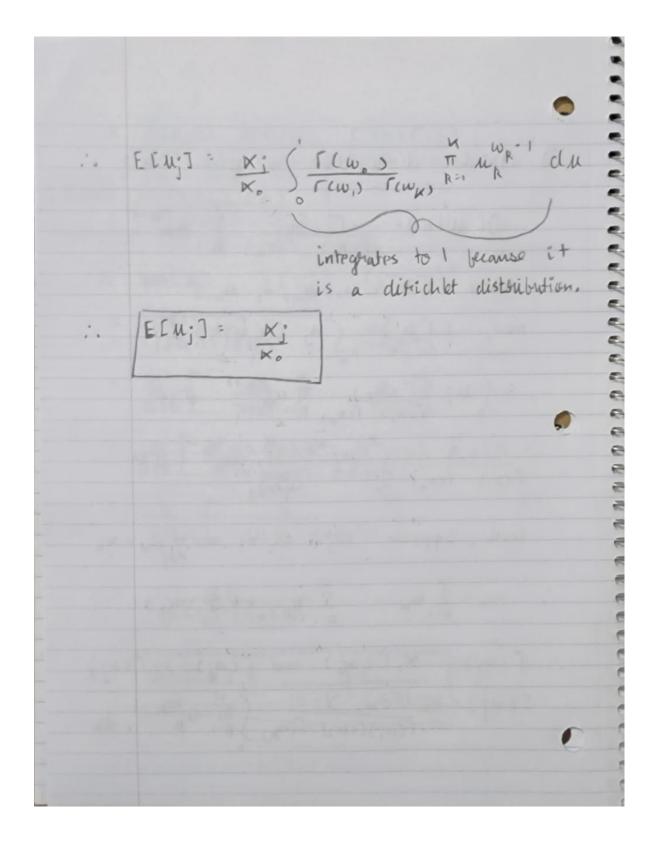
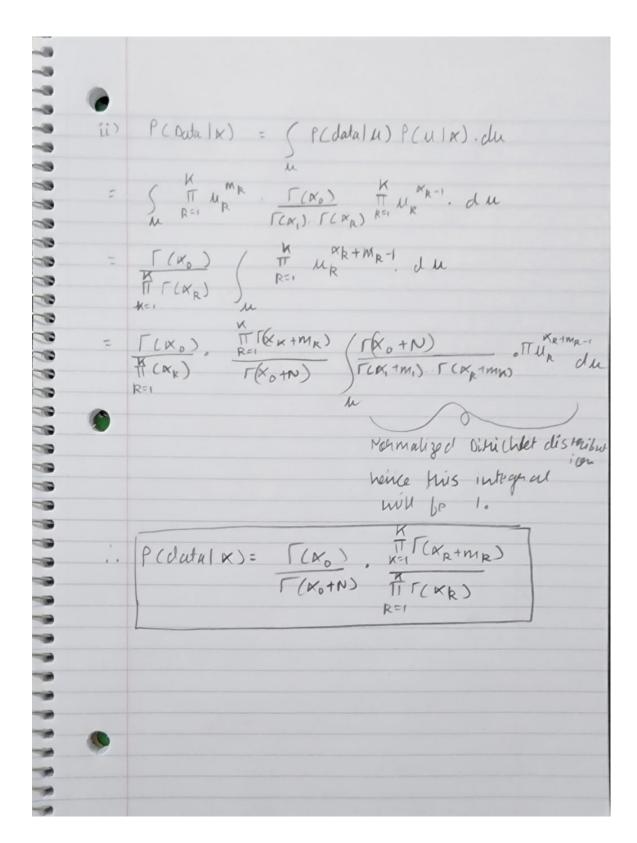
Soln 1 i) Now, Bota distribution is given as: B(u|a,b) = [(a+b) ua-1 (1-u)b-1 and E[11] = a a+b Now, Variance can be weiten as Van [u] = E[u] - (E[u]) me have EIM3, so let's find EIM23 E[u2] = S'M2 B(Ula, b) du = 5 u2 <u>r (a+b)</u> ua-'(1-u) du = [(a+b) ("(1-11)b-1 dx =  $\Gamma(a+b)$ .  $\Gamma(a+2)\Gamma(b)$   $\Gamma(a+b+2)$ .  $\Gamma(a+b+2)$ .  $\Gamma(a+b+2)$   $\Gamma(a+$  $= \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \frac{\Gamma(a+2)\Gamma(b)}{\Gamma(a+b+2)}$ 

1	Γ(α+b) (α+1)(α) Γ (α) Γ(b) Γ(α) Γ(b) (α+b+1) (α+b) Γ(α+b)
E[M2]=	(a+1) (a)
	(a+b+1) (a+b)
1.	VOOLUT = E[u2] - (E[u])2 MAIN
=	$\frac{(a+b+1)(a)}{(a+b+1)(a+b)} - \frac{a^2}{(a+b)^2}$
τ	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
-	$\frac{a}{a+b} \left[ \frac{a^2+a+ab+b-a^2-ab-a}{(a+b+1)(a+b)} \right]$
2 /2	a (a+b) (a+b+1)
11/2 =	$\frac{ab}{(a+b)^2(a+b+1)}$
· ·	$[vart u] = \frac{ab}{(a+b)^2 (a+b+1)}$
	FLANTED FLANES

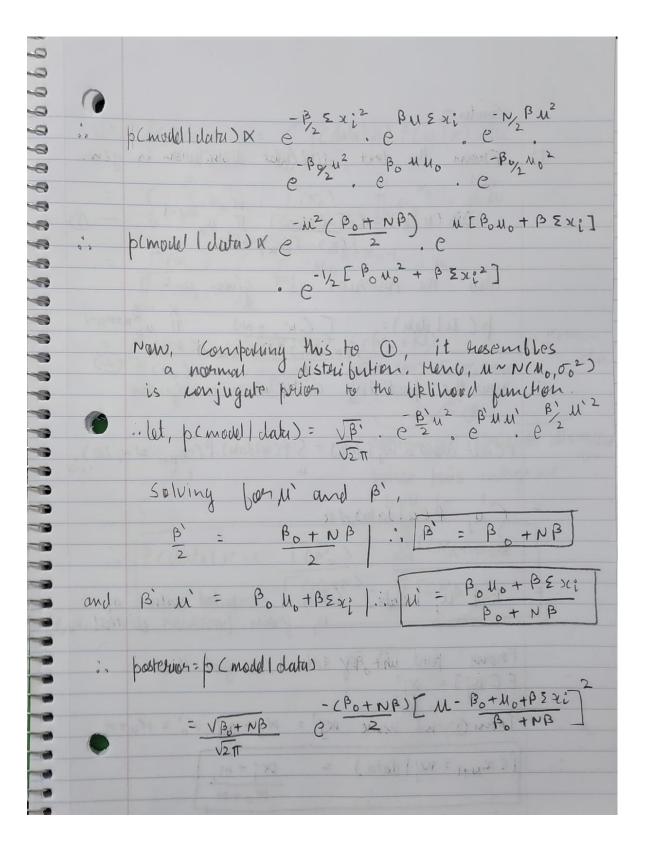
	District distribution is, given as:  D(U X) = \( \tag{C} \tag{X} \)  \[ \text{TT uk} \\ \text{TCK, ) \cdot \tag{C} \tag{K} \\ \text{Where } \( \text{u} = \tag{C} \tag{U_1}, \tag{U_2}, \tag{U_1} \\ \text{Now, } \( \text{E u i j} = \text{S u j D C U I X ) du} \)  = \( \text{U i j \tag{C} \tag{K} \tag{K} \\ \text{TCK, ) \cdot \tag{K} \\ \text{E u j j = \text{C} \tag{K} \\ \text{TCK, ) \cdot \text{TCK, } \text{R=1} \\ \text{TCK, ) \cdot \text{TCK, } \text{CCK, } \text{R=1} \\ \text{TCK, ) \cdot \text{TCK, } \text{CCK, } \te
0	NOW, Suppose $W_j^s = X_j^s + 1$ and $\forall W_k = x_k$ $W_0 = \sum_{k=1}^K w_k = \sum_{k=1}^K x_k + 1 = x_0 + 1$ $\Gamma(w_0) = X_0 \Gamma(w_0)$ and $\Gamma(w_j) = x_j^s \Gamma(x_j^s)$ $\Gamma(w_0) = X_0 \Gamma(w_0)$ and $\Gamma(w_j) = x_0^s \Gamma(x_j^s)$ $\Gamma(w_0) = X_0 \Gamma(w_0)$ and $\Gamma(w_0) = x_0^s \Gamma(x_0^s)$



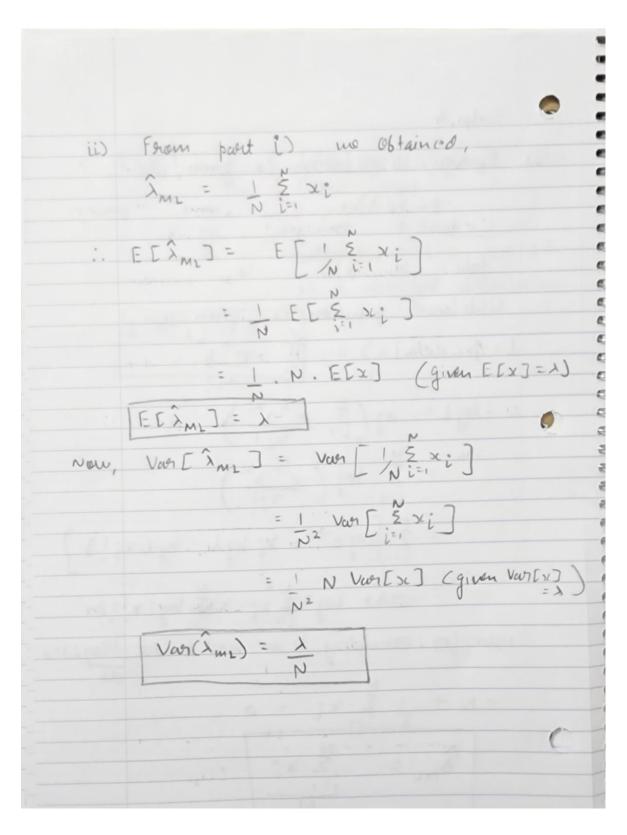
	From the tost, clivicalet distailution is given
D.	
	Din (u x) = $\int (x_0) \int \int u_x x_{x-1} - \int \int (x_1)^{-1} \int $
	also the PostPhior is given as:
	p(u)data) = [[[x, +N] II] ukr+mk-1]  [(x, +m,) = [(xk+mk) N=1 k]  Now, the predictive distribution is given as:
	P(next word is w) I data) = S P(wI duta). P(DCn+1 = w; IM)
=	Su; Pculdatas du
=	E [ U; I cluta] (This is expected value of uj from posterior distribution)
	From part ii) of soln I, we got that:
	Franco, me have x; = x;+m;, xo' = xo+n
	$P(x_{N+1} = w_j   data) = \frac{x_j + m_j}{x_{o} + N}$



Saln 3 Normal distribution can be written as: Liklihood function can be weather = p (data | model) = C e 2 2 PM Exi - NB 112 Now, Prior is also a normal distribution: p(model) = VP, e 2 e BANO - PONO2. Pastories & Liklihood x Prior. pc model data) x pc datal model ) x p (model)



(30)		
4		
-30		
-30		
9999	0	Soln 4
-3		Annual Company of the same of
-3	(3	Poisson distribution is given as.
-		
-		$  b(x \lambda) = \frac{e^{-\lambda}\lambda^{x}}{2c!}$
-		2/1
		4
20		data: x, xz, xp
3		and. Si, Xz, Zp
9		I blile al line in a di de anci
-	υ '	Liklihood function is given as:
-		$L =   \mathcal{L}(data   \lambda) = \prod_{i=1}^{N} \underbrace{e^{-\lambda}_{i}}_{X_{i}}^{X_{i}}$
1		L- pcdata (X) = 11 ex
		Y. J.
3		1 / N - X X 1
3	(8)	$\frac{1}{2} \cdot \log L = \log \left( \prod_{i=1}^{N} \frac{e^{-\lambda} \chi_{i}^{i}}{2\epsilon_{i}^{i}} \right)$
3		
-		$= \sum_{i=1}^{N} \log \left( \frac{e^{-\lambda_i x_i}}{x_{i}!} \right)$
3		$= \sum \log \left( \frac{e^{\lambda}}{\lambda} \right)$
3		
9		NI
		$= \sum_{i=1}^{N} \left[ \log e^{-\lambda} + x_i  \log \lambda - \log (x_{i}) \right]$
-		i=1 L 0
3		N
-		= -NX+ Logx & x: - \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \
-		
-		Now, for estimating I me need to do d(log L)=0
-		di
-		N
		-N+12x:=0
	-	7 2=1
		N
		1. \\ \( \) = \( \) \( \) \( \) \( \) = \( \) \(
		ML V=1
-		N



Som 5 i) To prove that gamma is conjugate to poisson, me need to prove that given a gamma prior, we obtain a gamma functional form of posterior as well. **S** Now, postedios can be whiten as: Posterios X Liklihood x Prios p ( ) | data) x p (data | ) x p ( ) row, from 50h 4 we got: -3 = e-NX , X1+X2 ... + XN K- e-NA. Exe (where K is constant) and we have,  $b(\lambda) = \frac{b_0}{\Gamma(\alpha_0)} \lambda^{\alpha_0 - 1} e^{-b_0 \lambda} \qquad (given)$ .: pc x Idata) x Ke-Nx x xi. 600 xa0-1 e-box

-> (N+bo) (Exe+ao)-1 Now, p (x 1 data) has some functioned form as that of its perior hence no can say that over perior is conjugate to little hood i. c gammer distribution is prior to poisson. distribution. pcx (data) = c e-x cN+60). (Ex: +00)-1 Compolung with gamma distribution to an = Exi + ao  $b_{N} = N + b_{0}$   $(\Sigma x_{i} + \alpha_{0}) = \lambda (N + b_{0})$   $(\Sigma x_{i}^{2} + \alpha_{0}) = \lambda (N + b_{0})$   $(\Sigma x_{i}^{2} + \alpha_{0}) = \lambda (N + b_{0})$   $(\Sigma x_{i}^{2} + \alpha_{0}) = \lambda (N + b_{0})$ 

```
ii) From result of i) let.
           \frac{(N+b_0)^{(\Sigma \times 1 + \alpha_0)}}{\Gamma(\Sigma \times 1 + \alpha_0)}
                                                  (constant)
                                                (mean)
      in Excision NSc
TO
         Substituting these me got,
b(x) | data) = C e^{-x c N + 603} x^{c N \overline{x}} + a_0 - 13
150
THE
TE
           To get MAP Estimate, we need to do
TO
TO
                d p(x) data) = 0
TO
TIP
           C (-(N+bo) e-x(N+bo) (N5+00-1) + (Nx +00-1) e (Nx+00-2)
           [-(N+bo) + (Nsc+a_0-1)] = 0
            \lambda_{MAP} = \frac{N\bar{x} + a_0 - 1}{N + b_0}
```