

# Methodology for Coordinated Multi-Regional Energy System Optimization

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## 1 Methodology

In modern energy systems, the increasing penetration of intermittent renewable energy sources (RES) such as solar and wind necessitates a more dynamic and regionally coordinated approach to system operation. A key concept in this framework is the *residual demand*, defined as the net load remaining after the contribution from renewable sources is subtracted from the total electricity demand. Residual demand reflects the portion of the load that must be met by dispatchable generation, storage discharge, demand response (DR), or interregional energy exchanges.

The modeling approach presented in this work is designed to optimize the operation of multi-regional energy systems by simultaneously considering:

- The dispatch of conventional generation,
- The dynamics of storage systems,
- The activation of demand response,
- The coordination of power exchanges between regions.

By explicitly modeling the residual demand, our approach accounts for the variability and uncertainty associated with renewable generation. It ensures that local flexibility resources (both generation and storage) are deployed efficiently while minimizing reliance on costly long-distance power transfers during periods of peak demand or renewable shortfall.

The framework is built upon a linear programming (LP) formulation that captures various operational constraints and economic trade-offs. This integrated model helps assess the benefits of regional flexibility, offering insights into how intra-national coordination can reduce adjustment costs and mitigate network congestion.

### 1.1 Conceptual Framework: Residual Demand and Modeling Variables

The cornerstone of our methodology is the *residual demand*  $RD_r(t)$  for each region  $r$  at time  $t$ , computed as:

$$RD_r(t) = \text{Demand}_r(t) - (\text{Solar}_r(t) + \text{Wind}_r(t)) - DR_r(t) \quad (1)$$

where:

- $\text{Demand}_r(t)$  represents the total electrical load in region  $r$ ,
- $\text{Solar}_r(t)$  and  $\text{Wind}_r(t)$  denote the available renewable generation,

- $DR_r(t)$  is the demand response activated to reduce the effective load.

The residual demand is met through a combination of dispatchable generation, storage operations, and interregional exchanges. The main decision variables used in the model are as follows:

### Dispatch Variables

For each dispatchable technology  $i$  in region  $r$ , the power output  $x_{r,i}(t)$  is bounded by:

$$0 \leq x_{r,i}(t) \leq \bar{P}_{r,i} \quad (2)$$

with ramping constraints:

$$x_{r,i}(t+1) - x_{r,i}(t) \leq RU_{r,i} \quad \forall t, \quad (3)$$

$$x_{r,i}(t) - x_{r,i}(t+1) \leq RD_{r,i} \quad \forall t. \quad (4)$$

### Renewable Flow and Curtailment Variables

For solar energy in region  $r$ :

$$s_r^{\text{flow}}(t) + s_r^{\text{spill}}(t) = \text{Solar}_r(t) \times (1 + \epsilon) \quad (5)$$

For wind energy:

$$w_r^{\text{flow}}(t) + w_r^{\text{spill}}(t) = \text{Wind}_r(t) \times (1 + \epsilon) \quad (6)$$

where  $\epsilon$  is the renewable uncertainty factor.

### Storage Variables

For each storage unit  $s$  in region  $r$ , the following variables are defined:

- Charging power:  $P_{r,s}^{\text{ch}}(t)$  with upper bound  $\bar{P}_{r,s}^{\text{ch}}$ .
- Discharging power:  $P_{r,s}^{\text{dis}}(t)$  with upper bound  $\bar{P}_{r,s}^{\text{dis}}$ .
- State-of-Charge (SoC):  $SOC_{r,s}(t)$ , bounded by the storage capacity  $E_{r,s}$ .

The storage dynamics are modeled as follows. At  $t = 0$ :

$$SOC_{r,s}(0) = SOC_{r,s}^0 + \eta_{r,s}^{\text{ch}} P_{r,s}^{\text{ch}}(0) \Delta t - \frac{1}{\eta_{r,s}^{\text{dis}}} P_{r,s}^{\text{dis}}(0) \Delta t \quad (7)$$

For  $t > 0$ :

$$SOC_{r,s}(t) = SOC_{r,s}(t-1) \times (1 - \delta_{r,s}) + \eta_{r,s}^{\text{ch}} P_{r,s}^{\text{ch}}(t) \Delta t - \frac{1}{\eta_{r,s}^{\text{dis}}} P_{r,s}^{\text{dis}}(t) \Delta t, \quad (8)$$

with

$$0 \leq SOC_{r,s}(t) \leq E_{r,s}. \quad (9)$$

Additional constraints limit the rate of change in charging and discharging:

$$|P_{r,s}^{\text{ch}}(t+1) - P_{r,s}^{\text{ch}}(t)| \leq \Delta P_{r,s}^{\text{ch}}, \quad (10)$$

$$|P_{r,s}^{\text{dis}}(t+1) - P_{r,s}^{\text{dis}}(t)| \leq \Delta P_{r,s}^{\text{dis}}. \quad (11)$$

## Interregional Exchange Variables

For authorized pairs of regions  $(r_1, r_2)$ , the exchange variable  $E_{r_1, r_2}(t)$  is bounded as:

$$-E^{\max} \leq E_{r_1, r_2}(t) \leq E^{\max}. \quad (12)$$

Dynamic pricing is applied to these exchanges, with higher costs during peak hours.

## Slack Variables

To ensure feasibility in the presence of imbalances, slack variables  $\lambda_r^{\text{shed}}(t)$  and  $\lambda_r^{\text{dump}}(t)$  are included in the balance constraints, with very high penalty costs.

## Demand Response Variables

Demand response is modeled through the variable  $DR_r(t)$  for each region  $r$ , bounded by:

$$0 \leq DR_r(t) \leq DR_r^{\max}, \quad (13)$$

and is associated with a variable cost  $c_r^{\text{DR}}$  (with an optional fixed cost component that can be incorporated via binary variables).

## 1.2 Model Formulation and Constraints

The optimization problem is formulated as a linear program aimed at minimizing the total operational cost while meeting the residual demand in every region and time step.

### Dispatch and Ramp Constraints

For each dispatchable unit  $i$  in region  $r$ :

$$0 \leq x_{r,i}(t) \leq \bar{P}_{r,i}, \quad (14)$$

with ramp constraints:

$$x_{r,i}(t+1) - x_{r,i}(t) \leq RU_{r,i} \quad \forall t, \quad (15)$$

$$x_{r,i}(t) - x_{r,i}(t+1) \leq RD_{r,i} \quad \forall t. \quad (16)$$

### Renewable Generation Balance

For solar and wind energy in region  $r$ :

$$s_r^{\text{flow}}(t) + s_r^{\text{spill}}(t) = \text{Solar}_r(t) \times (1 + \epsilon), \quad (17)$$

$$w_r^{\text{flow}}(t) + w_r^{\text{spill}}(t) = \text{Wind}_r(t) \times (1 + \epsilon). \quad (18)$$

### Storage Dynamics and Rate Constraints

The storage state-of-charge evolves as:

$$SOC_{r,s}(0) = SOC_{r,s}^0 + \eta_{r,s}^{\text{ch}} P_{r,s}^{\text{ch}}(0) \Delta t - \frac{1}{\eta_{r,s}^{\text{dis}}} P_{r,s}^{\text{dis}}(0) \Delta t, \quad (19)$$

$$SOC_{r,s}(t) = SOC_{r,s}(t-1) \times (1 - \delta_{r,s}) + \eta_{r,s}^{\text{ch}} P_{r,s}^{\text{ch}}(t) \Delta t - \frac{1}{\eta_{r,s}^{\text{dis}}} P_{r,s}^{\text{dis}}(t) \Delta t, \quad t > 0, \quad (20)$$

with the bound:

$$0 \leq SOC_{r,s}(t) \leq E_{r,s}, \quad (21)$$

and rate constraints:

$$|P_{r,s}^{\text{ch}}(t+1) - P_{r,s}^{\text{ch}}(t)| \leq \Delta P_{r,s}^{\text{ch}}, \quad (22)$$

$$|P_{r,s}^{\text{dis}}(t+1) - P_{r,s}^{\text{dis}}(t)| \leq \Delta P_{r,s}^{\text{dis}}. \quad (23)$$

### Interregional Exchange

For each authorized region pair  $(r_1, r_2)$ :

$$-E^{\max} \leq E_{r_1, r_2}(t) \leq E^{\max}, \quad (24)$$

with a dynamic pricing cost  $C^{\text{exchange}}(t)$  that may vary with time (e.g., higher during peak hours).

### Regional Balance (Residual Demand Constraint)

The net supply must match the residual demand in each region  $r$  at time  $t$ . This is captured by the following balance equation:

$$\begin{aligned} & \sum_{i \in \mathcal{T}_r} \alpha_{r,i} x_{r,i}(t) + \sum_{s \in \mathcal{S}_r} (P_{r,s}^{\text{dis}}(t) - P_{r,s}^{\text{ch}}(t)) \\ & + \sum_{(r,r') \in \mathcal{E}} \beta_{r,r'} E_{r,r'}(t) + \lambda_r^{\text{shed}}(t) - \lambda_r^{\text{dump}}(t) \\ & = \text{Demand}_r(t) - (s_r^{\text{flow}}(t) + w_r^{\text{flow}}(t)) - DR_r(t). \end{aligned} \quad (25)$$

Here,  $\alpha_{r,i}$  is +1 for generation units and -1 for export-type technologies, while  $\beta_{r,r'}$  depends on the direction of energy transfer.

### Slack and Demand Response Variables

To account for potential imbalances, slack variables  $\lambda_r^{\text{shed}}(t)$  and  $\lambda_r^{\text{dump}}(t)$  are included with high penalty costs. Demand response is incorporated via the variable  $DR_r(t)$ , constrained by:

$$0 \leq DR_r(t) \leq DR_r^{\max}. \quad (26)$$

## 1.3 Objective Function

The goal is to minimize the total system cost, which is composed of:

- **Dispatch Costs:**

$$\sum_{r,t} \sum_{i \in \mathcal{T}_r} c_{r,i} x_{r,i}(t)$$

- **Storage Operation Costs:**

$$\sum_{r,t} \sum_{s \in \mathcal{S}_r} c_{r,s}^{\text{storage}} P_{r,s}^{\text{dis}}(t)$$

- **Renewable Curtailment Penalties:**

$$\sum_{r,t} c^{\text{spill}} \left[ s_r^{\text{spill}}(t) + w_r^{\text{spill}}(t) \right]$$

- **Interregional Exchange Costs:**

$$\sum_{(r_1,r_2),t} C^{\text{exchange}}(t) E_{r_1,r_2}(t)$$

- **Penalty Costs for Slack Variables:**

$$\sum_{r,t} c^{\text{slack}} \left[ \lambda_r^{\text{shed}}(t) + \lambda_r^{\text{dump}}(t) \right]$$

- **Demand Response Costs:**

$$\sum_{r,t} c_r^{\text{DR}} DR_r(t)$$

Thus, the complete objective function is:

$$\begin{aligned} \min \quad & \sum_{r,t} \left\{ \sum_{i \in \mathcal{T}_r} c_{r,i} x_{r,i}(t) + \sum_{s \in \mathcal{S}_r} c_{r,s}^{\text{storage}} P_{r,s}^{\text{dis}}(t) \right. \\ & + c^{\text{spill}} \left[ s_r^{\text{spill}}(t) + w_r^{\text{spill}}(t) \right] + \sum_{(r_1,r_2) \in \mathcal{E}} C^{\text{exchange}}(t) E_{r_1,r_2}(t) \\ & \left. + c^{\text{slack}} \left[ \lambda_r^{\text{shed}}(t) + \lambda_r^{\text{dump}}(t) \right] + c_r^{\text{DR}} DR_r(t) \right\}. \end{aligned} \quad (27)$$

## 1.4 Computational Implementation

The linear programming model is implemented in Python using the PuLP library. The code defines the decision variables, sets up the constraints as described above, and solves the optimization problem using an appropriate solver (e.g., CBC). Following the optimization, key outputs such as dispatch schedules, storage operations, interregional exchanges, and demand response activations are generated for further analysis and visualization.

This comprehensive framework bridges the gap between localized flexibility and national-scale planning, providing insights into how intra-national coordination can optimize system performance, reduce adjustment costs, and enhance the integration of renewable energy resources.