

Sorting Basics

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Introduction

Sorting — Arranging the data in a particular order based on a parameter

Eg : 2 3 9 12 17 19 } ascending order

—————→
values increase

parameter = value

10 5 3 1

—————→

value decrease

} descending order

parameter = value

Quiz 1 13 9 6 12 ?

# factors	1	2	3	4	6
-----------	---	---	---	---	---

parameter = # of factor.

Note : Do not assume the data can only be sorted via value as parameter.

ie ascending or descending.

Minimize the cost to empty array

Given $\text{int}[n]$, minimize the cost to empty given array where

cost of removing an element = sum of elements left in the array

Eg $A[] = [2 \ 1 \ 4]$

way 1

Remove 1 \longrightarrow cost $2 + 1 + 4 = 7$

$[2 \ 4]$

Remove 4 \longrightarrow $2 + 4 = 6$

$[2]$

Remove 2 \longrightarrow $2 = 2$

Total cost
 $= 15$

$[2 \ 1 \ 4]$

Cost

way 2

Remove 4

$2 + 1 + 4 = 7$

$[2 \ 1]$

Remove 2

$2 + 1 = 3$

$[1]$

Remove 1

1

$= 11$

[4 6 1]

Cost

Remove 6

$$4 + 6 + 1 = 11$$

[4 1]

Remove 4

$$4 + 1 = 5$$

[1]

Remove 1

$$1 = 1$$

17

[3 5 1 -3]

Cost

Remove 5

$$3 + 5 + 1 - 3 = 6$$

[3 1 -3]

Remove 3

$$3 + -3 + 1 = 1$$

[1 -3]

Remove 1

$$1 - 3 = -2$$

[-3]

Remove -3

$$-3$$

2

$[a \ b \ c \ d]$ cost

Remove a

$$a + b + c + d$$

$[b \ c \ d]$

Remove b

$$b + c + d$$

$[c \ d]$

Remove c

$$c + d$$

$[d]$

Remove d

$$d$$

$$\underbrace{1.a + 2.b + 3.c + 4.d}_{\text{minimize}}$$

Note to minimize $1.a + 2.b + 3.c + 4.d$ the value of a should be max of $A[]$.

Algo steps

1) Sort the array in descending order

4	3	2	1
a	b	c	d

$$4 + 2 \cdot 3 + 3 \cdot 2 + 4 \cdot 1$$

$$\underline{\underline{20}}$$

NOTE \rightarrow Contribution of i^{th} value $\rightarrow \underline{\underline{i+1}}$

Pseudocode

[3 5 1 -3]

// sort the array in descending order

[5 3 1 -3]

0 1 2 3

cost = 0

```
for ( i → 0 to N-1 ) {  
    cost += (i+1) * A[i]  
}
```

print (cost)

$$1 \cdot 5 + 2 \cdot 3 + 3 \cdot 1 + 4 \cdot -3$$

$$5 + 6 + 3 - 12$$

$$14 - 12 = \underline{\underline{2}}$$

TC : $O(N + \text{TC for sorting}) = O(N \log N)$

SC : $O(1)$

$O(N \log N)$

Count Noble Integer

Given an $\text{int}[N]$ of distinct elements. Find the count of noble integers

$A[i]$ is noble if count of elements smaller than $A[i]$ is equal to $A[i]$

Eg $A[] = [1, -5, 3, 5, -10, 4]$
smaller 2 1 3 5 0 4

Output = 3

Eg $A[] = [-3, 0, 2, 5]$
smaller 0 1 2 3

output = 1

Bruteforce

For all element x . Iterate through array to find
of count of $< x = p$

if $x == p$

ans += 1

==

TC $O(N^2)$

SC $O(1)$

index	0	1	2	3
A[] =	-3	0	2	5
# smaller	0	1	2	3

Note \rightarrow If data is sorted then index == # of smaller

Pseudocode

// sort the A[] in ascending order

ans = 0

```

for i  $\rightarrow$  0 to N-1 {
    if (i == A[i]) {
        ans += 1
    }
}

```

print(ans)

TC : $O(N \log N + N)$ = $O(N \log N)$

SC : $O(1)$

Count Noble Integers

Given an $\text{int}[n]$ of elements. Find the count of noble integers

$A[i]$ is noble if count of elements smaller than $A[i]$ is equal to $A[i]$

	0	1	2	3	4
$A[]$	-10	1	1	3	10
# smaller	0	1	1	3	4

	0	1	2	3	4	5	6	7	8
$A[]$	-10	1	1	2	4	4	4	8	10
# smaller	0	1	1	3	4	4	4	7	8

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
$A[]$	-3	0	2	2	5	5	5	5	8	8	10	10	10	14
# small.	0	1	2	2	4	4	4	4	8	8	10	10	10	13

Observation

If $\text{pval} \neq \text{val}$ then # smaller = index

Pseudocode

// sort in ascending order.

smaller = 0

ans = 0

if (A[0] == 0) ans = 1 // A = [0]

for i \rightarrow 1 to N-1 {

 prev = A[i-1]

 val = A[i]

 if (prev != val) smaller = i

 if (val == smaller) {

 ans += 1

 }

}

print(ans)

ans = ~~1~~ ~~2~~ ~~3~~ ~~4~~

↓ ↓ ↓ ↓ ↓

[0 1 2 3 4]

0 1 1 3 10

smaller = ~~0~~ ~~1~~ ~~3~~ 4

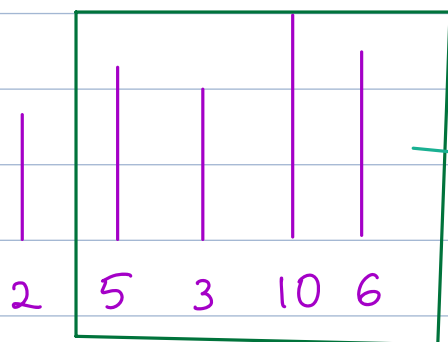
[0 1 2 3 4]

0 1 1 3 10

0 1 1 3 4

TC : $O(N \log N)$

SC : $O(1)$



check for the next
smallest in these students

values	5	2	3	10	6
index	0	1	2	3	4

Range [0 - 4]

minVal = ~~5~~ 2

minIndex = ~~0~~ 1

values	2	5	3	10	6
index	0	1	2	3	4

Range [1 - 4]

minVal = ~~2~~ 3

minIndex = ~~1~~ 2

values	2	3	5	10	6
index	0	1	2	3	4

minVal = 5

Range [2 - 4]

minIndex = 2

values	2	3	5	10	6
index	0	1	2	3	4

Range [3 - 4]

minVal = ~~10~~ 6

minIndex = ~~3~~ 4

values	2	3	5	6	10
index	0	1	2	3	4

Pseudocode { selection sort }

for $i \rightarrow 0$ to $N-1$

minVal = $A[i]$

minIndex = i

// Find min in range $[i$ to $N-1]$

```

for  $j \rightarrow i+1$  to  $N-1$  {
    if ( $A[j] < \text{minVal}$ ) {
        minVal =  $A[j]$ 
        minIndex =  $j$ 
    }
}

```

// swap index i with minIndex

temp = $A[i]$

$A[i] = A[\text{minIndex}]$

$A[\text{minIndex}] = \text{temp}$

3

TC : $O(N^2)$ SC : $O(1)$

Insertion Sort

$A[] = 3 \quad 10 \quad 6 \quad 2 \quad 4$

3 10 6 2 4

Iteration 1 $\rightarrow 3 \boxed{}$ 10 6 2 4

$\rightarrow 3 \quad 10 \boxed{}$ 6 2 4

$\rightarrow 3 \quad 6 \quad 10 \boxed{}$ 2 4

$\rightarrow 2 \quad 3 \quad \cancel{6} \quad 10 \boxed{}$ 4

4

$3 > 4$

$\rightarrow 6 \rightarrow 10$

Note

\downarrow all values greater than 4 will be shifted right.

Pseudo code

```
for  $i \rightarrow 0$  to  $N-1$  {
```

```
     $x = A[i]$ 
```

```
     $j = i - 1$ 
```

```
    // shifting logic
```

```
    while ( $j \geq 0$  &&  $A[j] > x$ ) {
```

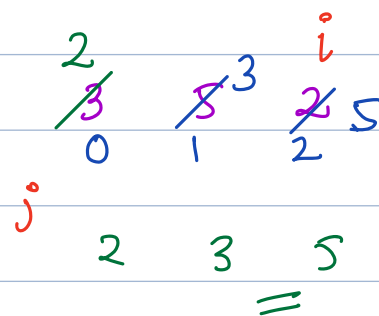
```
         $A[j+1] = A[j]$ 
```

```
         $j--$ 
```

```
    }
```

```
     $A[j+1] = x$ 
```

```
}
```



TC: $O(N^2)$

SC: $O(1)$

NOTE : Always use inbuilt sorting.