Lecture 5: Lecture 5

Fri 12 Sep 20

homework

1.

C. $\implies$ true

Beacause: in range $(-3, -2)$ have Max = -4, $x = 0$

B $\implies$ beacause: 16 not in range $(-3, -2)$

2. Can have multiple maxima: ± 1 are maxima $\implies$ -1 is MAX valued $\implies$ A true

3.

A true $\implies$ because $y = f(x)$ is increasing function above $(-\infty, 0]$ and $f(x) \implies$ range of x is $(-\infty, 0] \implies$ when x max $(x = 0) \implies f(0) = 1 \implies f(x) \leq f(0) = 1$ and $f(x) = 1$ $\forall x \in (-\infty, 0]$

6.

Distinguish between:

+ local maximum / local minimum: in a neighborhood, local

+ global maximum / global minimum: overall, global (all range)

C is incorrect because as $y \to \pm\infty$, there is no greatest or least value; there are only two local extrema: a local minimum at -2 and a local maximum at 2. Therefore, A is correct.

9.

$y = x \ln(x)$

$y' = \ln(x) + x \cdot \dfrac{1}{x}$

$y' = 0 \iff \ln(x) + 1 = 0$

$\iff \ln(x) = -1$

$\iff x = e^{-1} = \frac{1}{e}$

Variation table:

Min $y = -\frac{1}{e}$ when $x = \frac{1}{e}$, $x \in (0, e)$

11.

$D = \mathbb{R}$

$f(x) = \sin^4 x - 2 \cdot (1 - \sin^2 x) + 1[\sin^2 x + \cos^2 x = 1]$

$\iff \sin^4 x - 2 + 2\sin^2 x + 1$

Since $t = \sin^2 x \geq 0$ and $\sin^2 x \leq 1$, we have $t \in [0, 1]$.

Let $\sin^2 x = t \in [0, 1]$.

$g(t) = t^2 + 2t - 1$

$g'(t) = 2t + 2 = 0$

$\iff t = -1$

$f(0) = -1$

$f(1) = 2$

$\implies 2 - 1 = 1$

12.

$D = [-2; 2]$

$y' = \sqrt{4 - x^2} - \frac{2x(x+2)}{2\sqrt{4-x^2}}$

x	0		$\frac{1}{e}$		e
y'		$-$	0	$+$	
y					

Figure 1: Variation table 1

$\iff y' = \sqrt{4-x^2} - \frac{x(x+2)}{\sqrt{4-x^2}}$
$\iff 4 - x^2 = x^2 + 2x$
$\iff 2x^2 + 2x - 4 = 0 \iff \left[\begin{array}{l} x = 1 \\ x = -2 \end{array}\right.$
$f(-2) = 0$
$f(1) = 3\sqrt{3}$
$f(2) = 0$

14.
$D = [-2; 2]$
$y' = 3 - \frac{4x}{2\sqrt{4-x^2}}$
$\iff 3 - \frac{2x}{\sqrt{4-x^2}} = 0$
$\iff 3 = \frac{2x}{\sqrt{4-x^2}}$
$\iff 3\sqrt{4-x^2} = 2x$
$\iff \sqrt{4-x^2} = \frac{2x}{3}$
$\iff 4 - x^2 = \frac{4x^2}{9}$
$\iff 36 - 9x^2 = 4x^2$
$\iff 36 - 13x^2 = 0 \iff \left[\begin{array}{l} x = \frac{6\sqrt{13}}{13} \\ x = -\frac{6\sqrt{13}}{13} \end{array}\right.$

$\implies a = 13$
$\implies b = \frac{6\sqrt{13}}{13} \iff \frac{6}{13} \cdot \sqrt{13}$

$\iff \frac{6}{13} \cdot \frac{13}{\sqrt{13}}$, Note:$(\sqrt{x} = \frac{x}{\sqrt{x}}), (13 : 13 = 1)$
$\iff \frac{6}{\sqrt{13}}$
$\implies b = 6$
$\implies |a + b| = 13 + 6 = 19$

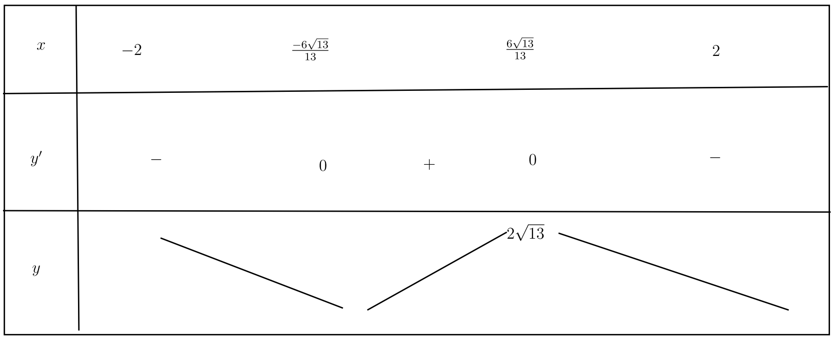

Figure 2: Variable table of ex14

21.
$D = \mathbb{R}/\{1\}$
$y' = \frac{3}{(x+1)^2}$
$y' = 0 \iff \frac{3}{(x+1)^2} > 0, \forall x \in \mathbb{R}$
-> a true

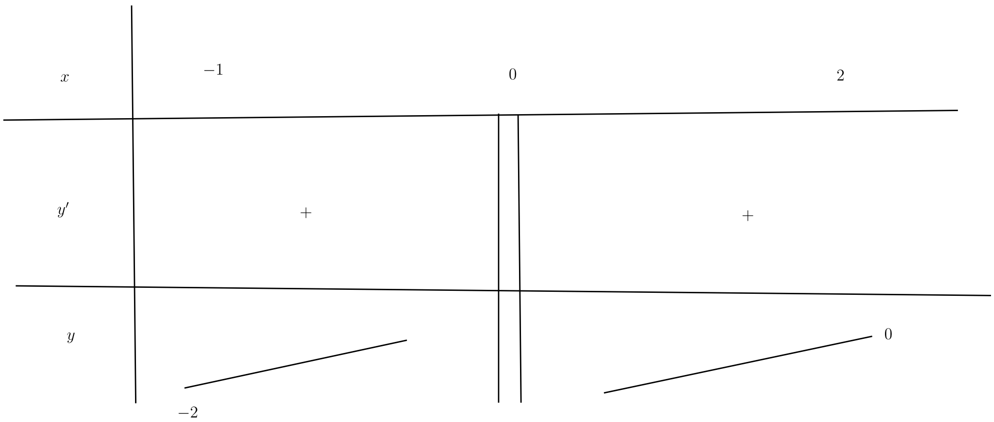

Figure 4: part b

-> b false

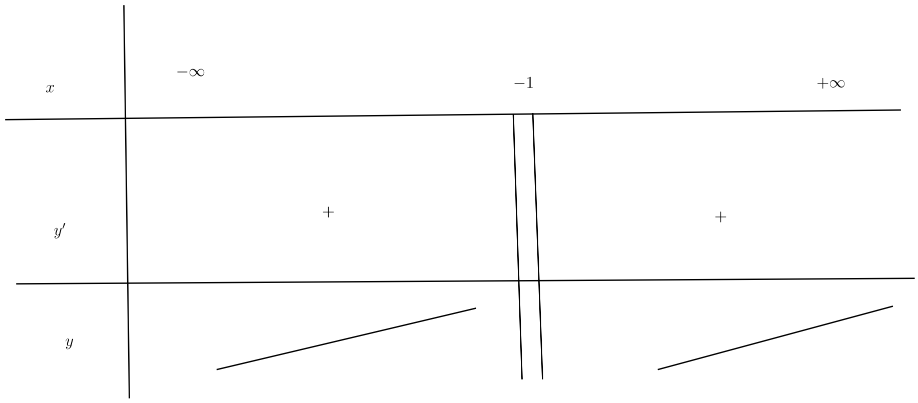

Figure 3: Variable of ex 21

$\implies$ c. true
d. False because y is a linear function that is not continuous at $x = -1$, and $(a; b)$ is an open interval.
22.
a)

$$\max_{[-3;2]} f(x) = f(2) = 6$$

$\implies$ false
b) $6 - (-5) = 11$
c) Have $-\infty$ should don't have min in range $[1; +\infty)$
d)
Let $t = 2\sin x - 1$
As we know range of $\sin x$ is $[-1; 1]$
$\implies -1 \leq \sin x \leq 1$
$\iff -2 \leq 2\sin x \leq 2$(multiply by 2)
$-3 \leq 2\sin x - 1 \leq 1$ (subtract 1)
$\implies t \in [-3; 1]$
$\implies$ Max in range $[-3; 1]$ is f(0) = 3
$\implies$ true.

23.
b) $\exists x \in +\infty$ the function $\implies$ has no maximum on $\mathbb{R}$
c) $5 + 3 = 8 \implies$ false
d)
$g(x) = f(4x - x^2) + \frac{1}{3}x^3 - 3x^2 + 8x + \frac{1}{3}$
$g'(x) = (4 - 2x) \cdot f'(4x - x^2) + x^2 - 6x + 8$
$= 2(2 - x) \cdot f'(4x - x^2) + (x - 2)(x - 4)$
$= 2(2 - x) \cdot f'(4x - x^2) - (2 - x)(x - 4) <*>$
$= (2 - x) \cdot 2 \cdot f'(4x - x^2) - x + 4$

$= (2 - x) \cdot 2 \cdot f'(4x - x^2) + 4 - x$
Note: $<*>$ is the crucial transformation step, changing the sign of $(x - 2)$ to $-(2 - x)$.
With $x \in [1; 3] \implies 4 - x \iff 4 - 3 = 1 \implies 4 - x > 0$
Let $h(x) = (4x - x^2) \iff h'(x) = 4 - 2x \iff x = 2$
Change the limits from x to u.
$\implies h(1) = 3$
$\implies h(2) = 4$
$\implies h(3) = 3$
Thus with $x \in [1; 3] \implies h(x) \in [3; 4]$ should $f'(4x - x^2) \geq 0$
$\implies 2f'(4x - x^2) + 4 - x \geq 0, \forall x \in [1; 3]$
We get $g'(x) = 0 \iff 2 - x = 0 \iff x = 2$
The variation table it as follows:

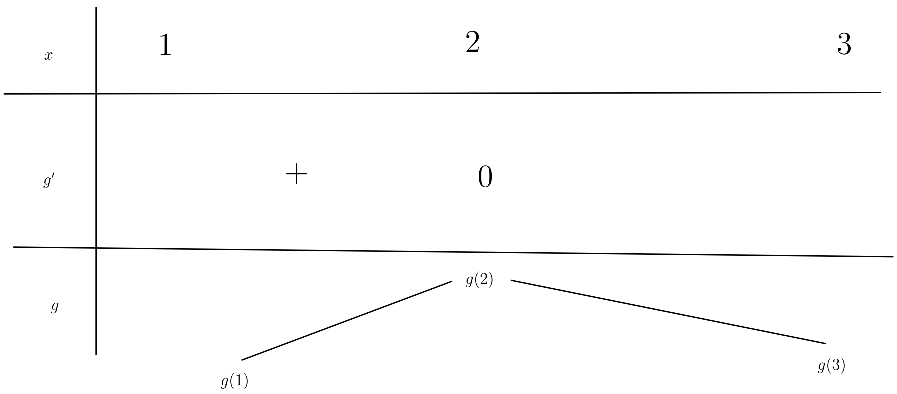

Figure 5: variation table of ex 23

$\implies \max\limits_{[1;3]} g(x) = g(2) = f(4) + 7 = 12$
$\implies$ true
24.
a)From the graph $f'(x)$, we obtain the variation table of $f(x)$ on the interval $[0; 5]$

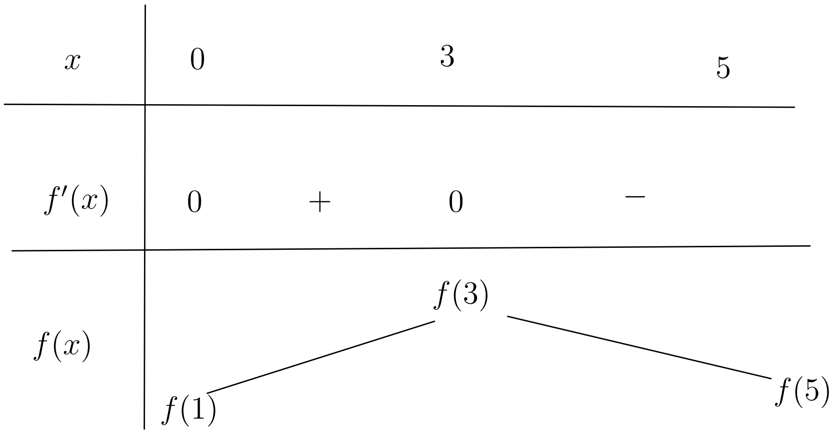

Figure 6: $\text{ex}_2 4$

Obtain: $\max\limits_{[0;5]} f(x) = f(3) \implies$ proposition false

b)Beacause function f decreasing on $[4;5]$, we have $\max\limits_{[4;5]} f(x)$ is $f(4)$

$\implies x_0 = 4$

$\iff 2 \cdot 4^2 + 4 = 36 \implies$ proposition true.

c)We get $f(x) \geq f(0), \forall x \in [0;1] \implies -f(x) \leq -f(0), \forall x \in [0;1]$ and $-f(x) = -f(0) \iff x = 0 \implies \max\limits_{[0;1]}[-f(x)] = -f(0) \implies$ proposition false.

d)We have :

$f(0) + f(1) - 2f(3) = f(5) - f(4)$

$\iff f(0) - f(5) = -f(4) - f(1) + 2f(3)$

we get $f(3) > f(1) \implies 2f(3) - f(1) > 0, \forall x \in [0;5]$

we get $f(4) < 0 \implies -f(4) > 0 \implies 2f(3) - f(4) > 0, \forall x \in [0;5]$

$\implies f(0) - f(5) > 0$

$\iff f(0) > f(5)$

$\implies \min\limits_{[0;5]} f(x) = f(5)$

$\implies$ Proposition true.