

"The key of persistence opens
all doors closed by resistance."

- John Di Lemme

Backtracking

TABLE OF CONTENTS

1. Dry. run of some recursive functions
2. Backtracking intro
3. Questions on backtracking



Notes

Advanced Contest Syllabus

- Arrays
- Bit Manipulation



Output based question →

```
1 int magicfun (int N){  
    if (N==0){return 0}  
    else{  
        return magicfun( N/2 ) * 10 + (N%2);  
    }  
}
```

$$f(n) = f(n/2) * 10 + (n \% 2)$$

↳ gives the binary of $(n)_{10}$.

$n=7$

```
if (N==0){return 0}  
else{  
    return magicfun( N/2 ) * 10 + (N%2);  
}
```

$= 11 * 10 + 1 = 111$
 $7 \% 2$

$n=3$

```
if (N==0){return 0}  
else{  
    return magicfun( N/2 ) * 10 + (N%2);  
}
```

$= 11$
 $3 \% 2$

$n=1$

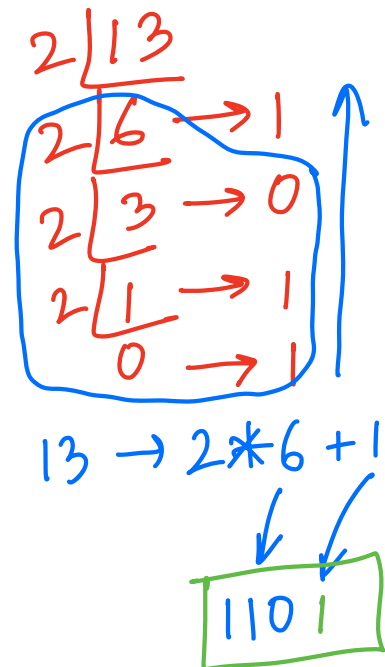
```
if (N==0){return 0}  
else{  
    return magicfun( N/2 ) * 10 + (N%2);  
}
```

$= 1$
 $1 \% 2$

$n=0$

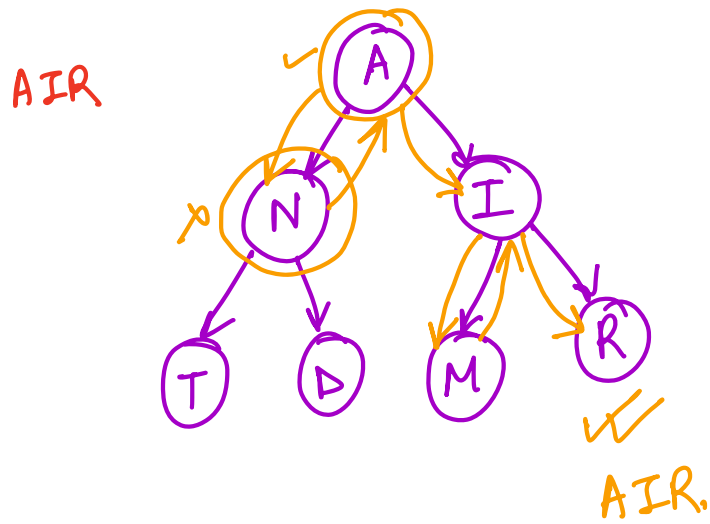
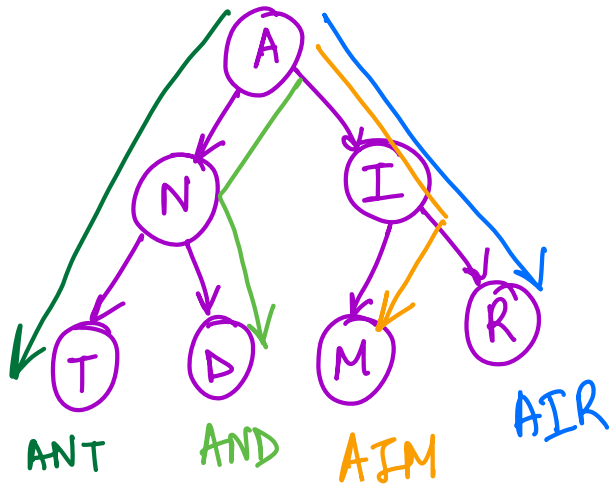
```
if (N==0){return 0}  
else{  
    return magicfun( N/2 ) * 10 + (N%2);  
}
```

$= 0$





Backtracking



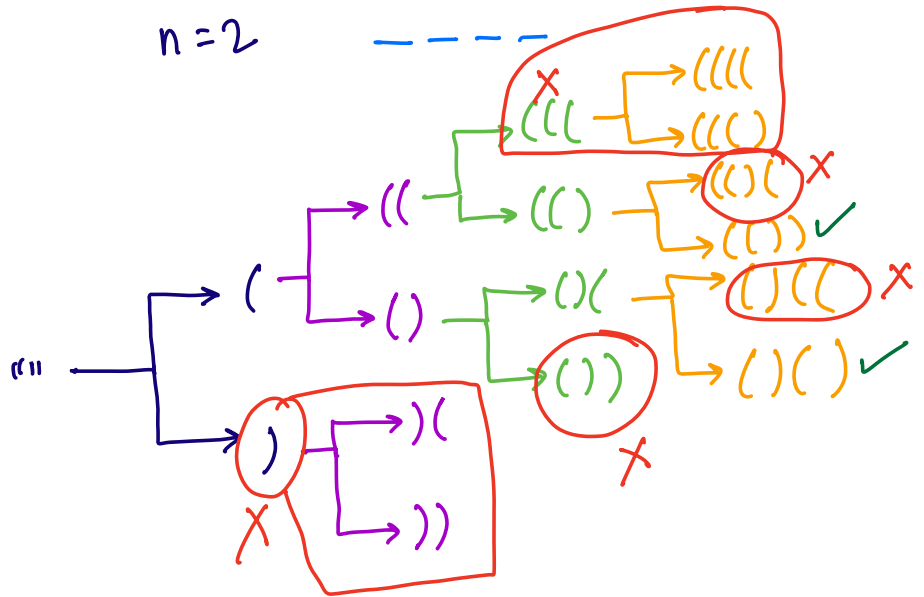
Valid Parenthesis

Generate all valid combinations of parenthesis of length $2N$.

$N=3$ ↴

$((()))$
 $()(())$
 $(())()$
 $()()()$
 $(())()$

$n=2$



Print Valid Parenthesis ($n, 0, 0$, new char $[2*n]$)

void Print Valid Parenthesis (int n , int opening, int closing, char[] str){

idx = opening + closing

if (idx == 2 * n)

// Print str

if (opening > closing) {

str[idx] = ')'

Print Valid Parenthesis (n , opening, closing + 1, str)

}

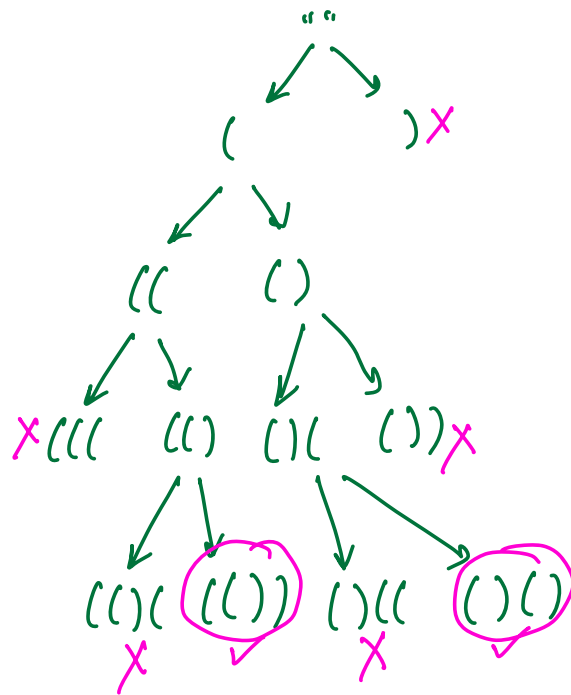
if (opening < n) {

str[idx] = '('

Print Valid Parenthesis (n , opening + 1, closing, str)

}

}



→ level 1 $\leq 2^1$

→ level 2 $\leq 2^2$

→ level 3 $\leq 2^3$

→ level 4 $\leq 2^4$.

$O(2^n)$ T.C

$O(n)$ S.C

Each subsequence of an array \Leftrightarrow A subset of the array.

$\{3, 5, 2, 9, 6, 4, 10, 15\}$

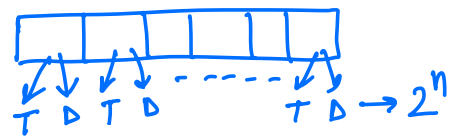


subarrays $\frac{n(n+1)}{2}$

$\{3, 5, 2, 9, 6, 4, 10, 15\}$

5 6 4 15
2 9 6 4 15

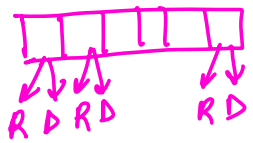
subsets
(2^n)



$\{3, 5, 2, 9, 6, 4, 10, 15\}$

~~3~~, ~~5~~, ~~2~~, 9, ~~6~~, ~~4~~, 10, 15 \rightarrow 5 9 10 15
~~3~~, ~~5~~, 2, 9, 6, 4, ~~10~~, 15 \rightarrow 2 9 6 4 15
~~3~~, ~~5~~, ~~2~~, ~~9~~, ~~6~~, 4, ~~10~~, ~~15~~ \rightarrow 4

subsequences



\downarrow
 2^n



② Generate all subsets of the given arr[]

$$2 * 2 * 2 = 2^3 = 8$$

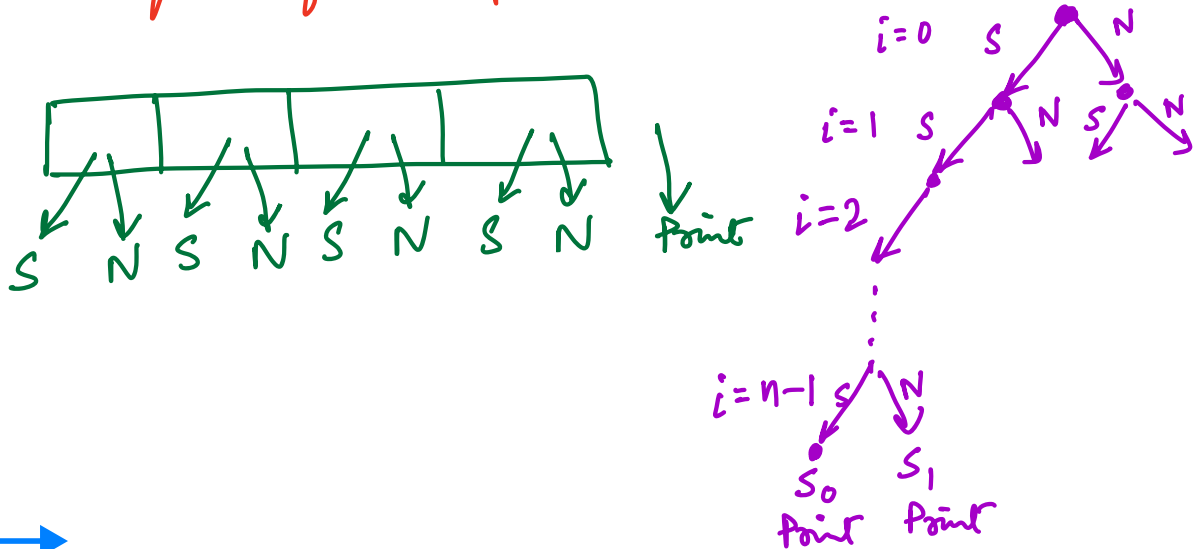
arr \rightarrow [10, 20, 30]
 0 1 2

Total no. of subsets =

{ }	{ 10, 20 }
{ 10 }	{ 10, 30 }
{ 20 }	{ 20, 30 }
{ 30 }	{ 10, 20, 30 }

arr[n] $\rightarrow 2^n$ subsets.

Each subsequence of an array \Leftrightarrow A subset of the array.



code \rightarrow

// Print all subsets of arr by selecting all combinations in [idx, n-1]

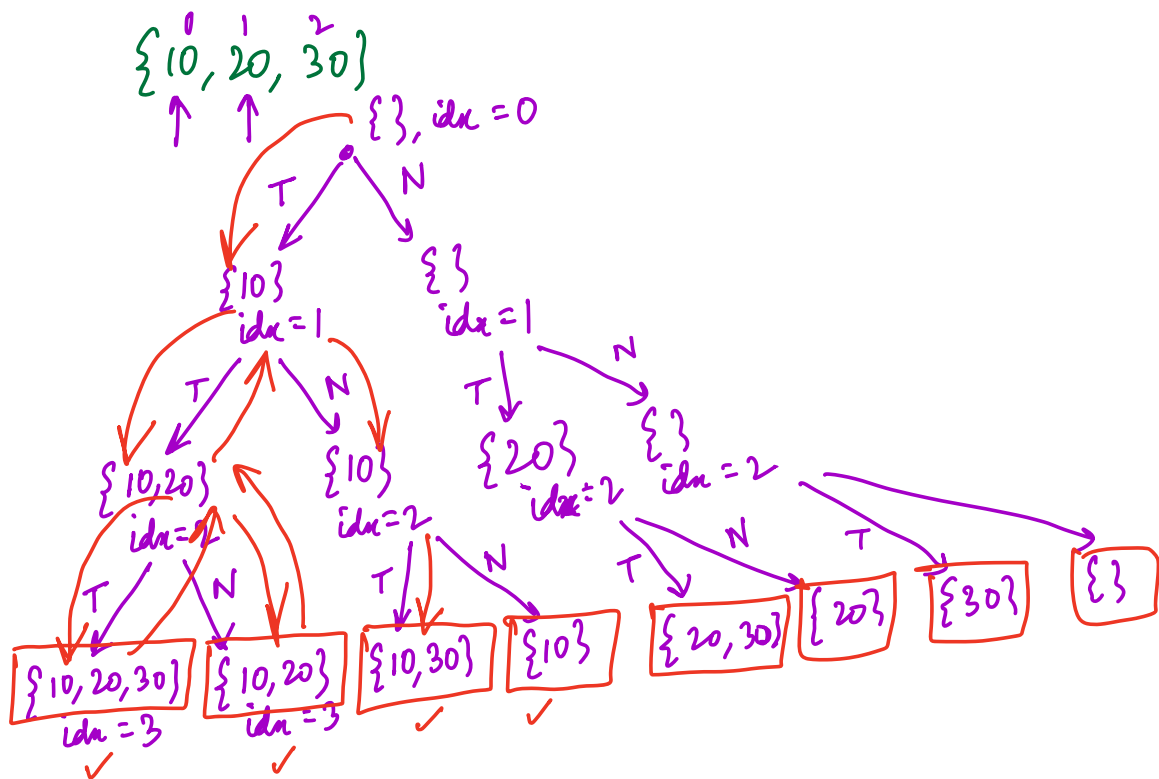
```
void subsets (int() arr, int idx, list<int> set){
```

```
    if (idx == arr.size()) {
        Print(set)
        return
```

```
    }
    set.add(arr[idx]);
    subsets(arr, idx+1, set);
    set.remove(set.size()-1);
    subsets(arr, idx+1, set);
```

```
}
```

[Break till 10:42 PM]



```
if (idx == arr.size()) {
    print(set)
    return
}
```

```
set.add(arr[idx]);
subsets(arr, idx + 1, set);
set.remove(set.size() - 1);
subsets(arr, idx + 1, set);
```



dry-run

 $\{10^0, 20^1, 30^2\}$ $idx = 0, set = \{\}$

```
set.add(arr[idx]); {10}
subsets(arr, idx + 1, set);
set.remove(set.size() - 1); {}
subsets(arr, idx + 1, set);
```

 $idx = 1, set = \{\}$ $idx = 1, set = \{10\}$
 $set.add(arr[idx]); \{10, 20\}$

```
subsets(arr, idx + 1, set);
set.remove(set.size() - 1); {10}
subsets(arr, idx + 1, set);
```

 $idx = 2, \{10, 20\}$
 $set.add(arr[idx]); \{10, 20, 30\}$

```
subsets(arr, idx + 1, set);
set.remove(set.size() - 1); {10, 20}
subsets(arr, idx + 1, set);
```

 $idx = 3, \{10, 20, 30\}$

```
if (idx == arr.size()) {
    print(set);
    return;
}
```

 $\{10, 20, 30\}$ $idx = 3, \{10, 20\}$

```
if (idx == arr.size()) {
    print(set);
    return;
}
```

 $\{10, 20\}$ $idx = 2, \{10\}$ $set.add(arr[idx]); \{10, 30\}$

```
subsets(arr, idx + 1, set);
set.remove(set.size() - 1); {10}
subsets(arr, idx + 1, set);
```

 $idx = 3, \{10, 30\}$

```
if (idx == arr.size()) {
    print(set);
    return;
}
```

 $\{10, 30\}$ $idx = 3, \{10\}$

```
if (idx == arr.size()) {
    print(set);
    return;
}
```

 $\{10\}$

Permutations

Given a character array with distinct elements, print all permutations of it without modifying it.

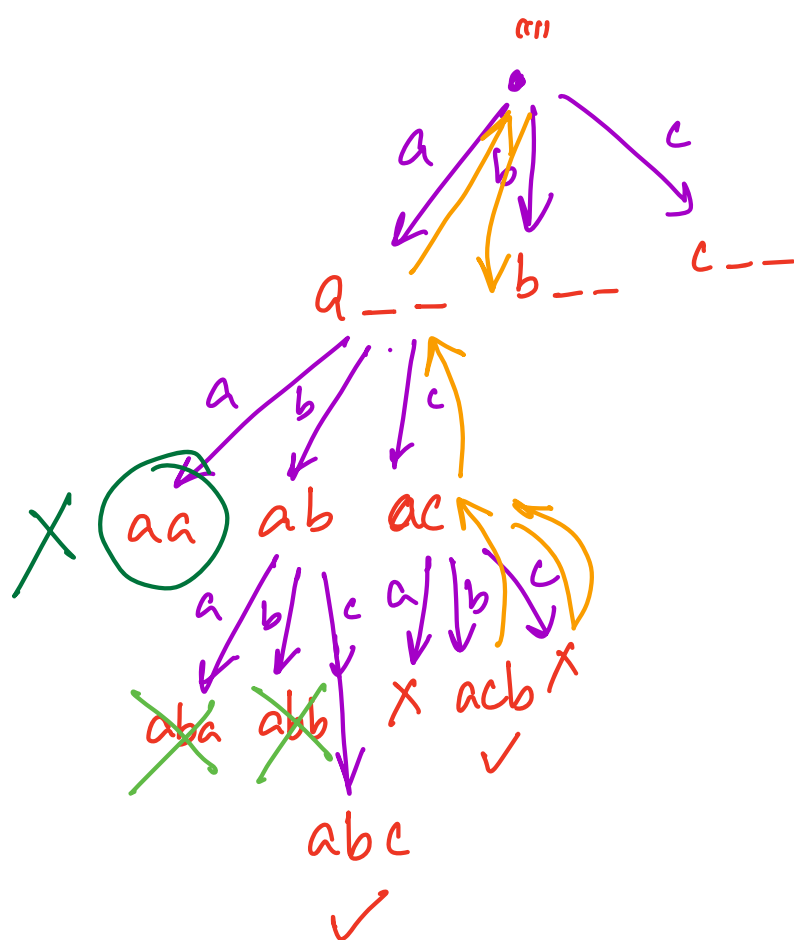
$A \rightarrow [a\ b\ c]$

abc cab
acb cba
bac
bca

$$\begin{array}{ccc} \text{---} & \text{---} & \text{---} \\ \downarrow & \downarrow & \downarrow \\ 3 & * & 2 & * & 1 = 3! = 6. \end{array}$$

$$\text{---} \text{---} \text{---} \text{---} \dots \text{---} \text{---} \text{---} \rightarrow n!$$

$n \quad n-1 \quad n-2 \quad n-3 \quad \dots \quad 3 \quad 2 \quad 1$

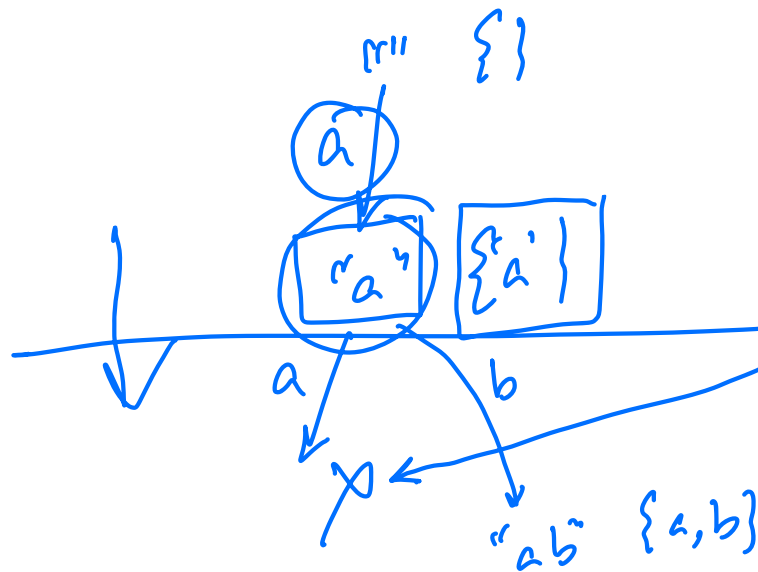


set (HashSet)
↓
to check if
an element
has been taken or
not.



pseudo-code →

```
permutations(str, n, "", new HashSet<X>);  
void permutations (String str, int n, String perm, HashSet set) {  
    if (perm.size() == n) {  
        print(perm)  
        return  
    }  
    for (i → 0 to n-1) {  
        if (!set.contains(str[i])) {  
            set.add(str[i])  
            permutations(str, n, perm+str[i], set)  
            set.remove(str[i])  
        }  
    }  
}
```



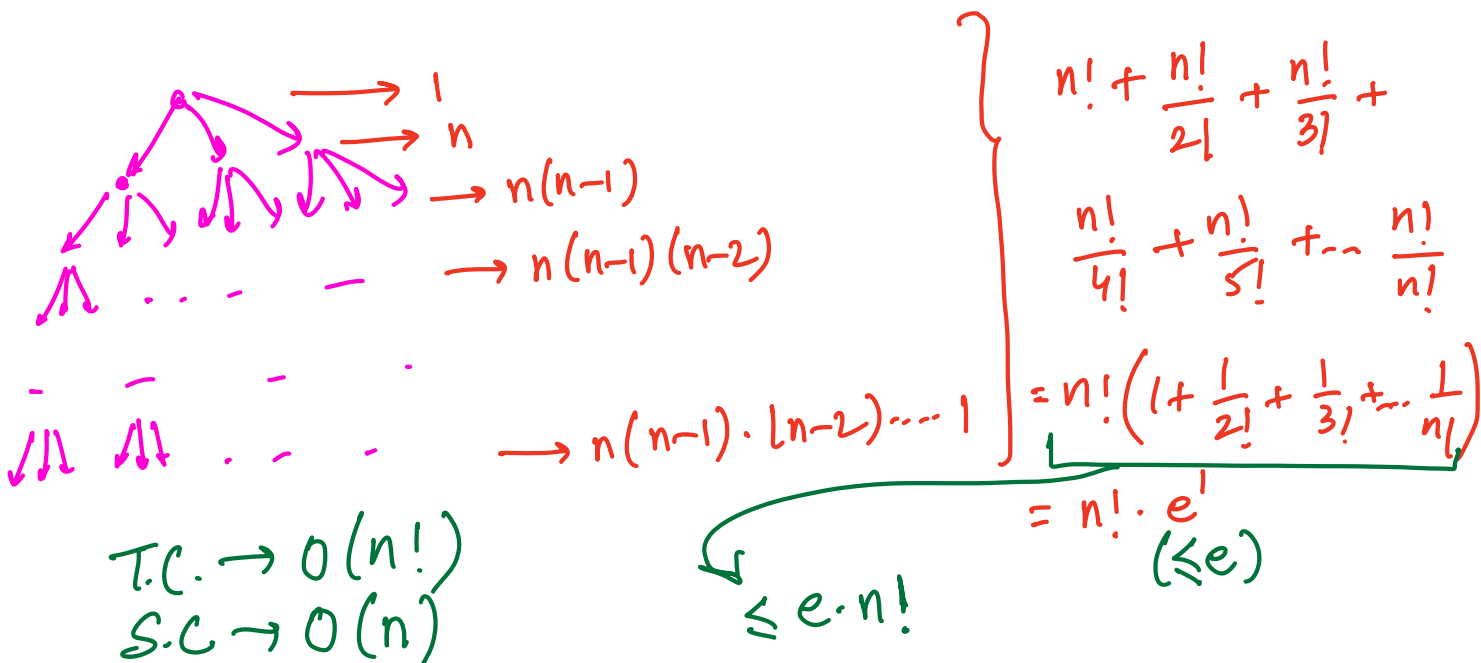
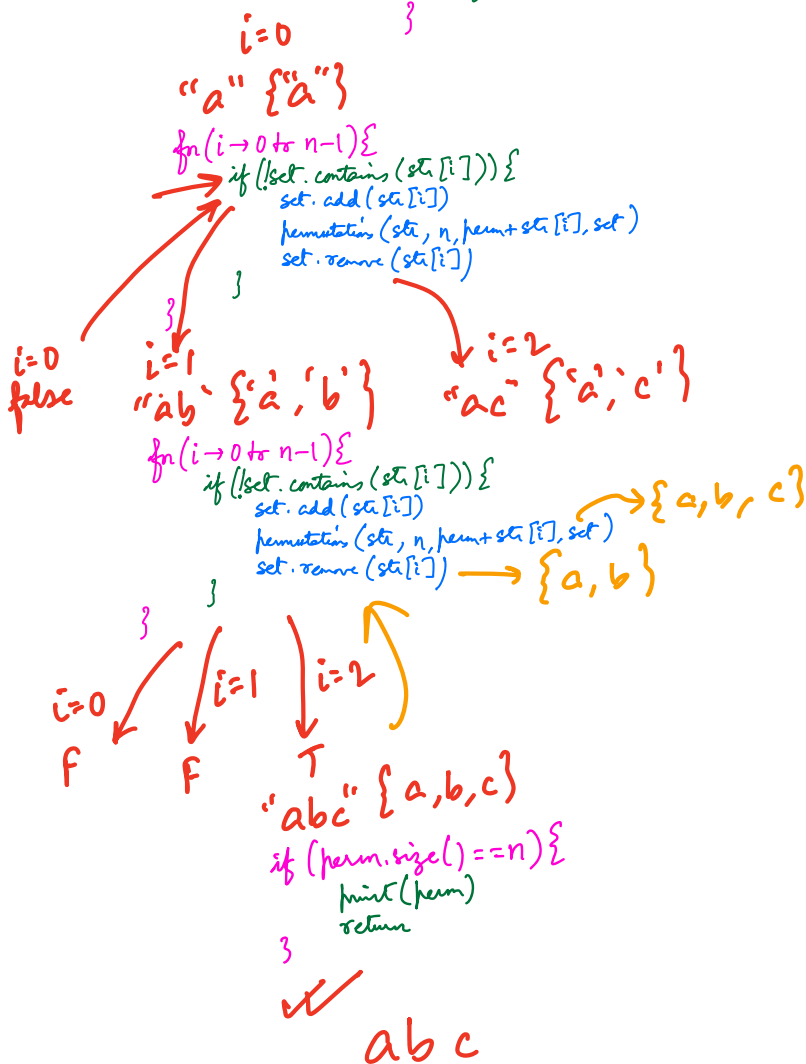


Time & Space Complexity Analysis →

^{0 1 2}
a b c
↑

“(” n=3 “a⁰b¹c²” {}

```
fn(i→0 to n-1){  
  if (!set.contains(str[i])){  
    set.add(str[i])  
    permutation(str, n, perm+str[i], set)  
    set.remove(str[i])  
  }  
}
```



$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \infty$$

$$1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots + \frac{x^n}{n!} \leq e^x$$

$$x=1 \Rightarrow 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \dots + \frac{1}{n!} \leq e^1$$