

Lecture 8: Lecture 8 (Mon 20 Oct 2025 18:58)

The asymptote of the graph of a function – Part 1

HOMEWORK LECTURE 08:

Sentence 03:

$$\lim_{x \rightarrow +\infty} \frac{3x-2}{4-x} = \lim_{x \rightarrow +\infty} = -3$$

$$\lim_{x \rightarrow -\infty} \frac{3x-2}{4-x} = \lim_{x \rightarrow -\infty} -3$$

$\implies y = -3$ is Horizontal Asymptote of function

Sentence 04:

$$\text{Domain : } 2x - 1 = 0 \implies x = \frac{1}{2}$$

$$\lim_{x \rightarrow \frac{1}{2}^+} \frac{2x+1}{2x-1} = +\infty$$

$\implies x = \frac{1}{2}$ is the vertical asymptote of the graph of the function

Sentence 7:

$$\text{Domain } x^2 - 3x + 2 = 0 \iff \begin{cases} x = 2 \\ x = 1 \end{cases}$$

$\lim_{x \rightarrow 2^+} \frac{\sqrt{x+3}-2}{x^2-3x+2} = +\infty \implies x = 2$ is the vertical asymptote of the graph of the function

$$\lim_{x \rightarrow 1^+} \frac{\sqrt{x+3}-2}{x^2-3x+2} = \lim_{x \rightarrow 1^+} \frac{(\sqrt{x+3}-2)(\sqrt{x+3}+2)}{x^2-3x+2(\sqrt{x+3}+2)} = \lim_{x \rightarrow 1^+} \frac{x-1}{(x-2)(x-1)(\sqrt{x+3}+2)} = \lim_{x \rightarrow 1^+} \frac{1}{(x-2)(\sqrt{x+3}+2)} = -\frac{1}{4} \implies \text{not have vertical asymptote}$$

Sentence 8:

$$\text{We have : } \lim_{x \rightarrow \infty} f(x) = 3$$

$$\lim_{x \rightarrow \infty} -3f(x) + 11 \iff -3 \cdot \lim_{x \rightarrow \infty} f(x) + 11 = -3 \cdot 3 + 11 = 2 \implies y = 2$$

Sentence 9:

$$\text{Domain } x^2 + x = 0 \iff \begin{cases} x = -1 \\ x = 0 \end{cases}$$

$\lim_{x \rightarrow -1^+} \frac{\sqrt{x+9}-3}{x^2+x} = \lim_{x \rightarrow -1^+} \frac{\sqrt{x+9}-3}{x(x+1)} = +\infty \implies x = -1$ is the vertical asymptote of the graph of the function.

$$\lim_{x \rightarrow 0^+} \frac{\sqrt{x+9}-3}{x^2+x} = \lim_{x \rightarrow 0^+} \frac{(\sqrt{x+9}-3)(\sqrt{x+9}+3)}{x^2+x(\sqrt{x+9}+3)} = \lim_{x \rightarrow 0^+} \frac{x}{x(x+1)(\sqrt{x+9}+3)} = \lim_{x \rightarrow 0^+} \frac{1}{(x+1)(\sqrt{x+9}+3)} = \frac{1}{6} \implies \text{not have the vertical asymptote of the graph of the function.}$$