Machine Intelligence 2

Exercise 06

Density Transformations & ICA

Group Members:
Xugang Zhou
Fangzhou Yang

Tutor:
Timm Lochmann

June 5, 2013

1 6.1 Density Transformations

Since

$$f(x)dx = f(x(u)) \mid det\frac{\partial x(u)}{\partial u} \mid du$$

$$u = u(x) = e^{-x}$$

$$P_x(x) = f(x) = e^{-x}$$

$$P_u(u) = f(x(u)) \mid det\frac{\partial x(u)}{\partial u} \mid$$

Thereforce

$$x = -\ln u$$

$$P_u(u) = x * \mid -\frac{1}{x} \mid = 1$$

2 6.2 Random Number Generation

Since

$$F(x) = \int_{-\infty}^{x} p(x)dx = \int_{-infty}^{x} \frac{1}{2b} e^{-\frac{|x-u|}{b}} dx$$

We get

$$z = F(x) = \begin{cases} 1/2e^{-\frac{u-x}{b}} & x \le u \\ \\ 1 - \frac{1}{2}e^{-\frac{x-u}{b}} & x > u \end{cases}$$

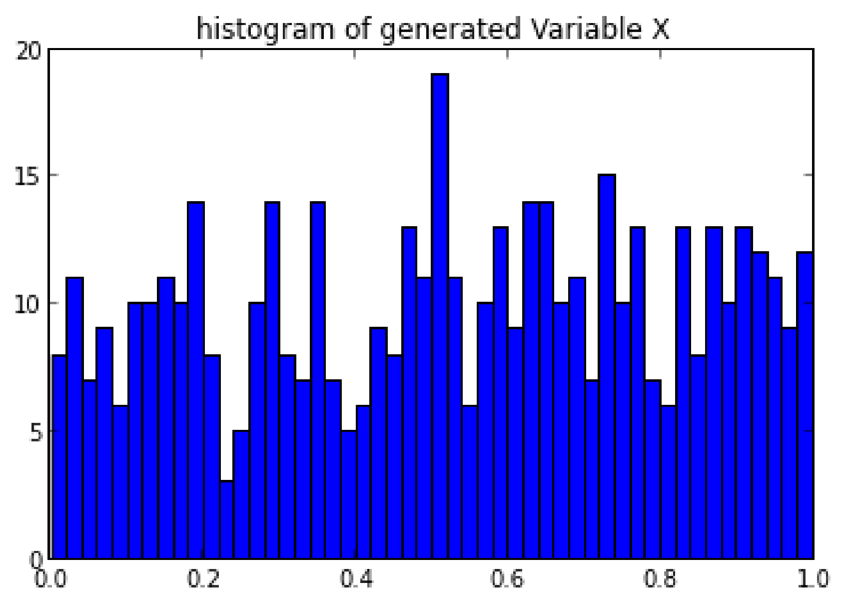
Thereforce

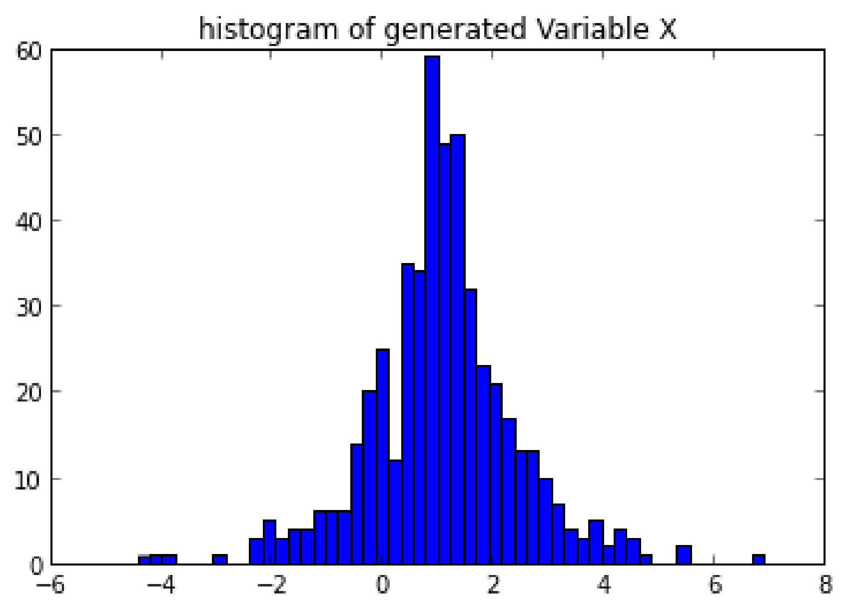
$$x = F^{-1}(z) = \begin{cases} u + b\ln(2z) & 0 \le z \le \frac{1}{2} \\ \\ u - b\ln(2-2z) & \frac{1}{2} < z \le 1 \end{cases}$$

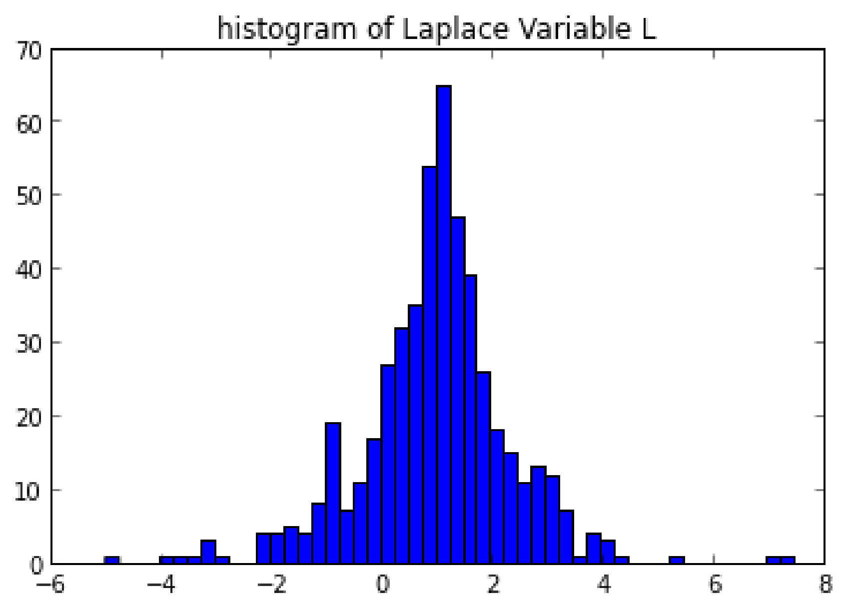
the random variable $z = F(x)$ is uniformly distributed on the interval [0,1]

```
import matplotlib.pyplot as plt
#data generation with the formula above
u = 1
b = 1
size = 500
vectorX = [0 for i in range(size)]
z = random.uniform(0,1,size)
for i in range(500):
    if(z[i] > 0.5):
        vectorX[i] = u - b*math.log(2-2*z[i])
    else:
        vectorX[i] = u + b*math.log(2*z[i])

plt.clf()
plt.hist(z, bins=50, color='blue')
plt.title('histogram of generated Variable Z')
```

```
plt.show()

plt.clf()
plt.hist(vectorX, bins=50, color='blue')
plt.title('histogram of generated Variable X')
plt.show()


vectorL = random.laplace(1,1,size)
plt.hist(vectorL, bins=50, color='blue')
plt.title('histogram of Laplace Variable L')
plt.show()
```





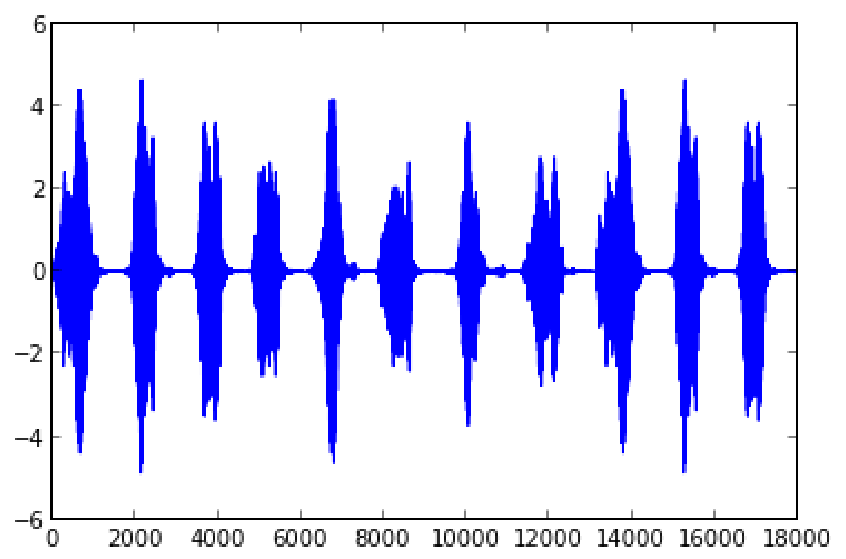


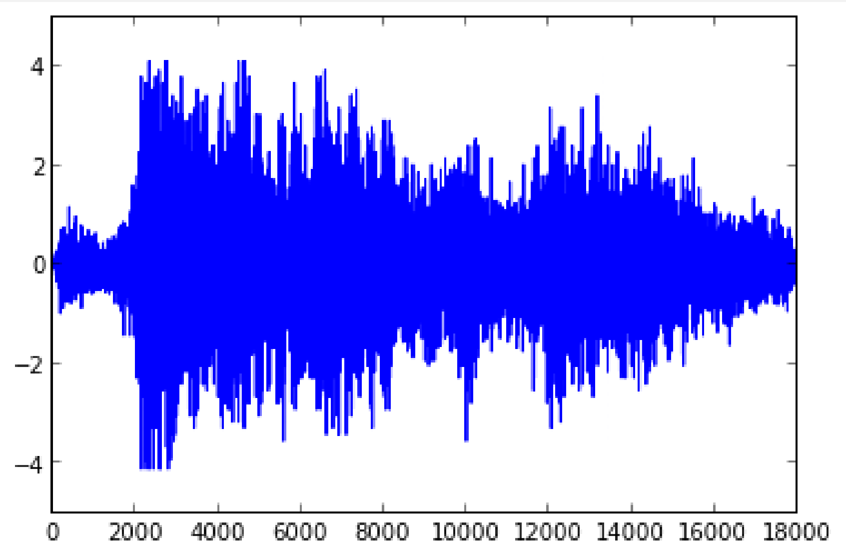
3 6.3 ICA

3.1 Initialization

```
from numpy import *
import random

#read data from pca2.csv
data01 = loadtxt('sounds/sound1.dat', unpack = True )
data02 = loadtxt('sounds/sound2.dat', unpack = True )
source = matrix([data01,data02])
length = source.shape[1]

#generate .wav files
import scipy.io.wavfile
def plotVoice(dataMatrix):
    length = dataMatrix.shape[1]
    fig1 = plt.figure(1)
    fig2 = plt.figure(2)
    ax1 = fig1.add_subplot(111)
    ax2 = fig2.add_subplot(111)
    ax1.plot(matrix.getA1(dataMatrix[0,:]))
    ax2.plot(matrix.getA1(dataMatrix[1,:]))
    plt.show()

def fileWriter(dataMatrix, file1, file2):
    normsig1 = asarray((2**16)*matrix.getA1(dataMatrix[0,:])/(max(matrix.getA1
        (dataMatrix[0,:]))-min(matrix.getA1(dataMatrix[0,:]))),int16) ##
        normalize before writing
```

```
    normsig2 = asarray((2**16)*matrix.getA1(dataMatrix[1,:])/(max(matrix.getA1
        (dataMatrix[1,:]))-min(matrix.getA1(dataMatrix[1,:]))),int16) ##
        normalize before writing
    scipy.io.wavfile.write(file1,8192,normsig1)
    scipy.io.wavfile.write(file2,8129,normsig2)

plotVoice(source)
fileWriter(source,'source1.wav','source2.wav')
```

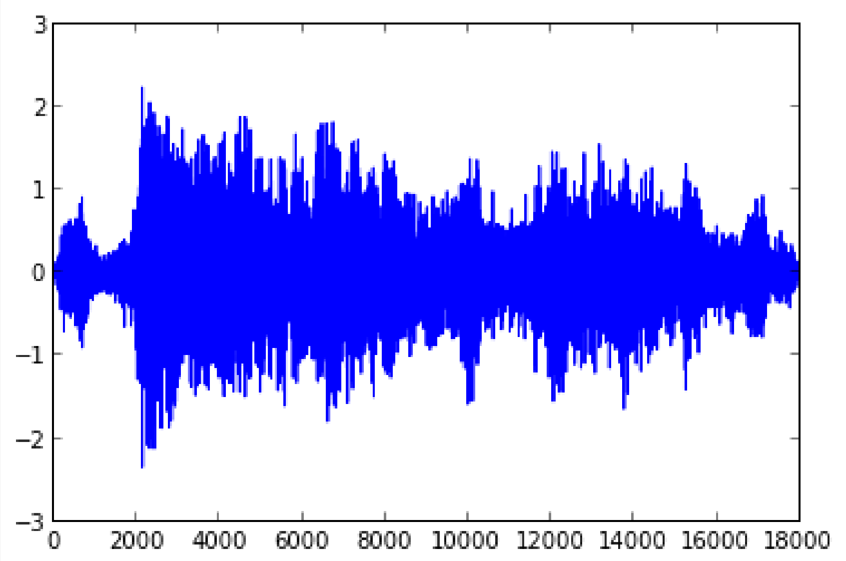


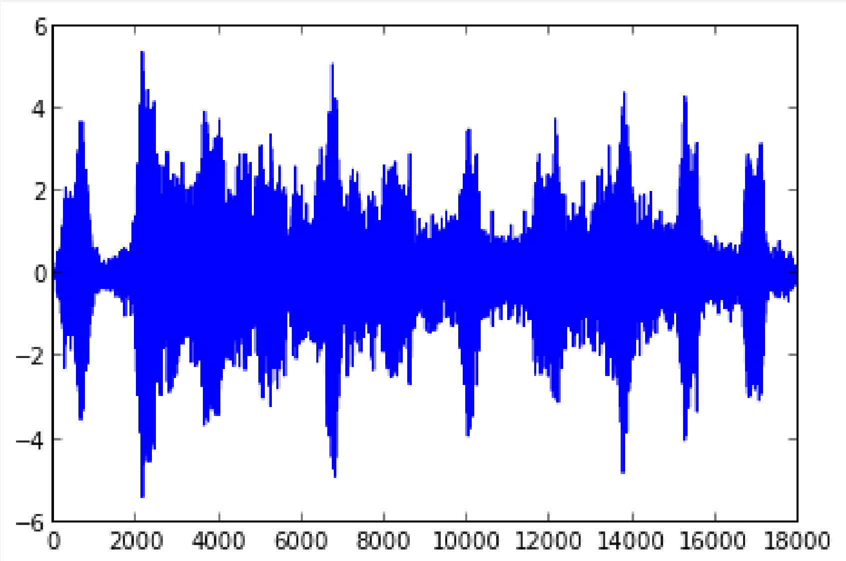



```
#create a random mixing matrix A and mix the sources: x = As
random.seed(100)
A = matrix([[random.random(),random.random()],[random.random(),random.random()
    ]])
print "Matrix A is:"
print A
mix = A * source
```

```
plotVoice(mix)
fileWriter(mix,'mix1.wav','mix2.wav')
```

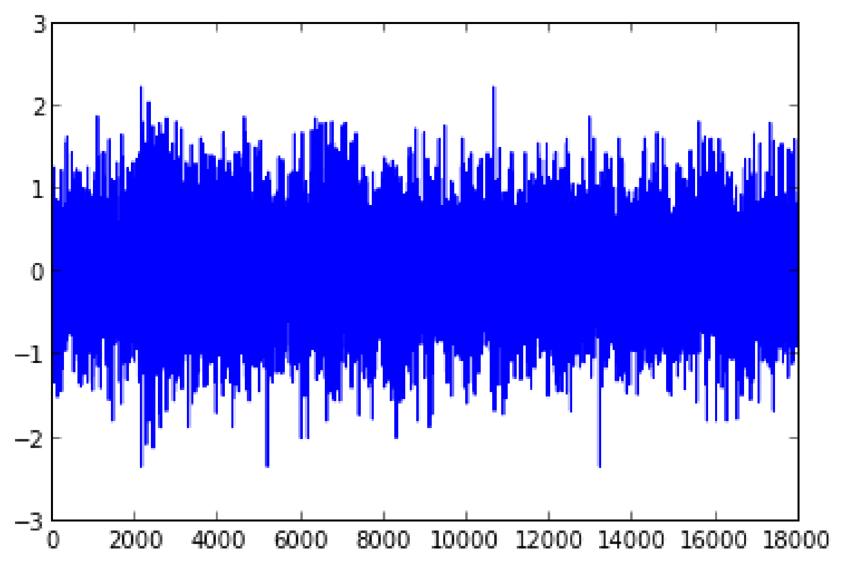
```
Matrix A is:
[[ 0.14566926  0.454927   ]
 [ 0.77078381  0.70551323]]
```

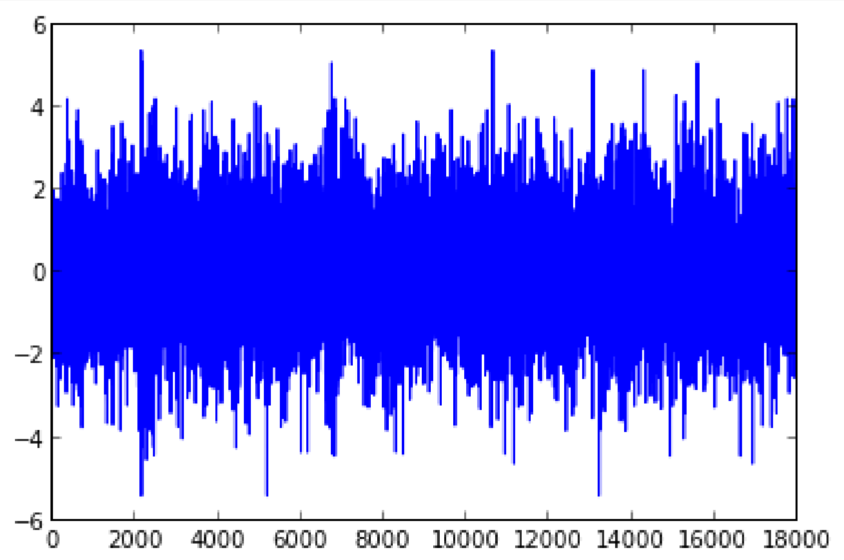





```
#permute the columns of N x p data matrix mix randomly
tmpCol = matrix([[0],[0]])
for i in range (length):
    colNo = random.randint(0, length-1)
    tmpCol = mix[:,colNo]
    mix[:,colNo] = mix[:,i]
    mix[:,i] = tmpCol

plotVoice(mix)
```

```
fileWriter(mix,'mix_per1.wav','mix_per2.wav')
```






```
#Calculate the correlations between the sources and the mixtures
correlation = matrix([[0.],[0.]])
correlation[0,0] = corrcoef([source[0,i] for i in range (length)] , [mix[0,j]
    for j in range (length)])[0,1]
correlation[1,0] = corrcoef([source[1,i] for i in range (length)] , [mix[1,j]
    for j in range (length)])[0,1]
print "Correlation between S and X:"
print correlation
```

```
Correlation between S and X:
[[ 0.11071182]
 [ 0.24131383]]
```

```
#center the data to zero man
X = matrix([data01,data02])
for i in range (length):
    X[:,i] = mix[:,i] - mix.mean(1)

#initialize the unmixing matrix W with random values
def init_W():
    random.seed(200)
    W = matrix([[random.random(), random.random()],[random.random(),random.
       random()]])
    return W
```

3.2 Optimization

$$f(x) = \frac{1}{1+e^{-x}}$$

$$f'(x) = f(x) * (1 - f(x))$$

$$f''(x) = f'(x) - 2f'(x)f(x)$$

```
#function f(x)
def f(x):
    return 1/(1+ math.exp(-x))

#function f''(x)/f'(x)
def fi(x):
    return 1 - 2*f(x)

#nomalization for matrix
def normalMatrix(m):
    for i in range(m.shape[0]):
        tmp = 0.
        for j in range(m.shape[1]):
            tmp += m[i,j] ** 2
        tmp = math.sqrt(tmp)
        for j in range(m.shape[1]):
            m[i,j] /= tmp
```

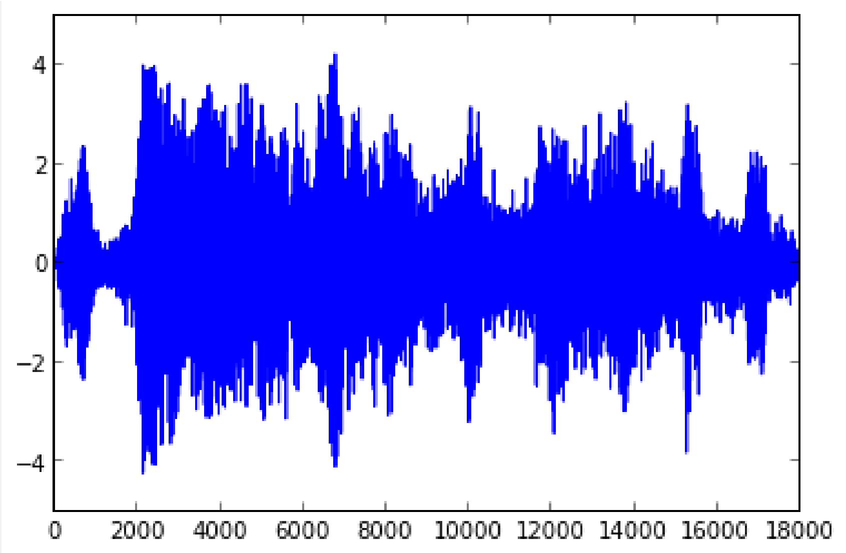
3.2.1 a.) Gradient Ascent

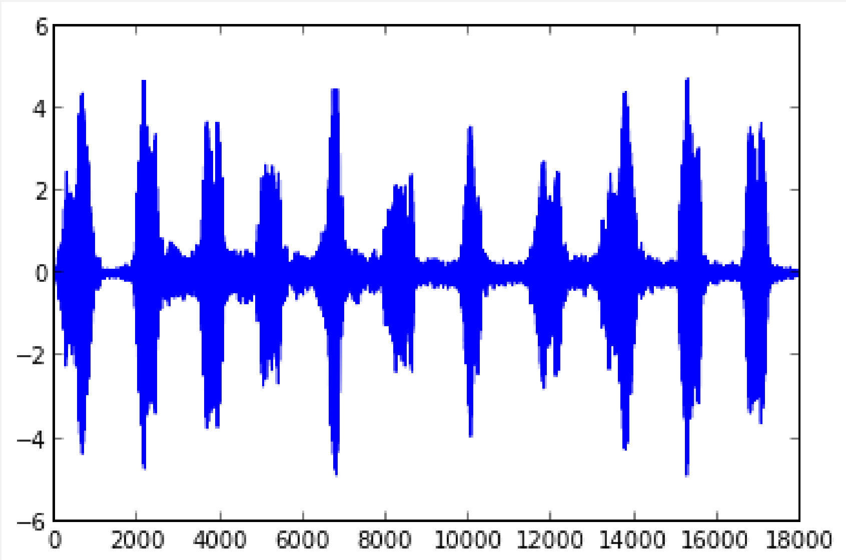
```
W = init_W()
print "Init-Matrix W: "
print W
iteration = length * 1
rate_init =0.5
```

```
delta_W = matrix([[0.,0.],[0.,0.]])
plot_DW = []
for t in range(1 , iteration):
    alpha = t % length
    rate = rate_init / t
    for i in range(2):
        sum_WikXk = 0.
        for k in range(2):
            sum_WikXk += W[i,k] * X[k,alpha]
        for j in range(2):
            inverse = W.getI()
            delta_W[i,j] = rate * ( inverse[j,i] + fi(sum_WikXk) * X[j,alpha] )
    W = W + delta_W
    if(t % 1000 == 0):
        tmp = delta_W[0,0]**2 + delta_W[1,1]**2 + delta_W[0,1]**2 + delta_W
            [1,0]**2
        plot_DW.append(tmp)
    #print W
print "matrix W is:"
print W
print "after normalization:"
normalMatrix(W)
print W
```

```
Init-Matrix W:
[[ 0.0456093   0.20344697]
 [ 0.70912398  0.1428485 ]]
matrix W is:

[[-1.23568822  2.24189833]
 [ 2.88958884 -0.50818057]]
after normalization:
[[-0.48271158  0.87577939]
 [ 0.98488529 -0.17320788]]
```

```
#recovery
ds = W * source

plotVoice(ds)
fileWriter(ds, 'recovery1.wav', 'recovery2.wav')
```



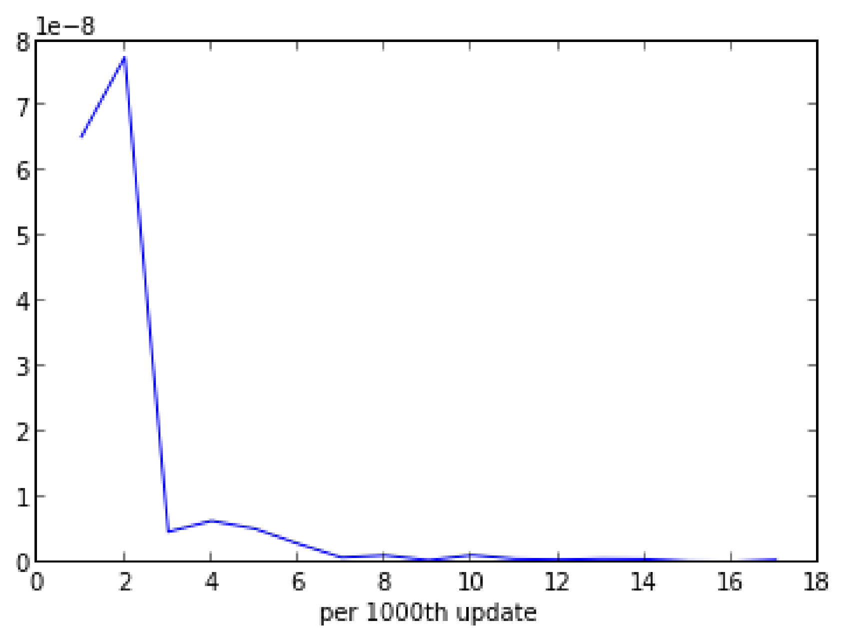



```
#Calculate the correlations between the true sources and the estimations
correlation = matrix([[0.],[0.]])
correlation[0,0] = corrcoef([source[0,i] for i in range (length)] , [ds[1,j]
    for j in range (length)])[0,1]
correlation[1,0] = corrcoef([source[1,i] for i in range (length)] , [ds[0,j]
    for j in range (length)])[0,1]
print "Correlation between the true sources and the estimations:"
print correlation
```

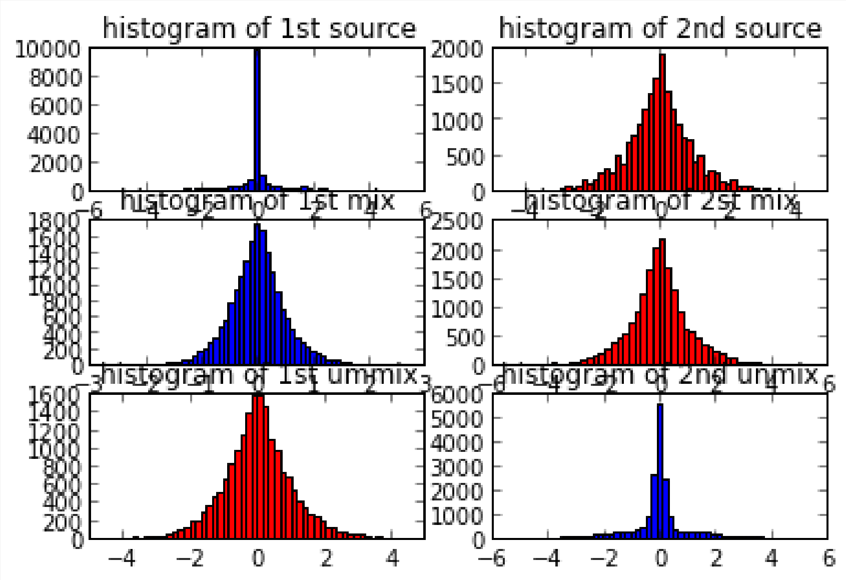
```
Correlation between S and X:
[[ 0.98486963]
 [ 0.87570078]]
```

```
#plot delta_W
fig = plt.figure()
ax = fig.add_subplot(111)
```

```
ax.set_xlabel('per 1000th update')
ax.plot([i+1 for i in range(len(plot_DW))],plot_DW)
plt.show()
```




```
#Plot the density of the mixed, unmixed, and true signals.
ax=plt.subplot(321)
ax.hist(matrix.getA1(source[0,:]), bins=50, color='blue')
ax.set_title('histogram of 1st source')

ax=plt.subplot(322)
ax.hist(matrix.getA1(source[1,:]), bins=50, color='red')
ax.set_title('histogram of 2nd source')

ax=plt.subplot(323)
ax.hist(matrix.getA1(mix[0,:]), bins=50, color='blue')
ax.set_title('histogram of 1st mix')

ax=plt.subplot(324)
ax.hist(matrix.getA1(mix[1,:]), bins=50, color='red')
ax.set_title('histogram of 2st mix')

ax=plt.subplot(325)
ax.hist(matrix.getA1(ds[0,:]), bins=50, color='red')
ax.set_title('histogram of 1st ummix')

ax=plt.subplot(326)
ax.hist(matrix.getA1(ds[1,:]), bins=50, color='blue')
ax.set_title('histogram of 2nd unmix')

plt.show()
```



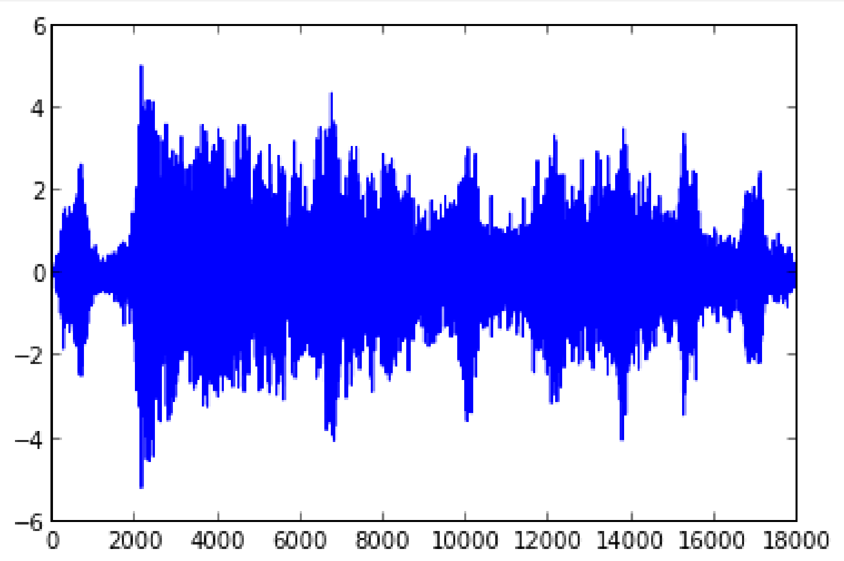
3.2.2 b.) Natural Gradient

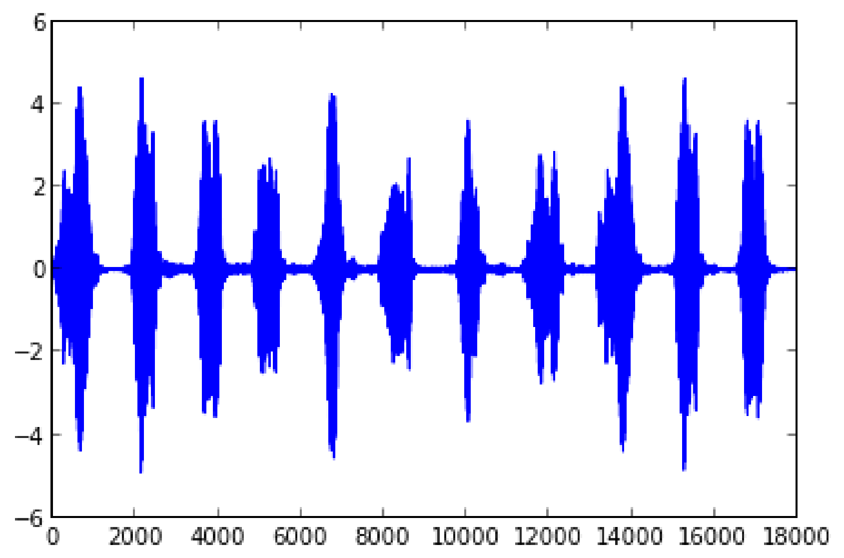
```
W = init_W()
print "Init-Matrix W: "
print W
iteration = length * 1
rate_init =0.5

delta_W = matrix([[0.,0.],[0.,0.]])
plot_DW_natural = []
for t in range(1 , iteration):
    alpha = t % length
    rate = rate_init / t
    for i in range(2):
        sum_WikXk = 0.
        for k in range(2):
            sum_WikXk += W[i,k] * X[k,alpha]
        for j in range(2):
            sum_Wlj = 0
            for l in range(2):
                if (l == i):
                    continue
                sum_WlkXk = 0
                for k in range (2):
                    sum_WlkXk += W[l,k] * X[k,alpha]
                sum_Wlj += sum_WlkXk
        delta_W[i,j] = rate * fi(sum_WikXk) * sum_Wlj
    W = W + delta_W
    if(t % 1000 == 0):
        tmp = delta_W[0,0]**2 + delta_W[1,1]**2 + delta_W[0,1]**2 + delta_W
            [1,0]**2
```

```
        plot_DW_natural.append(tmp)
    #print W
print "matrix W is:"
print W
print "after normalization:"
normalMatrix(W)
print W
```

```
Init-Matrix W:
[[ 0.0456093   0.20344697]
 [ 0.70912398  0.1428485 ]]
matrix W is:
[[ 0.0456093   0.08026234]
 [ 0.70912398  0.02462701]]
after normalization:
[[ 0.4940561   0.86943003]
 [ 0.9993975   0.03470785]]
```

```
#recovery
ds_natrual = W * source

plotVoice(ds_natrual)
fileWriter(ds_natrual, 'recovery_natual1.wav', 'recovery_natural2.wav')
```

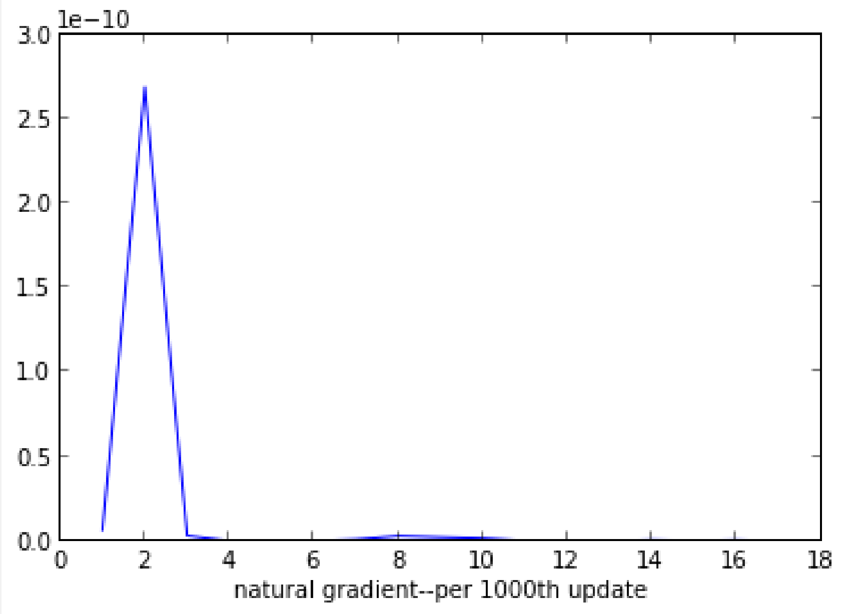




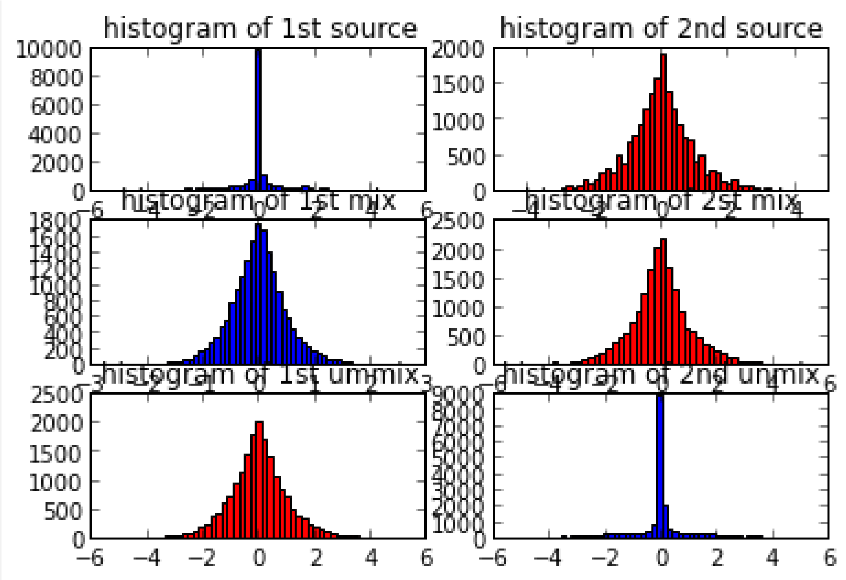

```
#Calculate the correlations between the true sources and the estimations
correlation_natural = matrix([[0.],[0.]])
correlation_natural[0,0] = corrcoef([source[0,i] for i in range (length)] , [
    ds_natrual[1,j] for j in range (length)])[0,1]
correlation_natural[1,0] = corrcoef([source[1,i] for i in range (length)] , [
    ds_natrual[0,j] for j in range (length)])[0,1]
print "Correlation between the true sources and the estimations:"
print correlation_natural
```

```
Correlation between the true sources and the estimations:
[[ 0.99939718]
 [ 0.86964748]]
```

```
#plot delta_W_natural
fig = plt.figure()
ax = fig.add_subplot(111)
ax.set_xlabel('natural gradient--per 1000th update')
ax.plot([i+1 for i in range(len(plot_DW_natural))],plot_DW_natural)
plt.show()
```

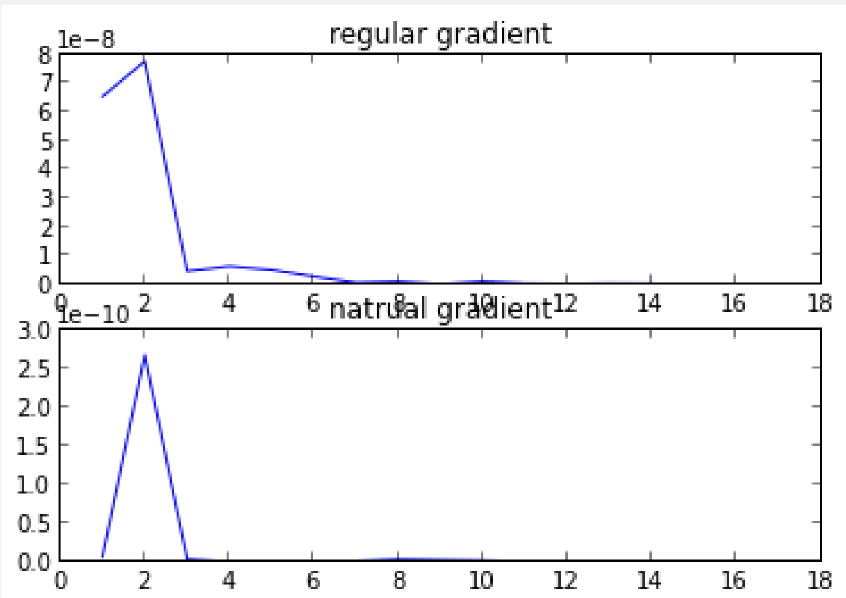



```
#Plot the density of the mixed, unmixed, and true signals.
ax=plt.subplot(321)
ax.hist(matrix.getA1(source[0,:]), bins=50, color='blue')
ax.set_title('histogram of 1st source')

ax=plt.subplot(322)
ax.hist(matrix.getA1(source[1,:]), bins=50, color='red')
ax.set_title('histogram of 2nd source')

ax=plt.subplot(323)
ax.hist(matrix.getA1(mix[0,:]), bins=50, color='blue')
ax.set_title('histogram of 1st mix')

ax=plt.subplot(324)
ax.hist(matrix.getA1(mix[1,:]), bins=50, color='red')
ax.set_title('histogram of 2st mix')

ax=plt.subplot(325)
ax.hist(matrix.getA1(ds_natrual[0,:]), bins=50, color='red')
ax.set_title('histogram of 1st ummix')

ax=plt.subplot(326)
ax.hist(matrix.getA1(ds_natrual[1,:]), bins=50, color='blue')
ax.set_title('histogram of 2nd unmix')

plt.show()
```



3.2.3 Comparison of two learning methods

```
ax=plt.subplot(211)
ax.set_title('regular gradient')
ax.plot([i+1 for i in range(len(plot_DW))],plot_DW)

ax=plt.subplot(212)
ax.set_title('natrual gradient')
ax.plot([i+1 for i in range(len(plot_DW_natural))],plot_DW_natural)
plt.show()
```



As we can see from the figure above, the natural gradient can get a convergence point much faster.

3.2.4 Comparison after whitening the Data

```
#function to get the covariance matrix
def get_CoMatrix(data, dimension):
    C = [[0. for i in range(dimension)] for j in range(dimension)]
    p = data.shape[1]
    m = mix.mean(1)
    for i in range(dimension):
        for j in range(dimension):
            for a in range(p):
                C[i][j] += ( (data[i,a] - m[i,0]) * (data[j,a] - m[j,0]) )/p
    return C

#function to get eigenvalues and eigenvectors
def get_PC(data, dimension, nume):
    C = get_CoMatrix(data, dimension)
    if nume == dimension:
        evals, evecs = np.linalg.eig(asmatrix(C))
    else:
        evals, evecs = sp.sparse.linalg.eigs(asmatrix(C), k = nume)
    return evals, evecs

evals, evecs = get_PC(X, 2, 2)
print evals
print evecs
E = matrix(evecs)
Dd = matrix(np.diag([ 1/math.sqrt(evals[i]) for i in range(2)]))
Z =(( X.T * E) * Dd).T
print 'after whitening:'
print Z
```

```
[ 0.05006899  1.35363586]
[[-0.92229698 -0.38648192]
 [ 0.38648192 -0.92229698]]
after whitening:
[[ 0.01834547  0.01844986  0.26840202 ..., -0.61824489  0.01844411
   0.01834547]
 [-0.00210848 -0.00199554  0.19219774 ...,  0.65723508  0.00203564
  -0.00210848]]
```

a.do Regular gradient with whitened data

```
W = init_W()
print "Init-Matrix W: "
print W
iteration = length * 1
rate_init =0.5

delta_W = matrix([[0.,0.],[0.,0.]])
```

```
plot_DW = []
for t in range(1 , iteration):
    alpha = t % length
    rate = rate_init / t
    for i in range(2):
        sum_WikXk = 0.
        for k in range(2):
            sum_WikXk += W[i,k] * Z[k,alpha]
        for j in range(2):
            inverse = W.getI()
            delta_W[i,j] = rate * ( inverse[j,i] + fi(sum_WikXk) * Z[j,alpha] )
    W = W + delta_W
    if(t % 1000 == 0):
        tmp = delta_W[0,0]**2 + delta_W[1,1]**2 + delta_W[0,1]**2 + delta_W
            [1,0]**2
        plot_DW.append(tmp)
    #print W
print "matrix W is:"
print W
print "after normalization:"
normalMatrix(W)
print W
```

```
Init-Matrix W:
[[ 0.0456093   0.20344697]
 [ 0.70912398  0.1428485 ]]
matrix W is:

[[-0.16409082  2.23398162]
 [ 1.78681701  0.03664664]]
after normalization:
[[-0.07325483  0.99731326]
 [ 0.99978975  0.02050514]]
```

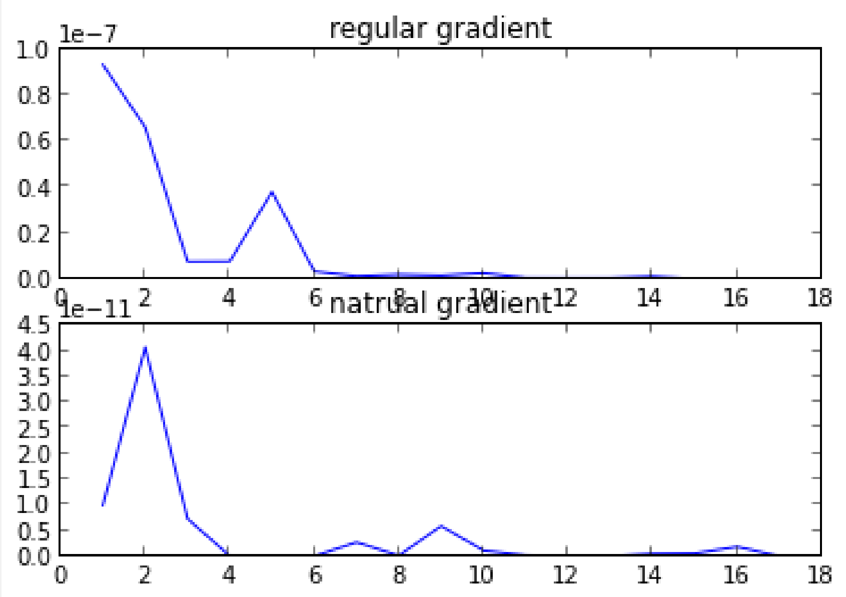
b.do Natural gradient with whitened data

```
W = init_W()
print "Init-Matrix W: "
print W
iteration = length * 1
rate_init =0.5

delta_W = matrix([[0.,0.],[0.,0.]])
plot_DW_natural = []
for t in range(1 , iteration):
    alpha = t % length
    rate = rate_init / t
    for i in range(2):
        sum_WikXk = 0.
        for k in range(2):
```

```
                sum_WikXk += W[i,k] * Z[k,alpha]
            for j in range(2):
                sum_Wlj = 0
                for l in range(2):
                    if (l == i):
                        continue
                    sum_WlkXk = 0
                    for k in range (2):
                        sum_WlkXk += W[l,k] * Z[k,alpha]
                    sum_Wlj += sum_WlkXk
            delta_W[i,j] = rate * fi(sum_WikXk) * sum_Wlj
        W = W + delta_W
        if(t % 1000 == 0):
            tmp = delta_W[0,0]**2 + delta_W[1,1]**2 + delta_W[0,1]**2 + delta_W
                [1,0]**2
            plot_DW_natural.append(tmp)
        #print W
print "matrix W is:"
print W
print "after normalization:"
normalMatrix(W)
print W
```

```
Init-Matrix W:
[[ 0.0456093   0.20344697]
 [ 0.70912398  0.1428485 ]]
matrix W is:
[[ 0.0456093   0.08508957]
 [ 0.70912398  0.03919878]]
after normalization:
[[ 0.47242747  0.88136955]
 [ 0.99847568  0.05519349]]
```

```
ax=plt.subplot(211)
ax.set_title('regular gradient')
ax.plot([i+1 for i in range(len(plot_DW))],plot_DW)

ax=plt.subplot(212)
ax.set_title('natrual gradient')
ax.plot([i+1 for i in range(len(plot_DW_natural))],plot_DW_natural)
plt.show()
```



As we can see, after data whitening, the natural gradient takes much longer time to get the convergence, while the convergence speed of regular gradient stays the same.