Quadratura de Gauss-Legendre para o cálculo do volume abaixo de uma superfície

Claudemir Woche e Márcio Barros

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1 O Problema

A região $U \in xy$ é $U = \left\{(x,y) \ \in \ \frac{x^2}{1600} + \frac{y^2}{400} \leq 1\right\}$

2 Mudança de variável 1

$$\int_{-40}^{40}\int_{-\frac{1}{2}\sqrt{40^2-x^2}}^{\frac{1}{2}\sqrt{40^2-x^2}} 0,2(x^2-y^2)\ dy\ dx$$

$x = \alpha R\cos(\beta) = 40\alpha\cos(\beta)$
$y = \alpha R\sin(\beta) = 20\alpha\sin(\beta)$

$|J| = 800\alpha$

3 Mudança de variável 2

$$\int_0^{2\pi}\int_0^1 0,2\big((\alpha 40)^2\cos^2\beta - (\alpha 20)^2\sin^2\beta\big)\ \alpha 800\ d\alpha\ d\beta$$

$\alpha = \frac{1}{2} + \frac{1}{2}h$
$\beta = \pi + \pi k$

$$|J| = \frac{\pi}{2}$$

4 Quadratura de Gauss-Legendre com 3 pontos em cada direção

$$\int_{-1}^{1}\int_{-1}^{1} 0,2\Bigg(\big((1/2+(1/2)h)40\cos(\pi+\pi k)\big)^2 - \big((1/2+(1/2)h)20\sin(\pi+\pi k)\big)^2\Bigg)$$
$$\Bigg(1/2+(1/2)(\pi+\pi k)\Bigg)800\frac{\pi}{2}\ dh\ dk$$

$$\varphi(h,k) = 0,2\Bigg(\big((1/2+(1/2)h)40\cos(k)\big)^2 - \big((1/2+(1/2)h)20\sin(k)\big)^2\Bigg)\Bigg(1/2+(1/2)k\Bigg)800\frac{\pi}{2}$$

Raízes = k = h = $\left\{ -\sqrt{\frac{3}{5}}; 0; \sqrt{\frac{3}{5}} \right\}$

$$\sum_{i=1}^{3}\sum_{j=1}^{3} w_i w_j \varphi(h_j, k_j) =$$

$$\frac{25}{81}\left(\varphi\left(-\sqrt{\frac{3}{5}}, -\sqrt{\frac{3}{5}}\right) + \varphi\left(-\sqrt{\frac{3}{5}}, \sqrt{\frac{3}{5}}\right) + \varphi\left(\sqrt{\frac{3}{5}}, -\sqrt{\frac{3}{5}}\right) + \varphi\left(\sqrt{\frac{3}{5}}, \sqrt{\frac{3}{5}}\right)\right) +$$

$$\frac{40}{81}\left(\varphi\left(-\sqrt{\frac{3}{5}}, 0\right) + \varphi\left(0, -\sqrt{\frac{3}{5}}\right) + \varphi\left(0, \sqrt{\frac{3}{5}}\right) + \varphi\left(\sqrt{\frac{3}{5}}, 0\right)\right) +$$

$$\frac{64}{81}\varphi(0,0) =$$

$$= \frac{25}{81}\left(271,245 + 271,245 + 132.367 + 132.367\right) +$$

$$\frac{40}{81}\left(575,64 + 23.685,4 + 23.685,4 + 280.911\right) +$$

$$\frac{64}{81}\left(50.205,5\right) = 283.990,148$$