Tópicos Avançados em Aprendizagem de Máquina - Trabalho 1 - Regressão Polinomial Bayesiana

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1 Importação de bibliotecas

[18]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
from matplotlib import rcParams
rcParams['figure.figsize'] = (10,10)
plt.style.use("seaborn-whitegrid")
np.set_printoptions(precision=5,suppress=True)
```

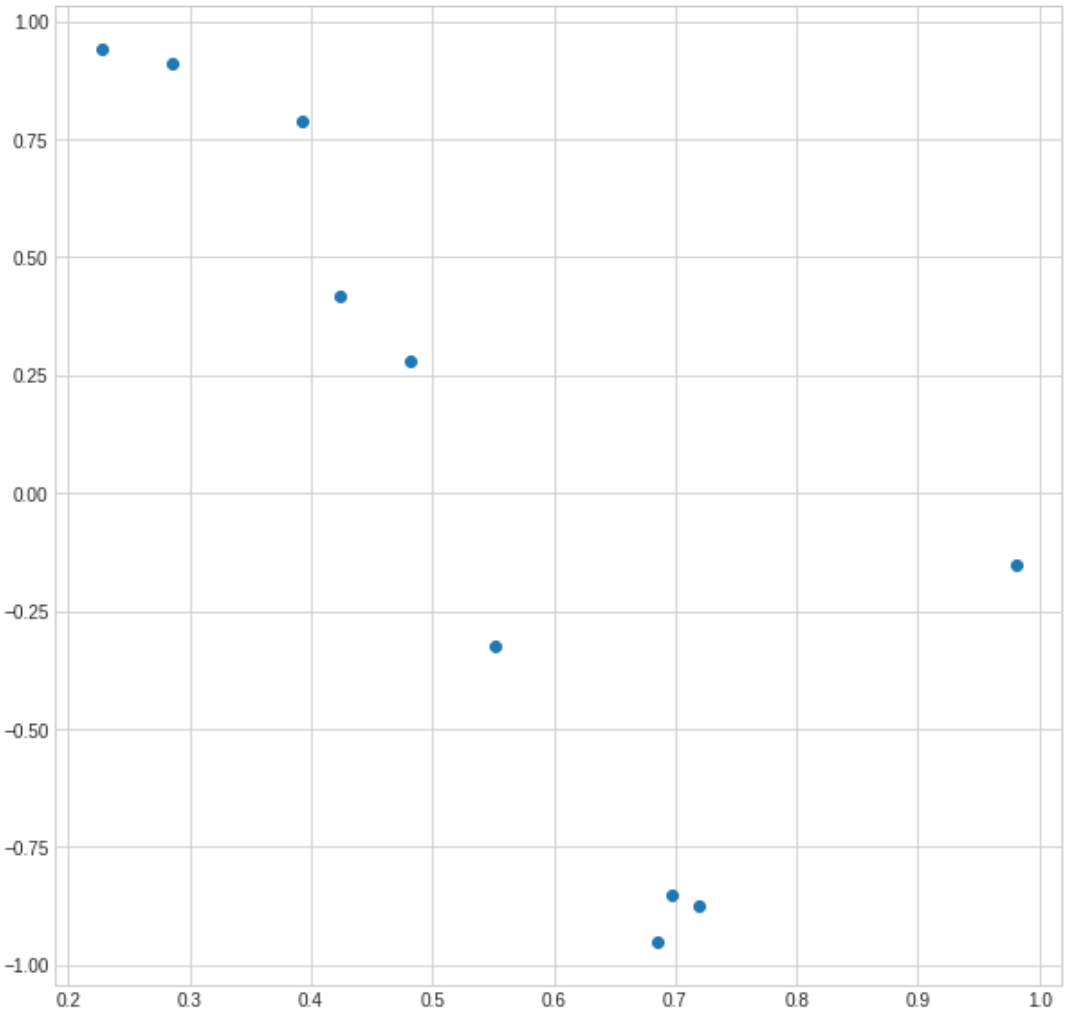
2 Importação e visualização dos dados

[2]:
```
dataset = pd.read_csv('polynomial_regression_data.csv',header=None).
 ↪rename(columns={0:'x',1:'y'})
dataset
```

[2]:

	x	y
0	0.696469	-0.851271
1	0.286139	0.910864
2	0.226851	0.939731
3	0.551315	-0.323798
4	0.719469	-0.872451
5	0.423106	0.417776
6	0.980764	-0.153078
7	0.684830	-0.949134
8	0.480932	0.281049
9	0.392118	0.787241

[19]:
```
plt.scatter(dataset['x'],dataset['y'])
plt.show()
```



3 Pré-processamento dos dados

[5]:
```
x = dataset['x'].values.reshape(-1,1)
y = dataset['y'].values.reshape(-1,1)
p = 6
x
```

[5]:
```
array([[0.69647],
       [0.28614],
       [0.22685],
       [0.55131],
       [0.71947],
       [0.42311],
       [0.98076],
       [0.68483],
       [0.48093],
```

```
        [0.39212]])
```

[6]:
```
X = np.ones(dataset.shape[0]).reshape(-1,1)
for i in range (1,p):
    arr = x ** i
    X = np.hstack([X,arr.reshape(-1,1)])

X
```

[6]:
```
array([[1.     , 0.69647, 0.48507, 0.33784, 0.23529, 0.16387],
       [1.     , 0.28614, 0.08188, 0.02343, 0.0067 , 0.00192],
       [1.     , 0.22685, 0.05146, 0.01167, 0.00265, 0.0006 ],
       [1.     , 0.55131, 0.30395, 0.16757, 0.09238, 0.05093],
       [1.     , 0.71947, 0.51764, 0.37242, 0.26795, 0.19278],
       [1.     , 0.42311, 0.17902, 0.07574, 0.03205, 0.01356],
       [1.     , 0.98076, 0.9619 , 0.9434 , 0.92525, 0.90745],
       [1.     , 0.68483, 0.46899, 0.32118, 0.21995, 0.15063],
       [1.     , 0.48093, 0.2313 , 0.11124, 0.0535 , 0.02573],
       [1.     , 0.39212, 0.15376, 0.06029, 0.02364, 0.00927]])
```

4 Passo de estimação

4.1 Definições a partir de conhecimentos/experimentos anteriores

Os momentos da priori $p(\mathbf{w}) = \mathcal{N}(\mathbf{w}|\mathbf{m_0}, \mathbf{S_0})$.

$\rightarrow$ Definirei $\mathbf{m_0} = [0\,0\,0\,0\,0\,0]$ e $\mathbf{S_0} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

A variância do ruído $p(\epsilon) = \mathcal{N}(\epsilon|0, \sigma^2)$

$\rightarrow$ Usarei $\sigma^2 = \{1\,, 5e-2\,, 1e-2\,, 1e-4\,, 1e-6\,, 1e-8\}$

[7]:
```
m0 = np.zeros(p).reshape(-1,1)
S0 = np.eye((p))
sigma_ruidos = [1,5E-2,1E-2,1E-4,1E-6,1E-8]
```

4.2 Cálculo da posteriori de w e da distribuição preditiva

Posteriori de w

$\rightarrow p(\mathbf{w}|\mathcal{D}) = \mathcal{N}(\mathbf{w}|\boldsymbol{\mu}, \boldsymbol{\Sigma})$

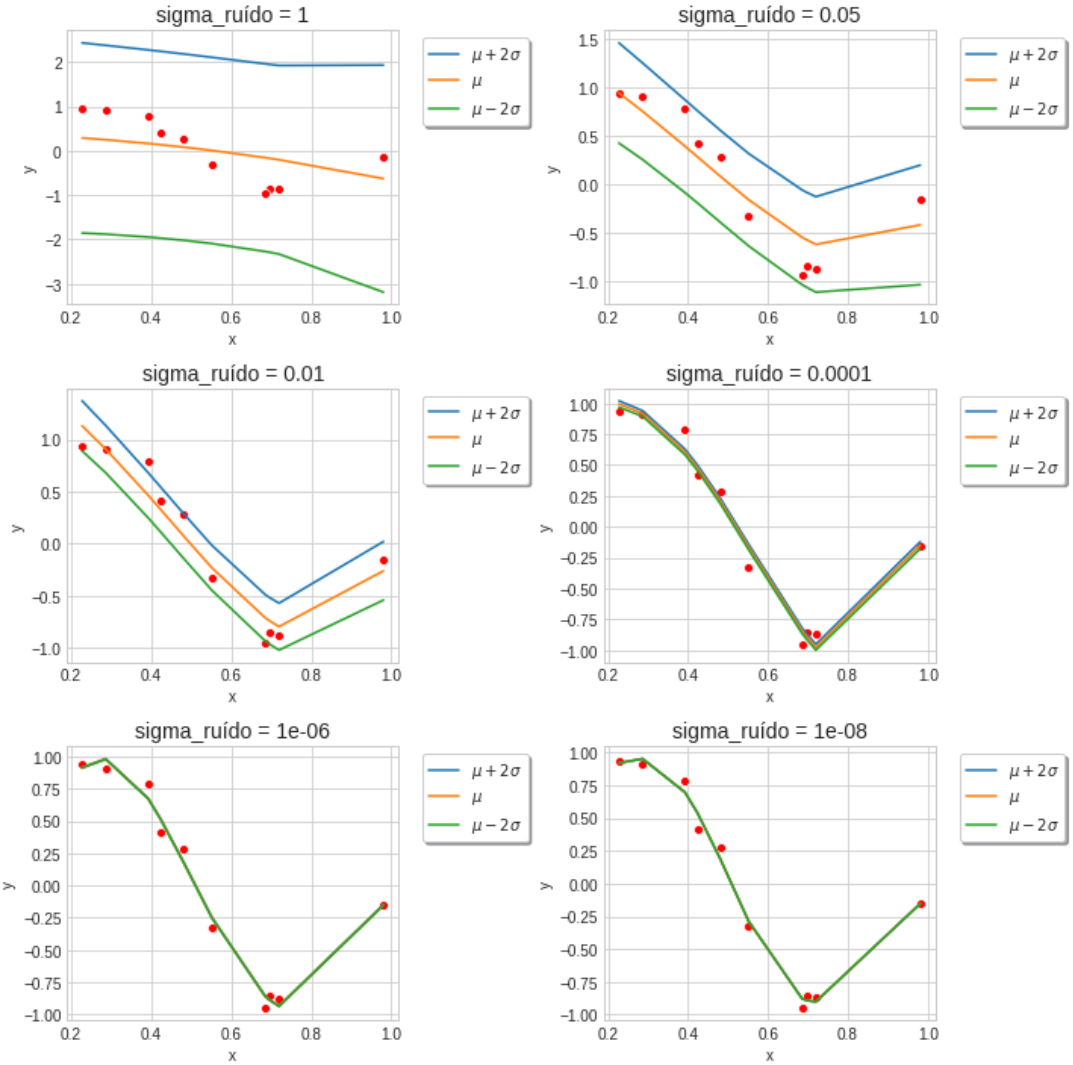
$\boldsymbol{\mu} = \mathbf{m_0} + (\mathbf{S_0}\mathbf{X}^T\mathbf{X} + \sigma^2\mathbf{I})^{-1}\,\mathbf{S_0}\mathbf{X}^T(\mathbf{y} - \mathbf{X}\mathbf{m_0})\,,$

$\boldsymbol{\Sigma} = \mathbf{S_0} - (\mathbf{S_0}\mathbf{X}^T\mathbf{X} + \sigma^2\mathbf{I})^{-1}\,\mathbf{S_0}\mathbf{X}^T\mathbf{X}\mathbf{S_0}$

Distribuição preditiva

$\rightarrow p(\mathbf{y}_*|\mathbf{X}_*) = \mathcal{N}(\mathbf{y}_*|\mathbf{X}_*\boldsymbol{\mu}\,,\, \mathbf{X}_*\boldsymbol{\Sigma}{\mathbf{X}_*}^T + \sigma^2\mathbf{I})$

[20]:
```
f, axes = plt.subplots(3, 2)
for i in range(3):
    for j in range(2):
        sigma_ruido = sigma_ruidos[i*2+j]

        u_1 = np.linalg.inv(S0.dot(X.T).dot(X) + np.eye(p)*sigma_ruido)
        u_2 = S0.dot(X.T).dot(y - X.dot(m0))
        u = m0 + u_1.dot(u_2)

        sigma_1 = u_1
        sigma_2 = S0.dot(X.T).dot(X).dot(S0)
        sigma = S0 - (sigma_1.dot(sigma_2))

        u_final = X.dot(u).flatten()
        sigma_final = np.diag(X.dot(sigma).dot(X.T) + np.
 ↪eye(10)*sigma_ruido )

        banda = 2 * np.sqrt(sigma_final).reshape(-1,1).flatten()
        banda_mais = u_final + banda
        banda_menos = u_final - banda

        axes[i,j].set_title(f"sigma_ruído = {sigma_ruido}", fontsize=14)
        sns.
 ↪scatterplot(dataset['x'],dataset['y'],ax=axes[i,j],color='r')
        sns.lineplot(dataset['x'],banda_mais,ax=axes[i,j],label='$\mu+2␣
 ↪\sigma$')
        sns.lineplot(dataset['x'],u_final,ax=axes[i,j],label='$\mu$')
        sns.
 ↪lineplot(dataset['x'],banda_menos,ax=axes[i,j],label='$\mu-2␣
 ↪\sigma$')
        axes[i,j].legend(bbox_to_anchor=(1.05, 1),␣
 ↪loc=2,shadow=True,frameon=True)

plt.tight_layout()
```


sigma_ruído = 1
sigma_ruído = 0.05
sigma_ruído = 0.01
sigma_ruído = 0.0001
sigma_ruído = 1e-06
sigma_ruído = 1e-08
$\mu + 2\sigma$
μ
$\mu - 2\sigma$
x
y