



A fuzzy-based customer clustering approach with hierarchical structure for logistics network optimization



Yong Wang^a, Xiaolei Ma^{b,*}, Yunteng Lao^b, Yinhai Wang^{b,*}

^a School of Management, Chongqing Jiaotong University, Chongqing 400074, China

^b Department of Civil and Environmental Engineering, University of Washington, Seattle, WA 98195 2700, USA

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ABSTRACT

Customer clustering is an essential step to reduce the complexity of large-scale logistics network optimization. By properly grouping those customers with similar characteristics, logistics operators are able to reduce operational costs and improve customer satisfaction levels. However, due to the heterogeneity and high-dimension of customers' characteristics, the customer clustering problem has not been widely studied. This paper presents a fuzzy-based customer clustering algorithm with a hierarchical analysis structure to address this issue. Customers' characteristics are represented using linguistic variables under major and minor criteria, and then, fuzzy integration method is used to map the sub-criteria into the higher hierarchical criteria based on the trapezoidal fuzzy numbers. A fuzzy clustering algorithm based on Axiomatic Fuzzy Set is developed to group the customers into multiple clusters. The clustering validity index is designed to evaluate the effectiveness of the proposed algorithm and find the optimal clustering solution. Results from a case study in Anshun, China reveal that the proposed approach outperforms the other three prevailing algorithms to resolve the customer clustering problem. The proposed approach also demonstrates its capability of capturing the similarity and distinguishing the difference among customers. The tentative clustered regions, determined by five decision makers in Anshun City, are used to evaluate the effectiveness of the proposed approach. The validation results indicate that the clustered results from the proposed method match the actual clustered regions from the real world well. The proposed algorithm can be readily implemented in practice to help the logistics operators reduce operational costs and improve customer satisfaction levels. In addition, the proposed algorithm is potential to apply in other research domains.

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1. Introduction

Logistics network optimization plays a critical role in contemporary logistics planning and supply chain network designs (Altıparmak, Gen, Lin, & Paksoy, 2006; Bidhandi, Yusuff, Ahmad, & Bakar, 2009). With a well-designed transportation and logistics network, logistics operators can significantly improve the entire freight system efficiency, and customers' needs are better accommodated in a timely manner (Wang, Ma, Wang, Mao, & Zhang, 2012). Under this circumstance, logistics operators and customers will achieve a win-win situation: logistics operators gain more revenues and customers are better served with a lower price due to the reduced operational and transportation costs (Wang, Ma, Lao, Wang, & Mao, 2013). Therefore, properly optimizing the logistics network has become a vital objective for logistics operators.

The typical logistics network optimization includes distribution center location selection, customer clustering and vehicle routing problem (VRP) (Lau, Jiang, Ip, & Wang, 2010; Manzini & Bindi, 2009; Sadjady & Davoudpour, 2012). The aim of logistics network optimizations is to design and allocate a set of logistics facilities for better satisfying the demands of customers (Taaffe, Geunes, & Romeijn, 2010). However, in reality, when the number of customers increases, the logistics network optimization problem become very challenging, and thus, customer clustering should be undertaken before conducting the vehicular dispatching (Jarrah & Bard, 2012). Customer clustering approach groups the customers with similar characteristics into the same category (Anzanello & Fogliatto, 2011). It not only improves the logistics system efficiency, but also reduces the operational costs. For instance, by categories these customers who require cold chain services, logistics companies can dispatch several refrigerated trucks to store and deliver temperature-sensitive goods with the same area. With the proper customer clustering strategy, a large logistics zone can be decomposed into smaller zones where customers share certain common features (i.e. geospatial location, demand etc.). Then, VRP is further

* Corresponding authors.

E-mail addresses: yongwx6@gmail.com (Y. Wang), xiaolm@uw.edu (X. Ma), laoy@uw.edu (Y. Lao), yinhai@uw.edu (Y. Wang).

simplified within each small zone. Thereby, it is necessary to understand customers' characteristics and conduct the customer clustering analysis before vehicle routing optimization in a large-scale network.

To reasonably cluster customers in the logistics network, multiple factors should be taken into account. These factors include customers' geospatial location, travel risk, accessibility, goods compatibility, transportation conditions, etc. However, certain customer attributes cannot be directly measured quantitatively. Considering the high dimensions and ambiguousness of customers' characteristics, traditional clustering algorithms may not be functional well. Therefore, it is desired to develop an innovative clustering approach incorporating both the customer's quantitative and qualitative attributes for logistics network optimization.

The remaining paper is organized as follows: relevant studies are firstly discussed, and then a hierarchical analysis structure for customer clustering is established based on the fuzzy comprehensive evaluation theory. With various definitions and notations documented in the clustering algorithm procedure, a proposed framework including fuzzy integration method, fuzzy clustering algorithm procedure, and clustering validity index is detailed. To evaluate the effectiveness of the proposed algorithm, a case study of customers clustering problem in Anshun, China is presented, followed by a thorough comparison with different prevailing approaches. In addition, the proposed algorithm is further validated using the actual clustered regions determined by local decision makers. Finally, conclusions are summarized at the end of this paper.

2. Literature review

With the advent of new technology such as electronic commerce and new data collection devices, customer behavior information becomes more and more available. Based on the customer's information, valuable knowledge can be extracted using appropriate data mining techniques (Chen, Chiang, Wu, & Chu, 2013; Ngai, Xiu, & Chau, 2009; Pishvaei, Rabbani, & Torabi, 2011). For example, Customer Relationship Management (CRM) has been recognized as a critical component in the business strategy development for companies (Hiziroglu, 2013; Ho, Ip, Lee, & Mou, 2012). Customer clustering categorizes the customers into multiple clusters. Within each cluster, customers share common behaviors. In this way, a company can develop the corresponding business strategy to retain the existing customers. Instead of taking care of each individual customer, the company can allocate their limited resources and efforts into certain clusters for cost savings. In the past decades, numerous business-related customer-clustering approaches were conducted. Wu and Chou (2011) established good customer relations and refined their marketing strategies to match customer expectations. They developed a latent mixed-class membership clustering approach to classify online customers based on the purchasing data across categories. Ren, Zheng, and Wu (2009) presented a clustering method based on genetic algorithm (GA) for telecommunication customer subdivision. Similarly, Ho et al. (2012) proposed a K-means clustering approach based on a robust GA to classify the existing customers. Different from the traditional K-means algorithm, their proposed algorithm is able to detect the optimal number of clusters. Huang, Chen, and Khoo (2012) took customers' voices and opinions into account, and developed a genetic clustering method for emotional design using a combined design structure matrix. Carpaneto, Chicco, Napoli, and Scutariu (2006) made special efforts to cluster electricity customers' representational load patterns on the frequency domain.

In the domain of logistics operations, clustering customers by their characteristics in a large-scale network is not an easy task.

Customer similarity is affected by various factors, such as the customer demand, local traffic condition, market environment, and the time window requirement, etc. Most of these attributes are difficult to measure in a quantitative form. This is because the majority of the above attributes are discrete, and traditionally obtained by human perception. Fuzzy set theory is considered as an appropriate countermeasure to tackle this vagueness and ambiguity. Many researchers have utilized fuzzy set theory to handle ambiguous scenarios in the decision-making procedure (D'Urso, Giovanni, Disegna, & Massari, 2013; Golmohammadi, 2011; Hu & Sheu, 2003; Selim, Araz, & Ozkarahan, 2008; Sheu, 2006; Wang, 2010; Zadeh, 1965). In Fuzzy set theory, linguistic terms are used to evaluate different subjective attributes (Wong & Lai, 2011; Chan, Kwong, & Hu, 2012), for example, "Very Low", "Low", "Medium", "High", "Very High", "Very Poor", "Poor", "Fair", "Good", "Very Good", etc. (Jacobsen, 2002; Lao, Wu, Wang, & McAllister, 2012; Li, Dai, & Tseng, 2011a; Liu & Jin, 2012; Wang et al., 2012). These linguistic variables are well suitable to transform into fuzzy numbers. There exist a variety of typical forms for fuzzy numbers, including trapezoidal fuzzy numbers, triangular fuzzy numbers and interval fuzzy numbers. Trapezoidal fuzzy numbers are considered as the general form of fuzzy numbers, and they are easy and accurate to process the linguistic variables (Liu & Jin, 2012).

Due to the inherent advantages of fuzzy set theory, many fuzzy systematic analysis methods are widely adopted into the logistics network operations and customer clustering process in different research fields. Sheu (2004) proposed a hybrid fuzzy-based method that combined fuzzy-AHP with fuzzy-MADM approaches for determining global logistics strategies. Sheu (2008) presented a hybrid neuro-fuzzy approach to choose appropriate global logistics operational modes for global supply chain management. Qin and Ji (2010) utilized the fuzzy programming tool to design a product recovery logistics network based on different criteria. Vahdani, Moghaddam, and Jolai (2013) presented a new solution approach combined fuzzy probabilistic programming and fuzzy multi-objective programming to address the bi-objective model for designing a reliable logistics network. Shin and Sohn (2004) proposed a fuzzy K-means clustering algorithm to group the stock trading customers into three tiers (Normal, Best and VIP level). The major criterion is the total trade amount over a 3-month period for each customer. Wang (2010) developed a clustering algorithm based on the fuzzy equivalence relation. The linguistic data sequences were firstly interpreted by fuzzy data sequences, and then the sequence with similar preference was classified into one cluster. The proposed clustering algorithm was successfully applied to mine the customer relationship. Chan et al. (2012) recently proposed a new methodology to perform market segmentation based on customers' requirements. In their paper, fuzzy compression technique was firstly used to reduce the high dimensions of customer requirements to two dimensions, and then, a hierarchical fuzzy clustering algorithm was applied to grouping customers with similar characteristics into same cluster based on the compressed data.

Very few relevant studies were conducted to address the customer clustering issues for logistics operations. As mentioned in the introduction, customer clustering is an intermediate stage during the optimization procedure for logistics distribution networks, and it is the critical premise on the vehicle routing planning issue. Hu and Sheu (2003) initially utilized fuzzy clustering to classify potential logistics customers into multiple groups based on the five major attributes: safety, transit time, transportation cost, accessibility and service quality. Based on the research by Hu and Sheu (2003), Sheu (2007) presented an integrated fuzzy-optimization framework to identify the customers with similar characteristics considering multiple attributes of customer demands. However, both of the above studies may suffer from the following issues: (1) The traditional clustering algorithm is not able to handle the

large-scale logistics network with considerable customers. (2) Due to the complexity of modern logistics systems, more attributes should be involved to quantify the similarities between customers. (3) In addition, ratings for different attributes may vary and impact the customer clustering. The heterogeneity among customer attributes should also be incorporated in the clustering procedure.

The aforementioned literature review does not identify a suitable solution for the customer-oriented modern city logistics distribution network optimization. To fill this research gap and provide an effective solution, a fuzzy clustering algorithm with the hierarchical analysis structure is proposed in this paper. Compared with the previous studies, the main contributions of this paper lie in: (1) Constructing a hierarchical analysis structure, where the rating for each criterion varies according to its level of importance; (2) Designing a fuzzy integration method based on sub-criteria, and proposing a fuzzy clustering algorithm to comprehensively evaluate customer attributes; (3) Utilizing the clustering validity index to determine the number of clusters and evaluate the effectiveness of the proposed algorithm. (4) Using real-world clustered customer regions to compare with the clustered results for validation purposes.

3. Methodology

The methodology is composed of three steps: (1) A hierarchical analysis structure is established containing major criteria and sub-criteria; (2) the linguistics variables for customer clustering evaluation are then defined and transformed into trapezoidal fuzzy numbers for further assessments; and (3) a customer clustering framework for the urban logistics distribution network is finally developed based on the fuzzified evaluation criteria.

3.1. Hierarchical analysis structure for customers clustering

This section aims to establish the hierarchical analysis structure for customer clustering in the urban logistics distribution network. As shown in Fig. 1, six major criteria (U_i) and thirteen sub-criteria (U_{ij}) are finalized to determine to capture the

characteristics of customers. These criteria are chosen based on previous studies (Hu & Sheu, 2003; Li et al., 2011a; Sheu, 2007) and are considered important through discussions with several transportation experts and business managers in the local transportation and business departments. It is noteworthy mentioning that selection of these criteria is rational since most of these criteria can accurately reflect the requirement of both logistics operators and governmental authorities. In addition, this hierarchical analysis structure can be expanded to accommodate more criteria from other perspectives.

Based on the above hierarchical analysis structure, each customer can be evaluated using linguistic terms by decision makers. By summarizing the sub-criteria evaluations, the score of a major criterion can be then calculated.

The above sub-criteria structure can be further explained as follows:

- U_{11} : Customers' demands may fluctuate significantly within a certain time period, or customers may request a large quantity of goods. The corresponding delivery strategy should be personalized to fulfill these customers' requirements.
- U_{12} : Customers may request that valuable goods should be delivered separately for safety. Consequently, specific security measures should be adopted.
- U_{13} : The products with the similar life cycle can be delivered together.
- U_{21} : This attribute is used to measure the similarity of the good ordered by a given customer group, e.g., seafood and electronic equipment should not be delivered in the same vehicle concurrently. The higher the external compatibility is, the more likely the bulk delivery service should be taken.
- U_{22} : The internal compatibility is to measure the similarity between the goods ordered by a specific customer, e.g., wine and furniture is less likely to be delivered to a customer simultaneously due to the heterogeneity of these two items. In reality, U_{21} and U_{22} are combined into analysis to determine whether multiple deliveries are needed for a given customer.

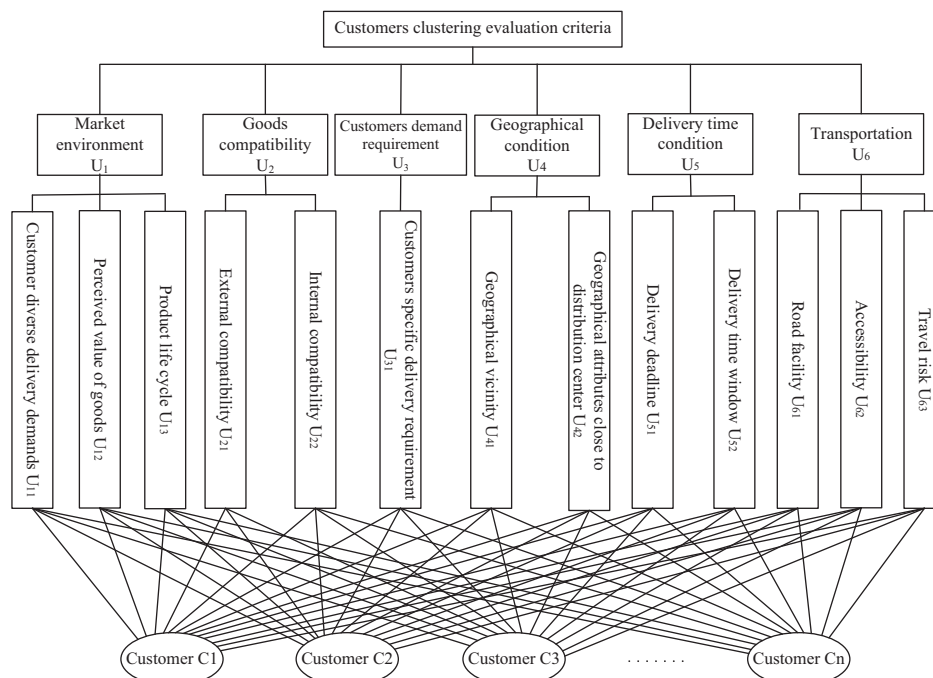


Fig. 1. Hierarchical Analysis Structure for Customers Clustering.

- U_{31} : Certain customers may request specific delivery services, such as vehicle type requirements, or special treatments to the goods, etc. A typical example is the cold chain delivery, where the temperature control is typically involved to keep the pharmaceutical drugs active.
- U_{41} : If customers are spatially adjacent with each other, then these customers can be served together in logistics distribution operations.
- U_{42} : If the distribution center is in close proximity to customers, and these customers can be considered to deliver together.
- U_{51} : Customers with close delivery deadlines tend to be served together for convenience.
- U_{52} : Customers with similar time windows tend to be served together for convenience.
- U_{61} : Customers are more likely to be grouped for delivery if their surrounding road facility conditions are alike.
- U_{62} : Customers with similar accessibility conditions are more likely to be served together.
- U_{63} : The hazardous goods (for example, firecracker, liquefied gas, etc.) tend to be treated separately as one delivery.

3.2. Linguistic variables fuzzification and related definitions

3.2.1. Linguistic variables fuzzification

Natural languages are used to evaluate the above characteristics of customers. In this study, we use the fuzzy set theory to convert natural languages into numerical inputs, and the trapezoidal fuzzy numbers are adopted in the study. Trapezoidal fuzzy number is represented as $\tilde{A} = (a, b, c, d)$ (Ban & Coroianu, 2011; Liu & Jin, 2012; Xu, Shang, Qian, & Shu, 2011), and the membership function $\mu_{\tilde{A}}(x)$ can be then calculated using the trapezoidal fuzzy number \tilde{A} :

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \leq a, \\ \frac{x-a}{b-a}, & a \leq x < b, \\ 1, & b \leq x < c, \\ \frac{d-x}{d-c}, & c \leq x < d, \\ 0, & x \geq d, \end{cases} \quad (1)$$

where a, b, c, d are real numbers, x is between b and c gives the maximal grade of $\mu_{\tilde{A}}(x)$, and x at a and d gives the minimal grade of $\mu_{\tilde{A}}(x)$.

Due to the uncertain nature of the customer clustering problem under the logistics distribution environment, criteria ratings and customer characteristics ratings are used as the linguistic variables. As shown in Table 1, a scale of AL-AH is applied to rate the level of importance for each sub-criterion compared with other sub-criteria, and AP-AG is applied to rate each sub-criterion of each customer. Table 1 presents the linguistic variables and trapezoidal fuzzy numbers for each criterion.

3.2.2. Related Definitions

Before conducting the customer clustering algorithm, several related definitions are needed and presented as follows.

Definition 1. Notations for the parameters are defined as follows: $D\{D_u|u=1,2,3,\dots,m'\}$ denotes the decision makers who evaluate the characteristics of customers; m' is the total number of decision makers.

$U^1\{U_t^1|t=1,2,3,\dots,r\}$ denotes the major criteria for clustering the customers; r is the total number of major criteria.

$U^2\{U_{t'}^2|t'=1,2,3,\dots,k\}$ denotes the sub-criteria for clustering the customers; k is the total number of sub-criteria.

$C\{C_i|i=1,2,3,\dots,n\}$ is the customer i in the logistics distribution network; n is the total number of customers.

Table 1

Relationship between linguistic terms and trapezoidal fuzzy numbers.

| Linguistic term | Abbreviation | Fuzzy numbers |
|---|-------------------|--------------------------|
| Absolutely Low (Absolutely Poor) | AL (AP) | (0.30, 0.33, 0.37, 0.39) |
| Very Low (Very Poor) | VL (VP) | (0.34, 0.37, 0.41, 0.44) |
| Between Very Low and Low (Between Very Poor and Poor) | B.VL& L (B.VP& P) | (0.39, 0.42, 0.45, 0.48) |
| Low (Poor) | L (P) | (0.42, 0.45, 0.49, 0.52) |
| Fairly Low (Fairly Poor) | FL (FP) | (0.49, 0.52, 0.55, 0.58) |
| Medium (Fairly) | M (F) | (0.52, 0.56, 0.60, 0.63) |
| Fairly High (Fairly Good) | FH (FG) | (0.60, 0.63, 0.66, 0.70) |
| High (Good) | H (G) | (0.63, 0.67, 0.72, 0.75) |
| Between High and Very High (Between Good and Very Good) | B.H& VH (B.G& VG) | (0.72, 0.75, 0.81, 0.85) |
| Very High (Very Good) | VH (VG) | (0.82, 0.86, 0.92, 0.95) |
| Absolutely High (Absolutely Good) | AH (AG) | (0.92, 0.95, 0.98, 1) |

$W_{u,t}^2(u=1,2,3,\dots,m';t=1,2,3,\dots,r)$ denotes the evaluation value for sub-criterion l of major criterion t by decision maker u , and also called fuzzy attribute weights in the form of trapezoidal fuzzy number as $W_{u,t}^2 = (\theta_{u,t}^2, h_{u,t}^2, g_{u,t}^2, k_{u,t}^2)$;

$X_{u,i,t}^2(u=1,2,3,\dots,m';i=1,2,3,\dots,n;t=1,2,3,\dots,r)$ denotes the evaluation value for customer i under sub-criterion l of major criterion t by decision maker u , and it can be expressed by trapezoidal fuzzy number as $X_{u,i,t}^2 = (a_{u,i,t}^2, b_{u,i,t}^2, c_{u,i,t}^2, d_{u,i,t}^2)$;

$O = \{o_\gamma|\gamma=1,2,3,\dots,c\}Z_{t,i}^1(t=1,2,3,\dots,r;i=1,2,3,\dots,n)$ denotes the comprehensive evaluation index from all of experts for customer i under major criterion t , and it can be expressed by trapezoidal fuzzy number as $Z_{t,i}^1 = (T_{t,i}^1, Q_{t,i}^1, H_{t,i}^1, G_{t,i}^1)$;

$\mu_{t,i}^1(t=1,2,3,\dots,r;i=1,2,3,\dots,n)$ denotes the membership degree integrated from sub-criteria to major criteria;

$p'\{P_{r'}|r'=1,2,3,\dots,c'\}$ represents each initial cluster, and c' is the total number of initial clusters;

$p\{P_\gamma|\gamma=1,2,3,\dots,c\}$ represents each final cluster, and c is the total number of final clusters;

$O = \{o_\gamma|\gamma=1,2,3,\dots,c\}$ represents the set of sample attribute center in each cluster, and c is the total number of final clusters;

$X = \{x_i|i=1,2,3,\dots,n\}$ denotes the sample set of customers;

$F = \{f_t|t=1,2,3,\dots,r\}$ denotes the attribute set of X .

Definition 2. $x_{it} = \mu_t(x_i)$ denotes the membership degree of the sample x_i with attribute f_t ; In the clustering process, the attribute f_t can be further separated as $m_{t,1}, m_{t,2}, \dots, m_{t,s_t}$, where m_{t,s_t} denotes the s_t^{th} sub-attribute of attribute f_t , thereby, the attribute set F can be expressed as $F = M = \{m_{1,1}, m_{1,2}, \dots, m_{1,s_1}, \dots, m_{t,1}, m_{t,2}, \dots, m_{t,s_t}, \dots, m_{r,1}, m_{r,2}, \dots, m_{r,s_r}\}$, which is defined as the fuzzy attribute (concept) set.

Definition 3. According to Axiomatic Fuzzy Set (AFS) theory (Liu, 1998a, 1998b; Liu, Wang, & Chai, 2005; Zhang, Liang, & Tong, 2004a, Zhang, Liang, & Tong, 2004b), (M, τ, X) is called an Axiomatic Fuzzy Set (AFS) structure, X is called a sample set, M is called an attribute (concept) set, τ is called structure; Let R be the binary relation of the sample set X , and the binary relation contains sub-preference relations and preference relations, and the corresponding concept of sub-preference relation is called simple concept, and the preference relation is called complex concept.

Definition 4. Let m be the simple fuzzy concept (Zhang et al., 2004b), and $m \in \tau(x, y)$.

$\rho_m: X \rightarrow R^+ = [0, \infty)$, in addition, if ρ_m satisfies the following conditions:

- (1) $\rho_m(x) = 0 \Leftrightarrow (x, x) \neq R_m, x \in X$;
- (2) $(x, y) \in R_m \Rightarrow \rho_m(x) \geq \rho_m(y), x, y \in X$;

ρ_m is called the membership degree of simple fuzzy concept m .

Definition 5. Let B is fuzzy concept set, and $C \subseteq X, B \subseteq M, X \subseteq R, \bar{m} \subseteq \bar{B}$, where $\bar{B}(x)$ denotes the set of membership degrees which are less than or equal to $\rho_m(x)$ in the fuzzy concept set B , then $J_m(x)$ is the measure of sample x belonging to the simple fuzzy concept m , and $\mu_B(x)$ is the membership degree of fuzzy concept B at sample x , they can be expressed as follows:

$$\bar{B}(x) = \{y | y \in \rho_m(x), m \subseteq B, \forall x \in C\}, \quad (2)$$

$$J_m(x) = \frac{\sum_{x \in \bar{m}(x)} \rho_m(x)}{\sum_{x \in X} \rho_m(x)}, \quad (3)$$

$$\mu_B(x) = \inf_{m \in B} (J_m(x)) \in [0, 1], \quad x \in X. \quad (4)$$

Definition 6. Let $\eta(x_i)$ be the simple fuzzy concept set, s_t is the number of sub-attributes of f_t , that is, $\eta(x_i) = \{m_{t,s'} | t = 1, 2, \dots, r; s' = 1, 2, \dots, s_t; i = 1, 2, \dots, n\}$, let ζ_{C_i} be the fuzzy description of each sample C_i , then,

$$\zeta_{C_i} = \arg \min \{ \prod m_{t,s'} | m_{t,s'} \in \eta(x_i), \quad t = 1, 2, \dots, r; s' = 1, 2, \dots, s_t; \quad i = 1, 2, \dots, n \}. \quad (5)$$

Definition 7. Let ξ_{P_j} be the weighted fuzzy description, $\xi_{P_j} = \{S_{P_j}, w_{P_j}\}$, where S_{P_j} denotes the fuzzy description of cluster P_j , and w_{P_j} is the weight of attributes belong to cluster P_j , in addition, let \wedge be the fuzzy characteristic and \wedge can be used to combine the fuzzy description of samples, for example, $\zeta_{C_1} = \{m_1\}$, $\zeta_{C_2} = \{m_2\}$, then, $\zeta_{C_1} \wedge \zeta_{C_2} = \{m_1, m_2, m_1 m_2\}$.

Definition 8. Let X be the sample set of customers, M be the simple fuzzy concept set of X , $B \subseteq M$, then the membership information entropy function (Shannon, 2001) $E(B)$ and the membership distribution coefficient function $D(B)$ are defined as follow:

$$E(B) = - \sum_{x \in X} (\mu_B(x) \ln(\mu_B(x))), \quad (6)$$

$$D(B) = - \left(\left(\sum_{x \in X} \mu_B(x) / n \right) \ln \left(\sum_{x \in X} \mu_B(x) / n \right) \right). \quad (7)$$

When $E(B)$ decreases, the membership degree of the sample x within fuzzy concept B becomes closer to the two ends of the interval $[0, 1]$, and the boundaries between 0 and 1 are clearer. When $D(B)$ decreases, the membership degree of the sample x within fuzzy concept B becomes closer to the left end or right end of the interval $[0, 1]$ (Wang et al., 2012). Therefore, the evaluation index $V = E(B)/D(B)$ is defined for comprehensively evaluating the membership information entropy and distribution coefficient. When V is smaller, it is more reasonable for the fuzzy concept B to describe the sample X .

3.3. The proposed framework for customers clustering

The framework contains three major steps. In the first step, the evaluation sub-criteria by m' decision makers should be properly mapped into the higher hierarchical criteria based on the fuzzy

integration method. Then, clustering algorithm is undertaken to group the customers into different clusters. Finally, the clustering validity index is designed to determine the reasonable number of clusters and the customers' clustering units. These steps are presented in detail as follows:

3.3.1. Fuzzy integration method based on sub-criteria

According to Definition 1, X_{u,i,t_l}^2 is the evaluation value for customer i under sub-criterion l of major criterion t by decision maker u , W_{u,t_l}^2 is the evaluation value for sub-criterion l of major criterion t by decision maker u , and they are expressed by trapezoidal fuzzy number as:

$X_{u,i,t_l}^2 = (a_{u,i,t_l}^2, b_{u,i,t_l}^2, c_{u,i,t_l}^2, d_{u,i,t_l}^2)$ and $W_{u,t_l}^2 = (\theta_{u,t_l}^2, h_{u,t_l}^2, g_{u,t_l}^2, k_{u,t_l}^2)$ respectively. The comprehensive evaluation index from all of decision makers for customer i under major criterion t can be expressed as

$$Z_{t,i}^1 = \frac{1}{m' \times t_s} \otimes \sum_{u=1}^{m'} ((X_{u,i,t_1}^2 \otimes W_{u,t_1}^2) \oplus (X_{u,i,t_2}^2 \otimes W_{u,t_2}^2) \oplus \dots \oplus (X_{u,i,t_s}^2 \otimes W_{u,t_s}^2)) \quad (8)$$

Where \otimes and \oplus denote the vector multiplication the vector addition respectively. $Z_{t,i}^1$ is the comprehensive trapezoidal fuzzy number for evaluating the customer i under major criterion t , m' is the number of experts, and t_s is the number of sub-criteria s under major criterion t .

Let $Y = (a, b, c, d)$ be a trapezoidal fuzzy number. The integrated representation of trapezoidal fuzzy number Y is defined as $P(Y) = \frac{1}{6}(a + 2b + 2c + d)$ (Chou, 2009; Chou, Chang, & Shen, 2008; Kahraman, Ruan, & Doğan, 2003; Liu & Jin, 2012). Thereby, according to Definition 1, $Z_{t,i}^1$ can be expressed as $Z_{t,i}^1 = (T_{t,i}^1, Q_{t,i}^1, H_{t,i}^1, G_{t,i}^1)$, then the integrated membership degree $\mu_{t,i}^1$ of the customer i under major criterion t is described as

$$\mu_{t,i}^1 = \frac{1}{6} (T_{t,i}^1 + 2Q_{t,i}^1 + 2H_{t,i}^1 + G_{t,i}^1) \quad (9)$$

3.3.2. Fuzzy clustering algorithm procedure

The integrated membership degree $\mu_{t,i}^1$ calculated from the fuzzy integration method is used as the input of clustering approach. The next step is to conduct clustering analysis for dividing the customers into different clusters. Axiomatic fuzzy set (AFS) theory logic has been proven as an effective approach for tackling human perception related clustering problem (Liu, 1998a; Zhang et al., 2004a, 2004b). Since the traditional AFS method is not appropriate to include a large number of criteria (Liu et al., 2005), an improved AFS theory logic algorithm is introduced as follows:

Step 1. According to Definition 2, the attribute f_t can be transformed into four sub-attributes: $m_{t,1}, m_{t,2}, m_{t,3}, m_{t,4}$, and their membership degree can be expressed as $\rho_{m_{t,1}}, \rho_{m_{t,2}}, \rho_{m_{t,3}}, \rho_{m_{t,4}}$ respectively, and let $\mu_t(x_i) = \mu_{t,i}^1$, and then, the membership degrees can be written as

$$\begin{aligned} \rho_{m_{t,1}}(x_i) &= \mu_t(x_i), \\ \rho_{m_{t,2}}(x_i) &= h_{t,1} - \mu_t(x_i), \\ \rho_{m_{t,3}}(x_i) &= |\mu_t(x_i) - h_{t,2}|, \\ \rho_{m_{t,4}}(x_i) &= h_{t,3} - |\mu_t(x_i) - h_{t,2}|, \\ h_{t,1} &= \max\{\mu_t(x_1), \mu_t(x_2), \dots, \mu_t(x_n)\}, \\ h_{t,2} &= \frac{\mu_t(x_1) + \mu_t(x_2) + \dots + \mu_t(x_n)}{n}, \\ h_{t,3} &= \max_{1 \leq i \leq n}\{|\mu_t(x_i) - h_{t,2}| + h_{t,2}\}, \\ t &= 1, 2, \dots, r. \end{aligned} \quad (10)$$

Step 2. Calculate the fuzzy attributes of each sample.

Step 2.1 Calculate $J_{m_{t,1}}(x_i), J_{m_{t,2}}(x_i), J_{m_{t,3}}(x_i), J_{m_{t,4}}(x_i)$ by following step 1 and the Eq. (3).

Step 2.2 Let $\mu_{m_{t,1}}(x_i) = J_{m_{t,1}}(x_i), \mu_{m_{t,2}}(x_i) = J_{m_{t,2}}(x_i), \mu_{m_{t,3}}(x_i) = J_{m_{t,3}}(x_i), \mu_{m_{t,4}}(x_i) = J_{m_{t,4}}(x_i)$, then, fuzzy concept of the maximum of membership fuzzy values can be expressed as:

$$\begin{aligned} \eta_{t,s'}(x_i) &= \{m_{t,s'} | \mu_{m_{t,s'}}(x_i) \\ &= \max\{\mu_{m_{t,1}}(x_i), \mu_{m_{t,2}}(x_i), \mu_{m_{t,3}}(x_i), \mu_{m_{t,4}}(x_i)\}, \\ t &= 1, 2, \dots, r, s' = 1, 2, 3, 4, i = 1, 2, \dots, n\}. \end{aligned} \quad (11)$$

Step 2.3 Calculate the ratio of membership information entropy and distribution coefficient function. The evaluation index of each sample attribute of step 2.2 is calculated as

$$V_{\eta_{t,s'}(x_i)} = E(\eta_{t,s'}(x_i)) / D(\eta_{t,s'}(x_i)). \quad (12)$$

Step 2.4 Let $\eta(x_i)$ be the simple fuzzy concept set, that is, $\eta(x_i)$ can be expressed as follows:

$$\eta(x_i) = \{\eta_{t,s'}(x_i) | t = 1, 2, \dots, r; s' = 1, 2, 3, 4; i = 1, 2, \dots, n\}. \quad (13)$$

Step 2.5 Select the smallest value $\mu_{\alpha}(x_i)$ corresponding to attribute α ; select the second smallest value $\mu_{\beta}(x_i)$ corresponding to attribute β ; $\mu_{\alpha}(x_i)$ is defined as

$$\mu_{\alpha}(x_i) = \inf_{\sigma \in \eta(x_i)} (J_{\sigma}(x_i)) \in [0, 1]. \quad (14)$$

Let $\eta(x_i)' = \{\eta(x_i) - \{\alpha\}\}$, and then $\mu_{\beta}(x_i)$ is defined as:

$$\mu_{\beta}(x_i) = \inf_{\sigma \in \eta(x_i)'} (J_{\sigma}(x_i)) \in [0, 1]. \quad (15)$$

Step 2.6 Find the evaluation index $V_{\eta_{t,s'}(x_i)}$ corresponding to attribute α , and the evaluation index $V_{\eta_{t,s'}(x_i)'}$ corresponding to attribute β , and compare $V_{\eta_{t,s'}(x_i)}$ and $V_{\eta_{t,s'}(x_i)'}$.

Step 2.7 If $V_{\eta_{t,s'}(x_i)'} \geq V_{\eta_{t,s'}(x_i)}$, then eliminate attribute α , and the remaining sample attributes are $\eta(x_i)' = \{\eta(x_i) - \{\alpha\}\}$, and return to step 2.5, continue the recursion until $V_{\eta_{t,s'}(x_i)'} < V_{\eta_{t,s'}(x_i)}$.

Step 2.8 Return the final remaining attributes $\eta(x_i)'$ for each sample x_i , the remaining attributes can be used to describe the sample x_i , thereby, we get the fuzzy attributes of each sample from step 2.1–step 2.8.

Step 3 Calculate the initial clusters based on the fuzzy attributes of each sample

Step 3.1 Given that the fuzzy description of each sample C_i is ζ_{C_i} , and we construct the fuzzy sample relation $q_{ij} = \min\{\mu_{\zeta_{C_i} \wedge \zeta_{C_j}}(C_i), \mu_{\zeta_{C_i} \wedge \zeta_{C_j}}(C_j)\}$, Liu (1998b) has demonstrated that the integer θ exists for $(Q_{\wedge}^{\theta})^2 = (Q_{\wedge}^{\theta})$, thereby, we can deduct the equivalence relation at universe of discourse C based on the fuzzy sample relation matrix $Q = Q_{\wedge}^{\theta} = (q_{ij}^{\theta})_{n \times n}$.

Step 3.2 Denote the diagonal element of fuzzy sample relation matrix as q_{ii}^{θ} . It can be verified as $q_{ij}^{\theta} = q_{ji}^{\theta} = q_{ii}^{\theta}$, thereby, we can identify the different membership degree based

on the diagonal elements and other elements in the matrix. The diagonal elements are expressed as α_e ($e = 1, 2, \dots, g$), and these values gradually increase according to the sequence of e . Find $q_{ij}^{\theta} = q_{ji}^{\theta} = q_{ii}^{\theta}$ in the fuzzy sample relation matrix; the corresponding samples can be grouped into one or multiple clusters, and the remaining samples where $q_{ij}^{\theta} > \alpha_e$ can be grouped as another cluster. This procedure will recursively continue until $e = g$, then, we can obtain the initial clusters $P_1, P_2, \dots, P_{c'}$.

Step 4 Complete the final clusters based on the weighted fuzzy description of clusters

Step 4.1 The samples are divided into different clusters according to different α_e values. We obtain the initial clusters $P_1, P_2, \dots, P_{c'}$ based on α_e , and calculate the weights of fuzzy description of $P_{r'} : \zeta_{P_{r'}} = \{S_{P_{r'}}, w_{P_{r'}}\}$, $\zeta_x = \{m_{t,s'} | t \in \{1, 2, \dots, r\}, s' = 1, 2, 3, 4, x \in P_{r'}\}$, ζ_x denotes the fuzzy attributes set of $P_{r'}$, $S_{P_{r'}} = \{m | m \in \zeta_x, x \in P_{r'}\}$, $w_{P_{r'}} = \{w_m | m \in S_{P_{r'}}\}$, $w_m = \frac{|P_{r'}^m|}{\sum_{x \in P_{r'}} |\zeta_x|}$, $|P_{r'}^m| = |\{x | x \in P_{r'}, m \in \zeta_x\}|$, where $|\cdot|$ is defined as the number of elements, and $w_m \in w_{P_{r'}}$ is the weight of $m \in S_{P_{r'}}$.

Step 4.2 For the each sample $x \in X$, calculate the weighted membership degree $\mu_{(c,w,\gamma^*)}(x) = \sum_{m \in S_{P_{r'}}} w_m J_m(x)$, where $J_m(x)$ is the measure of initial sample, calculate $\mu_{(c,w,\gamma^*)}(x) = \arg \max_{1 \leq \gamma' \leq c'} \{\mu_{(c,w,\gamma')}(x)\}$, then we can obtain $x \in P_{r'}$ and finally we obtain the result of clusters $P_1, P_2, \dots, P_{c'}$, and simultaneously we calculate the fuzzy attributes set and as $\zeta_x = \{m_{t,s'} | t \in \{1, 2, \dots, r\}, s' = 1, 2, 3, 4, x \in P_{r'}\}$, $S_{P_{r'}} = \{m | m \in \zeta_x, x \in P_{r'}\}$.

3.3.3. Clustering validity index

Clustering validity index (Wang, 2010; Li, Liu, & Chen, 2011b) is used to evaluate the clustering result. According to different thresholds $\alpha_e \in [0, 1]$, we can select the optimal result from the clustering validity index. Some notations are introduced to express the validity index. N is the number of samples, P_1, P_2, \dots, P_c are the cluster results, c is the number of clusters, $O = \{o_{\gamma} | \gamma = 1, 2, 3, \dots, c\}$ is the set of sample attribute center in each cluster, $o_{\gamma} = \{o_{\gamma}^m = \sum_{x \in P_{\gamma}} \rho_m(x) / n_{P_{\gamma}} | m \in S_{P_{\gamma}}\}$, $1 \leq \gamma \leq c$, where ρ_m is the membership degree of simple fuzzy concept m , and $n_{P_{\gamma}}$ is the number of samples in cluster γ . Thereby, the clustering validity index I_{α} can be expressed as follows:

$$I_{\alpha} = \frac{c \times (c-1) \sum_{\gamma=1}^c \sum_{x \in P_{\gamma}} \sum_{m \in S_{P_{\gamma}}} \|\rho_m(x) - o_{\gamma}^m\|^2}{2 \times \alpha_e \times \left(\sum_{\gamma=1}^c \sum_{k=1, \gamma \neq k}^c \sum_{m \in S_{P_{\gamma}}} \|o_{\gamma}^m - o_k^m\|^2 \right)}. \quad (16)$$

Where $\frac{\sum_{\gamma=1}^c \sum_{k=1, \gamma \neq k}^c \sum_{m \in S_{P_{\gamma}}} \|o_{\gamma}^m - o_k^m\|^2}{\frac{c \times (c-1)}{2}}$ describes the dispersion degree between clusters by comparing their attributes, which can be used for merging different clusters, and $\sum_{\gamma=1}^c \sum_{x \in P_{\gamma}} \sum_{m \in S_{P_{\gamma}}} \|\rho_m(x) - o_{\gamma}^m\|^2$ denotes the closeness of attributes from different samples within each cluster. The clustering validity index I_{α} becomes smaller when the closeness within each cluster becomes smaller and the dispersion degree between clusters becomes greater. A cluster becomes the most distinct from other clusters when I_{α} reaches the smallest value.

4. Implementation and comparisons

4.1. Data source

A case study in Anshun, China is utilized to evaluate the effectiveness of the proposed customer clustering algorithm. Anshun City is considered as an important transportation hub of Guizhou Province, and it acts as a key transportation center for the entire province. A distribution center, owned by a local logistics company in Anshun city, aims to accommodate customers' requirements in an effective and efficient fashion. To ease the difficulty of distribution region partition problem and vehicle routing problem, customer clustering should be undertaken in the entire distribution network. As shown in Fig. 2, 40 representative customers (expressed as C1, C2, ..., C40) have been chosen for clustering analysis. On the map, the red circle indicates the Central Business District (CBD) and the blue lines are the major arterials of Anshun City.

To ensure the accuracy and reliability of clustering analysis, five decision makers $D = \{D_1, D_2, D_3, D_4, D_5\}$ were invited to evaluate the criteria and characteristics of customers. Two are from the department of market management in Anshun City, another decision maker is from the third party logistics company in Anshun City, and the other two are from the bureau of commerce in Anshun City. All five have many years management experience, and very familiar with the characteristics of these customers and local economic condition.

After interviewing these top decision makers, we obtained the ratings for 13 sub-criteria, as shown in Table 2. Due to the space limit, the language variable used for evaluating the characteristics of customers is not listed here, but according to Eqs. (8) and (9), we can obtain the comprehensive evaluation values by integrating the sub-criteria into the major criteria. The evaluation values are presented in Table 3.

4.2. Result analysis

The aggregated evaluation matrix is used as the inputs in the clustering algorithm procedure. Following Steps 1 through 3 in the clustering algorithm procedure, the diagonal elements α_e in steps 3.2 can be calculated. Using threshold α to represent the value of α_e in each different location of the matrix, then we can obtain the initial clustering results and the clustering validity index as follows:

- (1) When threshold $\alpha = 0.6474$, $I_\alpha = 5.65$, there are two clusters:

$$P'_1 = \{C26, C27, C28, C29, C30\},$$

$$P'_2 = \{C1, C2, C3, C4, C5, C6, C7, C8, C9, C10, C11, C12, C13, C14, C15, C16, C17, C18, C19, C20, C21, C22, C23, C24, C25, C31, C32, C33, C34, C35, C36, C37, C38, C39, C40\}.$$

- (2) When threshold $\alpha = 0.7288$, $I_\alpha = 5.17$, there are three clusters:

$$P'_1 = \{C26, C27, C28, C29, C30\},$$

$$P'_2 = \{C31, C33, C34, C35, C36, C39\},$$

$$P'_3 = \{C1, C2, C3, C4, C5, C6, C7, C8, C9, C10, C11, C12, C13, C14, C15, C16, C17, C18, C19, C20, C21, C22, C23, C24, C25, C32, C37, C38, C40\}.$$

- (3) When threshold $\alpha = 0.7472$, $I_\alpha = 4.05$, there are four clusters:

$$P'_1 = \{C26, C27, C28, C29, C30\},$$

$$P'_2 = \{C31, C33, C34, C35, C36, C39\},$$

$$P'_3 = \{C7, C8, C9, C10, C12, C14, C16\},$$

$$P'_4 = \{C1, C2, C3, C4, C5, C6, C11, C13, C15, C17, C18, C19, C20, C21, C22, C23, C24, C25, C32, C37, C38, C40\}.$$

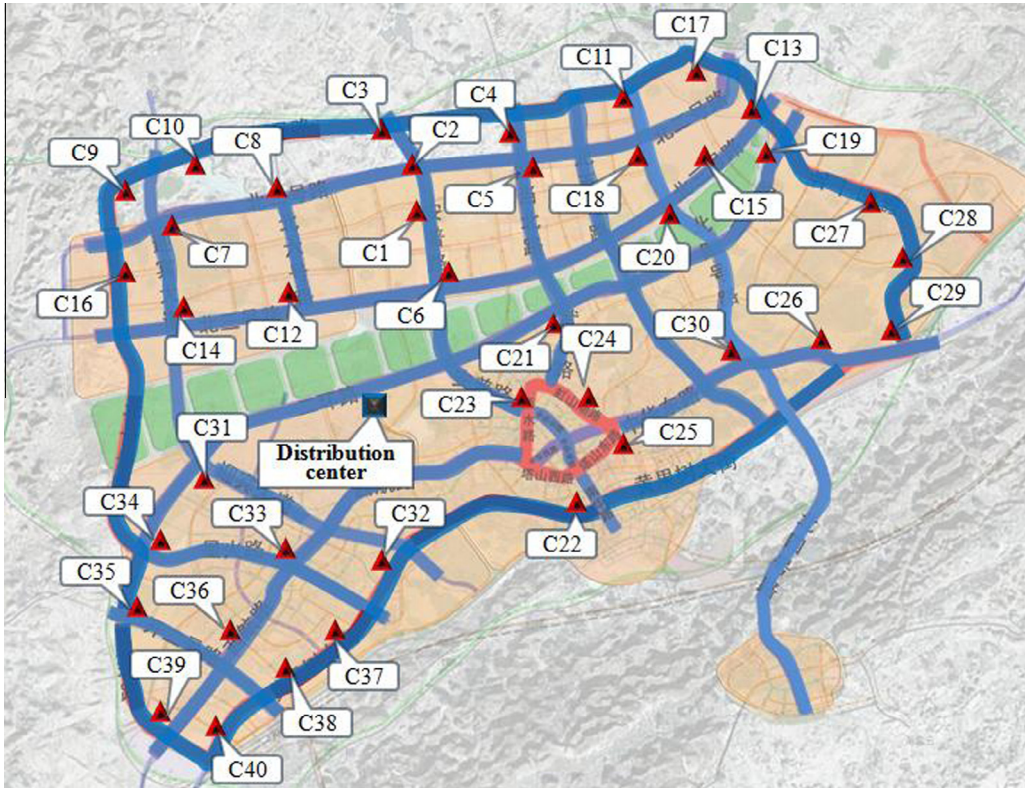


Fig. 2. Customer Distribution Map.

Table 2
Linguistic Ratings for the Sub-criteria.

| Sub-criteria | Decision makers | | | | |
|-----------------|-----------------|----------------|----------------|----------------|----------------|
| | D ₁ | D ₂ | D ₃ | D ₄ | D ₅ |
| U ₁₁ | F | FG | F | FG | F |
| U ₁₂ | F | P | FP | P | FP |
| U ₁₃ | P | FP | F | FP | P |
| U ₂₁ | F | FP | F | F | FP |
| U ₂₂ | VP | B.VP& P | VP | AP | VP |
| U ₃₁ | F | FG | F | FG | F |
| U ₄₁ | FP | F | FG | FP | F |
| U ₄₂ | B.VP& P | P | FP | B.VP& P | P |
| U ₅₁ | FG | G | B.G& VG | FG | G |
| U ₅₂ | F | FG | FG | G | F |
| U ₆₁ | VG | B.G& VG | G | B.G& VG | G |
| U ₆₂ | G | FG | B.G& VG | AG | VG |
| U ₆₃ | AG | VG | B.G& VG | G | VG |

(4) When threshold $\alpha = 0.7715$, $I_\alpha = 3.24$, there are five clusters:

$$\begin{aligned}
 P'_1 &= \{C26, C27, C28, C29, C30\}, \\
 P'_2 &= \{C31, C33, C34, C35, C36, C39\}, \\
 P'_3 &= \{C7, C8, C9, C10, C12, C14, C16\}, \\
 P'_4 &= \{C11, C13, C15, C17, C18, C19, C20\}, \\
 P'_5 &= \{C1, C2, C3, C4, C5, C6, C21, C22, C23, C24, C25, C32, \\
 &\quad C37, C38, C40\}.
 \end{aligned}$$

(5) When threshold $\alpha = 0.7981$, $I_\alpha = 2.52$, there are six clusters:

$$\begin{aligned}
 P'_1 &= \{C26, C27, C28, C29, C30\}, \\
 P'_2 &= \{C31, C33, C34, C35, C36, C39\}, \\
 P'_3 &= \{C7, C8, C9, C10, C12, C14, C16\}, \\
 P'_4 &= \{C11, C13, C15, C17, C18, C19, C20\}, \\
 P'_5 &= \{C3, C4, C5, C6\}, \\
 P'_6 &= \{C1, C4, C21, C22, C23, C24, C25, C32, C37, C38, C40\}.
 \end{aligned}$$

(6) When threshold $\alpha = 0.8251$, $I_\alpha = 3.17$, there are seven clusters:

$$\begin{aligned}
 P'_1 &= \{C26, C27, C28, C29, C30\}, \\
 P'_2 &= \{C31, C33, C34, C35, C36, C39\}, \\
 P'_3 &= \{C7, C8, C9, C10, C12, C14, C16\}, \\
 P'_4 &= \{C11, C13, C15, C17, C18, C19, C20\}, \\
 P'_5 &= \{C3, C4, C5, C6\}, \\
 P'_6 &= \{C32, C37, C38, C40\}, \\
 P'_7 &= \{C1, C4, C21, C22, C23, C24, C25\}.
 \end{aligned}$$

(7) When threshold $\alpha = 0.8416$, $I_\alpha = 3.49$, there are eight clusters:

$$\begin{aligned}
 P'_1 &= \{C26, C27, C28, C29, C30\}, \\
 P'_2 &= \{C31, C33, C34, C35, C36, C39\}, \\
 P'_3 &= \{C7, C8, C9, C10, C12, C14, C16\}, \\
 P'_4 &= \{C11, C13, C15, C17, C18, C19, C20\}, \\
 P'_5 &= \{C3, C4, C5, C6\}, \\
 P'_6 &= \{C32, C37, C38, C40\}, \\
 P'_7 &= \{C1, C4\}, \\
 P'_8 &= \{C21, C22, C23, C24, C25\},
 \end{aligned}$$

(8) When threshold $\alpha = 0.8733$, $I_\alpha = 4.11$, there are nine clusters:

$$\begin{aligned}
 P'_1 &= \{C26, C27, C28, C29, C30\}, \\
 P'_2 &= \{C31, C33, C34, C35, C36, C39\}, \\
 P'_3 &= \{C7, C8, C9, C10, C12, C14, C16\}, \\
 P'_4 &= \{C11, C13, C15, C17, C18, C19, C20\}, \\
 P'_5 &= \{C3, C4, C5, C6\}, \\
 P'_6 &= \{C32, C37, C38, C40\}, \\
 P'_7 &= \{C1, C4\}, \\
 P'_8 &= \{C21, C22\}, \\
 P'_9 &= \{C23, C24, C25\}.
 \end{aligned}$$

Table 3
Aggregate Evaluation Matrix for Clustering.

| Criteria | Aggregate evaluation for clustering | | | | | | | | | |
|----------------|-------------------------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 | C10 |
| U ₁ | 0.4930 | 0.4900 | 0.4958 | 0.4958 | 0.4991 | 0.4979 | 0.4169 | 0.4192 | 0.4171 | 0.4194 |
| U ₂ | 0.1765 | 0.1830 | 0.1728 | 0.1815 | 0.1752 | 0.1775 | 0.4514 | 0.4479 | 0.4438 | 0.4490 |
| U ₃ | 0.3171 | 0.3195 | 0.3171 | 0.3404 | 0.3046 | 0.3171 | 0.3751 | 0.3636 | 0.3751 | 0.3636 |
| U ₄ | 0.3283 | 0.3218 | 0.3220 | 0.3201 | 0.3283 | 0.3307 | 0.2781 | 0.2855 | 0.2851 | 0.2691 |
| U ₅ | 0.4995 | 0.4739 | 0.5084 | 0.4932 | 0.5011 | 0.4756 | 0.2474 | 0.2469 | 0.2468 | 0.2469 |
| U ₆ | 0.3416 | 0.3442 | 0.3431 | 0.3421 | 0.3429 | 0.3438 | 0.3416 | 0.3442 | 0.3429 | 0.3420 |
| C11–C20 | | | | | | | | | | |
| U ₁ | 0.3333 | 0.4197 | 0.3320 | 0.4158 | 0.3339 | 0.4165 | 0.3320 | 0.3339 | 0.3320 | 0.3330 |
| U ₂ | 0.2210 | 0.4479 | 0.2194 | 0.4562 | 0.2210 | 0.4479 | 0.2171 | 0.2151 | 0.2176 | 0.2208 |
| U ₃ | 0.3226 | 0.3898 | 0.3226 | 0.3751 | 0.3226 | 0.3767 | 0.3395 | 0.3320 | 0.3395 | 0.3320 |
| U ₄ | 0.1975 | 0.2781 | 0.1982 | 0.2812 | 0.1975 | 0.2812 | 0.2013 | 0.1971 | 0.2012 | 0.1971 |
| U ₅ | 0.6184 | 0.2468 | 0.6173 | 0.2469 | 0.6176 | 0.2468 | 0.6165 | 0.6184 | 0.6165 | 0.6184 |
| U ₆ | 0.5741 | 0.3431 | 0.5750 | 0.3429 | 0.5733 | 0.3426 | 0.5749 | 0.5820 | 0.5726 | 0.5786 |
| C21–C30 | | | | | | | | | | |
| U ₁ | 0.2663 | 0.2664 | 0.2669 | 0.2649 | 0.2674 | 0.2602 | 0.2565 | 0.2602 | 0.2586 | 0.2590 |
| U ₂ | 0.2711 | 0.2700 | 0.2717 | 0.2697 | 0.2714 | 0.3607 | 0.3594 | 0.3657 | 0.3579 | 0.3717 |
| U ₃ | 0.2185 | 0.2104 | 0.2275 | 0.2356 | 0.2104 | 0.5640 | 0.5515 | 0.5640 | 0.5515 | 0.5800 |
| U ₄ | 0.3588 | 0.3509 | 0.3588 | 0.3581 | 0.3501 | 0.2742 | 0.2695 | 0.2684 | 0.2862 | 0.2742 |
| U ₅ | 0.2762 | 0.2702 | 0.2762 | 0.2751 | 0.2762 | 0.4009 | 0.4080 | 0.4097 | 0.4080 | 0.4009 |
| U ₆ | 0.6983 | 0.6965 | 0.6985 | 0.6937 | 0.7078 | 0.2989 | 0.3015 | 0.3046 | 0.2992 | 0.3082 |
| C31–C40 | | | | | | | | | | |
| U ₁ | 0.3915 | 0.3903 | 0.3910 | 0.3942 | 0.3899 | 0.3927 | 0.3929 | 0.3899 | 0.3966 | 0.3916 |
| U ₂ | 0.2459 | 0.2434 | 0.2492 | 0.2439 | 0.2459 | 0.2501 | 0.2434 | 0.2459 | 0.2497 | 0.2434 |
| U ₃ | 0.2625 | 0.2442 | 0.2538 | 0.2356 | 0.2528 | 0.2452 | 0.2538 | 0.2442 | 0.2528 | 0.2452 |
| U ₄ | 0.5021 | 0.4942 | 0.4898 | 0.4943 | 0.4955 | 0.4898 | 0.4955 | 0.4942 | 0.4898 | 0.4947 |
| U ₅ | 0.2474 | 0.2463 | 0.2468 | 0.2429 | 0.2468 | 0.2429 | 0.2423 | 0.2474 | 0.2468 | 0.2469 |
| U ₆ | 0.4641 | 0.4659 | 0.4682 | 0.4664 | 0.4762 | 0.4638 | 0.4763 | 0.4623 | 0.4662 | 0.4709 |

According to the above initial clustering results, we apply Step 4 in the clustering algorithm procedure to adjust the samples between the clusters. Then we are able to calculate the clustering validity index to determine the final clustering results as follows:

(1) When threshold $\alpha = 0.6474, I_\alpha = 5.65$, there are two clusters:

$$\begin{aligned} P_1 &= \{C26, C27, C28, C29, C30\}, \\ P_2 &= \{C1, C2, C3, C4, C5, C6, C7, C8, C9, C10, C11, C12, C13, \\ &\quad C14, C15, C16, C17, C18, C19, C20, C21, C22, C23, C24, \\ &\quad C25, C31, C32, C33, C34, C35, C36, C37, C38, C39, C40\}. \end{aligned}$$

(2) When threshold $\alpha = 0.7288, I_\alpha = 4.41$, there are three clusters:

$$\begin{aligned} P_1 &= \{C26, C27, C28, C29, C30\}, \\ P_2 &= \{C31, C32, C33, C34, C35, C36, C37, C38, C39, C40\}, \\ P_3 &= \{C1, C2, C3, C4, C5, C6, C7, C8, C9, C10, C11, C12, C13, \\ &\quad C14, C15, C16, C17, C18, C19, C20, C21, C22, C23, C24, C25\}. \end{aligned}$$

(3) When threshold $\alpha = 0.7472, I_\alpha = 3.59$, there are four clusters:

$$\begin{aligned} P_1 &= \{C26, C27, C28, C29, C30\}, \\ P_2 &= \{C31, C32, C33, C34, C35, C36, C37, C38, C39, C40\}, \\ P_3 &= \{C7, C8, C9, C10, C12, C14, C16\}, \\ P_4 &= \{C1, C2, C3, C4, C5, C6, C11, C13, C15, C17, C18, C19, \\ &\quad C20, C21, C22, C23, C24, C25\}. \end{aligned}$$

(4) When threshold $\alpha = 0.7715, I_\alpha = 2.96$, there are five clusters:

$$\begin{aligned} P_1 &= \{C26, C27, C28, C29, C30\}, \\ P_2 &= \{C31, C32, C33, C34, C35, C36, C37, C38, C39, C40\}, \\ P_3 &= \{C7, C8, C9, C10, C12, C14, C16\}, \\ P_4 &= \{C11, C13, C15, C17, C18, C19, C20\}, \\ P_5 &= \{C1, C2, C3, C4, C5, C6, C21, C22, C23, C24, C25\}. \end{aligned}$$

(5) When threshold $\alpha = 0.7981, I_\alpha = 1.85$, there are six clusters:

$$\begin{aligned} P_1 &= \{C26, C27, C28, C29, C30\}, \\ P_2 &= \{C31, C32, C33, C34, C35, C36, C37, C38, C39, C40\}, \\ P_3 &= \{C7, C8, C9, C10, C12, C14, C16\}, \\ P_4 &= \{C11, C13, C15, C17, C18, C19, C20\}, \\ P_5 &= \{C1, C2, C3, C4, C5, C6\}, \\ P_6 &= \{C21, C22, C23, C24, C25\}. \end{aligned}$$

(6) When threshold $\alpha = 0.8733, I_\alpha = 3.13$, there are seven clusters:

$$\begin{aligned} P_1 &= \{C26, C27, C28, C29, C30\}, \\ P_2 &= \{C31, C32, C33, C34, C35, C36, C37, C38, C39, C40\}, \\ P_3 &= \{C7, C8, C9, C10, C12, C14, C16\}, \\ P_4 &= \{C11, C13, C15, C17, C18, C19, C20\}, \\ P_5 &= \{C1, C2, C3, C4, C5, C6\}, \\ P_6 &= \{C21, C22\}, \\ P_7 &= \{C23, C24, C25\}. \end{aligned}$$

As shown in Fig. 3, comparing the above final clustering results, we can find when $\alpha = 0.7981, I_\alpha = 1.85$ is the smallest value, indicating that the effectiveness of customer clustering is optimal in this situation.

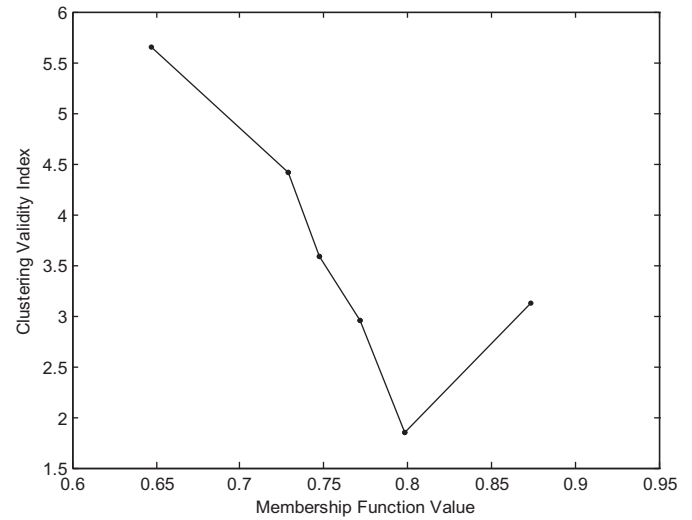


Fig. 3. Clustering Validity Index Diversification Diagram.

4.3. Algorithm comparisons

To further demonstrate the superiority of the proposed approach, three recent fuzzy clustering algorithms were compared with the proposed approach. Wang (2010) presented a clustering method based on the fuzzy equivalence relation for customer relationship management. Li et al. (2011b) proposed another comprehensive methodology to cluster logistics distribution centers using fuzzy comprehensive evaluation theory. Wang et al. (2012) developed a fuzzy clustering algorithm to study multiple distribution center locations problem, in their algorithm, the aggregated major attributes are transformed into three sub-attributes during clustering. Although the clustering algorithms proposed by Li et al. (2011b) and Wang et al. (2012) was initially used for multiple distribution center locations selection, these algorithms can still be applied to cluster the logistics customers. For comparison purposes, we implemented Wang's approach (2010), Wang et al.'s algorithm (2012) and Li's algorithm (2011b) into our case study in the same context for clustering these forty customers. The results are shown in Figs. 4a–d.

Through Wang's approach (2010), we can finally obtain six clusters. We were then able to calculate the clustering validity index as $I_\alpha = 3.25$. The clustering results are shown as follows:

$$\begin{aligned} P_1 &: \{C1, C2, C3, C6, C7, C8, C9, C10, C12, C14, C16\}, \\ P_2 &: \{C21, C22, C23, C24, C25\}, \\ P_3 &: \{C4, C5, C11, C13, C15, C17, C18, C19, C20\}, \\ P_4 &: \{C31, C32, C33, C34, C37\}, \\ P_5 &: \{C35, C36, C38, C39, C40\}, \\ P_6 &: \{C26, C27, C28, C29, C30\}. \end{aligned}$$

Li's approach generated four clusters, where the clustering validity index is $I_\alpha = 3.79$. The clustering results are shown as follows:

$$\begin{aligned} P_1 &: \{C1, C2, C3, C6, C7, C8, C9, C10, C12, C14, C16\}, \\ P_2 &: \{C4, C5, C11, C13, C15, C17, C18, C19, C20\}, \\ P_3 &: \{C21, C22, C23, C24, C25, C26, C27, C28, C29, C30\}, \\ P_4 &: \{C31, C32, C33, C34, C35, C36, C37, C38, C39, C40\}. \end{aligned}$$

Similarly, seven clusters are obtained based on Wang et al.'s algorithm (2012). The clustering validity index is $I_\alpha = 2.83$. The clustering results are shown as follows:

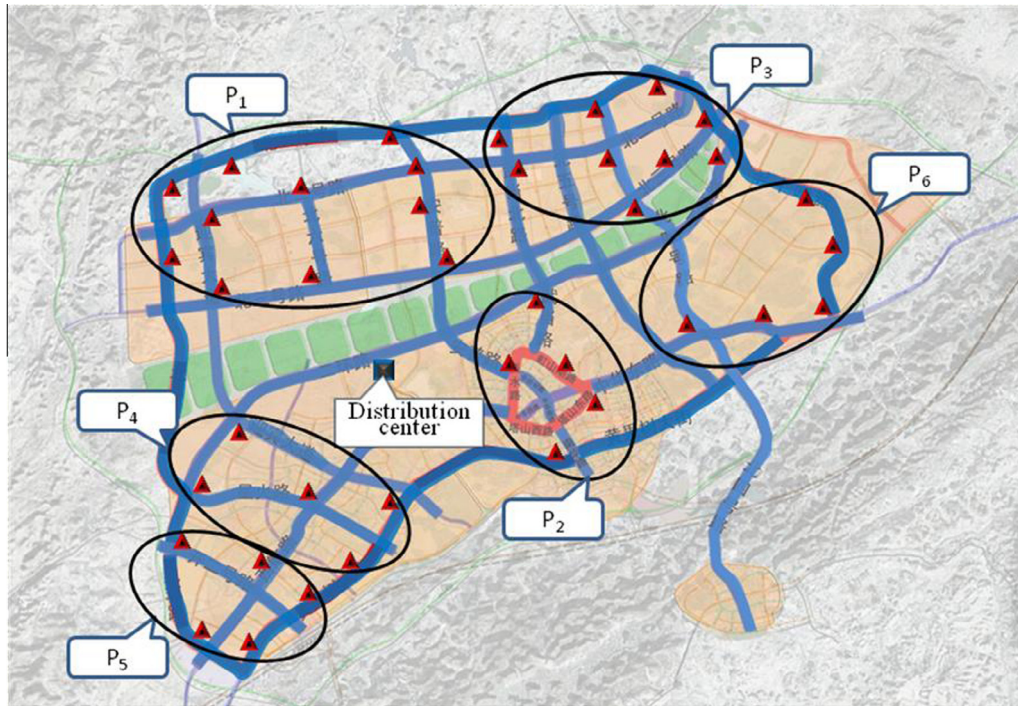


Fig. 4a. Clustering results of Wang (2010)'s approach.

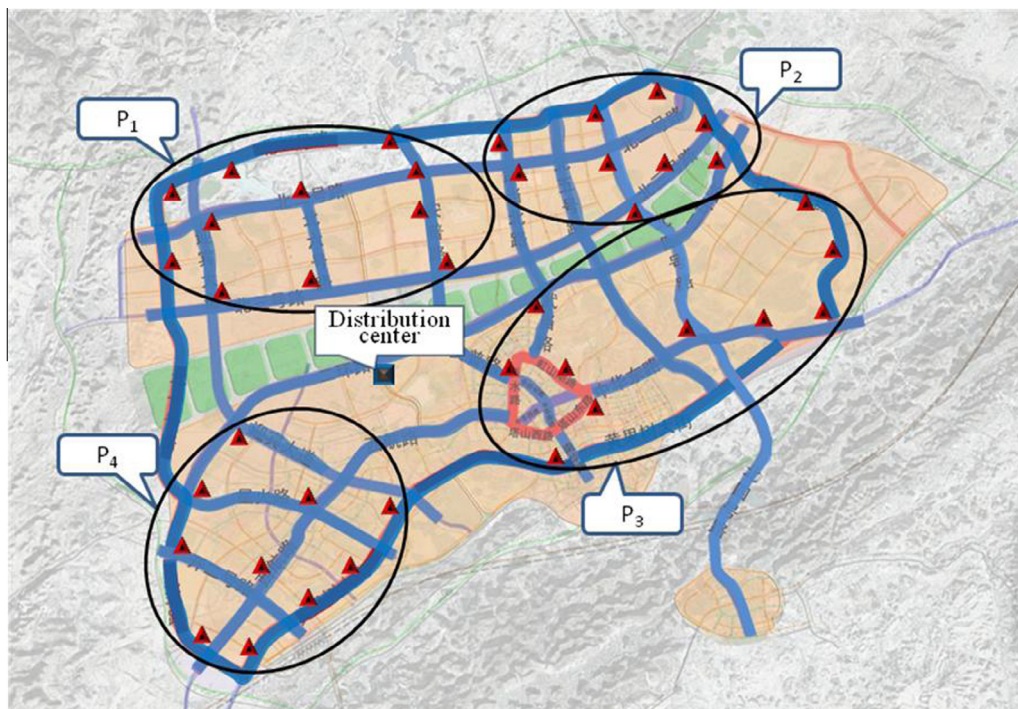


Fig. 4b. Clustering results of Li et al. (2011b)'s approach.

- $P_1 : \{C1, C2, C3, C6\},$
 $P_2 : \{C7, C8, C9, C10, C12, C14, C16\},$
 $P_3 : \{C4, C5, C11, C13, C15, C17, C18, C19, C20\},$
 $P_4 : \{C21, C22, C23, C24, C25, C30\},$
 $P_5 : \{C26, C27, C28, C29\},$
 $P_6 : \{C31, C33, C34, C35, C36, C39\},$
 $P_7 : \{C32, C37, C38, C40\}.$

As introduced in the Methodology section, the clustering validity index takes into account both the closeness within each cluster and the dispersion degree between other clusters, and thus it can be served to measure “goodness” of clustering results. The smaller the clustering validity index is, the better a clustering algorithm performs. From this standpoint, the proposed algorithm outperforms the other three algorithms with the lowest clustering validity index $I_\alpha = 1.85$. One reason that the proposed method performs

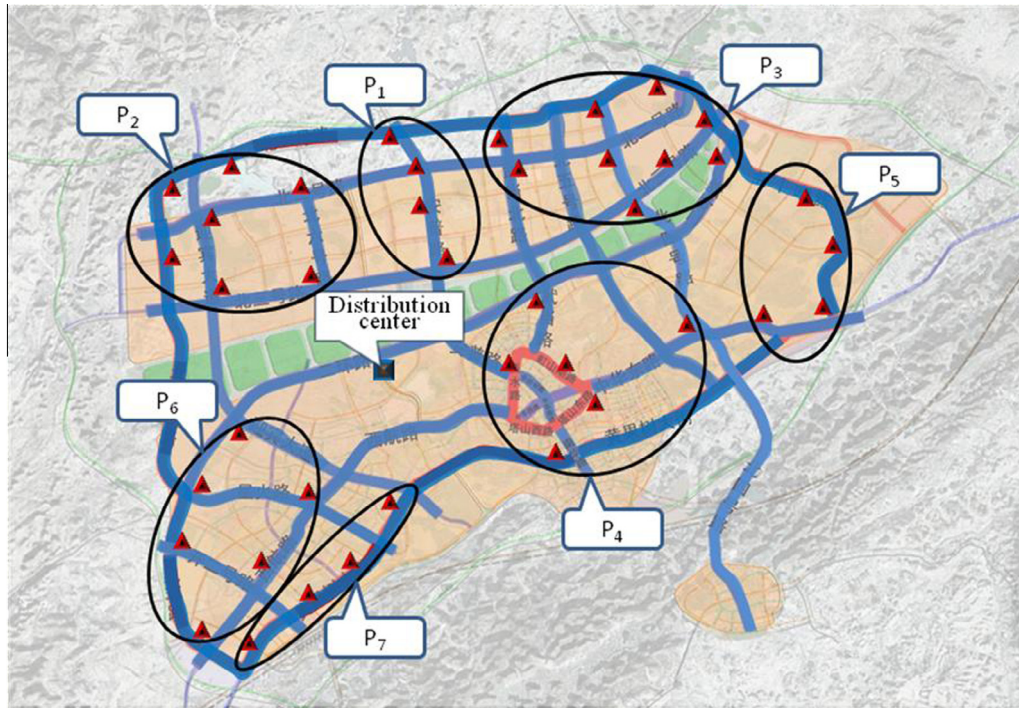


Fig. 4c. Clustering results of Wang et al.'s (2012) algorithm.

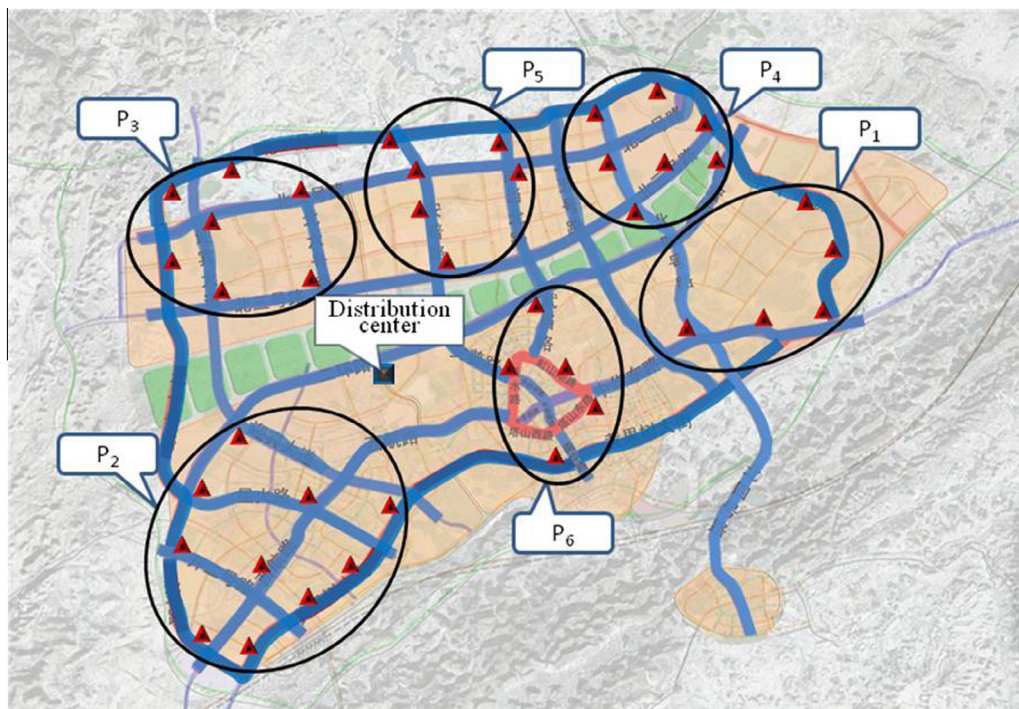


Fig. 4d. Clustering results of the proposed algorithm.

better is that it can handle more attributes and be able to split the characteristics of each customer into finer attributes in the clustering procedure. These finer attributes increase the heterogeneity between customers. Therefore, the proposed method is able to better distinguish the difference between customers than previous approaches. The other reason is that the customer characteristics

ratings are included in the clustering process. These ratings help the method capture the level of importance for each attribute and the similarity among customers.

For evaluating the effectiveness of the customer clustering results, further investigation was conducted. Five decision makers with many years management experience are required to perform

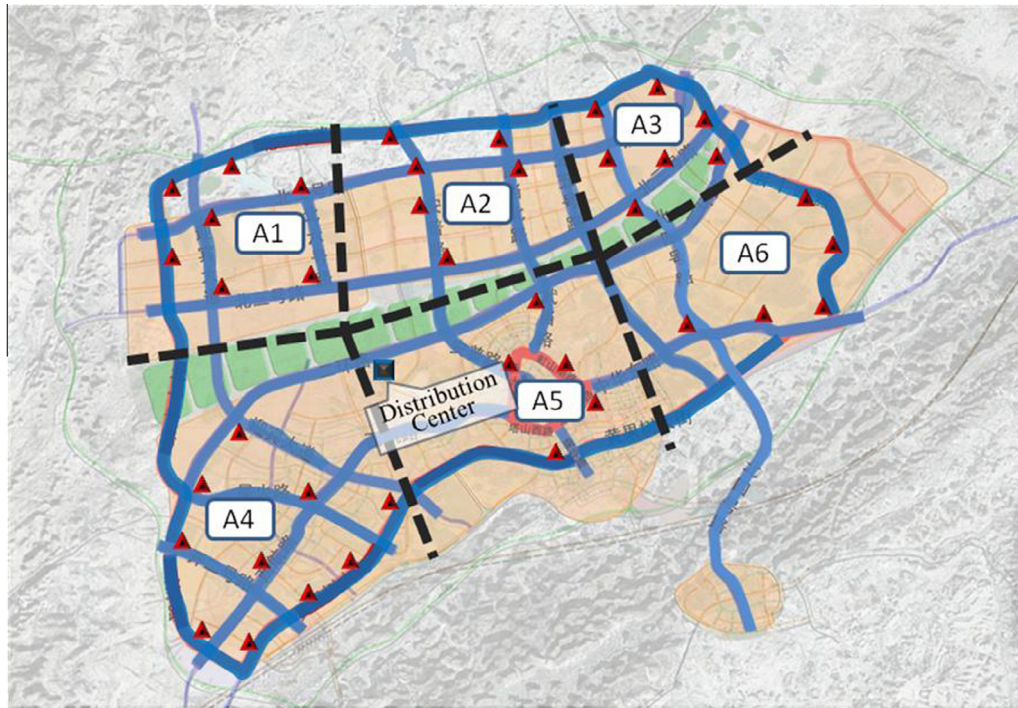


Fig. 5. Business areas within Anshun City.

customer clustering for the business planning purpose in Anshun City. The decision makers identified 6 distinct areas for clustering. These 6 areas are shown in Fig. 5:

- A1: Electronic business district
- A2: Vegetable market area
- A3: Restaurant area
- A4: Bulk material district
- A5: Living material shopping district
- A6: Apparel trading area

Based on the actual clustered results, the following findings are presented:

- The proposed method can capture the similarity of important attributes and group the customers with these attributes together. For example, specific requirement on the delivery time is an important attribute for the customers in A2 (Vegetable market area). The proposed method groups these customers into P5, whereas Wang's algorithm (2010) separated them into P1 and P3, Li's algorithm (2011b) separated them into P1 and P2, and Wang et al.'s algorithm (2012) separated them into P1 and P3.
- The proposed method can better distinguish the difference between customers than previous approaches. One example is that Li's algorithm (2011b) groups A5 (Living material shopping district) and A6 (Apparel trading area) into one group whereas the proposed method splits them into two clusters. Another example is that Wang's algorithm (2010) groups A1 (Electronic business district) and some customers from A2 (vegetable market) together whereas the proposed method separates them.

In summary, the proposed algorithm yields more reasonable results by considering more attributes associated with each customer. In reality, the logistics operators can arrange the corresponding vehicles to serve the customers within each cluster, and conduct an in-depth vehicle routing algorithm to better

optimize the entire logistics network. Proper customer clustering strategies can ultimately improve logistics companies by reducing operational costs, improving customer satisfaction degree, and reducing urban traffic congestion and air pollution. In addition, customer clustering can greatly reduce the complexity of VRP formulation (Hu & Sheu, 2003). In the metropolitan logistics network with thousands of customers, traditional approach for VRP may not be effective to cope with such a sophisticated scenario (Lee, Dong, & Bian, 2010; Qi, Lin, Li, & Miao, 2012).

5. Conclusions

Customer clustering is of critical significance for logistics distribution network planning. It is considered as an effective countermeasure to enhance system efficiency, lower operational cost, increase customer satisfaction, and reduce the complexity of vehicle routing problem. Thus, it is particularly suitable for large-scale metropolitan logistics distribution network optimization. However, few studies have been specifically done to address this issue. There are numerous factors influencing the clustering procedure, and the importance of each factor varies with each other. This paper provides a novel approach to cluster customers with similar characteristics under a hierarchical analysis structure. The hierarchical analysis structure is able to depict each customer's attributes by major and minor criteria, and then, each criterion is represented by a linguistic variable. Based on the fuzzy set theory, each linguistic variable is further translated as the corresponding trapezoidal fuzzy numbers. To quantitatively consider the impacts from all criteria, fuzzy integration algorithm is proposed. An improved AFS approach based on fuzzy clustering algorithm is used to group customers with similar characteristics into multiple clusters. To evaluate the effectiveness of clustering performance, a clustering validity index is utilized to measure both dispersion degree between clusters and closeness within each cluster.

The proposed method has been successfully applied to assist logistics operators clustering customers for a logistics company

in Anshun City, China. For the comparison purpose, this research also implemented three recently proposed methods: a clustering method based on the fuzzy equivalence relation from Wang (2010), a fuzzy comprehensive evaluation method from Li et al. (2011b) and a fuzzy clustering algorithm for location selection from Wang et al. (2012). Compared with these three prevailing algorithms, the proposed customer clustering algorithm better captures the similarity between customers, and has demonstrated its capability to conduct the complex customer clustering in a modern city logistics distribution network. Two possible reasons illustrate the superiority of the proposed approach. One reason is that it can handle more attributes and be able to split the characteristics of each customer into finer attributes in the clustering procedure. The finer attributes help to better distinguish the differences between customers than previous approaches. The other reason is that the clustering process in the proposed method includes the customer characteristics ratings, which contributes to capture the level of importance for each attribute and the similarity among customers.

Further model validation from five decision makers in Anshun City was conducted. The results showed that the clustered results from the proposed method matched the actual customer regions in the real world well. The proposed algorithm can also be extended to solve other relevant clustering problems in other domains. Several potential applications include: (1) E-supply chain efficiency maximization for e-commerce (Bidhandi et al., 2009); (2) transportation travel pattern and regularity clustering (Taaffe et al., 2010); (3) transit market segmentation analysis (Ma, Wu, Wang, Chen, & Liu, 2013); (4) customer payment behavior clustering in the field of telecommunication (Chen et al., 2013); and (5) retail customer relationship management (CRM) (Cheng & Chen, 2009), etc.

To enhance the usability and depth of this study, further research can be conducted to incorporate more factors into the logistics network design, such as modeling the dynamic and uncertain characteristics of customers' demands, clustering customers with multiple distribution centers, and integrating both vehicle routing (VRP) problem and logistics network optimization problem.

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