Cyber-Physical Systems (CSC.T431), 2020 Assignment (2)
Upload your answers (PDF) via OCW-i by Dec. 9, 2020.

1. Consider the transition system corresponding to Peterson's mutual exclusion protocol. The set of state variables for this system contains the variables $P_1.mode$, $P_2.mode$, *turn*, $flag_1$, and $flag_2$. Draw the reachable subgraph of this transition system. How many states are reachable?

2. In Peterson's mutual exclusion protocol, the process P_1, when it wants to enter the critical section, first sets the register $flag_1$ to 1 and then sets the register *turn* to 1. Suppose we switch the order in which these two steps are executed. That is, consider a modified version of Peterson's protocol in which the process P_1, when it wants to enter the critical section, first sets the register *turn* to 1 and then sets the register $flag_1$ to 1; symmetrically, the process P_2, when it wants to enter the critical section, first sets the register *turn* to 2 and then sets the register $flag_2$ to 1. Everything else stays the same. Does the modified protocol satisfy the requirement of mutual exclusion? If yes, give a brief justification; if no, show a counter example.

3. Consider the model of the simple pendulum. Suppose the external torque u is set to 0 and the rod length l is 1 meter. Analyze the motion of the pendulum if the initial angular displacement is set to $-15\pi/16$ radians, that is, slightly displaced from the vertically upward position. Plot the resulting response signal $\bar{\varphi}$ using your favorite tools or libraries (e.g., MATLAB, Julia, Python with Matplotlib, etc.).

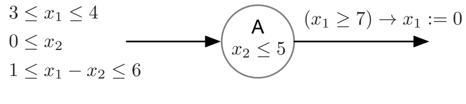
4. Consider a two-dimensional dynamical system whose dynamics is given by:

$$\dot{s}_1 = 3s_1 + 4s_2,$$
$$\dot{s}_2 = 2s_1 + s_2.$$

Find the equilibria of this system. For each equilibrium, analyze if the equilibrium is (a) asymptotically stable, (b) stable but not asymptotically stable, or (c) unstable.

5. For the Fischer's timing-based mutual exclusion protocol, consider the starvation-freedom requirement "if a process P enters the mode Test, then it will eventually enter the mode Crit." Does the system satisfy this requirement? If yes, give a brief justification; if no, show a counter example.

6. Suppose a timed automaton with two clock variables x_1 and x_2. Suppose that the constraints on the clock variables $3 \leq x_1 \leq 4$, $x_2 \geq 0$, and $1 \leq x_1 - x_2 \leq 6$ hold before entering a mode A.



(a) Compute the canonical DBM corresponding to the given constraints.

(b) The clock invariant of A is $x_2 \leq 5$. Compute the canonical DBM that captures the set of clock values that can be reached as the process waits in A.

(c) Consider the mode-switch out of A shown above. Compute the canonical DBM that captures the set of clock values that are possible after taking this transition.