no new atomic sentences are derived
2. Consider the final state as a model m , assigning true/false to symbols
3. Every clause in the original KB is true in m
Proof: Suppose a clause $a_1 \wedge \dots \wedge a_k \Rightarrow b$ is false in m
Then $a_1 \wedge \dots \wedge a_k$ is true in m and b is false in m
Therefore the algorithm has not reached a fixed point!
4. Hence m is a model of KB
5. If $KB \models q$, q is true in **every** model of KB , including m

General idea: construct any model of KB by sound inference, check α

Backward chaining

Idea: work backwards from the query q :

- to prove q by BC,

- check if q is known already, or

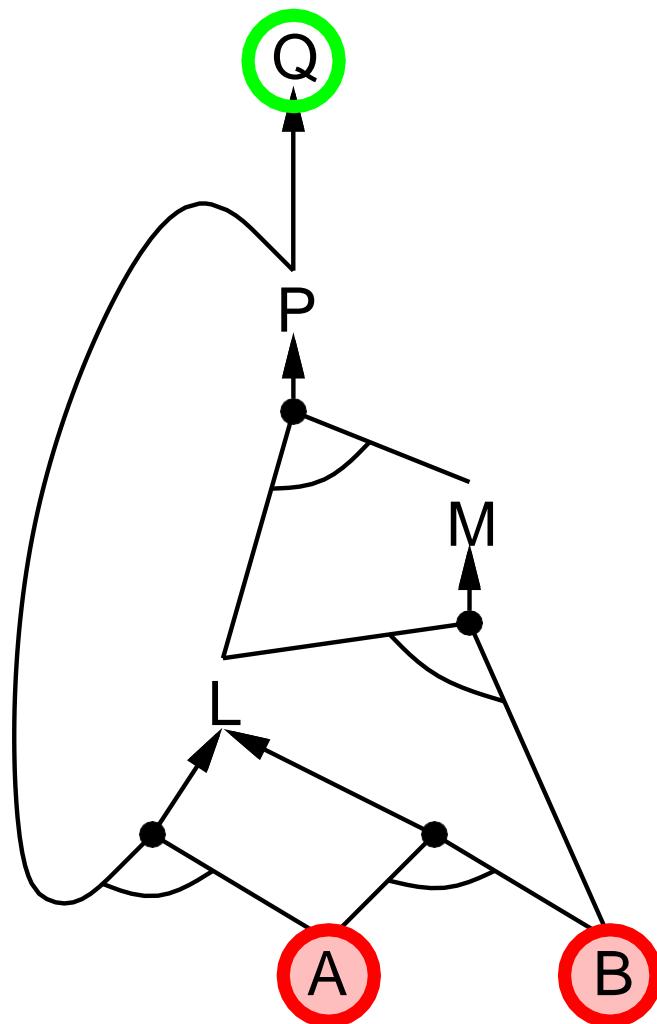
- prove by BC all premises of some rule concluding q

Avoid loops: check if new subgoal is already on the goal stack

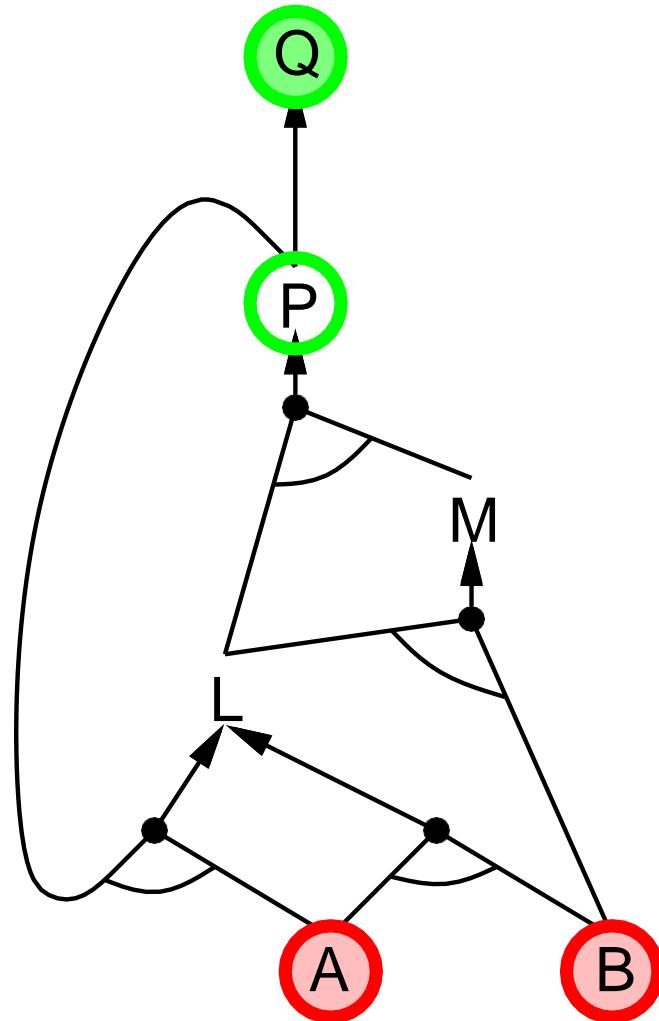
Avoid repeated work: check if new subgoal

- 1) has already been proved true, or
- 2) has already failed

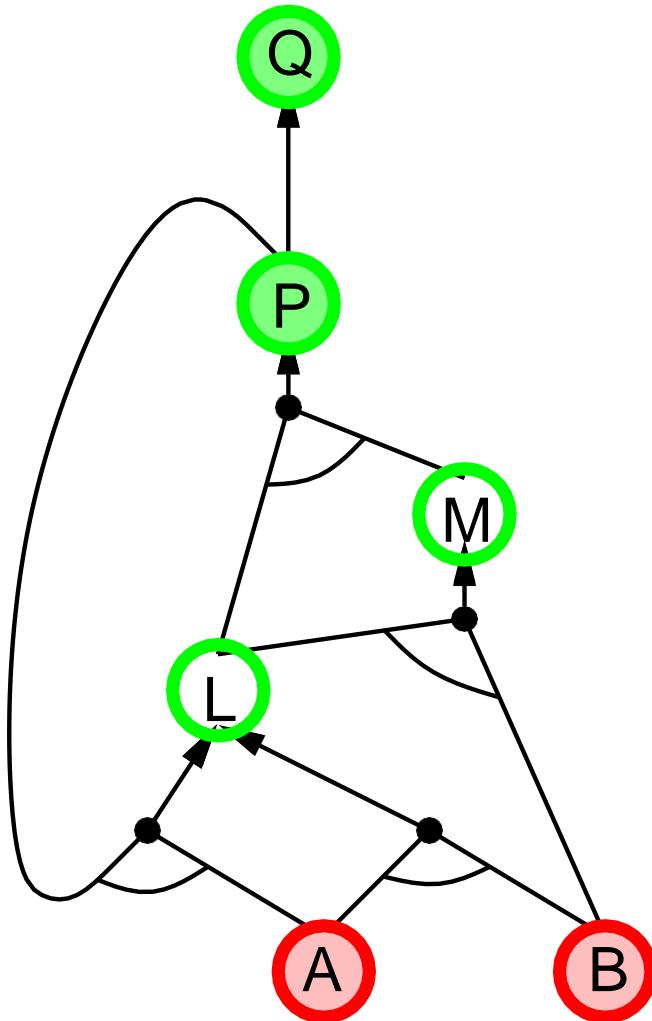
Backward chaining example



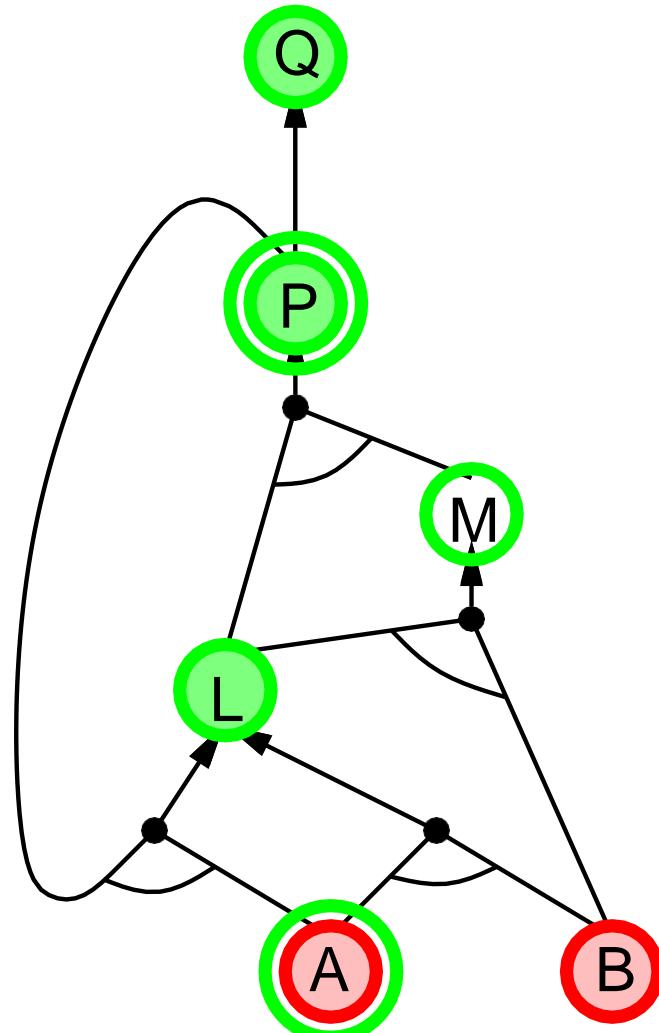
Backward chaining example



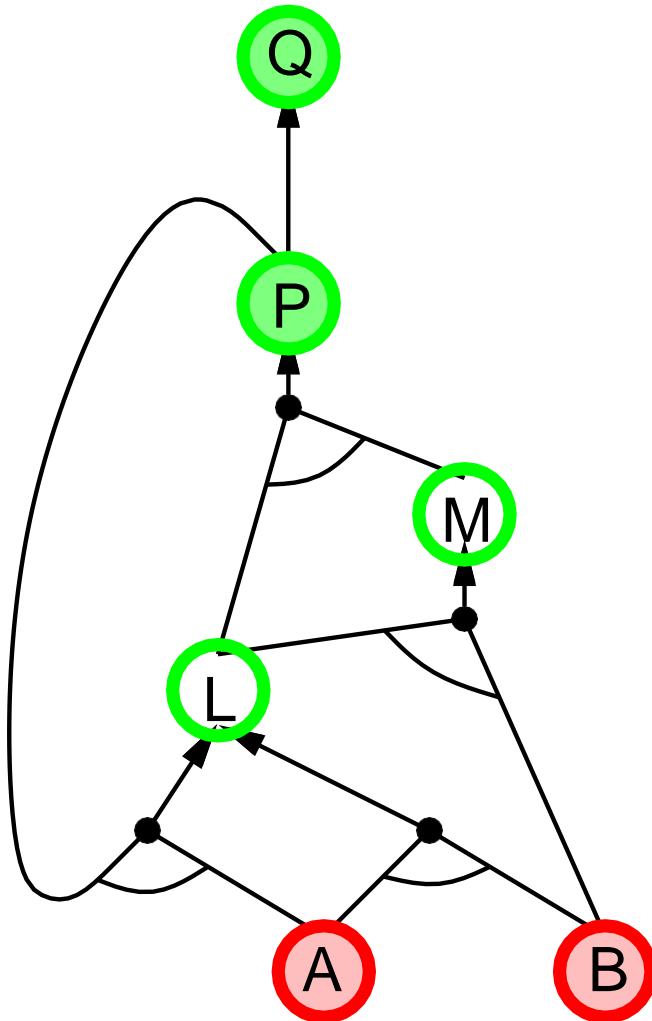
Backward chaining example



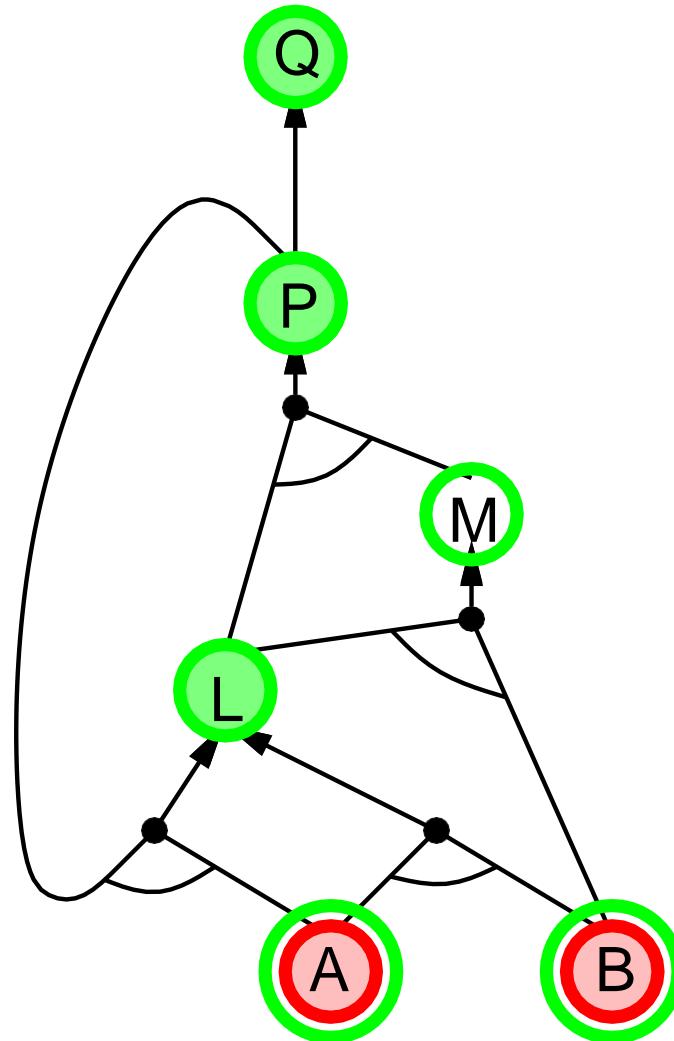
Backward chaining example



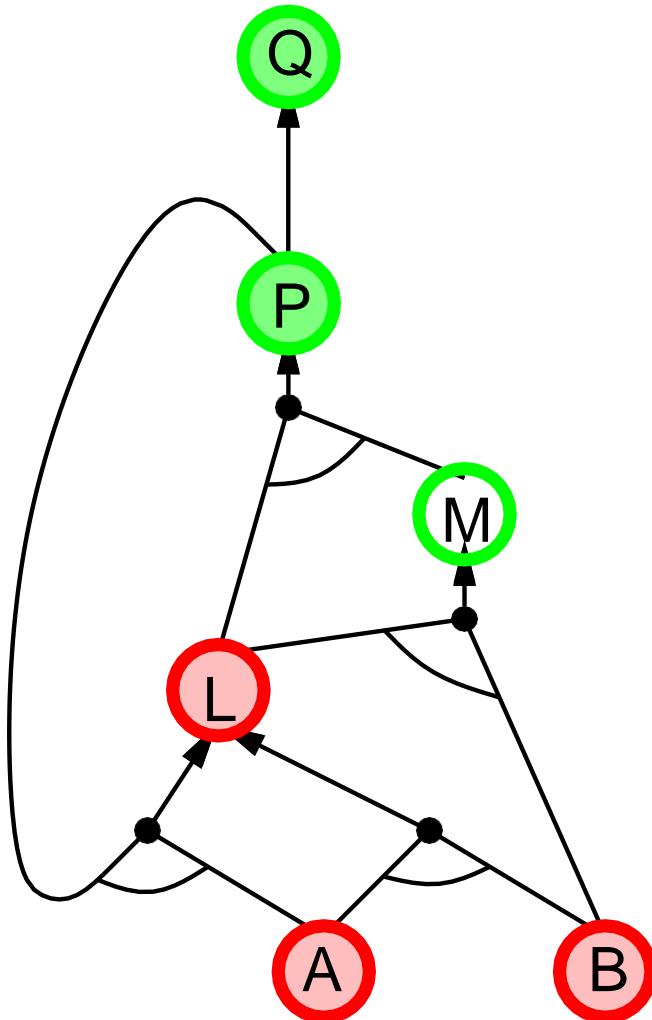
Backward chaining example



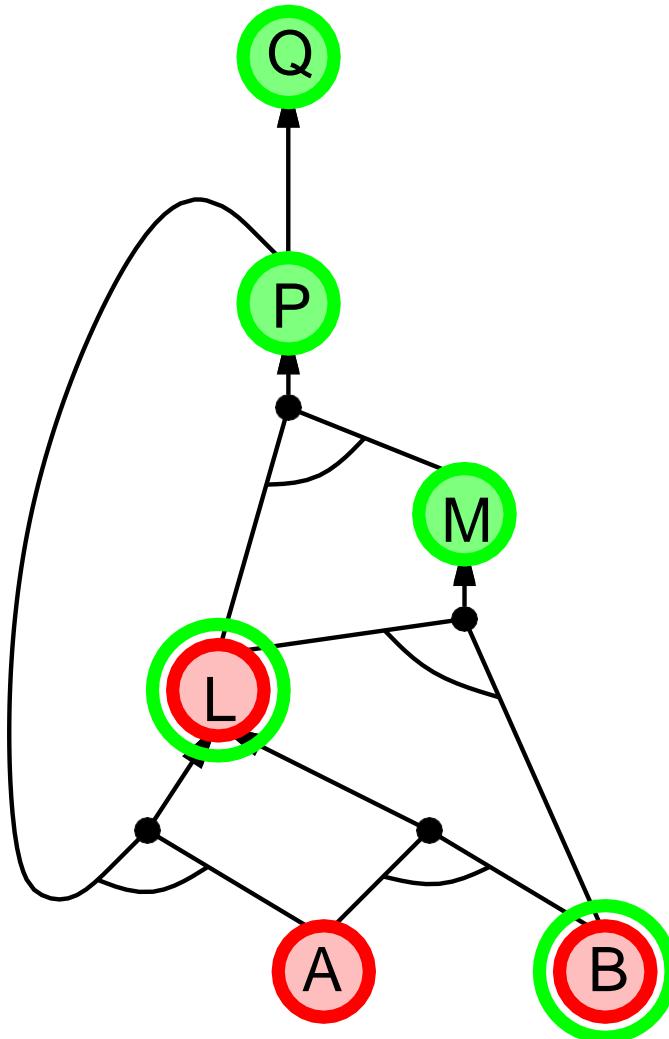
Backward chaining example



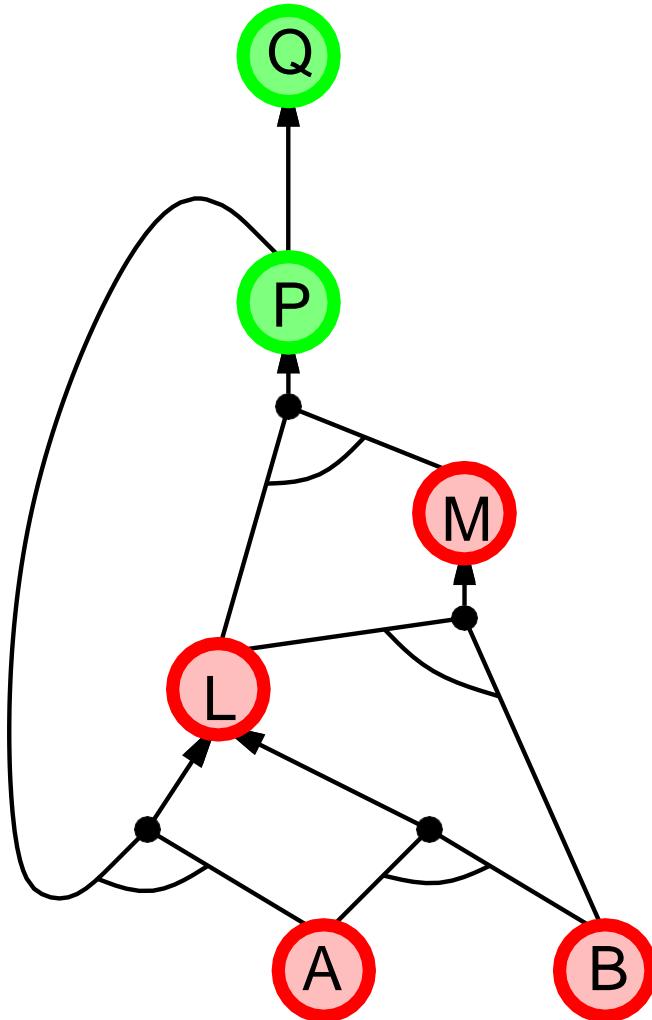
Backward chaining example



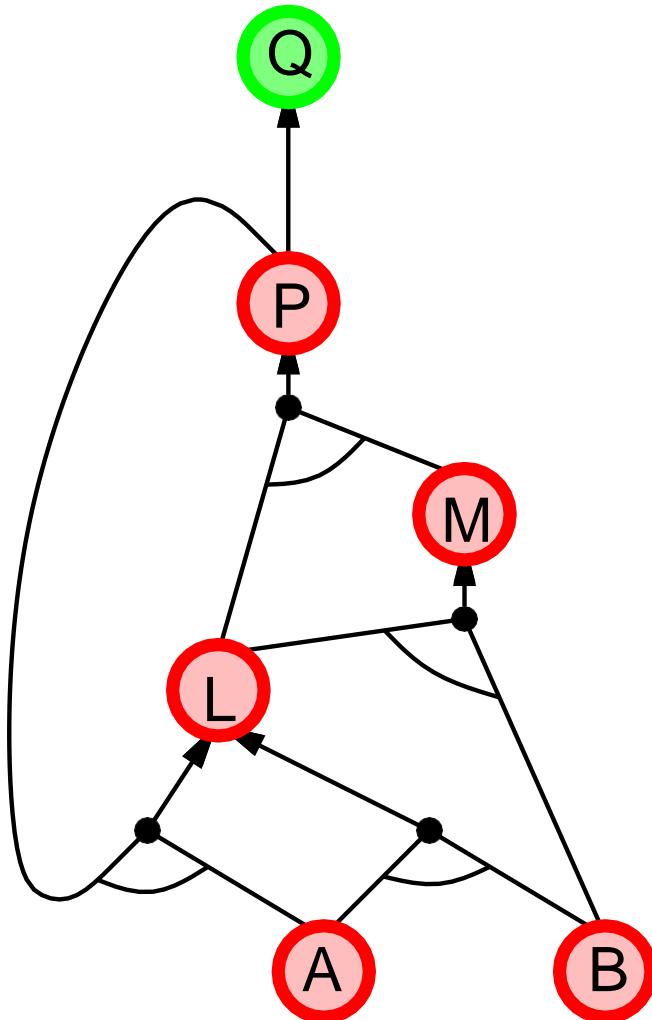
Backward chaining example



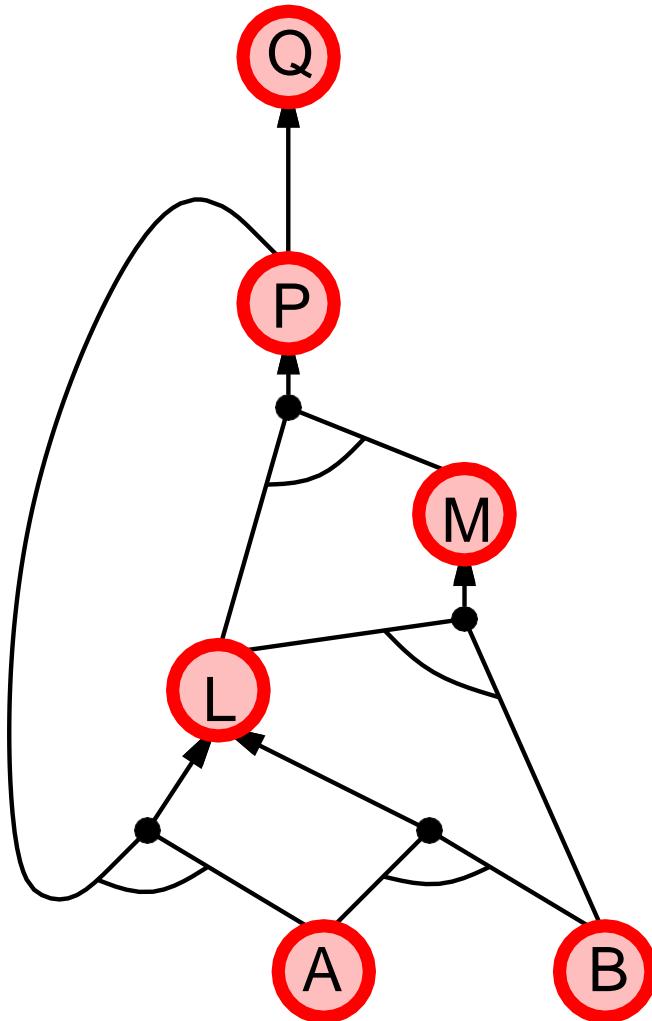
Backward chaining example



Backward chaining example



Backward chaining example



Forward vs. backward chaining

FC is **data-driven**, cf. automatic, unconscious processing,
e.g., object recognition, routine decisions

May do lots of work that is irrelevant to the goal

BC is **goal-driven**, appropriate for problem-solving,
e.g., Where are my keys? How do I get into a PhD program?

Complexity of BC can be **much less** than linear in size of KB

Resolution

Conjunctive Normal Form (CNF—universal)

conjunction of disjunctions of literals
| $\{z\}$ }
clauses

E.g., $(A \vee \neg B) \wedge (B \vee \neg C \vee \neg D)$

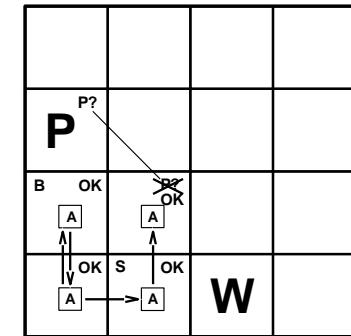
Resolution inference rule (for CNF): complete for propositional logic

$$\frac{f_1 \vee \dots \vee f_k, \quad m_1 \vee \dots \vee m_n}{f_1 \vee \dots \vee f_{i-1} \vee f_{i+1} \vee \dots \vee f_k \vee m_1 \vee \dots \vee m_{j-1} \vee m_{j+1} \vee \dots \vee m_n}$$

where f_i and m_j are complementary literals. E.g.,

$$\frac{P_{1,3} \vee P_{2,2}, \quad \neg P_{2,2}}{P_{1,3}}$$

Resolution is sound and complete for propositional logic



Conversion to CNF

$$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$$

1. Eliminate \Leftrightarrow , replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$.

$$(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$$

2. Eliminate \Rightarrow , replacing $\alpha \Rightarrow \beta$ with $\neg \alpha \vee \beta$.

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg(P_{1,2} \vee P_{2,1}) \vee B_{1,1})$$

3. Move \neg inwards using de Morgan's rules and double-negation:

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge ((\neg P_{1,2} \wedge \neg P_{2,1}) \vee B_{1,1})$$

4. Apply distributivity law (\vee over \wedge) and flatten:

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$$

Resolution algorithm

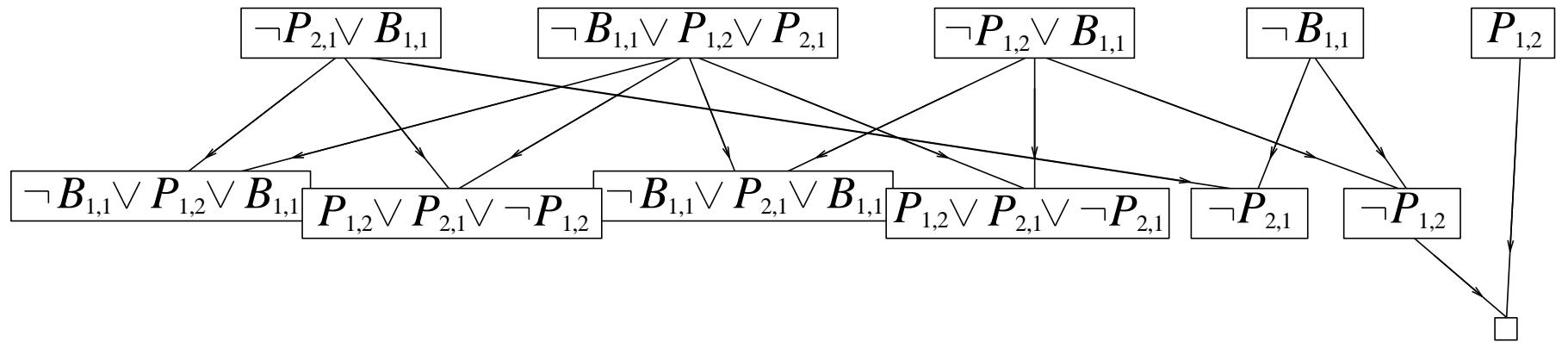
Proof by contradiction, i.e., show $KB \wedge \neg\alpha$ unsatisfiable

```
function PL-Resolution( $KB, \alpha$ ) returns true or false
    inputs:  $KB$ , the knowledge base, a sentence in propositional logic
             $\alpha$ , the query, a sentence in propositional logic

     $clauses \leftarrow$  the set of clauses in the CNF representation of  $KB \wedge \neg\alpha$ 
     $new \leftarrow \{ \}$ 
    loop do
        for each  $C_i, C_j$  in  $clauses$  do
             $resolvents \leftarrow$  PL-Resolve( $C_i, C_j$ )
            if  $resolvents$  contains the empty clause then return true
             $new \leftarrow new \cup resolvents$ 
        if  $new \subseteq clauses$  then return false
         $clauses \leftarrow clauses \cup new$ 
```

Resolution example

$$KB = (B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})) \wedge \neg B_{1,1} \quad \alpha = \neg P_{1,2}$$



Summary

Logical agents apply inference to a knowledge base
to derive new information and make decisions

Basic concepts of logic:

- syntax: formal structure of sentences
- semantics: truth of sentences wrt models
- entailment: necessary truth of one sentence given another
- inference: deriving sentences from other sentences
- soundness: derivations produce only entailed sentences
- completeness: derivations can produce all entailed sentences

Wumpus world requires the ability to represent partial and negated information, reason by cases, etc.

Forward, backward chaining are linear-time, complete for Horn clauses

Resolution is complete for propositional logic

Propositional logic lacks expressive power

ToDo

Code for a logical agent for the Wumpus world is available at various places on the Web (including a SWISH notebook)

The most natural language for encoding this agent is Prolog

Download existing Prolog code for a logical agent for the Wumpus world and try to understand its working

Prolog uses a backward-chaining inference engine to derive consequences

Demonstrate your Wumpus agent in SWISH