Tutorial Exercises Week 5

Question 1

Write a function `roots(a,b,c)` that takes as input three numbers a, b and c and outputs a list with as two elements the roots

$$x_\ell = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x_r = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

of the quadratic function $f(x) = ax^2 + bx + c$. As an example, the command `roots(1,2,-1)` should return `[-2.414213562373095, 0.41421356237309515]`.

Question 2

Write a function called `maximum()` that takes as input a mathematical function $f : \mathbb{R} \to \mathbb{R}$ and an initial guess `guess`. It should output an x that maximizes the function f. Your function should make use of `fmin()` with initial guess `guess`.

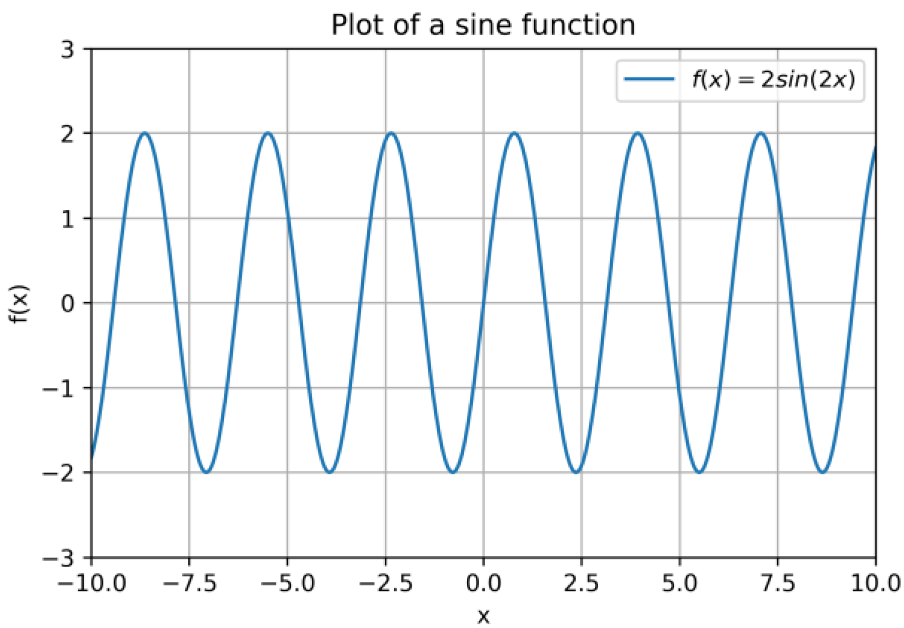
Hint: An x maximizes the function $f(x)$ if and only if it minimizes the function $-f(x)$.

Define the function $f(x) = -x^2 - 3x + 1$ as a Python function and test your function `maximum()` on it with initial guess `guess = 10`. The maximum should be attained at $x = -1.5$.

Question 3

In this exercise, we will plot a sine function and try to find all the roots that it has.

a) Replicate the figure below in Python, i.e., plot the function $f(x) = 2sin(2x)$ on the interval $[-10, 10]$ with the specified figure requirements. You can compute the sine function of a number y with `np.sin(y)`.



The function f has many roots, i.e., x-values that satisfy $2\sin(2x) = 0$, on the interval $[-10, 10]$. We will try to find all of them and indicate them in the figure of part a).

b) Create a vector $guess = [-10, -9.5, -9, \dots, -1, -0.5, 0, 0.5, 1, 1.5, 2, \dots, 10]$ using `np.linspace()`

c) Create a for-loop that executes for every choice of initial guess in the vector `guess` the function `fsolve()` with the chosen guess. The roots that are found should be stored in a list called `roots`.

Hint: Define an empty list `roots = []` and append the found roots to it. The output should be

[-9.424777960769301, -9.42477796076938, -9.42477796076938, -9.42477796076938, -7.853981633974483, -7.853981633974483, -3.141592653589793, -6.283185307179586, -6.283185307179586, -513.6503988619313, -4.71238898038469, -4.71238898038469, -14.137166941154069, -3.141592653589793, -3.141592653589793, -3.141592653589793, -1.5707963267948966, -1.5707963267948966, -1.5707963267948966, 0.0, 0.0, 0.0, 1.5707963267948966, 1.5707963267948966, 1.5707963267948966, 3.141592653589793, 3.141592653589793, 3.141592653589793, 14.137166941154069, 4.71238898038469, 4.71238898038469, 782.2565707438586, 6.283185307179586, 6.283185307179586, 3.141592653589793, 7.853981633974483, 7.853981633974483, 9.42477796076938, 9.42477796076938, 9.42477796076938, 9.424777960769301]

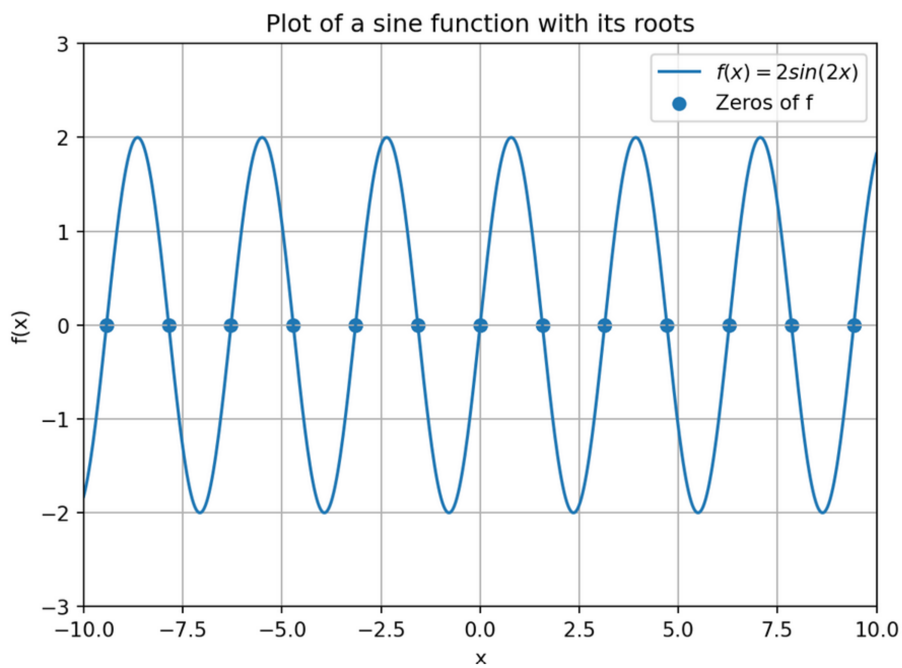
It is important to observe that different roots of f might be found depending on the intitial guess chosen for `fsolve()`. You can also see that sometimes for different guesses the same root is found; you could reduce this list to only include the unique elements (`using np.unique()`) but you don't have to worry about this for now.

Next we will plot the roots in the figure of part a) as points.

d) Create a vector $zeros = [0, 0, \dots, 0]$ whose number of zeros is the same as the number of elements in the vector `roots` (i.e., the length of `roots`)

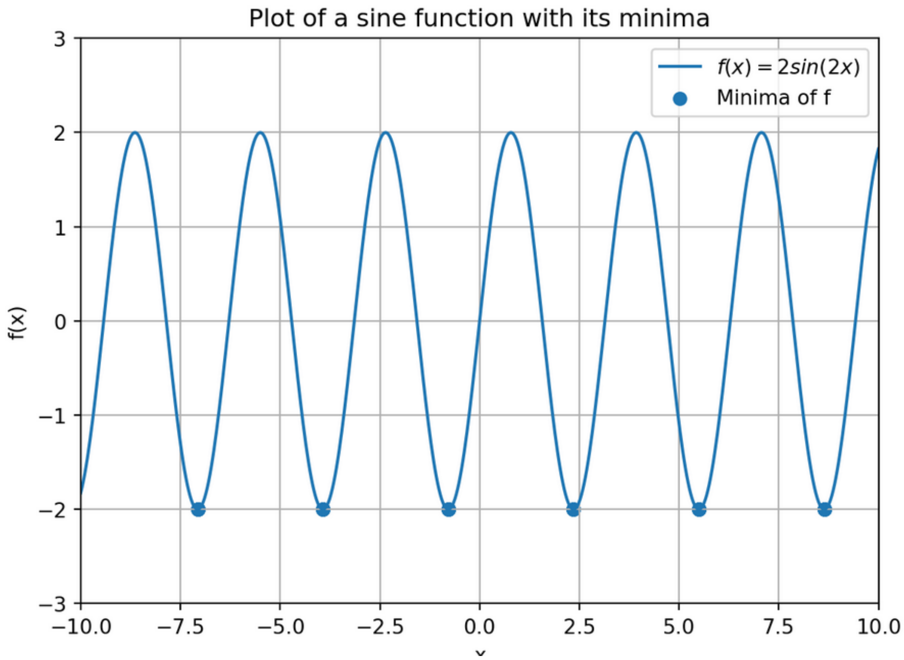
Hint: You can do this with a for-loop or you might have a look at the`zeros()` function from Numpy yourself.

e) Plot the vectors `roots` and `zeros` against each other using a scatter plot (so that the combinations appear as points) in the figure generated in part a). Make the necessary adjustments so that the output looks like the figure below.



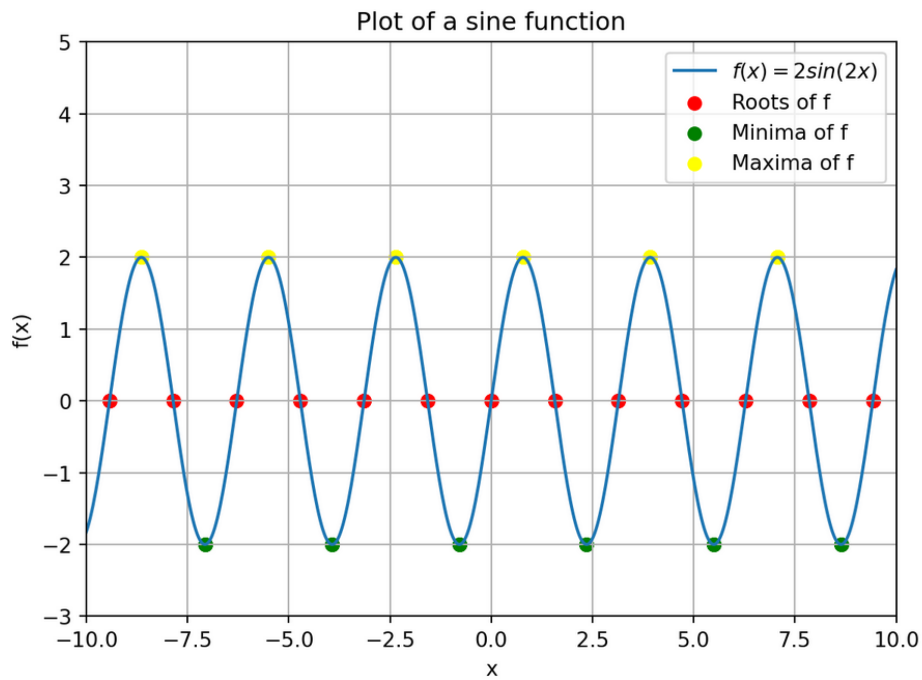
Question 4

Create a similar figure as in Question 3, but now with the minima indicated in the figure instead of the roots of f. Your figure should look like this.



Bonus question

Combining all the previous questions, create one figure that contains all the roots (as red points), minima (as green points) and maxima (as yellow points) of the function f. Your figure should look like this:



Hint: To create the different colors, you can use the argument `c='[color]'` of `plt.scatter()` with [color] replaced by the desired color.