EX 10

$X_1 \sim \mathcal{N}(165, 6^2)$

$X_2 \sim \mathcal{N}(175; 11^2)$

1/ $P(153 < X_1 < 177)$

$= P\left(\frac{153-165}{6} < \frac{X_1 - 165}{6} < \frac{177-165}{6}\right)$

$= \phi(2) - \phi(-2)$

$= 2\phi(2) - 1 \simeq 0{,}954$

2) $P(X_2 < 190 / X_2 > 180) =$

$= \frac{P(180 < X_2 < 190)}{P(X_2 > 180)}$

$\cdot\ P(180 < X_2 < 190) = P\left(\frac{180-175}{11} < \frac{190-175}{11}\right)$

$= \phi\left(\frac{15}{11}\right) - \phi\left(\frac{5}{11}\right)$

$\ 1{,}36 \qquad 0{,}45$

$\simeq 0{,}239$

$P(X_2 > 180) = 1 - \phi\left(\frac{5}{11}\right) \simeq 0{,}326$

$P(X_2 < 190 / X_2 > 180) = \frac{0{,}239}{0{,}326} = 0{,}733$

3) a) A = la personne choisie mesure plus de 170 cm.

$P(A)$?

$$P(A) = P(A \mid F)\,P(F) + P(A \mid H)\,P(H)$$
$$= P(X_1 > 170)\,P(F) + P(X_2 > 170)\,P(H)$$
$$= \left(1 - \Phi\left(\tfrac{5}{6}\right)\right)(0{,}52) + \left(1 - \Phi\left(\tfrac{5}{11}\right)\right) \times (0{,}48)$$

b) $P(F / A) = \dfrac{P(A \mid F)\,P(F)}{P(A)}$ (Bayes)

Ex 11

$X \sim \mathcal{N}(m, \sigma^2)$

$P(X > 200) = 0{,}2 \quad (1)$

$P(X > 300) = 0{,}01 \quad (2)$

$(1) : P\left(\frac{X-m}{\sigma} > \frac{200-m}{\sigma}\right) = 0{,}2 \Leftrightarrow 1 - \phi\left(\frac{200-m}{\sigma}\right) = 0{,}2$

$(2) : P\left(\frac{X-m}{\sigma} > \frac{300-m}{\sigma}\right) = 0{,}01 \Leftrightarrow 1 - \phi\left(\frac{300-m}{\sigma}\right) = 0{,}01$

$$\Rightarrow \begin{cases} \phi\left(\frac{200-m}{\sigma}\right) = 0{,}8 \\ \phi\left(\frac{300-m}{\sigma}\right) = 0{,}99 \end{cases} \Leftrightarrow \begin{cases} 200 - m = \sigma\, \phi^{-1}(0{,}8) & (1) \\ 300 - m = \sigma\, \phi^{-1}(0{,}99) & (2) \end{cases}$$

$(2) - (1) : \sigma = \frac{100}{\phi^{-1}(0{,}99) - \phi^{-1}(0{,}8)} \simeq 67{,}36$

2) $P(100 < X < 147)$

3) $Y \sim \mathcal{B}(10 ; P(X > 200) = p)$

4) $\bar{X}_n = \frac{1}{n} \sum_{i=1}^{n} X_i \quad , X_i \sim \mathcal{N}(m, \sigma^2)$

$E(\bar{X}_n) = \frac{1}{n} \sum_{i=1}^{n} E(X_i) = m$

$V(\bar{X}_n) = 1/n^2 \sum_{i=1}^{n} V(X_i) = \frac{\sigma^2}{n}$

$\bar{X}_n$ est une combinaison linéaire des v.a indép suivant une loi normale, alors $\bar{X}_n$ est suit une loi normale.

$\bar{X}_n \sim \mathcal{N}\left(m, \left(\frac{\sigma}{\sqrt{n}}\right)^2\right)$

c) n? $P(|\bar{X}_n - m| \leq 10) \geq 0{,}99$

$P(|\bar{X}_n - m| \leq 10) \geq 0{,}99$

$\Leftrightarrow P(-10 \leq \bar{X}_n - m \leq 10) \geq 0{,}99$

$\Leftrightarrow P\left(-\frac{10\sqrt{n}}{\sigma} \leq \frac{\bar{X}_n - m}{\sigma/\sqrt{n}} \leq \frac{10\sqrt{n}}{\sigma}\right) \geq 0{,}99$

$\Leftrightarrow \phi\left(\frac{10\sqrt{n}}{\sigma}\right) - \phi\left(\frac{-10\sqrt{n}}{\sigma}\right) \geq 0{,}99$

$\Leftrightarrow 2\phi\left(\frac{10\sqrt{n}}{\sigma}\right) \geq 1{,}99$

$\Rightarrow \phi\left(\frac{10\sqrt{n}}{\sigma}\right) \geq 0{,}995$

$\Rightarrow \frac{10\sqrt{n}}{\sigma} \geq \phi^{-1}(0{,}995)$

$\Leftrightarrow n \geq \underbrace{\frac{\sigma^2}{100}\left(\phi^{-1}(0{,}995)\right)^2}_{300\,?}$

3) **Ex 12 :**

$X \sim N(100\,;(16)^2)$

1) $P(92 < X < 112)$

2) $N \sim B(n, p)$; $p = P(X > 116)$.

b) $P(N=0)$; $P(N \geq 1) = 1 - P(N=0)$

c) $n = 100$.

$P(N > 20)$?

$N \sim B(100, \overset{\to 0{,}15}{p})$ $\quad p = P(X > 116)$ $\quad q = 1-p$ $\quad$ T.L.C

$$P(X > 116) = P\left(\frac{X-100}{116} > \frac{116-100}{16}\right) = 1 - \phi(1) \simeq 0{,}15 \quad \to 0{,}8413$$

$$P(N > 20) = P\left(\frac{N-100}{\sqrt{npq}} > \frac{20-100}{\sqrt{100pq}}\right) \simeq 1 - \phi\left(\frac{20-100}{\sqrt{100pq}}\right)$$

$N \sim \mathcal{B}(100\,;\,0{,}15)$

$E(N) = 15\,;\; V(N) = 15 \times 0{,}85 = 12{,}75 = (3{,}57)^2$

$P(N > 20) = P\left(\frac{N-15}{3{,}57} > \underbrace{\frac{20-15}{3{,}57}}_{1{,}4}\right)$

T.C.L

$\simeq 1 - \phi(1{,}4) \simeq 0{,}081$

$\Leftrightarrow \phi\left(\frac{10\sqrt{n}}{\sigma}\right) - \phi\left(\frac{-10\sqrt{n}}{\sigma}\right) \geq 0{,}99$

$\Leftrightarrow 2\phi\left(\frac{10\sqrt{n}}{\sigma}\right) \geq 1{,}99$

$\Rightarrow \phi\left(\frac{10\sqrt{n}}{\sigma}\right) \geq 0{,}995$

$\Rightarrow \frac{10\sqrt{n}}{\sigma} \geq \phi^{-1}(0{,}995)$

$\Leftrightarrow n \geq \underbrace{\frac{\sigma^2}{100}\left(\phi^{-1}(0{,}995)\right)^2}_{300\,?}$

3) $\bar{V} = \frac{1}{36} \sum_{i=1}^{36} X_i \qquad X_i \sim N(100; (16)^2)$

a) $\bar{V} \sim N\left(100, \left(\frac{16}{6}\right)^2\right)$

$P(\bar{V} > 104) = P\left(\frac{\bar{V} - 100}{16/6} > \frac{104 - 100}{16/6}\right)$

$= 1 - \phi(1,5)$,

$V(\bar{V}) = \frac{1}{(36)^2} \sum_{1}^{36} V(X_i)$

$= \frac{36 \times (16)^2}{(36)^2} =$

$= \frac{16^2}{36} = \left(\frac{16}{6}\right)^2$