

**Cyber-Physical Systems (CSC.T431), 2020 Assignment (1)**

Upload your answers (PDF) via OCW-i by Nov. 8, 2020.

1. We want to design a reactive component with three Boolean variables  $x$ ,  $y$ , and  $reset$  and a Boolean output variable  $z$ . The desired behavior is the following. The component wait until it has encountered a round in which the input variable  $x$  is 1 and a round in which the input variable  $y$  is 1, and as soon as both of these have been encountered, it sets the output  $z$  to 1. By default the output  $z$  is 0. For instance, if  $x$  is 1 in rounds 2, 3, 7, 12,  $y$  is 1 in rounds 5, 6, 10, and  $reset$  is 1 in round 9, then  $z$  should be 1 in rounds 5 and 12. Design a synchronous reactive component that captures this behavior. Use the extended-state machine notation.

2. Consider the synchronous reactive component shown in Figure 1. List all the possible reactions<sup>1</sup> of the component. Does the output  $y$  await  $x$ ? Does the output  $z$  await  $x$ ?

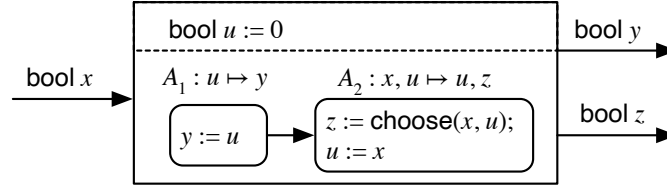


Figure 1: A Synchronous Reactive Component

3. Consider the component DoubleSplitDelay defined as

$$(\text{SplitDelay}[out \mapsto temp] \parallel \text{SplitDelay}[in \mapsto temp]) \setminus temp.$$

This component is similar to the component DoubleDelay except we use instances of the component SplitDelay instead of Delay. Show the “compiled” version of DoubleSplitDelay, that is, list its state, input, output, and local variables, and precedence constraints<sup>2</sup>. What are the await dependencies among output and input variables for DoubleSplitDelay?

4. The composed system RailroadSystem1 has four state variables,  $east$ ,  $west$ , each of which can take two values, and  $mode_W$  and  $mode_E$ , each of which can take three values. Thus, RailroadSystem1 has 36 states. How many of these 36 states are reachable?

<sup>1</sup>Write one reaction like  $u \xrightarrow{x/yz} u'$ , by filling variables with Boolean values (0 or 1).

<sup>2</sup>Show your answer as a diagram with a task graph, like Figure 1.

5. Given two natural numbers  $m$  and  $n$ , consider the program Mult that multiplies the input numbers using two variables  $x$  and  $y$ , of type nat, as shown in Figure 2. Describe the transition system  $\text{Mult}(m, n)$  that capture the behavior of this program on input numbers  $m$  and  $n$ , that is, describe the states, initial states, and transitions. Argue that when the value of the variable  $x$  is 0, the value of the variable  $y$  must equal the product of the input numbers  $m$  and  $n$ , that is, the following property is an invariant of this transition system:

$$(mode = \text{stop}) \rightarrow (y = m \times n)$$

$$(x > 0) \rightarrow \{x := x - 1; y := y + n\}$$

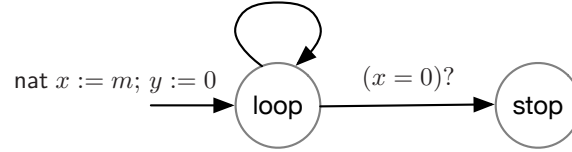


Figure 2: Multiplication

6. Describe an asynchronous process AsyncAnd that models an asynchronous *And* gate with two Boolean input channels  $in_1$  and  $in_2$  and a Boolean output channel  $out$ . The process can be described as an extended-state machine with three modes as in the case of the process AsyncNot and with three Boolean state variables.