Trabalho 1 - Tópicos Avançados em Aprendizagem de Máquina - Regressão Linear Bayesiana

Claudemir Woche Vasconcelos Carvalho

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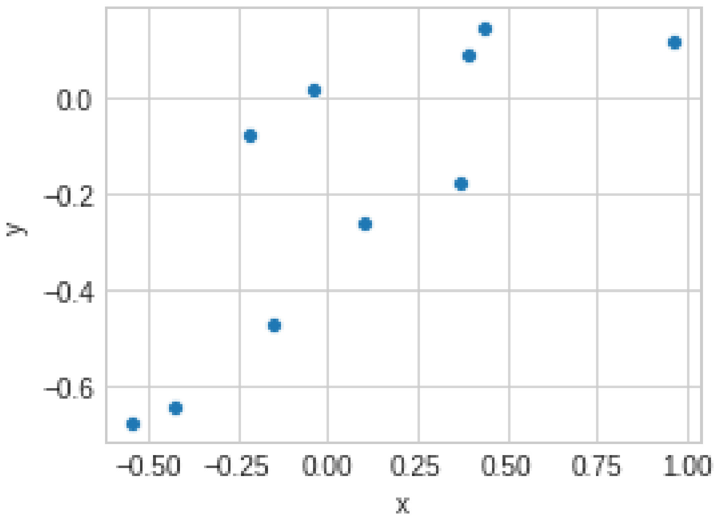
1 Importação de bibliotecas

[1]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
from matplotlib import rcParams
rcParams['figure.figsize'] = (4,3)
plt.style.use("seaborn-whitegrid")
```

2 Importação e visualização dos dados

[2]:
```
dataset = pd.read_csv('linear_regression_data.csv',header=None).
    ↪rename(columns={0:'x',1:'y'})
```

[3]:
```
sns.scatterplot(dataset['x'],dataset['y'])
plt.show()
```



3 Pré-processamento dos dados

[4]:
```
X = np.hstack([np.ones(dataset.shape[0]).reshape(-1,1),dataset['x'].
    ↪values.reshape(-1,1)])
y = dataset['y'].values.reshape(-1,1)
n_lin,n_col = X.shape
print('X = \n',X)
print('\ny = \n',y)
```

```
X =
 [[ 1.          0.39293837]
 [ 1.         -0.42772133]
 [ 1.         -0.54629709]
 [ 1.          0.10262954]
 [ 1.          0.43893794]
 [ 1.         -0.15378708]
 [ 1.          0.9615284 ]
 [ 1.          0.36965948]
 [ 1.         -0.0381362 ]
 [ 1.         -0.21576496]]

y =
 [[ 0.08635962]
 [-0.64387173]
 [-0.67498147]
 [-0.26289158]
 [ 0.14317741]
 [-0.47272884]
 [ 0.1141669 ]
 [-0.18032295]
 [ 0.01182141]
 [-0.07986457]]
```

4 Passo de estimação

4.1 Definições a partir de conhecimentos/experimentos anteriores

Os momentos da priori $p(\mathbf{w}) = \mathcal{N}(\mathbf{w}|\mathbf{m_0}, \mathbf{S_0})$

$\rightarrow$ Definirei $\mathbf{m_0} = [0\ 0]$ e $\mathbf{S_0} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

A variância do ruıdo $p(\epsilon) = \mathcal{N}(\epsilon|0, \sigma^2)$

$\rightarrow$ Definirei $\sigma^2 = 1$

[5]:
```
m0 = np.zeros(n_col).reshape(-1,1)
S0 = np.diag((1,)*n_col)
sigma_ruido = 1
```

4.2 Cálculo da posteriori de w

$p(\mathbf{w}|\mathcal{D}) = \mathcal{N}(\mathbf{w}|\boldsymbol{\mu}, \boldsymbol{\Sigma})$

$\boldsymbol{\mu} = \mathbf{m_0} + (\mathbf{S_0}\mathbf{X}^T\mathbf{X} + \sigma^2\mathbf{I})^{-1}\ \mathbf{S_0}\mathbf{X}^T(\mathbf{y} - \mathbf{X}\mathbf{m_0})$

[6]:
```
u_1 = np.linalg.inv(S0.dot(X.T).dot(X) + np.eye(n_col)*sigma_ruido)
u_2 = S0.dot(X.T).dot(y - X.dot(m0))
```

```
u = m0 + u_1.dot(u_2)
print(u)
```

```
[[-0.20591704]
 [ 0.34610423]]
```

$\mathbf{\Sigma} = \mathbf{S_0} - (\mathbf{S_0}\mathbf{X}^T\mathbf{X} + \sigma^2\mathbf{I})^{-1}\ \mathbf{S_0}\mathbf{X}^T\mathbf{X}\mathbf{S_0}$

[7]:
```
sigma_1 = u_1
sigma_2 = S0.dot(X.T).dot(X).dot(S0)
sigma = S0 + sigma_1 * sigma_2
print(sigma)
```

```
[[ 1.93135423 -0.02448966]
 [-0.02448966  1.67975609]]
```

5 Passo de predição

Distribuição preditiva

$\rightarrow p(\mathbf{y}_*|\mathbf{X}_*) = \mathcal{N}(\mathbf{y}_*|\mathbf{X}_*\boldsymbol{\mu}\ ,\ \mathbf{X}_*\mathbf{\Sigma}\mathbf{X}_*{}^T + \sigma^2\mathbf{I})$

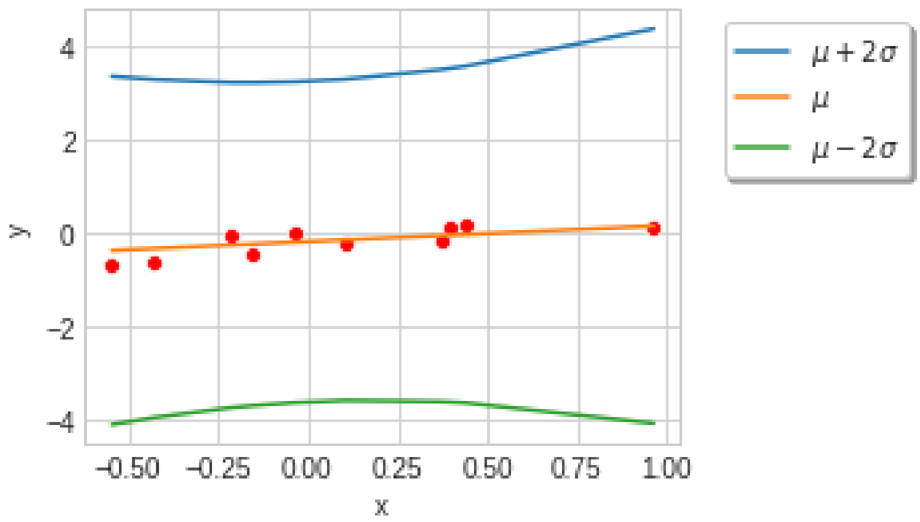
[8]:
```
u_final = X.dot(u).flatten()
sigma_final = np.diag(X.dot(sigma).dot(X.T) + sigma_ruido * np.
  ↪eye(n_lin))
print('u_final = \n',u_final)
print('\nsigma_final = \n',sigma_final)
```

```
u_final =
 [-0.06991941 -0.3539532  -0.39499277 -0.17039652 -0.05399876 -0.2591434
  0.126872   -0.07797633 -0.21911614 -0.28059421]

sigma_final =
 [3.17146367 3.25960761 3.45941876 2.94402008 3.23348811 2.97861363
 4.43725565 3.14278409 2.9356651  3.02012249]
```

[9]:
```
banda = 2 * np.sqrt(sigma_final).reshape(-1,1).flatten()
banda_mais = u_final + banda
banda_menos = u_final - banda
```

[10]:
```
sns.scatterplot(dataset['x'],dataset['y'],color='r')
sns.lineplot(dataset['x'],banda_mais,label='$\mu+2 \sigma$')
sns.lineplot(dataset['x'],u_final,label='$\mu$')
sns.lineplot(dataset['x'],banda_menos,label='$\mu-2 \sigma$')
plt.legend(bbox_to_anchor=(1.05, 1), loc=2,shadow=True,frameon=True)
plt.show()
```



4
2
0
−2
−4
y
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
x
$\mu + 2\sigma$
μ
$\mu - 2\sigma$