Previous lecture Summary:

(1) CNN operators.

* Convolution (cross-correlation)
* Pooling

  - Stride
  - padding

Multiple channels

* Inputs
* output

1X1 convolution.

→ Nothing to do with spatial relations

→ Output channels.

256 channel → [1x1] → 64 channel output.

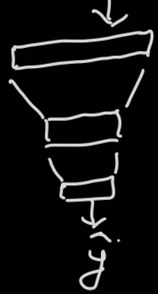
Forward Prop

Input → [C] → [C] → [C]

[P] [P] [P]

Lab

1D



Today:

Backprop errors in CNN.

↓

Gradient of Loss is computed.

(conv layer)

⟶

⟵

Notations

- (1) Inputs are $\underline{X}$: X_{ij}
- (2) Parameters in a filter (Kernel) 2D → $\underline{W}$: W_{ij}
- (3) Hidden layer activation : $\underline{H}$: H_{ij}
- (4) outputs $\underline{O}$: O_{ij}
- (5) '$*$' denote convolution (cross-correlation)

$$\underline{X} * \underline{W} = \underline{O}$$

inputs (images)

(or)

$$\underline{H} * \underline{W} = \underline{O}$$

activations (op of prev. conv. --)

$\underline{X}$: 3×3, $\underline{W}$: 2×2, stride = 1, padding = none

$$\begin{bmatrix} \overset{h_{11}^{(l-1)}}{\underline{\underline{x_{11}}}} & \overset{h_{12}^{(l-1)}}{x_{12}} & \overset{h_{13}^{(l-1)}}{x_{13}} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & \overset{h_{33}^{(l-1)}}{x_{33}} \end{bmatrix} * \begin{bmatrix} \underline{\underline{w_{11}^{(l)}}} & w_{12}^{(l)} \\ w_{21}^{(l)} & w_{22}^{(l)} \end{bmatrix} = \begin{bmatrix} \overset{z\,\cancel{h}_{11}^{(l)}}{\underline{\underline{o_{11}}}} & \overset{z\,\cancel{h}_{12}^{(l)}}{o_{12}} \\ \overset{z\,\cancel{h}_{21}^{(l)}}{o_{21}} & \overset{z\,\cancel{h}_{22}^{(l)}}{o_{22}} \end{bmatrix} =$$

$$\underline{H}^{(l-1)} * \underline{W}^{(l)} = \sigma\left(\underline{H}^{(l)}\right)$$

non linear activation (ReLU).

Target outputs:

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \leftarrow$$

Loss function: $\rightarrow$

$$L\left(\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}, \begin{bmatrix} o_{11} & o_{12} \\ o_{21} & o_{22} \end{bmatrix}\right)$$

chain rule

$$O_{11} = \underline{w_{11} x_{11}} + w_{12} x_{12} + w_{21} x_{21} + w_{22} x_{22}$$

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} * \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix}$$

$$O_{12} = w_{11} x_{12} + \underline{w_{12}} x_{13} + w_{21} x_{22} + w_{22} x_{23}$$

$$O_{21} = w_{11} x_{21} + w_{12} x_{22} + w_{21} x_{31} + w_{22} x_{32}$$

$$\underline{W}\,\underline{x} \rightarrow \sigma(.) \rightarrow \underline{h}$$

$$O_{22} = w_{11} x_{22} + w_{12} x_{23} + w_{21} x_{32} + w_{22} x_{33}$$

Gradients

(1) $\boxed{\dfrac{\partial L}{\partial \underline{W}}}$ (2) $\dfrac{\partial L}{\partial \underline{h}^{(l)}}$ at any (l)

$\boxed{\dfrac{\partial L}{\partial \underline{X}}}$

$\dfrac{\partial L}{\partial \underline{W}}$:

$$y_{11} \quad y_{12} \quad y_{21} \quad y_{22} \rangle L$$

$$\underline{W} \leftarrow O_{11} \quad O_{12} \quad O_{21} \quad O_{22}$$

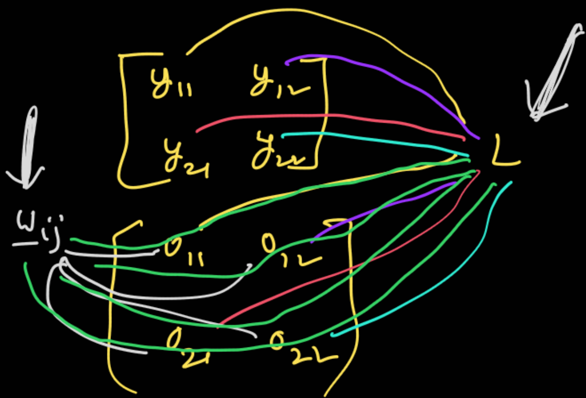
$$L = \sum (y_{ij} - O_{ij}).$$

Path 1 Path 2

$$\frac{\partial L}{\partial w_{11}} = \frac{\partial L}{\partial O_{11}} \frac{\partial O_{11}}{\partial w_{11}} + \frac{\partial L}{\partial O_{12}} \frac{\partial O_{12}}{\partial w_{11}} + \frac{\partial L}{\partial O_{21}} \frac{\partial O_{21}}{\partial w_{11}} + \frac{\partial L}{\partial O_{22}} \frac{\partial O_{22}}{\partial w_{11}}$$

Path 3 Path 4

$$\frac{\partial L}{\partial w_{12}} = \frac{\partial L}{\partial O_{11}} \frac{\partial O_{11}}{\partial w_{12}} + \frac{\partial L}{\partial O_{12}} \frac{\partial O_{12}}{\partial w_{12}} + \frac{\partial L}{\partial O_{21}} \frac{\partial O_{21}}{\partial w_{12}} + \frac{\partial L}{\partial O_{22}} \frac{\partial O_{22}}{\partial w_{12}}$$



$$\frac{\partial L}{\partial w_{21}} \quad , \quad \frac{\partial L}{\partial w_{22}}$$

$$\frac{\partial L}{\partial w_{11}} : \frac{\partial L}{\partial o_{11}} x_{11} + \frac{\partial L}{\partial o_{12}} x_{12} + \frac{\partial L}{\partial o_{21}} x_{21} + \frac{\partial L}{\partial o_{22}} x_{22}$$

$$\frac{\partial L}{\partial w_{12}} : \frac{\partial L}{\partial o_{11}} x_{12} + \frac{\partial L}{\partial o_{12}} x_{13} + \frac{\partial L}{\partial o_{21}} x_{22} + \frac{\partial L}{\partial o_{22}} x_{23}$$

$$\frac{\partial L}{\partial w_{21}} : \frac{\partial L}{\partial o_{11}} x_{21} + \frac{\partial L}{\partial o_{12}} x_{22} + \frac{\partial L}{\partial o_{21}} x_{31} + \frac{\partial L}{\partial o_{22}} x_{32}$$

$$\frac{\partial L}{\partial w_{22}} : \frac{\partial L}{\partial o_{11}} x_{22} + \frac{\partial L}{\partial o_{12}} x_{23} + \frac{\partial L}{\partial o_{21}} x_{32} + \frac{\partial L}{\partial o_{22}} x_{33}$$

$$\begin{bmatrix} \frac{\partial L}{\partial w_{11}} & \frac{\partial L}{\partial w_{12}} \\ \frac{\partial L}{\partial w_{21}} & \frac{\partial L}{\partial w_{22}} \end{bmatrix} = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} * \begin{bmatrix} \frac{\partial L}{\partial o_{11}} & \frac{\partial L}{\partial o_{12}} \\ \frac{\partial L}{\partial o_{21}} & \frac{\partial L}{\partial o_{22}} \end{bmatrix}$$

$$\frac{\partial L}{\partial \underline{w}} = \boxed{\text{conv}\left(\underline{X} , \frac{\partial L}{\partial \underline{O}} \right)}$$

(cross-correlation)

$$\boxed{\frac{\partial L}{\partial \underline{w}^{(l)}} = \text{conv}\left(\text{inputs } \underline{H}^{(l-1)} , \frac{\partial L}{\partial H^{(l)}} \right)}$$

$$\frac{\partial L}{\partial X} : \quad \boxed{w_{11}}\, x_{11} + w_{12}x_{12} + w_{21}x_{21} + w_{22}x_{22}$$

$$\frac{\partial o_{11}}{\partial x_{11}} = w_{11}$$

zero

$$\frac{\partial L}{\partial x_{11}} = \frac{\partial L}{\partial o_{11}} w_{11} + \frac{\partial L}{\partial o_{12}} 0 + \frac{\partial L}{\partial o_{21}} 0 + \frac{\partial L}{\partial o_{22}} 0$$

$$\frac{\partial L}{\partial x_{12}} = \frac{\partial L}{\partial o_{11}} w_{12} + \frac{\partial L}{\partial o_{12}} w_{11} + \frac{\partial L}{\partial o_{21}} 0 + \frac{\partial L}{\partial o_{22}} 0$$

$$\frac{\partial L}{\partial x_{13}} = \frac{\partial L}{\partial o_{11}} 0 + \frac{\partial L}{\partial o_{12}} w_{12} + \frac{\partial L}{\partial o_{21}} 0 + \frac{\partial L}{\partial o_{22}} 0$$

$$\frac{\partial L}{\partial x_{21}} = \frac{\partial L}{\partial o_{11}} w_{21} + \frac{\partial L}{\partial o_{12}} 0 + \frac{\partial L}{\partial o_{21}} w_{11} + \frac{\partial L}{\partial o_{22}} 0$$

$$\frac{\partial L}{\partial x_{22}} = \frac{\partial L}{\partial o_{11}} w_{22} + \frac{\partial L}{\partial o_{12}} w_{21} + \frac{\partial L}{\partial o_{21}} w_{12} + \frac{\partial L}{\partial o_{22}} w_{11}$$

$$\frac{\partial L}{\partial x_{23}} = \frac{\partial L}{\partial o_{11}} 0 + \frac{\partial L}{\partial o_{12}} w_{22} + \frac{\partial L}{\partial o_{21}} 0 + \frac{\partial L}{\partial o_{22}} w_{11}$$

$$\frac{\partial L}{\partial x_{31}} = \frac{\partial L}{\partial o_{11}} 0 + \frac{\partial L}{\partial o_{12}} 0 + \frac{\partial L}{\partial o_{21}} w_{21} + \frac{\partial L}{\partial o_{22}} 0$$

$$\frac{\partial L}{\partial x_{32}} = \frac{\partial L}{\partial o_{11}} 0 + \frac{\partial L}{\partial o_{12}} 0 + \frac{\partial L}{\partial o_{21}} w_{22} + \frac{\partial L}{\partial o_{22}} w_{21}$$

$$\frac{\partial L}{\partial x_{33}} = \frac{\partial L}{\partial o_{11}} 0 + \frac{\partial L}{\partial o_{12}} 0 + \frac{\partial L}{\partial o_{21}} 0 + \frac{\partial L}{\partial o_{22}} w_{22}$$

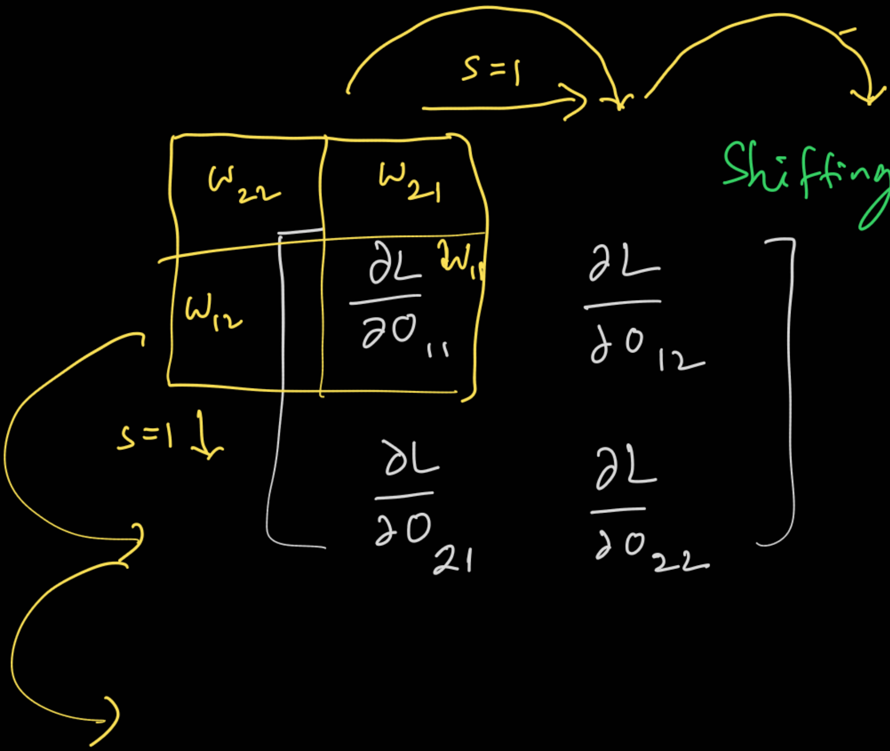
Matrix form:

$$\begin{bmatrix} \frac{\partial L}{\partial x_{11}} & \frac{\partial L}{\partial x_{12}} & \frac{\partial L}{\partial x_{13}} \\ \frac{\partial L}{\partial x_{21}} & \frac{\partial L}{\partial x_{22}} & \frac{\partial L}{\partial x_{23}} \\ \frac{\partial L}{\partial x_{31}} & \frac{\partial L}{\partial x_{32}} & \frac{\partial L}{\partial x_{33}} \end{bmatrix} = \begin{bmatrix} \frac{\partial L}{\partial O_{11}} & \frac{\partial L}{\partial O_{12}} \\ \frac{\partial L}{\partial O_{21}} & \frac{\partial L}{\partial O_{22}} \end{bmatrix} \circledast \begin{bmatrix} W_{22} & W_{21} \\ W_{12} & W_{11} \end{bmatrix}$$

90°

$$\begin{matrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{matrix}$$

90°



Shifting 1 cell at a time

Shifting 1 cell at a time.

$$\frac{\partial L}{\partial \underline{X}} = \cancel{\text{full}}\_\text{Convolution}\left(\frac{\partial L}{\partial \underline{O}}, \underline{\text{Rotate180}(\underline{W})}\right)$$

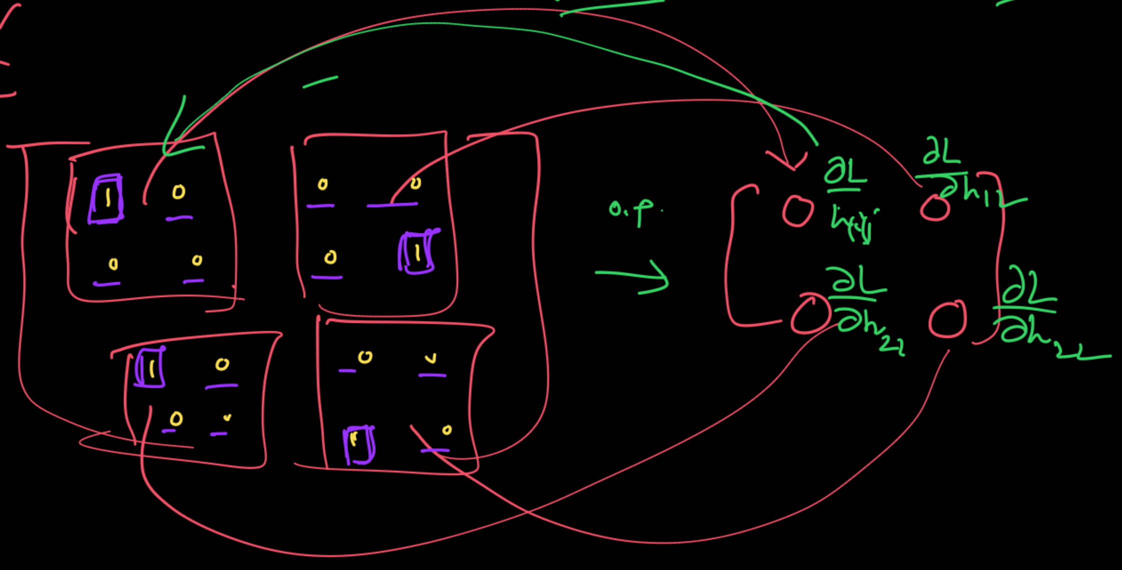
= conv.

$$\boxed{\frac{\partial L}{\partial \underline{H}^{(l-1)}} = \text{Full\_Conv}\left(\frac{\partial L}{\partial \underline{H}^{(l)}}, \text{Rotate180}\left(\underline{W}^{(l)}\right)\right)}$$

#1

Pooling layer

No Parameters



Mask