"Although the world is full of suffering,
it is full also of the overcoming of it"
— Helen Keller

a	b	a & b	a | b	a ^ b	~a
0	0	0	0	0	1
0	1	0	1	1	1
1	0	0	1	1	0
1	1	1	1	0	0

Truth Table

19 & 15

$$
\begin{array}{r}
10011 \\
\& \; 01111 \\
\hline
00011
\end{array}
\longrightarrow (3)_{10}
$$

$(0.5)_{10} \longrightarrow (0.1)_2$

$(0.25)_{10} \longrightarrow (0.01)_2$

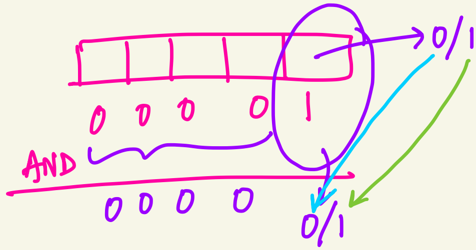
$\downarrow * 4$ $\downarrow * 4$

1

Properties of AND

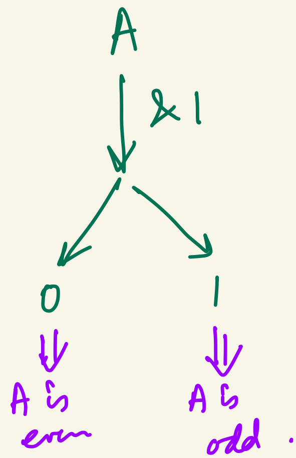
→ If A is odd, then LSB(A) = 1.

$A \& 1 = 1$ (if A is odd)



$A = 181$

```
  10110101
         1
  --------
  00000001  → (1)₁₀
```



→ If A is even, then LSB(A) = 0

$A \& 1 = 0$ (if A is even)

$A = 180$

```
     10110100
     00000001
AND  --------
     00000000  → (0)₁₀
```

→ $A \& 0 = 0$

→ $A \& A = A$

Properties of OR

→ $A \mid 0 = A$

→ $A \mid A = A$

Properties of XOR

→ $A \wedge 0 = A$

→ $A \wedge A = 0$

$$
\begin{array}{lcccccc}
A \rightarrow & 1 & 0 & 1 & 1 & 0 \\
A \rightarrow & 1 & 0 & 1 & 1 & 0 \\
\hline
\text{XOR} & 0 & 0 & 0 & 0 & 0
\end{array}
$$

Commutative Properties

$A \& B = B \& A$

$A \mid B = B \mid A$

$A \wedge B = B \wedge A$

Associative Properties

$(A \& B) \& C = A \& (B \& C)$

$(A \mid B) \mid C = A \mid (B \mid C)$

$(A \wedge B) \wedge C = A \wedge (B \wedge C)$

$$a \wedge b \wedge a \wedge d \wedge b$$

$$= (a \wedge a) \wedge (b \wedge b) \wedge d$$

$$= (0 \wedge 0) \wedge d$$

$$= 0 \wedge d$$

$$= d$$

$$\cancel{1} \wedge \cancel{3} \wedge \cancel{5} \wedge \cancel{3} \wedge 2 \wedge \cancel{1} \wedge \cancel{5}$$

$$= 2$$

Left Shift Operator (<<)

8-bit form

a = 10	⇒	00001010	
a<<0	⇒	00001010	→ 10 = $10 * 2^0$
a<<1	⇒	~~0~~00010100	→ 20 = $10 * 2^1$
a<<2	⇒	000101000	→ 40 = $10 * 2^2$
a<<3	⇒	001010000	→ 80 = $10 * 2^3$
a<<4	⇒	010100000	→ 160 = $10 * 2^4$
a<<5	⇒	~~1~~01000000	→ 64 (overflow, MSB is lost)

$$\boxed{\begin{aligned} a \ll n &= a * 2^n \\ 1 \ll n &= 2^n \end{aligned}}$$

~~1010~~ | 1 1 0110 . . . 0101110 . . . 0

32 bit

Right Shift Operator (>>)

8-bit form

$a = 10 \Rightarrow 00001010$

$a >> 0 \Rightarrow 00001010 \rightarrow 10 = 10/2^0$

$a >> 1 \Rightarrow 00000101\cancel{0} \rightarrow 5 = 10/2^1$

$a >> 2 \Rightarrow 00000010\cancel{1} \rightarrow 2 = 10/2^2$

$a >> 3 \Rightarrow 00000001 \rightarrow 1 = 10/2^3$

$a >> 4 \Rightarrow 00000000 \rightarrow 0 = 10/2^4$

$$a >> n = a/(2^n)$$

$$1 >> n = 1/(2^n)$$

Quiz:- $1 << 3 = 1 * 2^3$

$= 8$

$$
\begin{array}{rl}
 & \begin{array}{cccccc} 5 & 4 & 3 & 2 & 1 & 0 \end{array} \\
45 \longrightarrow & \begin{array}{cccccc} 1 & \boxed{0} & 1 & 1 & 0 & 1 \end{array} \\
\text{OR} & \begin{array}{cccccc} 0 & \boxed{1} & 0 & 0 & 0 & 0 \end{array} \longrightarrow (1<<4) \\
\hline
 & \begin{array}{cccccc} 1 & \boxed{1} & 1 & 1 & 0 & 1 \end{array}
\end{array}
$$

Set the 4^{th} bit $\rightarrow$ $n \mid (1<<4)$

→ Set the i^{th} bit of a number if unset, and do nothing if already set.

$$n = (n \mid (1<<i))$$

$i = 2$

$$
\begin{array}{rl}
 & 1\;0\;1\;1\;0\;1 \\
\text{OR} & 0\;0\;0\;1\;0\;0 \leftarrow (1<<2) \\
\hline
 & 1\;0\;1\;1\;0\;1
\end{array}
$$

$i = 3$

$$
\begin{array}{rl}
 & 1\;1\;0\;1\;0\;1 \\
\text{OR} & 0\;0\;1\;0\;0\;0 \leftarrow (1<<3) \\
\hline
 & 1\;1\;1\;1\;0\;1
\end{array}
$$

→ Flip the i^{th} bit of a number n.

$$n = (n \wedge (1<<i))$$

$i = 2$

$$
\begin{array}{rl}
 & 1\;0\;1\;\boxed{1}\;0\;1 \\
 & 0\;0\;0\;\boxed{1}\;0\;0 \leftarrow (1<<i) \\
\text{xor} & \\
\hline
 & 1\;0\;1\;\boxed{0}\;0\;1
\end{array}
$$

$$\boxed{0} \wedge 1 = \boxed{1}$$

$$\boxed{1} \wedge 1 = \boxed{0}$$

→ Check if i^{th} bit is set or not

```
    101101        i = 2
    000100
AND
------------
    000100        > 0
```

```
    101101        i = 4
    010000
AND
------------
    000000        = 0.
```

$n \,\&\, (1 << i)$

> 0 → i^{th} bit was set

$== 0$ → i^{th} bit was not set.

[Break till 10:32 PM]

Q1) Check whether i^{th} bit of n is set or not.

$$\text{if}((n \,\&\, (1<<i)) > 0) \begin{cases} \to \text{set} \\ \to \text{unset} \end{cases}$$

$$\text{if}(((n>>i) \,\&\, 1 == 0) \begin{cases} \to \text{unset} \\ \to \text{set.} \end{cases}$$

101101

$i = 2$

$(1011\boxed{01} >> 2) \,\&\, 1$

$= 101\underline{1} \,\&\, \underline{1} = 1$

$i = 4$

$(10\boxed{1101} >> 4) \,\&\, 1$

$= 10 \,\&\, 1 = 0$

```
fn checkBit(n, i) {
    if(((n>>i) & 1) == 0)
        return false
    return true
}
```

$((n>>i) \,\&\, 1) == 0 \iff ((n \,\&\, (1<<i)) == 0)$

$TC \to O(1)$

$SC \to O(1)$

Q2) Count the nr. of set bits in n.

$n = 12 \Rightarrow (1100)_2$

ans = 2

Approach 1

```
fn countBits(n) {
    cnt = 0
    for(i -> 0 to 31)
        if(checkBit(n, i))
            cnt += 1

    return cnt
}
```

32 iterations.

Approach 2

```
fn countBits(n) {
    cnt = 0
    while(n>0) {
        if((n&1)==1)
            cnt+=1
        n=(n>>1)
    }
    return cnt
}
```

$O(\log n)$ T.C.

$O(1)$ S.C.

$n = 13 \Rightarrow 1101$ cnt = 1

$n = n>>1 = 6 \Rightarrow 110$ cnt = 1

$n = n>>1 = 3 \Rightarrow 11$ cnt = 2

$n = n>>1 = 1 \Rightarrow 1$ cnt = 3

$n = n>>1 = 0 \Rightarrow 0$ ✗

Q3) Unset the i^{th} bit of n if i^{th} bit is set.

$n = 6, \ i = 2$

$$110 \downarrow 010 \rightarrow 2.$$

```
fn unsetBit(n, i) {
    if (checkBit(n, i)) {
        n = (n ^ (1 << i))
    }
    return n
}
```

O(1) T.C.
O(1) S.C.

Q4) You need to create a binary number which has A 0's followed by B 1's followed by C 0's. You need to return the decimal equivalent. Can you write a fn to find the nr.?

$0 <= A, B, C <= 20$

$A = 4, B = 3, C = 2$

8 7 6 5 4 3 2 1 0

$\underbrace{0000}\underbrace{111}\underbrace{00} \longrightarrow (28)_{10}$.

$[0, C-1]$

$[C, C+B-1] \longrightarrow$ set each of these bits.

```
fn solve(A, B, C){
    ans = 0
    fn(i -> C to C+B-1){
        ans = (ans | (1<<i))
    }
    return ans
}
```

O(B) T.C.

O(1) S.C

$100000 = (1<<5)$

-1

$11111 = (1<<5)-1$

$\underbrace{1111\ldots1}_{i} = (1<<i)-1$

$\overbrace{111\ldots1}^{B}\overbrace{00\ldots000}^{C}$

$\Downarrow$

$(((1<<B)-1)<<C)$

```
fn solve(A, B, C){
    return (((1<<B)-1)<<C)
}
```

$a = 110\,1\,0$
$a-1 = 110\,\boxed{01}$
$\sim(a-1) = 001\,1\,0$

$\boxed{a \& \sim(a-1)}$

The number with only the rightmost set bit of a as set.

$a = 1100$
$a-1 = 1011$
$\sim(a-1) = 0100$

AND

$$\begin{array}{r} 1100 \\ \& \; 0100 \\ \hline 0100 \end{array}$$

$$\begin{array}{r} 1100 \\ 0100 \\ \hline 0100 \end{array}$$

$\underset{a}{110100} \xrightarrow[\text{}]{\text{AND } (001100)} \begin{array}{r} 110100 \\ 000100 \end{array}$

sub/xor

1 step
cnt=1

$110000 \xrightarrow{\text{AND } (010000)} \begin{array}{r} 110000 \\ 010000 \end{array}$

sub/xor

1 step cnt=2

100000
AND
$$\begin{array}{r} 100000 \\ 100000 \\ \hline 100000 \end{array}$$

sub/xor

1 step
cnt=3

$\boxed{000000}$

$$\boxed{\begin{array}{l} a = a \wedge (a \& \sim(a-1)) \\ a = a \& (a-1) \end{array}}$$

O(set bits) T.C.

$4 \Rightarrow \quad 00000100$

$-4 \Rightarrow \quad 11111100$

$11111000 \longrightarrow \boxed{-8}$

$00001000 \longrightarrow 8$