Forecasting the Price of Silver

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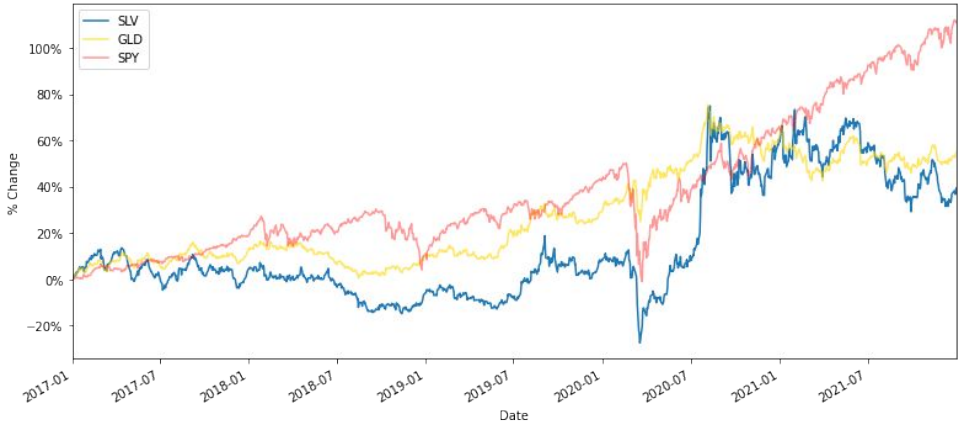
Investment Outlook

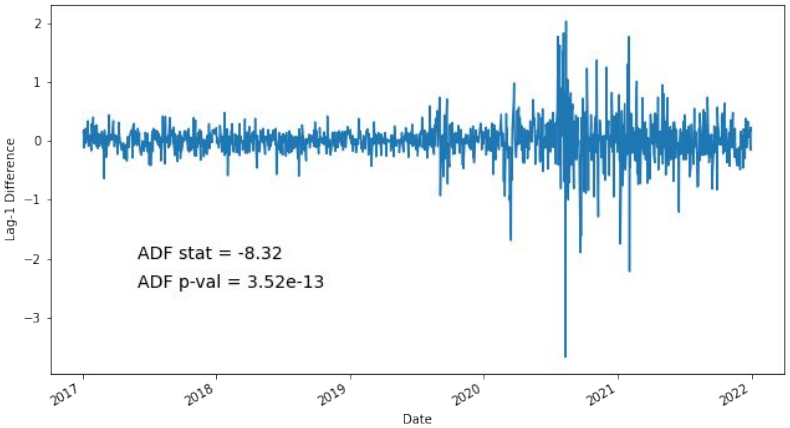
- Silver has long played a significant role in society
  - Currency
  - Precious Metal
  - Industrial Component
- Silver has also been considered a safe haven in periods of economic uncertainty
- With the emergence of cryptocurrency, does silver remain a solid investment?
- Stakeholders include private and professional investors, speculators

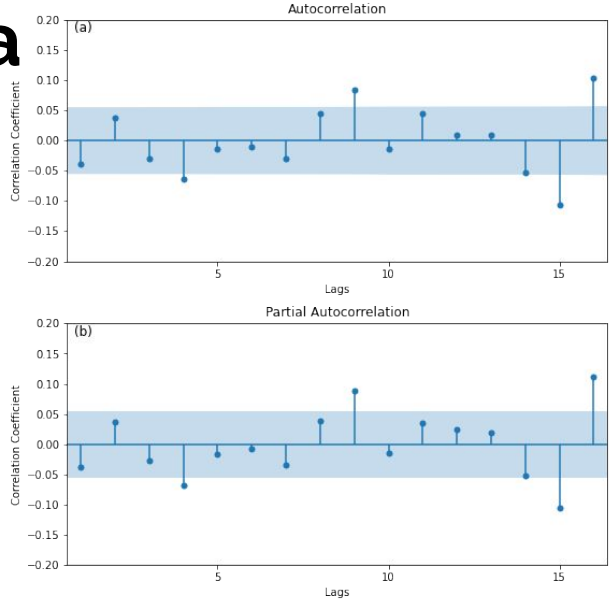
Data Overview



- Study uses exchange-traded fund SLV as a proxy for price of silver from 2017–2021
- SLV has traded within two ranges over the past five years
- Slightly lagging other precious metal gold (GLD) but showing more volatility
- Precious metals have trailed the broad market (SPY) in performance

Exploring the Data





- Lag-1 differencing produces highly stationary dataset
- Low ADF p-value indicates second-differencing unnecessary

- ACF plot show significant autocorrelation at lags 4, 9, 15, 16 - perhaps weekly periodicity
- PACF plot shows almost identical autocorrelations - mixture of AR, MA

ARIMA Modeling

SLV Modeling Procedure

- Data is chronologically split 80/20
  - Training Set: 2017–2020
  - Test Set: 2021
- Model is trained via training set
- Models are chosen based on AIC/BIC score, MAE/RMSE metrics
- Predictions are generated over the test set and models are evaluated
- Selected model is used to generate 2022 forecast

Model-Order Fitting

- Grid search of first-differenced (d=1) ARIMA models is conducted over p (AR) and q (MA) orders
- ARIMA(5,1,4) model shows lowest AIC but significantly higher BIC than ARIMA(3,1,2), 2nd lowest AIC
- Log-transformation is performed to reduce heteroscedasticity
- Here, ARIMA(3,1,2), (2,1,3) show nearly identical AIC/BIC
- ARIMA(3,1,2) is selected as the chosen model

Seasonality (SARIMA)

- Autocorrelation suggests some type of seasonality
- Grid search of Seasonal ARIMA (SARIMA) models reveal SARIMA(0,1,0)(2,0,2)3 as having the lowest AIC/BIC scores - lower than ARIMA(3,1,2)

SARIMAX Results

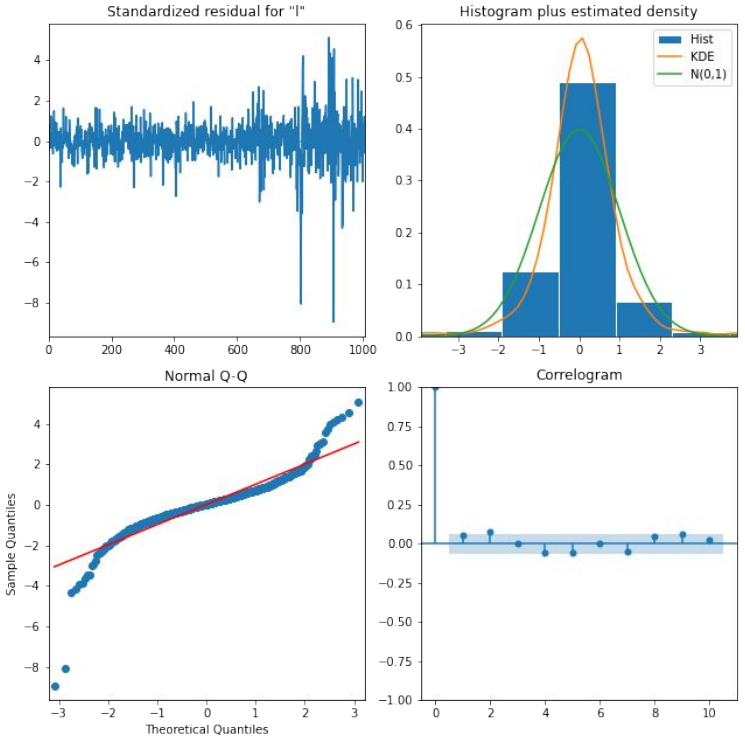
Dep. Variable:	Close	No. Observations:	1007
Model:	ARIMA(0, 1, 0)x(2, 0, [1, 2], 3)	Log Likelihood	2703.794
Date:	Mon, 10 Jan 2022	AIC	-5397.589
Time:	07:26:39	BIC	-5373.020
Sample:	0	HQIC	-5388.254
	- 1007		
Covariance Type:	opg		

	coef	std err	z	P>|z|	[0.025	0.975]
ar.S.L3	-1.0081	0.015	-69.397	0.000	-1.037	-0.980
ar.S.L6	-0.9748	0.016	-61.977	0.000	-1.006	-0.944
ma.S.L3	1.0380	0.016	64.222	0.000	1.006	1.070
ma.S.L6	0.9634	0.019	50.706	0.000	0.926	1.001
sigma2	0.0003	4.68e-06	58.217	0.000	0.000	0.000

Ljung-Box (L1) (Q):	3.15	Jarque-Bera (JB):	8267.02
Prob(Q):	0.08	Prob(JB):	0.00
Heteroskedasticity (H):	5.44	Skew:	-1.04
Prob(H) (two-sided):	0.00	Kurtosis:	16.89

SARIMA Model

- Reduced but persistent heteroscedasticity in residual plot
- Extreme values veer from normal line in quantile–quantile plot
- Correlogram indicates no significant autocorrelation in residuals
- Plots similar to those of ARIMA(3,1,2)



Prediction & Forecasting

In-Sample Prediction

- One–step-ahead predictions of the test set are generated
  - Model is trained on training data
  - Prediction of first test data point
  - Actual value is added to training set
  - Model is retrained on new training set
  - Prediction of next data point
  - Steps 3–5 are repeated

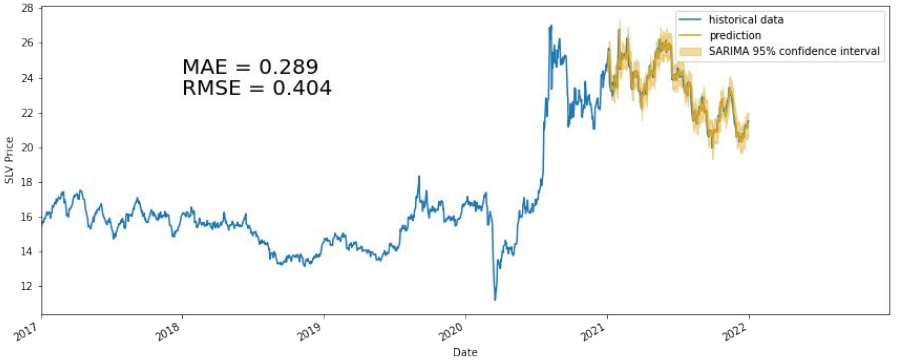
Out-Of-Sample (OOS) Forecast

- Selected model is used to generate forecast for 2022
  - Model is trained on entire dataset
  - Forecast of first OOS data point
  - Forecasted value is added to training set
  - Model is retrained on new training set
  - Forecast of next OOS data point
  - Steps 3–5 are repeated

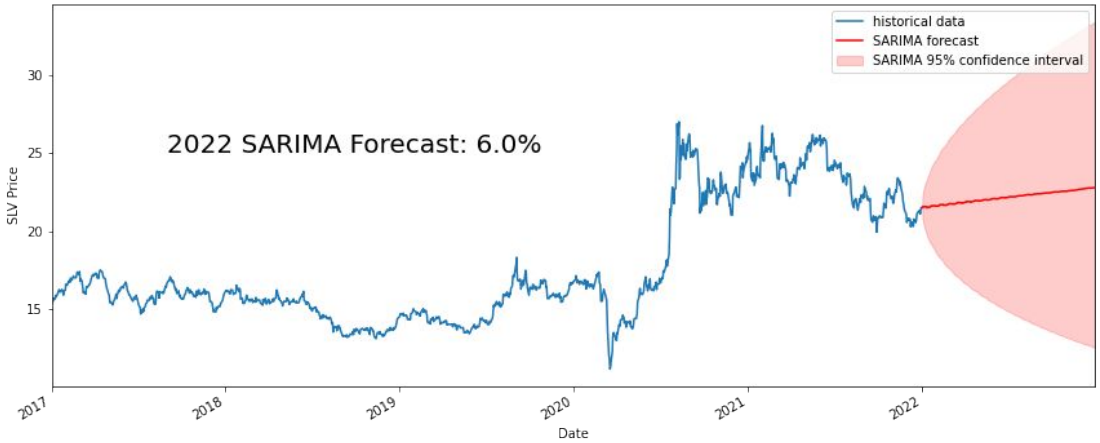
Prediction Results

- Both base and log-transformed ARIMA(3,1,2) and SARIMA(0,1,0)(2,0,2)3 models are evaluated against baseline
- Baseline: today's price = tomorrow's
- Metrics
  - MAE - mean absolute error
  - RMSE - root–mean–square error
- Log-transformed models consistently exhibit worse metrics
- Only base SARIMA outperforms baseline

Model	Training Set	Test Set
Base ARIMA(3,1,2)	MAE: 0.1848 RMSE: 0.3132	MAE: 0.2997 RMSE: 0.4126
Log-transformed ARIMA(3,1,2)	MAE: 0.1843 RMSE: 0.3155	MAE: 0.3000 RMSE: 0.4157
Base SARIMA(0,1,0)(2,0,2)3	MAE: 0.1854 RMSE: 0.3110	MAE: 0.2886 RMSE: 0.4041
Log-transformed SARIMA(0,1,0)(2,0,2)3	MAE: 0.1835 RMSE: 0.3162	MAE: 0.2914 RMSE: 0.4057
Baseline	MAE: 0.1830 RMSE: 0.3184	MAE: 0.2895 RMSE: 0.4054



Forecast



- Base SARIMA model is the selected model
- Generates a 2022 SLV forecast of 6.0% appreciation
- Price projected to fall within interval of (12.56, 33.42) by the end of 2022 with 95% probability

Conclusions

- SLV price is made stationary by lag-1 first-order differencing
- Significant autocorrelation evident at lags 4, 9, 15, suggesting weekly seasonality
- Best models to describe SLV price are non-seasonal ARIMA(3,1,2) and seasonal SARIMA(0,1,0)(2,0,2)3 with lag-3 seasonality
- SLV data show significant heteroscedasticity, which is not resolved by log-transformation
- SARIMA model barely outclasses baseline model, which predicts next-day value as equal to present-day value
- Selected SARIMA models forecasts a 6.0% appreciation in price of silver for 2022
- Results may be improved by coupling selected SARIMA model with GARCH model, which models volatility