Quadratura de Gauss-Legendre para o cálculo da área de um superfície

Claudemir Woche e Márcio Barros

29 de Maio de 2020

1 O Problema

A região $U \in xy$ é $U = \Big\{(x, y) \ \in \ \frac{x^2}{1600} + \frac{y^2}{1600} \leq 1\Big\}$

2 Mudança de variável 1

$$\int_{-40}^{40} \int_{-\sqrt{40^2-x^2}}^{\sqrt{40^2-x^2}} \sqrt{(0,4x)^2 + (0,4y)^2 + 1} \ dy \ dx$$

$x = \alpha R \cos{(\beta)} = 40\alpha \cos{(\beta)}$
$y = \alpha R \sin{(\beta)} = 40\alpha \sin{(\beta)}$

$|J| = R^2\alpha = 1600\alpha$

3 Mudança de variável 2

$$\int_{0}^{1} \int_{0}^{2\pi} \sqrt{256\alpha^2 + 1} \ 1600\alpha \ d\beta \ d\alpha$$

$\alpha = \pi + \pi h$
$\beta = \frac{1}{2} + \frac{1}{2}k$

$$|J| = \frac{\pi}{2}$$

4 Quadratura de Gauss-Legendre com 3 pontos em cada direção

$$\int_{-1}^{1} \int_{-1}^{1} 800\pi \sqrt{256\big(1/2 + (1/2)k\big)^2 + 1} \ \left(\frac{1}{2} + \frac{1}{2}k\right) \ dk \ dh$$

$$\varphi(k) = 800\pi \sqrt{256\big(1/2 + (1/2)k\big)^2 + 1} \ \left(\tfrac{1}{2} + \tfrac{1}{2}k\right)$$

$$\text{Raízes} = \text{k} = \left\{ -\sqrt{\tfrac{3}{5}}; 0; \sqrt{\tfrac{3}{5}} \right\}$$

$$\varphi(-\sqrt{3/5}) = -16\sqrt{2585 - 640\sqrt{15}} \ \big(\sqrt{15} - 5\big)\pi \approx 584.05$$

$$\varphi(0) = 400\sqrt{65}\pi \approx 10131.33$$

$$\varphi(\sqrt{3/5}) = 800\left(1/2 + (\sqrt{3/5})/2\right)\sqrt{1 + 256\left(1/2 + (\sqrt{\tfrac{3}{5}})/2\right)^2}\pi \approx 31737.59$$

$$\sum_{i=1}^{3}\sum_{j=1}^{3} w_i w_j \varphi(k_j) =$$

$$\frac{25}{81}\varphi(-\sqrt{3/5}) + \frac{40}{81}\varphi(0) + \frac{25}{81}\varphi(\sqrt{3/5}) +$$

$$\frac{40}{81}\varphi(-\sqrt{3/5}) + \frac{64}{81}\varphi(0) + \frac{40}{81}\varphi(\sqrt{3/5}) +$$

$$\frac{25}{81}\varphi(-\sqrt{3/5}) + \frac{40}{81}\varphi(0) + \frac{25}{81}\varphi(\sqrt{3/5}) = \frac{90}{81}\varphi(-\sqrt{3/5}) + \frac{144}{81}\varphi(0) + \frac{90}{81}\varphi(\sqrt{3/5})$$

$$\approx 53924,183$$