## **Methodology**

### **1. Introduction**

In modern energy systems, the increasing penetration of intermittent renewable energy sources (RES) such as solar and wind necessitates a more dynamic and regionally coordinated approach to system operation. A key concept in this framework is the **residual demand**, defined as the net load remaining after the contribution from renewable sources is subtracted from the total electricity demand. Residual demand reflects the portion of the load that must be met by dispatchable generation, storage discharge, demand response (DR), or interregional energy exchanges.

The modeling approach presented in this work is designed to optimize the operation of multi-regional energy systems by simultaneously considering the dispatch of conventional generation, the dynamics of storage systems, the activation of demand response, and the coordination of power exchanges between regions. By explicitly modeling the residual demand, our approach accounts for the variability and uncertainty associated with renewable generation. It ensures that local flexibility resources (both generation and storage) are deployed efficiently while minimizing reliance on costly long-distance power transfers during periods of peak demand or renewable shortfall.

The framework is built upon a linear programming (LP) formulation that captures various operational constraints and economic trade-offs. This integrated model helps to assess the benefits of regional flexibility, offering insights into how intra-national coordination can reduce adjustment costs and mitigate network congestion.

### **2. Conceptual Framework: Residual Demand and Modeling Variables**

The cornerstone of our methodology is the **residual demand** (RDr(t)RD\_r(t)RDr​(t)) for each region rrr at time ttt, computed as:

RDr(t)=Demandr(t)−(Solarr(t)+Windr(t))−DRr(t)RD\_r(t) = \text{Demand}\_r(t) - \left( \text{Solar}\_r(t) + \text{Wind}\_r(t) \right) - DR\_r(t)RDr​(t)=Demandr​(t)−(Solarr​(t)+Windr​(t))−DRr​(t)

where:

* Demandr(t)\text{Demand}\_r(t)Demandr​(t) represents the total electrical load in region rrr.
* Solarr(t)\text{Solar}\_r(t)Solarr​(t) and Windr(t)\text{Wind}\_r(t)Windr​(t) denote the available renewable generation.
* DRr(t)DR\_r(t)DRr​(t) is the demand response activated to reduce the effective load.

The residual demand is then met through a combination of dispatchable generation, storage operations, and interregional exchanges. The following decision variables are defined in the model:

* **Dispatch Variables** (xr,i(t)x\_{r,i}(t)xr,i​(t)): Represent the power output from dispatchable technology iii in region rrr at time ttt. Each technology is characterized by a maximum capacity (P‾r,i\overline{P}\_{r,i}Pr,i​), a minimum capacity (P‾r,i\underline{P}\_{r,i}P​r,i​), and a marginal cost (cr,ic\_{r,i}cr,i​). Furthermore, ramp-up (RUr,iRU\_{r,i}RUr,i​) and ramp-down (RDr,iRD\_{r,i}RDr,i​) constraints limit how quickly the output can change between time steps.
* **Renewable Flow and Curtailment Variables**:
  + srflow(t)s\_{r}^{\text{flow}}(t)srflow​(t) and srspill(t)s\_{r}^{\text{spill}}(t)srspill​(t) for solar energy.
  + wrflow(t)w\_{r}^{\text{flow}}(t)wrflow​(t) and wrspill(t)w\_{r}^{\text{spill}}(t)wrspill​(t) for wind energy.
* These variables ensure that the sum of the used and curtailed renewable energy equals the forecasted available renewable energy, adjusted by an uncertainty factor ϵ\epsilonϵ:  
  srflow(t)+srspill(t)=Solarr(t)×(1+ϵ)s\_{r}^{\text{flow}}(t) + s\_{r}^{\text{spill}}(t) = \text{Solar}\_r(t) \times (1+\epsilon)srflow​(t)+srspill​(t)=Solarr​(t)×(1+ϵ) wrflow(t)+wrspill(t)=Windr(t)×(1+ϵ)w\_{r}^{\text{flow}}(t) + w\_{r}^{\text{spill}}(t) = \text{Wind}\_r(t) \times (1+\epsilon)wrflow​(t)+wrspill​(t)=Windr​(t)×(1+ϵ)
* **Storage Variables**: For each storage unit sss in region rrr, the model defines:
  + **Charging power** Pr,sch(t)P^{\text{ch}}\_{r,s}(t)Pr,sch​(t) with an upper bound P‾r,sch\overline{P}^{\text{ch}}\_{r,s}Pr,sch​.
  + **Discharging power** Pr,sdis(t)P^{\text{dis}}\_{r,s}(t)Pr,sdis​(t) with an upper bound P‾r,sdis\overline{P}^{\text{dis}}\_{r,s}Pr,sdis​.
  + **State-of-Charge (SoC)** SOCr,s(t)SOC\_{r,s}(t)SOCr,s​(t), which evolves according to the charging and discharging operations and is bounded by the storage capacity Er,sE\_{r,s}Er,s​.
* The storage dynamic is modeled as:  
  SOCr,s(t)={SOCr,s0+ηr,sch Pr,sch(t) Δt−1ηr,sdis Pr,sdis(t) Δt,t=0SOCr,s(t−1)×(1−δr,s)+ηr,sch Pr,sch(t) Δt−1ηr,sdis Pr,sdis(t) Δt,t>0SOC\_{r,s}(t) = \begin{cases} SOC^0\_{r,s} + \eta^{\text{ch}}\_{r,s} \, P^{\text{ch}}\_{r,s}(t) \, \Delta t - \frac{1}{\eta^{\text{dis}}\_{r,s}} \, P^{\text{dis}}\_{r,s}(t) \, \Delta t, & t = 0 \\ SOC\_{r,s}(t-1) \times (1 - \delta\_{r,s}) + \eta^{\text{ch}}\_{r,s} \, P^{\text{ch}}\_{r,s}(t) \, \Delta t - \frac{1}{\eta^{\text{dis}}\_{r,s}} \, P^{\text{dis}}\_{r,s}(t) \, \Delta t, & t > 0 \end{cases}SOCr,s​(t)={SOCr,s0​+ηr,sch​Pr,sch​(t)Δt−ηr,sdis​1​Pr,sdis​(t)Δt,SOCr,s​(t−1)×(1−δr,s​)+ηr,sch​Pr,sch​(t)Δt−ηr,sdis​1​Pr,sdis​(t)Δt,​t=0t>0​  
  where Δt\Delta tΔt is the time resolution in hours.  
  Additionally, to capture operational limitations, the model includes constraints on the rate of change of charging and discharging:  
  ∣Pr,sch(t+1)−Pr,sch(t)∣≤ΔPr,sch\left| P^{\text{ch}}\_{r,s}(t+1) - P^{\text{ch}}\_{r,s}(t) \right| \leq \Delta P^{\text{ch}}\_{r,s}​Pr,sch​(t+1)−Pr,sch​(t)​≤ΔPr,sch​ ∣Pr,sdis(t+1)−Pr,sdis(t)∣≤ΔPr,sdis\left| P^{\text{dis}}\_{r,s}(t+1) - P^{\text{dis}}\_{r,s}(t) \right| \leq \Delta P^{\text{dis}}\_{r,s}​Pr,sdis​(t+1)−Pr,sdis​(t)​≤ΔPr,sdis​
* **Interregional Exchange Variables** (Er1,r2(t)E\_{r\_1,r\_2}(t)Er1​,r2​​(t)): These variables model the energy traded between regions r1r\_1r1​ and r2r\_2r2​ within predetermined interconnection limits:  
  −Emax⁡≤Er1,r2(t)≤Emax⁡-E^{\max} \leq E\_{r\_1,r\_2}(t) \leq E^{\max}−Emax≤Er1​,r2​​(t)≤Emax  
  Dynamic pricing is applied to these exchanges, with higher costs imposed during peak hours.
* **Slack Variables** (λrshed(t)\lambda^{\text{shed}}\_{r}(t)λrshed​(t) and λrdump(t)\lambda^{\text{dump}}\_{r}(t)λrdump​(t)): To guarantee feasibility in the face of potential imbalances, high-penalty slack variables are included in the balance constraints.
* **Demand Response Variables** (DRr(t)DR\_r(t)DRr​(t)): These continuous variables represent the amount of load reduction activated in response to grid conditions. They are subject to an upper bound DRrmax⁡DR^{\max}\_rDRrmax​ and are associated with a variable cost crDRc^{\text{DR}}\_rcrDR​ (with an optional fixed cost component activated via binary variables, if needed).

### **3. Model Formulation and Constraints**

The optimization problem is formulated as a linear program that minimizes the total operational cost while ensuring that the residual demand is met at all times.

#### **3.1. Dispatch and Ramp Constraints**

For each dispatchable unit iii in region rrr:

0≤xr,i(t)≤P‾r,i0 \leq x\_{r,i}(t) \leq \overline{P}\_{r,i}0≤xr,i​(t)≤Pr,i​

The ramp rate constraints enforce:

xr,i(t+1)−xr,i(t)≤RUr,i∀ tx\_{r,i}(t+1) - x\_{r,i}(t) \leq RU\_{r,i} \quad \forall \, txr,i​(t+1)−xr,i​(t)≤RUr,i​∀t xr,i(t)−xr,i(t+1)≤RDr,i∀ tx\_{r,i}(t) - x\_{r,i}(t+1) \leq RD\_{r,i} \quad \forall \, txr,i​(t)−xr,i​(t+1)≤RDr,i​∀t

#### **3.2. Renewable Generation Balance**

For each renewable technology (solar and wind) in region rrr:

srflow(t)+srspill(t)=Solarr(t)×(1+ϵ)s\_{r}^{\text{flow}}(t) + s\_{r}^{\text{spill}}(t) = \text{Solar}\_r(t) \times (1+\epsilon)srflow​(t)+srspill​(t)=Solarr​(t)×(1+ϵ) wrflow(t)+wrspill(t)=Windr(t)×(1+ϵ)w\_{r}^{\text{flow}}(t) + w\_{r}^{\text{spill}}(t) = \text{Wind}\_r(t) \times (1+\epsilon)wrflow​(t)+wrspill​(t)=Windr​(t)×(1+ϵ)

#### **3.3. Storage Dynamics and Rate Constraints**

The state-of-charge (SoC) for each storage unit evolves according to:

SOCr,s(0)=SOCr,s0+ηr,sch Pr,sch(0) Δt−1ηr,sdis Pr,sdis(0) ΔtSOC\_{r,s}(0) = SOC^0\_{r,s} + \eta^{\text{ch}}\_{r,s} \, P^{\text{ch}}\_{r,s}(0) \, \Delta t - \frac{1}{\eta^{\text{dis}}\_{r,s}} \, P^{\text{dis}}\_{r,s}(0) \, \Delta tSOCr,s​(0)=SOCr,s0​+ηr,sch​Pr,sch​(0)Δt−ηr,sdis​1​Pr,sdis​(0)Δt SOCr,s(t)=SOCr,s(t−1)×(1−δr,s)+ηr,sch Pr,sch(t) Δt−1ηr,sdis Pr,sdis(t) Δt∀ t>0SOC\_{r,s}(t) = SOC\_{r,s}(t-1) \times (1 - \delta\_{r,s}) + \eta^{\text{ch}}\_{r,s} \, P^{\text{ch}}\_{r,s}(t) \, \Delta t - \frac{1}{\eta^{\text{dis}}\_{r,s}} \, P^{\text{dis}}\_{r,s}(t) \, \Delta t \quad \forall\, t>0SOCr,s​(t)=SOCr,s​(t−1)×(1−δr,s​)+ηr,sch​Pr,sch​(t)Δt−ηr,sdis​1​Pr,sdis​(t)Δt∀t>0

with:

0≤SOCr,s(t)≤Er,s0 \leq SOC\_{r,s}(t) \leq E\_{r,s}0≤SOCr,s​(t)≤Er,s​

and rate-of-change limits:

∣Pr,sch(t+1)−Pr,sch(t)∣≤ΔPr,sch\left| P^{\text{ch}}\_{r,s}(t+1) - P^{\text{ch}}\_{r,s}(t) \right| \leq \Delta P^{\text{ch}}\_{r,s}​Pr,sch​(t+1)−Pr,sch​(t)​≤ΔPr,sch​ ∣Pr,sdis(t+1)−Pr,sdis(t)∣≤ΔPr,sdis\left| P^{\text{dis}}\_{r,s}(t+1) - P^{\text{dis}}\_{r,s}(t) \right| \leq \Delta P^{\text{dis}}\_{r,s}​Pr,sdis​(t+1)−Pr,sdis​(t)​≤ΔPr,sdis​

#### **3.4. Interregional Exchange**

For authorized region pairs (r1,r2)(r\_1, r\_2)(r1​,r2​), the energy exchange is constrained by:

−Emax⁡≤Er1,r2(t)≤Emax⁡- E^{\max} \leq E\_{r\_1,r\_2}(t) \leq E^{\max}−Emax≤Er1​,r2​​(t)≤Emax

Dynamic pricing is applied to these exchanges, where the exchange cost Cexchange(t)C^{\text{exchange}}(t)Cexchange(t) may vary with the time of day (e.g., higher during peak hours).

#### **3.5. Regional Balance (Residual Demand Constraint)**

The operational balance for each region is enforced by equating the net supply to the residual demand. For each region rrr and time ttt, the balance constraint is formulated as:

∑i∈Trαr,i xr,i(t)+∑s∈Sr(Pr,sdis(t)−Pr,sch(t))+∑(r,r′)∈Eβr,r′ Er,r′(t)+λrshed(t)−λrdump(t)\sum\_{i \in \mathcal{T}\_r} \alpha\_{r,i}\, x\_{r,i}(t) + \sum\_{s \in \mathcal{S}\_r} \left( P^{\text{dis}}\_{r,s}(t) - P^{\text{ch}}\_{r,s}(t) \right) + \sum\_{(r,r') \in \mathcal{E}} \beta\_{r,r'}\, E\_{r,r'}(t) + \lambda^{\text{shed}}\_{r}(t) - \lambda^{\text{dump}}\_{r}(t)i∈Tr​∑​αr,i​xr,i​(t)+s∈Sr​∑​(Pr,sdis​(t)−Pr,sch​(t))+(r,r′)∈E∑​βr,r′​Er,r′​(t)+λrshed​(t)−λrdump​(t) =Demandr(t)−(srflow(t)+wrflow(t))−DRr(t)\quad = \text{Demand}\_r(t) - \left( s\_{r}^{\text{flow}}(t) + w\_{r}^{\text{flow}}(t) \right) - DR\_r(t)=Demandr​(t)−(srflow​(t)+wrflow​(t))−DRr​(t)

where:

* αr,i\alpha\_{r,i}αr,i​ is set to +1+1+1 for generation technologies and −1-1−1 for export-type technologies.
* βr,r′\beta\_{r,r'}βr,r′​ takes values of +1+1+1 or −1-1 −1 depending on the direction of energy transfer.

This equation ensures that the combination of dispatchable production, net storage output, and interregional exchanges fully compensates for the residual demand in each region.

#### **3.6. Slack and Demand Response Variables**

To avoid infeasibility in cases where the residual demand cannot be perfectly met, slack variables λrshed(t)\lambda^{\text{shed}}\_{r}(t)λrshed​(t) and λrdump(t)\lambda^{\text{dump}}\_{r}(t)λrdump​(t) are introduced with very high penalty costs. Demand response is modeled through the continuous variable DRr(t)DR\_r(t)DRr​(t) (bounded by DRrmax⁡DR^{\max}\_rDRrmax​), which represents the extent of load reduction applied in region rrr at time ttt.

### **4. Objective Function**

The optimization goal is to minimize the total system cost, which includes the following components:

* **Dispatch Costs:** ∑r,t∑i∈Trcr,i xr,i(t)\sum\_{r,t}\sum\_{i \in \mathcal{T}\_r} c\_{r,i}\, x\_{r,i}(t)r,t∑​i∈Tr​∑​cr,i​xr,i​(t)
* **Storage Operation Costs:** ∑r,t∑s∈Srcr,sstorage Pr,sdis(t)\sum\_{r,t}\sum\_{s \in \mathcal{S}\_r} c^{\text{storage}}\_{r,s}\, P^{\text{dis}}\_{r,s}(t)r,t∑​s∈Sr​∑​cr,sstorage​Pr,sdis​(t)
* **Renewable Curtailment Penalties:** ∑r,tcspill[srspill(t)+wrspill(t)]\sum\_{r,t} c^{\text{spill}}\Bigl[ s\_{r}^{\text{spill}}(t) + w\_{r}^{\text{spill}}(t) \Bigr]r,t∑​cspill[srspill​(t)+wrspill​(t)]
* **Interregional Exchange Costs:** ∑(r1,r2),tCexchange(t) Er1,r2(t)\sum\_{(r\_1,r\_2),t} C^{\text{exchange}}(t)\, E\_{r\_1,r\_2}(t)(r1​,r2​),t∑​Cexchange(t)Er1​,r2​​(t)
* **Penalty Costs for Slack Variables:** ∑r,tcslack[λrshed(t)+λrdump(t)]\sum\_{r,t} c^{\text{slack}}\Bigl[ \lambda^{\text{shed}}\_{r}(t) + \lambda^{\text{dump}}\_{r}(t) \Bigr]r,t∑​cslack[λrshed​(t)+λrdump​(t)]
* **Demand Response Costs:** ∑r,tcrDR DRr(t)\sum\_{r,t} c^{\text{DR}}\_r\, DR\_r(t)r,t∑​crDR​DRr​(t)

Thus, the complete objective function is given by:

min⁡∑r,t{∑i∈Trcr,i xr,i(t)+∑s∈Srcr,sstorage Pr,sdis(t)+cspill[srspill(t)+wrspill(t)]\min \quad \sum\_{r,t} \Bigg\{ \sum\_{i \in \mathcal{T}\_r} c\_{r,i}\, x\_{r,i}(t) + \sum\_{s \in \mathcal{S}\_r} c^{\text{storage}}\_{r,s}\, P^{\text{dis}}\_{r,s}(t) + c^{\text{spill}} \left[ s\_{r}^{\text{spill}}(t) + w\_{r}^{\text{spill}}(t) \right] minr,t∑​{i∈Tr​∑​cr,i​xr,i​(t)+s∈Sr​∑​cr,sstorage​Pr,sdis​(t)+cspill[srspill​(t)+wrspill​(t)] +∑(r1,r2)∈ECexchange(t) Er1,r2(t)+cslack[λrshed(t)+λrdump(t)]+crDR DRr(t)}+ \sum\_{(r\_1,r\_2) \in \mathcal{E}} C^{\text{exchange}}(t)\, E\_{r\_1,r\_2}(t) + c^{\text{slack}} \left[ \lambda^{\text{shed}}\_{r}(t) + \lambda^{\text{dump}}\_{r}(t) \right] + c^{\text{DR}}\_r\, DR\_r(t) \Bigg\}+(r1​,r2​)∈E∑​Cexchange(t)Er1​,r2​​(t)+cslack[λrshed​(t)+λrdump​(t)]+crDR​DRr​(t)}

### **5. Computational Implementation**

The linear programming model is implemented using the PuLP library in Python. The code defines the decision variables, sets up the constraints based on the equations described above, and solves the optimization problem using a suitable solver (e.g., CBC). Following optimization, the model outputs key results—including dispatch schedules, storage operations, interregional flows, and DR activations—which can then be analyzed and visualized to assess the economic and operational impacts of coordinated regional flexibility.

This comprehensive framework bridges the gap between localized flexibility and national-scale planning, providing insights into how intra-national coordination can optimize system performance, reduce adjustment costs, and enhance the integration of renewable energy resources.