Lecture 6: Lecture 6

Mon 22 Sep 2

Lecture 6 : the maximum and minium values of a function (part 2)
I.Theory
1. Find max, min of function on segment
Suppose the funtion $f(x)$ is continuous on the interval $[a, b]$ and differentiable on the open interval (a, b). Then, the rule for finding the maximum and minium values of $f(x)$ on $[a; b]$ is as follows:
Step 1: Find the points x_1, x_2, x_n in the interval (a, b) where the derivaive of the function is zero or does not exist (in simple terms, solve the equation $f'(x) = 0$).
Step 2: Calculate $f(x_1), f(x_2), \ldots, f(x_n), f(a), f(b)$ (here $f(a)$ and $f(b)$ are values at the endpoints of the interval).
Step 3: Compare all the values obtained in the step 2.
The largest of these values is maximum value of $f(x)$ on $[a, b]$ and the smallest is the minimum value of $f(x)$ on $[a, b]$
Example 1.
a)
$f(x)$ continuos on interval $[-2; 2]$
$f'(x) = 3x^2 - 6x + 9x + 10$
$f'(x) = 0 \iff 3x^2 - 6x + 9x + 10 = 0 \iff \left[\begin{array}{l} x = 3 \neq [-2; 2] \implies (not - pick) \\ x = -1 \in [-2; 2] \implies (pick) \end{array}\right.$
v $f(-1) = 15 \implies$ max
$f(-2) = 8$
$f(2) = -12 \implies$ min
$\implies \max\limits_{[-2;2]} = 15$ at $x = 1$
$\implies \min\limits_{[-2;2]} = -12$ at $x = 2$
b)
$f(x)$ continuos on interval $[1; e^2]$
$f'(x) = \frac{(\ln x)' \cdot x - x' \cdot \ln x}{x^2} = \frac{1 - \ln x}{x^2}$
$f'(x) = 0 \implies 1 - \ln x = 0 \iff 1 = \ln x \iff x = e \in [1; e^2]$
$f(1) = 0$ at $x = 1$
$f(e) = \frac{\ln(e)}{e} = \frac{1}{e}$
$f(e^2) = \frac{2}{e^2}$
$\min\limits_{[1;e^2]} f(x) = 0$ at $x = 1$
$\max\limits_{[1;e^2]} f(x) = \frac{1}{e}$ at $x = e$
Note:
$(\ln(x))' = \frac{1}{x}$
$\ln(e) = 1$ $\ln(x) = y \iff e^y = x$
$\ln(x) = 1 \iff e^1 = x \iff x = e$
c)
$f(x)$ continuos on interval $[0; \ln 2]$
$f'(x) = e^x + xe^x \iff e^x(x + 1)$
$f'(x) = 0 \iff e^x(x + 1) = 0 \iff 1 + x = 0 \iff x = -1 \notin [0; \ln 2]$
$f(0) = 0$
$f(\ln 2) = \ln 2 \cdot e^{\ln(2)} = 2 \cdot \ln 2$
$\implies \min\limits_{[0;\ln 2]} f(x) = 0$ at $x = 0$

$\Longrightarrow \max\limits_{[0;\ln 2]} f(x)$ at $x = \ln 2$

Note: $e^{\ln 2} = 2$

II.Apply

Example 2.

Note 1: Find $\max v(t), t \in [0;6]$

Note 2: $v(t) = s'(t)$

According to the physical meaning of the derivative, we have the instantaneous velocity of a particle:

$v(t) = s'(t) = -3t^2 + 8t^2 + 2t + 2$

Consider $v(t)$ on $[0;6]$; we get $v(t)$ continuous on the interval $[0;6]$

$v'(t) = -6t + 16$

$v'(t) = 0 \iff t = \frac{8}{3} \in [0;6]$

$v(0) = 2; v(6) = -10, v(\frac{8}{3}) = \frac{70}{3}$

$\Longrightarrow \max\limits_{[0;6]} v(t) = \frac{70}{3}$ (m/s) at $t = \frac{8}{3}$ (s)

Example 3.

Note: find $\max\limits_{[0;30]} F(x)$

Note: find x

$F'(x) = 0,05x(30 - x) - \frac{1}{40}x^2$

$= \frac{3}{2}x - 0,05x^2 - \frac{1}{40}x^2$

$= \frac{3}{2}x - \frac{3}{40}x^2$

$F'(x) = 0 \iff \left[\begin{array}{l} x = 20 \in [0;30] \\ x = 0 \in [0;30] \end{array} \right.$

$F(0) = 0$ at $x = 0$

$F(30) = 0$ at $x = 30$

$F(20) = 100$ at $x = 20$

$\Longrightarrow \max\limits_{[0;30]} F(x) = 100$ at $x = 20$(miligam)

Example 4.

Sumarize :

- if 35 - 1 = 34 $\Longrightarrow$ then increase by 50 motorbike
- if 35 + 1 = 36 $\Longrightarrow$ decrease by 50 motorbike
- know cost price = 30 million / one
- bonus distcount 8% in cost price

Solve

+) Profit = (revenue - cost)

+) revenue = price * quantity sold |Note: if + 1 $\Longrightarrow -50 \Longrightarrow +x \Longrightarrow -50x$

+) price = $(35 + x) \Longrightarrow x < 0$ price increase, $x > 0$ price decrease

+) Quantity sold = $(1000 - 50x)$

+) profit of one motorbike: $(35 + x) - 30 + 30 \cdot 8\% = 7,4 + x$

+) Let sum profit is $P(x) \Longrightarrow P(x) = (7,4 + x) \cdot (1000 - 50x)$ **Note:This is sum profit = profit * quantity sold**

$= 7400 - 370x + 1000x - 50x^2$

$= -50x^2 + 630x - 7400$

$P'(x) = -1000x + 630$

$\Longrightarrow P(x) = 0 \iff x = 6,3 \Longrightarrow x > 0 \Longrightarrow$ **price decrease**

variation table:

x	$-\infty$		$6,3$		$+\infty$
$P'(x)$		$+$	0		
$P(x)$		$\nearrow$	9384,5	$\searrow$	

Figure 1: figure 4 of example 4

$\implies$ Price $= 35 + 6,3 = 41,3$ million $\implies D$

Example 5.

Note: s = v * t(general formula), t = s / v, find x -> find equation timer

Sumarize:

+) AB = 4 km
+) AM = v = 6km/h(1)
+) v of MC = 10km/h(2)
+) (1) and (2) is v

Solve:

$AC = AM + MC \rightarrow (0 \leq x \leq 7$ beacause: if MC = MC $\implies$ x = x = 0, else $MC = MB \implies 7 - x \implies x = 7$

Let $MC = x \rightarrow S_1$(distane)

$\iff AC = AM + x$

$AM^2 = AB^2 + BM^2$ Note : $BM = BC - MC(MC = x \iff BM = 7 - x$

$AM^2 = 4^2 + (7 - x)^2 \iff AM = \sqrt{16 + (7 - x)^2} \rightarrow S_2$(distance)

We get equation timer:

$AM = \frac{\sqrt{16+(7-x)^2}}{6}, MC = \frac{x}{10}$

$AC = AM + MC = \frac{\sqrt{16+(7-x)^2}}{6} + \frac{x}{10}$

Let $AC = T(x) \implies T(x) = \frac{\sqrt{16+(7-x)^2}}{6} + \frac{x}{10}$ Note: $(A - B)^2 = A^2 - 2AB + B^2$

Consider: $T(x) = \frac{\sqrt{x^2-14x+65}}{6} + \frac{x}{10}$

$T'(x) = \frac{2x-14}{12\sqrt{x^2-14x+65}} + \frac{1}{10}$ |Note: LCM(12,10) = 60

$= \frac{10x-70}{60\sqrt{x^2-14x+65}} + \frac{6\sqrt{x^2-14x+65}}{60\sqrt{x^2-14x+65}}$

$= \frac{10x-70+6\sqrt{x^2-14x+65}}{60\sqrt{x^2-14x+65}}$

$T'(x) = 0 \iff 10x - 70 + 6\sqrt{x^2 - 14x + 65} = 0$

$\iff 5x - 35 + 3\sqrt{x^2 - 14x + 65} = 0$

$\iff 5x - 35 = -3\sqrt{x^2 - 14x + 65}$

$\iff (5x-35)^2 = 9(x^2-14x+65)$
$25x^2 - 350x + 1225 = 9x^2 - 126x + 585$
$16x^2 - 224x + 640 = 0 \iff$
$\left[\begin{array}{l} x = 10 \notin [0;7] \\ x = 4 \in [0;7] \end{array}\right. \quad T(0) = \frac{\sqrt{65}}{6}$
$T(4) = \frac{37}{30}$
$T(7) = \frac{41}{30}$
$\implies \min\limits_{[0;7]} T(x) = \frac{37}{30}$,at,$x = 4$

III.Homework

Sentence 1. $\mathbb{D} = \mathbb{R} \implies f(x)$ continuous on the interval $[-1;2]$
$f'(x) = -4x^3 + 24x^2$
$f'(x) = 0 \iff \left[\begin{array}{l} x = 6 \notin [-1;2] \\ x = 0 \in [-1;2] \end{array}\right.$
$f(-1) = 12$
$f(2) = 33$
$f(0) = 1$
$\implies \max\limits_{[-1;2]} f(x) = 33$ at $x = 2$

Sentence2. $\mathbb{D} = \mathbb{R} \implies f(x)$ continuous on the interval $[2;19]$
$f'(X) = 3x^2 - 24$
$f'(x) = 0 \iff \left[\begin{array}{l} x = 2\sqrt{2} \in [2;19] \\ x = -2\sqrt{2} \notin [2;19] \end{array}\right.$
$f(2) = -40$
$f(19) = 6403$
$f(2\sqrt{2}) = -32\sqrt{2}$
$\implies \min\limits_{[2;19]} f(x) = -32\sqrt{2}$ at $x = 2\sqrt{2}$

Sentence 3.$\mathbb{D} = \mathbb{R}$ $f(x)$ continuous on the interval $[\frac{1}{2};2]$
$y' = 2x - \frac{2}{x^2}$
$y' = 0 \iff \left[\begin{array}{l} x = 1 \in [\frac{1}{2};2] \\ x = 0 \notin [\frac{1}{2};2] \end{array}\right.$
$y(\frac{1}{2}) = \frac{17}{4}$
$y(2) = 5$
$y(1) = 3$
$\implies \min\limits_{[\frac{1}{2};2]} y = 3$ at $x = 1 \implies m = 3$

Sentence 4. $\mathbb{D} = \mathbb{R} \implies f(x)$ continuous on the interval $[0;\frac{\pi}{2}]$
$f'(x) = 1 - \sqrt{2}\sin x$
$f'(x) = 0 \iff 1 = \sqrt{2}\sin x$
$\implies \sin x = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$(Note: multiply with $\sqrt{2}$)
$\iff \sin x = \frac{\sqrt{2}}{2}$
$\iff x = \frac{\pi}{4} \in [0;\frac{\pi}{2}](Note:$ we know $\sin(\frac{\pi}{4}) = \frac{2\sqrt{2}}{2})$
$f(0) = \sqrt{2}$
$f(\frac{\pi}{2}) = \frac{\pi}{2}$
$f(\frac{\pi}{4}) = \frac{\pi}{4} + 1 \implies \min\limits_{[0;\frac{\pi}{2}]} f(x) = \sqrt{2}$ at $x = 0$

Sentence 5.
Note: m is a constant $\mathbb{D} = \mathbb{R} \implies f(x)$ continuous on the interval $[-1;1]$
$y' = x^2 - x$

$y' = 0 \iff \left[\begin{array}{l} x = 1 \in [-1;1] \\ x = 0 \in [-1;1] \end{array}\right.$

$f(-1) = -\frac{5}{6} + 2m = \frac{1}{6} \implies m = \frac{1}{2} \implies \text{with } m = \frac{1}{2} \implies \min\limits_{[-1;1]} y = \frac{1}{6}$

Sentence 6.

$\mathbb{D} = \mathbb{R} \setminus \{3\} \implies$ continuous on the interval $[0;2]$

$y' = -\frac{3}{(x-3)^2} \implies y' < 0, \forall x \in \mathbb{R}/\{3\}$

$y(0) = 0$

$y(2) = -2$

$\implies \max\limits_{[0;2]} y = 0$ at $x = 0$

Sentence 7.

$\mathbb{D} = \mathbb{R} \implies f(x)$ continuous on the interval $[0;\pi]$

$f'(x) = 4(\sin x)^2 \cdot \cos x - \cos x$

$= \cos x(4(\sin x)^2 - 1) = 0 \iff \left[\begin{array}{l} \cos x = 0 \iff x = \frac{\pi}{2} + k\pi \\ 4(\sin x)^2 = 0 \iff (\sin x)^2 = \frac{1}{4} \iff \sin x = \pm\frac{1}{2} \end{array}\right.$

$\iff \left[\begin{array}{l} x = \frac{\pi}{2} + k\pi \\ x = \frac{\pi}{6} + 2k\pi \\ x = \frac{5\pi}{6} + 2k\pi \\ x = \frac{7\pi}{6} + 2k\pi \notin [0;\pi] \\ x = \frac{11\pi}{6} + 2k\pi \notin [0;\pi] \end{array}\right.$

$f(\frac{\pi}{2}) = \frac{4}{3}$

$f(\frac{\pi}{6}) = \frac{2}{3}$

$f(\frac{5\pi}{6}) = \frac{2}{3}$

$\implies \frac{4}{3} + \frac{2}{3} = 2$

Sentence 8.

$\mathbb{D} = \mathbb{R} \implies y$ continuous on the interval $[-4;-1]$

$y' = \frac{2x-4}{2\sqrt{x^2-4x}}$

$y' = \frac{x-4}{\sqrt{x^2-4x}}$

$y' = 0 \iff \frac{x-4}{\sqrt{x^2-4x}} = 0 \iff \left[\begin{array}{l} x - 4 = 0 \\ x^2 - 4x \geq 0 \end{array}\right. \iff \left[\begin{array}{l} x = 4 \notin [-1;-4] \\ x = 0 \notin [-1;-4] \end{array}\right.$

$f(-4) = 4\sqrt{2}$

$f(-1) = \sqrt{5}$

$\implies (4\sqrt{2})^2 + (\sqrt{5})^2 = 32 + 5 = 37$

Sentence 9.

$\mathbb{D} = \mathbb{R} \implies y$ continuous on the interval $[-4;4]$

$y' = 3x^2 - 9$

$y' = 0 \iff 3x^2 - 6x - 9 = 0 \iff \left[\begin{array}{l} x = 3 \\ x = -1 \end{array}\right.$

$f(-4) = -41$

$f(4) = 15$

$f(3) = 8$

$f(-1) = 40$

$\implies M = 40, m = -41 \implies m + M = -41 + 40 = -1$

Sentence 10.

$\mathbb{D} = \mathbb{R} \setminus \{1\} \implies y$ continuous on the interval $[2;4]$

$y' = \frac{x^2-2x-3}{(x-1)^2}$

$y' = 0 \iff x^2 - 2x - 3 = 0 \iff \left[\begin{array}{l} x = 3 \\ x = -1 \notin [2;4] \end{array}\right.$

$y(2) = 7$
$y(4) = \frac{19}{3}$
$y(3) = 6$
$\Longrightarrow \max\limits_{[2;4]} y = 7$ at $x = 2$
Sentence 11.
$\mathbb{D} = \mathbb{R} \implies y$ continuous on the interval $[1;3]$
$y' = 3x^2 - 4x + 3$
$y' = 0 \iff$ no solution
$f(1) = -2$
$f(3) = 14$
$\Longrightarrow 14 - (-2) = 16$
Sentence 12.
$\mathbb{D} = \mathbb{R} \implies$ y continuous on the interval $[1;3]$
$y' = 3x^2 - 6x - 9$
$y' = 0 \iff$ no solution
$f(-4) = -76$
$f(6) = 54$
$\Longrightarrow 54 - (-76) = 130$
Sentence 13.Note: Be careful with the absolute value sign
$$y = |(x-2)(x-3)| \iff \left[\begin{array}{l} y = (x-2)(x-3) \\ y = -(x-2)(x-3) \end{array}\right. \iff \left[\begin{array}{l} y = x^2 - 6x + 6 \\ y = -x^2 + 6x - 6 \end{array}\right. \iff$$
$$\left[\begin{array}{l} y' = 2x - 6 \\ y' = -2x + 6 \end{array}\right. \iff \left[\begin{array}{l} y' = 0 \iff x = 3 \in [0;3] \\ y' = 0 \iff x = x = \in [0;3] \end{array}\right.$$
$y(0) = 6$
$y(3) = 0$
$\Longrightarrow \max\limits_{[0;3]} y = 6$ at $x = 0$
Sentence 14.
$\mathbb{D} = \mathbb{R} \implies y$ continuous on the interval $[-2;3]$
$y' = 4x^3 - 8x$
$$y' = 0 \iff \left[\begin{array}{l} x = 0 \in [-2;3] \\ x = \pm\sqrt{2} \in [-2;3] \end{array}\right.$$
$y(0) = 9$
$y(-2) = 9$
$y(3) = 54$
$y(\sqrt{2}) = 5$
$y(-\sqrt{2}) = 5$
$\Longrightarrow \max\limits_{[-2;3]} y = 54$ at $x = 3$
Sentence 15.
$\mathbb{D} = \mathbb{R} \implies f(x)$ continuous on the interval $[-3;3]$
$f'(x) = 3x^2 - 3x$
$$f'(x) = 0 \iff \left[\begin{array}{l} x = 1 \in [-3;3] \\ x = 0 \in [-3;3] \end{array}\right.$$
$f(-3) = -16$
$f(3) = 20$
$f(0) = 2$
$f(1) = 0$

$\implies \min\limits_{[-3;3]} f(x) = -16$ at $x = -3$

Sentence 16.

$\mathbb{D} = \mathbb{R} \setminus \{-2\}$

$y' = \frac{x^2+4x-5}{(x+2)^2}$

$y' = 0 \iff \left[\ x^2 + 4x - 5 = 0 \right. \iff \left[\begin{array}{l} x = 1 \in [0;3] \\ x = -5 \notin [0;3] \end{array} \right.$

$y(0) = \frac{1}{2}$

$y(3) = \frac{4}{5}$

$y(1) = 0$

$\implies \max\limits_{[0;3]} y = \frac{4}{5}$ at $x = 3$

Sentence 17.

$\mathbb{D} = \mathbb{R} \setminus \{0\} \implies y$ continuous on the interval $[1;4]$

$y' = 1 - \frac{9}{x^2}$

$y' = 0 \iff 1 = \frac{9}{x^2}$

$\iff x^2 - 9 = 0 \iff \left[\begin{array}{l} x = 3 \in [1;4] \\ x = -3 \notin [1;4] \end{array} \right.$

$f(1) = 10$

$f(4) = \frac{25}{4}$

$f(3) = 6$

$\implies 6 + 10 = 16$

Sentence 18.

a) true beacause :

one print machine = 12 USD $\implies$ x print machine = $12x$ USD

b) false beacause :

cost 1h = 9 USD $\implies$ cost x hours = $9 \cdot x$

c)true beacause :

we know :

workload : 3000

cost x machine : $12x$

Productivity x machine : $30x$

$\implies$ Completion time = workload / productivity $\iff \frac{3000}{30x} = \frac{100}{x}$

$\implies$ Monitoring cost = cost 1 hour * completion time $\iff 9 \cdot \frac{100}{x} = \frac{900}{x}$

$\implies$ Total cost = $\frac{900}{x} + 12x$

d)true beacause:

Do same as question c), except the workload is 4000 and other problems

$\implies \frac{4000}{30x} = \frac{400}{3x}$

$\implies 9 \cdot \frac{400}{3x} = \frac{1200}{x}$

$\implies \frac{1200}{x} + 12x$

Let $c(x) = \frac{1200}{x} + 12x$

$c'(x) = -\frac{1200}{x^2} + 12$

$c'(x) = 0 \iff x = 10$

we know $x \in [1;14]$

$c(1) = 1212$

$c(14) = \frac{1776}{7}$

$c(10) = 240$

$\implies \min\limits_{[1;14]} c(x) = 240$ at $x = 10$

$\Longrightarrow$ The minium production cost to print all the received publlication is : 240 (USD)
Sentence 19.
a) True beacause :
30 minute = 0,5 half an hour $\Longrightarrow c(0,5) = \frac{0,5}{(0,5)^2+1} = 0,4$
b) false beacause :
$\frac{t}{t^2+1} = 0,3$
$\Longleftrightarrow 0,3t^2 - t + 0,3 = 0 \Longleftrightarrow \left[\begin{array}{l} t = \frac{1}{3} \\ t = 3 \end{array}\right.$
$\Longrightarrow t_{min} = \frac{1}{3}$
c) true beacause:
$t > 0 \Longrightarrow t \in (0; +\infty)$
$c'(t) = \frac{1-t^2}{(t^2+1)^2}$
$c'(t) = 0 \Longleftrightarrow \left[\begin{array}{l} t = -1 \notin (0; +\infty) \\ t = 1 \in (0; +\infty) \end{array}\right.$

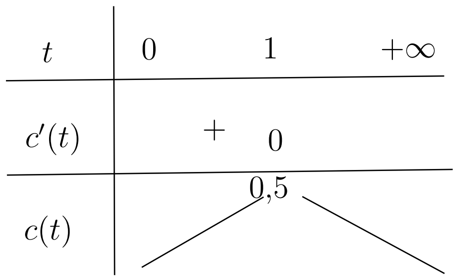

Figure 2: figuare

$\Longrightarrow \max\limits_{[0;+\infty]} c(t) = 0,5$ at $t = 1$
d) true beacause with $t = 1 \Longrightarrow c(1) = 0,5$
Sentence 20:
Note: You can use a Casio calculator to test each answer.
Solve:
constraint: x $>$ 5 $\Longrightarrow x \in (5; +\infty)$
$c'(x) = 2 - \frac{2}{(x-6)^2}$
$c'(x) = 0 \Longleftrightarrow \left[\begin{array}{l} x = 5 \notin (5; +\infty) \\ x = 7 \in (5; +\infty) \end{array}\right.$
$\Longrightarrow c(7) = 20$
$\Longrightarrow \min\limits_{(5;+\infty)} c(x) = 20$ at $x = 7$
Sentence 21:

Let SB $= x \implies SA = 4 - x (0 < x < 4)$
$SC = \sqrt{1+x^2}$(Hypotenuse of triangle SBC)
We know : $AC = SC + SA$
Assume that: SA goes underground
Let function cost is $C(x)$
We have:
$C(x) = 5000\sqrt{1+x^2} + 3000(4-x)$
$C'(x) = \frac{5000x}{\sqrt{1+x^2}} - 3000$
$C'(x) = 0 \iff \left[\begin{array}{l} x = \frac{3}{4} \in (0;4) \\ x = -\frac{3}{4} \notin (0;4) \end{array} \right.$
Variation table:

x	0		$\frac{3}{4}$		4
$C'(x)$			0	+	
$C(x)$		$\searrow$	16000	$\nearrow$	

Figure 3: varvar

$\implies \min\limits_{(0;4)} C(x) = 16000$ at $x = \frac{3}{4}$
$\implies SA = 4 - \frac{3}{4} = \frac{13}{4}$
Sentence 22:
Sumize:
give : 180m materials
Method 1: Using AM-GM inequality
Let x is width of rectangle
let y is length of rectangle
$\implies x + 2y = 180$
$\implies x = 180 - 2y$
$\implies S = (180-2y) \cdot y = \frac{1}{2} \cdot 2y \cdot (180-2y) \leq \frac{1}{2} \cdot \frac{(2y+180-2y)^2}{4} = \frac{180^2}{8} = 4050$ [**Note:using technique AM-GM inequatity**]
S to max when inequality "=" $\implies 2y = 180 - 2y \implies y = 45$
Method 2:using derivatives
constraint: $0 < y < 90$

Let $S(y)$ is function area
$\implies S(y) = (180 - 2y) \cdot y$
$\iff S(y) = -2y^2 + 180y$
$S'(y) = -4y + 180$
$S'(y) = 0 \iff y = 45$
Variation table:

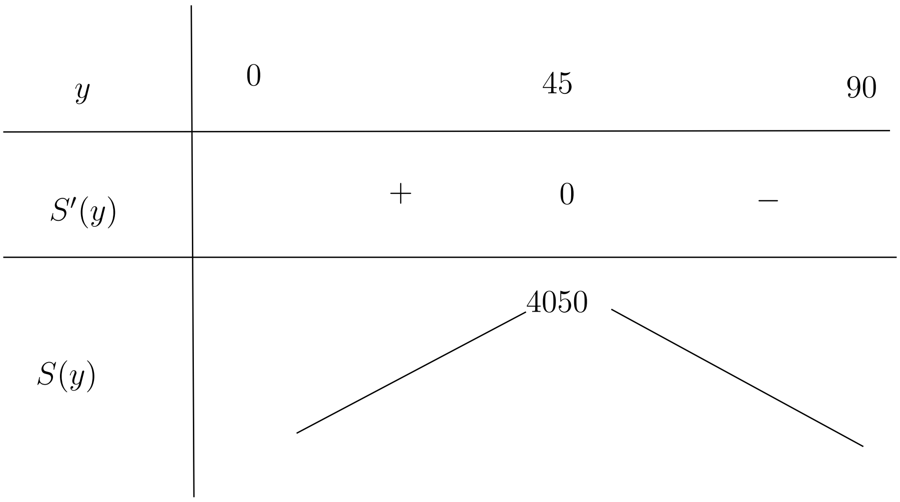

Figure 4: var21

$\implies \max\limits_{(0;90)} S(y) = 4050$ at $y = 45$
Sentence 23:
Method 1 : Using derivatives
Constraint: $a, x > 0$
We know: $V = a^2 \cdot x \implies a = \sqrt{\frac{V}{x}}$
$S = S_p + S_t = 4 \cdot a \cdot h + 2 \cdot a^2 = 4 \cdot \sqrt{\frac{V}{x}} \cdot x + \frac{2V}{x}$
$S = f(x) = 4 \cdot \sqrt{Vx} + \frac{2V}{x}$ **Note:** $\frac{1}{\sqrt{x}} \cdot x = \sqrt{x}, \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$
Consider $f(x) = 4 \cdot \sqrt{Vx} + \frac{2V}{x}$
$f'(x) = \frac{2V}{2\sqrt{Vx}} - \frac{2V}{x^2}$
$f'(x) = 0 \iff \frac{2V}{2\sqrt{Vx}} - \frac{2V}{x^2} = 0$
$\iff \frac{2V}{2\sqrt{Vx}} = \frac{2V}{x^2}$
$\iff \frac{V}{\sqrt{Vx}} = \frac{V}{x^2}$ **Note: ÷ both sides by 2**
$\iff Vx^2 = V\sqrt{Vx}$
$\iff x^2 = \sqrt{Vx}$ **Note: ÷ both sides by V**
$\iff x^4 = Vx$ **Note: square both sdes**
$\iff x = V^{1/3}$ **Note:** $a^n = b \implies a = b^{1/n}$
+)Sign alnalys tips (chapter 1 - lecture 1):

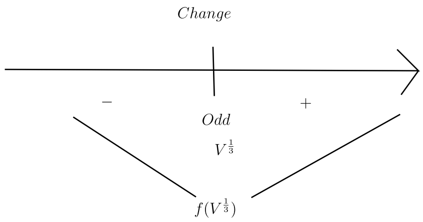

Figure 5: trick table

Method 2: Using the AM-GM inequality
We have:
$4 \cdot \sqrt{Vx} + \frac{2V}{x} = \frac{2V}{x} + 2\sqrt{Vx} + 2\sqrt{Vx} \geq 6\sqrt[3]{V^2}$
The equality " = " holds when $\frac{2V}{x} = 2\sqrt{Vx} \implies x = \sqrt[3]{V}$
Note:
1. As b = c, it suffices to compare a with b.)
2. $\sqrt{x} = x^{\frac{1}{2}}$
Sentence 24:
Method 1: Using the deratives
$\rightarrow$ Make a rectangular gift box (with a lid) $\implies$ need to calculate $S = S_p + S_t$
$\rightarrow V = 200(cm^3) \iff 0,0002(m^3)$
$\rightarrow h = 2(cm) \iff h = 0,02(m)$
$\rightarrow l = x(cm) \iff \frac{x}{100}(m)(x > 0)$
$\rightarrow V = w \cdot h \cdot l \implies w = \frac{V}{l \cdot h} = \frac{0,0002}{\frac{x}{100} \cdot 0,02} = \frac{1}{x}$
$\rightarrow S = S_p + S_t = 2(lw + lh + wh)$
$\rightarrow$ Let $S = f(x)$:
We consider $f(x) = 2(\frac{x}{100} \cdot \frac{1}{x} + \frac{x}{100} \cdot 0,02 + \frac{1}{x} \cdot 0,02)$
$= 0,02 + 0,0004x + \frac{0,04}{x}$
$f'(x) = 0,0004 - \frac{0,04}{x^2}$
$f'(x) = 0 \iff \left[\begin{array}{l} x = 10 \in (0; +\infty) \\ x = -10 \notin (0; +\infty) \end{array} \right.$

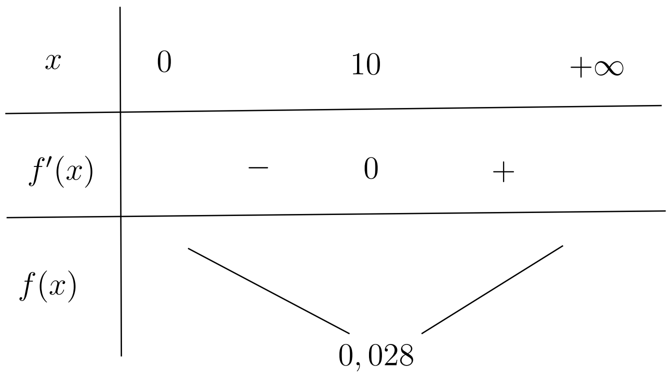

Figure 6: VARIATION TABLE

Method 2: Using the AM-GM inequality

We have: $A = 0,02 + 0,0004x + \frac{0,04}{x}$

(Do not select the const) $\implies 0,0004x + \frac{0,04}{x} \geq 2\sqrt{0,0004x \cdot \frac{0,04}{x}} = 0,008$

Thus: $A = 0,02 + 0,0004x + \frac{0,04}{x} \geq 0,02 + 0,008 = 0,028$

The equality "=" holds when : $0,0004x = \frac{0,04}{x} \implies x = 10$ (beacause $x > 0$)