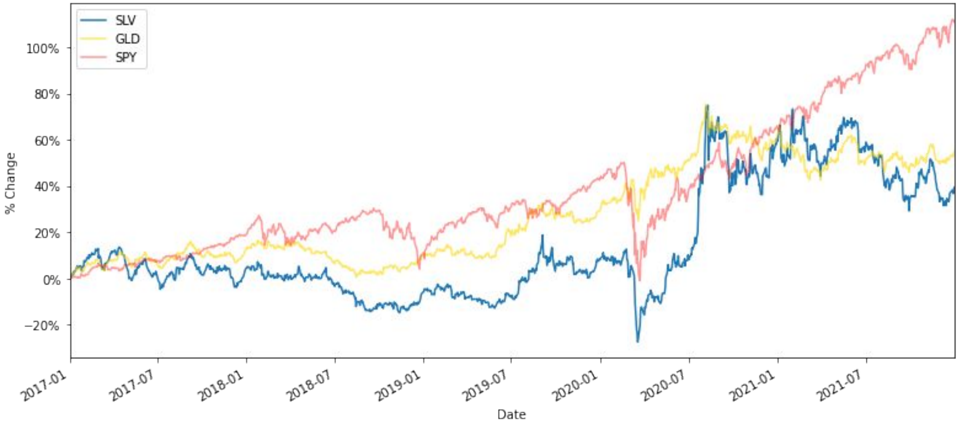
Forecasting the Price of Silver

Eric H. Wilson

Investment Outlook

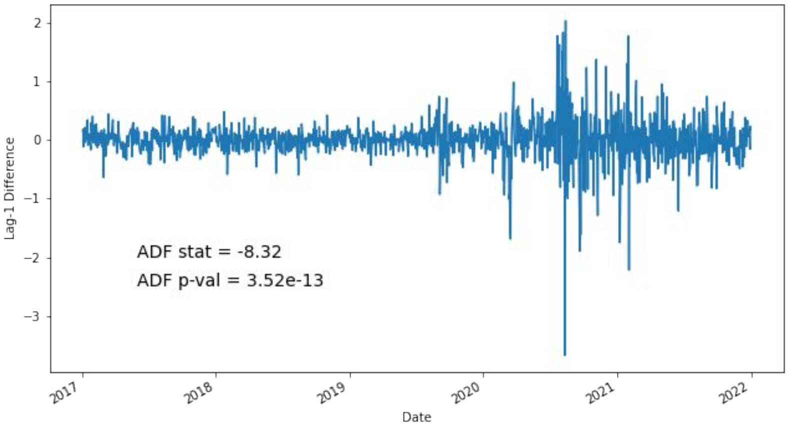
- Silver has long played a significant role in society
    - Currency
    - Precious Metal
    - Industrial Component
- Silver has also been considered a safe haven in periods of economic uncertainty
- With the emergence of cryptocurrency, does silver remain a solid investment?
- Stakeholders include private and professional investors, speculators

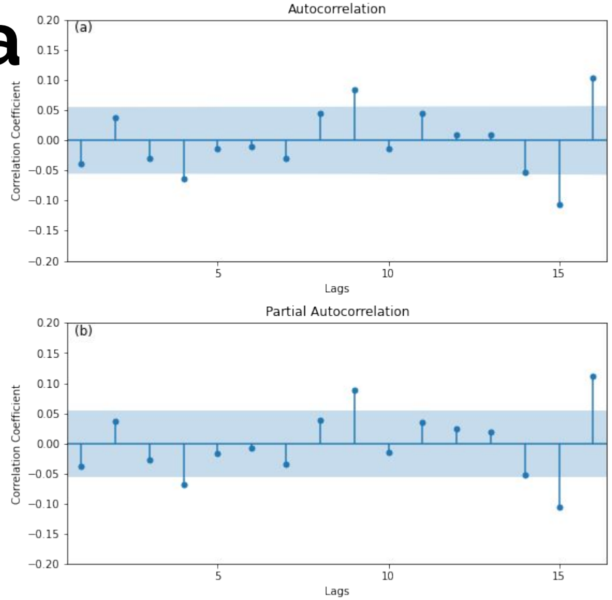
Data Overview



- Study uses exchange-traded fund SLV as a proxy for price of silver from 2017–2021
- SLV has traded within two ranges over the past five years
- Slightly lagging other precious metal gold (GLD) but showing more volatility
- Precious metals have trailed the broad market (SPY) in performance

Exploring the Data





- Lag-1 differencing produces highly stationary dataset
- Low ADF p-value indicates second-differencing unnecessary

- ACF plot show significant autocorrelation at lags 4, 9, 15, 16 - perhaps weekly periodicity
- PACF plot shows almost identical autocorrelations - mixture of AR, MA

ARIMA Modeling

SLV Modeling Procedure

- Data is chronologically split 80/20
  - Training Set: 2017–2020
  - Test Set: 2021
- Model is trained via training set
- Models are chosen by grid search based on AIC/BIC score
- Predictions are generated over the test set and models are evaluated over MAE/RMSE metrics
- Selected model is used to generate 2022 forecast

Model-Order Fitting

- Grid search of first-differenced (d=1) ARIMA models is conducted over p (AR) and q (MA) orders
- ARIMA(3,1,2) model shows lowest BIC, 2nd lowest AIC
  Log-transformation is performed to reduce heteroscedasticity
- Here, ARIMA(3,1,2), (2,1,3) show nearly identical AIC/BIC
- ARIMA(3,1,2) is selected as the chosen model

Seasonality (SARIMA)

- Autocorrelation suggests some type of seasonality
- Grid search of Seasonal ARIMA (SARIMA) models reveal SARIMA(0,1,0)(2,0,2)[3] as having the lowest AIC/BIC scores - lower than ARIMA(3,1,2)

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                                    SARIMAX Results
==========================================================================================
Dep. Variable:                                 Close   No. Observations:             1007
Model:             ARIMA(0, 1, 0)x(2, 0, [1, 2], 3)   Log Likelihood            2703.794
Date:                               Mon, 10 Jan 2022   AIC                      -5397.589
Time:                                       07:26:39   BIC                      -5373.020
Sample:                                            0   HQIC                     -5388.254
                                              - 1007
Covariance Type:                                 opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ar.S.L3       -1.0081      0.015    -69.397      0.000      -1.037      -0.980
ar.S.L6       -0.9748      0.016    -61.977      0.000      -1.006      -0.944
ma.S.L3        1.0380      0.016     64.222      0.000       1.006       1.070
ma.S.L6        0.9634      0.019     50.706      0.000       0.926       1.001
sigma2         0.0003   4.68e-06     58.217      0.000       0.000       0.000
===================================================================================
Ljung-Box (L1) (Q):                   3.15   Jarque-Bera (JB):              8267.02
Prob(Q):                              0.08   Prob(JB):                         0.00
Heteroskedasticity (H):               5.44   Skew:                            -1.04
Prob(H) (two-sided):                  0.00   Kurtosis:                        16.89
===================================================================================
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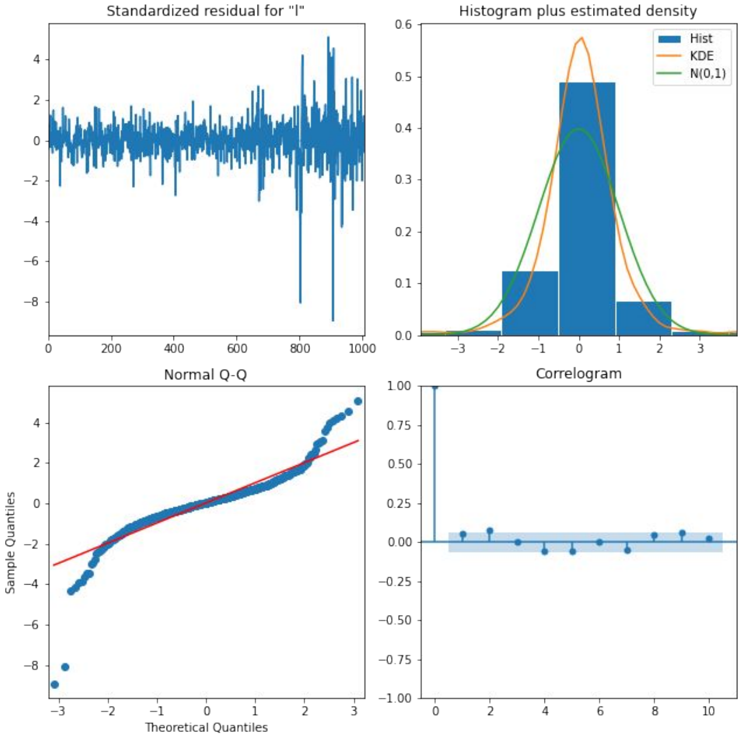
SARIMA Model

- Reduced but persistent heteroscedasticity in residual plot
- Extreme values veer from normal line in quantile–quantile plot
- Correlogram indicates no significant autocorrelation in residuals
- Plots similar to those of ARIMA(3,1,2)



Prediction & Forecasting

In-Sample Prediction

- One–step-ahead predictions of the test set are generated
  - Model is trained on training data
  - Prediction of first test data point
  - Actual value is added to training set
  - Model is retrained on new training set
  - Prediction of next data point
  - Steps 3–5 are repeated

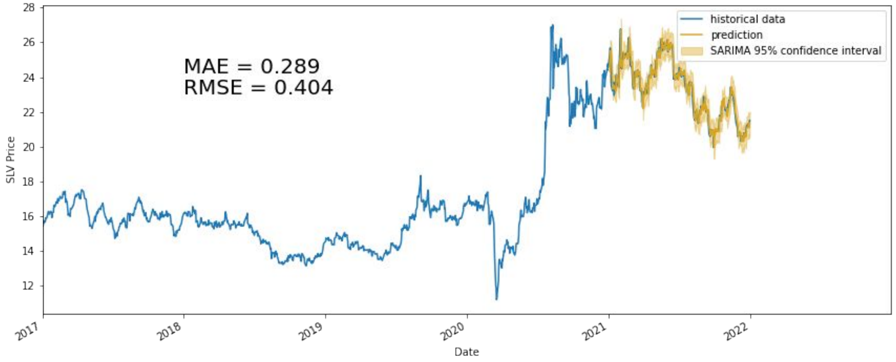
Out-Of-Sample (OOS) Forecast

- Selected model is used to generate forecast for 2022
  - Model is trained on entire dataset
  - Forecast of first OOS data point
  - Forecasted value is added to training set
  - Model is retrained on new training set
  - Forecast of next OOS data point
  - Steps 3–5 are repeated

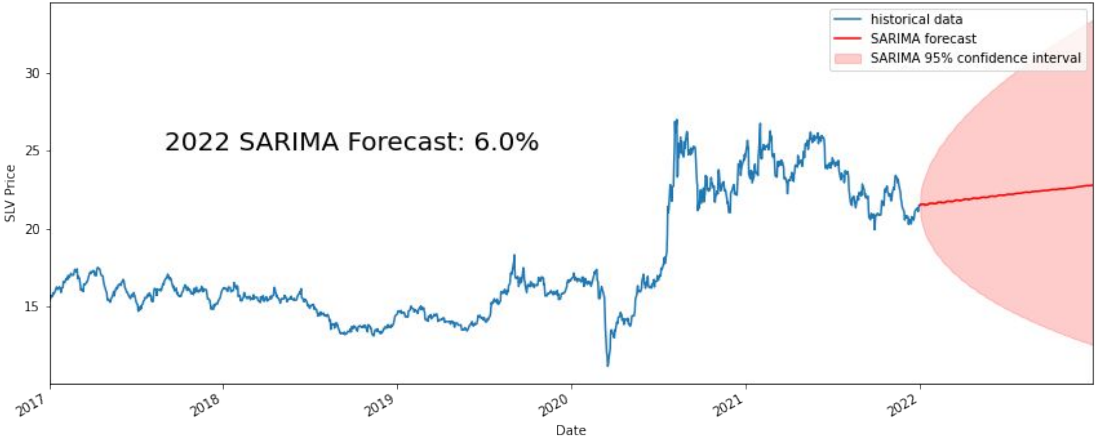
Prediction Results

- Both base and log-transformed ARIMA(3,1,2) and SARIMA(0,1,0)(2,0,2)[3] models are evaluated against baseline
- Baseline: today's price = tomorrow's
- Metrics
  - MAE - mean absolute error
  - RMSE - root–mean–square error
- Log-transformed models consistently exhibit worse metrics
- Only base SARIMA outperforms baseline but insignificantly (p=0.86)

Model	Training Set	Test Set
Base ARIMA(3,1,2)	MAE: 0.1848 RMSE: 0.3132	MAE: 0.2997 RMSE: 0.4126
Log-transformed ARIMA(3,1,2)	MAE: 0.1843 RMSE: 0.3155	MAE: 0.3000 RMSE: 0.4157
Base SARIMA(0,1,0)(2,0,2)3	MAE: 0.1854 RMSE: 0.3110	MAE: 0.2886 RMSE: 0.4041
Log-transformed SARIMA(0,1,0)(2,0,2)3	MAE: 0.1835 RMSE: 0.3162	MAE: 0.2914 RMSE: 0.4057
Baseline	MAE: 0.1830 RMSE: 0.3184	MAE: 0.2895 RMSE: 0.4054



Forecast



- Base SARIMA model is the selected model
- Generates a 2022 SLV forecast of 6.0% appreciation
- Price projected to fall within interval of (12.56, 33.42) by the end of 2022 with 95% probability

Conclusions

- SLV price is made stationary by lag-1 first-order differencing
- Significant autocorrelation evident at lags 4, 9, 15, suggesting weekly seasonality
- Best models to describe SLV price are non-seasonal ARIMA(3,1,2) and seasonal SARIMA(0,1,0)(2,0,2)[3] with lag-3 seasonality
- SLV data show significant heteroscedasticity, which is not resolved by log-transformation
- SARIMA model barely outclasses baseline model, which predicts next-day value as equal to present-day value
- Selected SARIMA models forecasts a 6.0% appreciation in price of silver for 2022
- Results may be improved by coupling selected SARIMA model with GARCH model, which models volatility