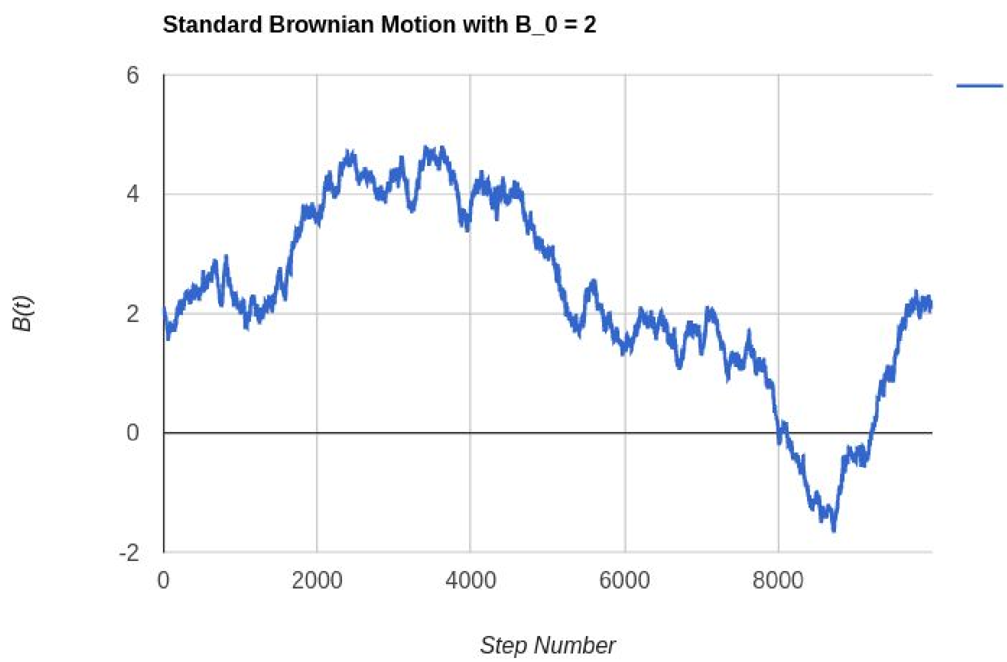
Abstract
For this exercise, we simulate stochastic processes including scaled random walk, standard Brownian Motion(BM) and geometric Brownian motion, and then check some of the properties of these processes including expected value and variance/ volatility. Plots of the simulations are attached in this report.

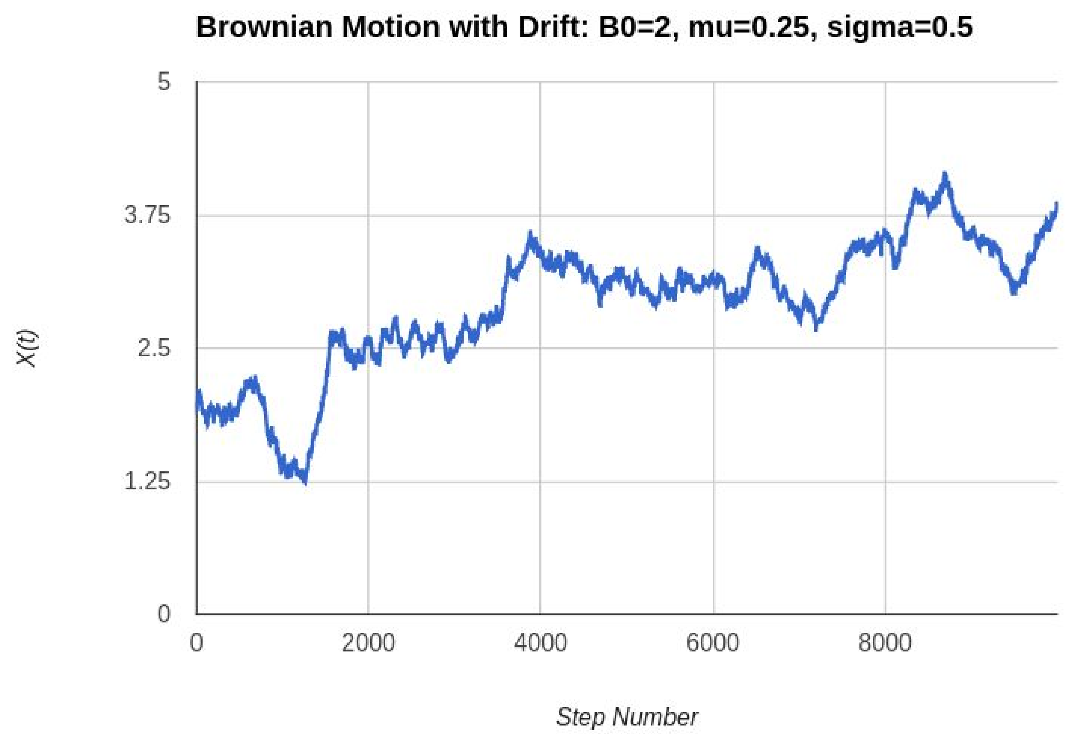
Execution Instruction
To run the code, please execure **./runBrownian.sh** at the project root. This script encapsulates the sequence of compiling and running the program. You will receive a prompt as to what exercise to run. Each question in the homework is matched with a number in the program. For instance, to run question 1, select 1.

Tasks:

1. Scaled Random Walk: Generated data can be found in *ascaledrandomwalk.xls* at the root of the project directory.
2. Standard BM: Generated data can be found in *astandardbrownianmotion.xls* at the root of the project directory. Here is a sample excel plot:



3. BM with Drift: Generated data can be found in *abrownianmotion.xls* at the root of the project directory. Here is a sample excel plot:



4. Geometric BM: Generated data can be found in *ageometricbrownianmotion.xls* at the root of the project directory. Here is a sample excel plot:

5. Checking properties of BM without drift
   For standard BM $B(t_i) = B(t_{i-1}) + \sqrt{t_i - t_{i-1}}U_i$, each increment is normally distributed with expected value, 0 and variance t_i. To check this, we run 100000 BM simulations for time 0 to 10 partitioned into 1000 steps. This means $dt = t_i - t_{i-1} = 0.01$. The code is contained in *BrownianMotion#checkMeanAndVarianceWithoutDrift.* Here is a sample result:

```
Enter number of simulations: 100000
Enter number of steps : 1000
Enter time to check. Must be before or at T: 10
Enter time, T: 10

---------------------------------------------------------------------------
Theoretical E[B(t)] = 0; Empirical E[B(t)] = -0.00257759
Theoretical Var[B(t)] = 10; Empirical Var[B(t)] = 9.9085
---------------------------------------------------------------------------
```

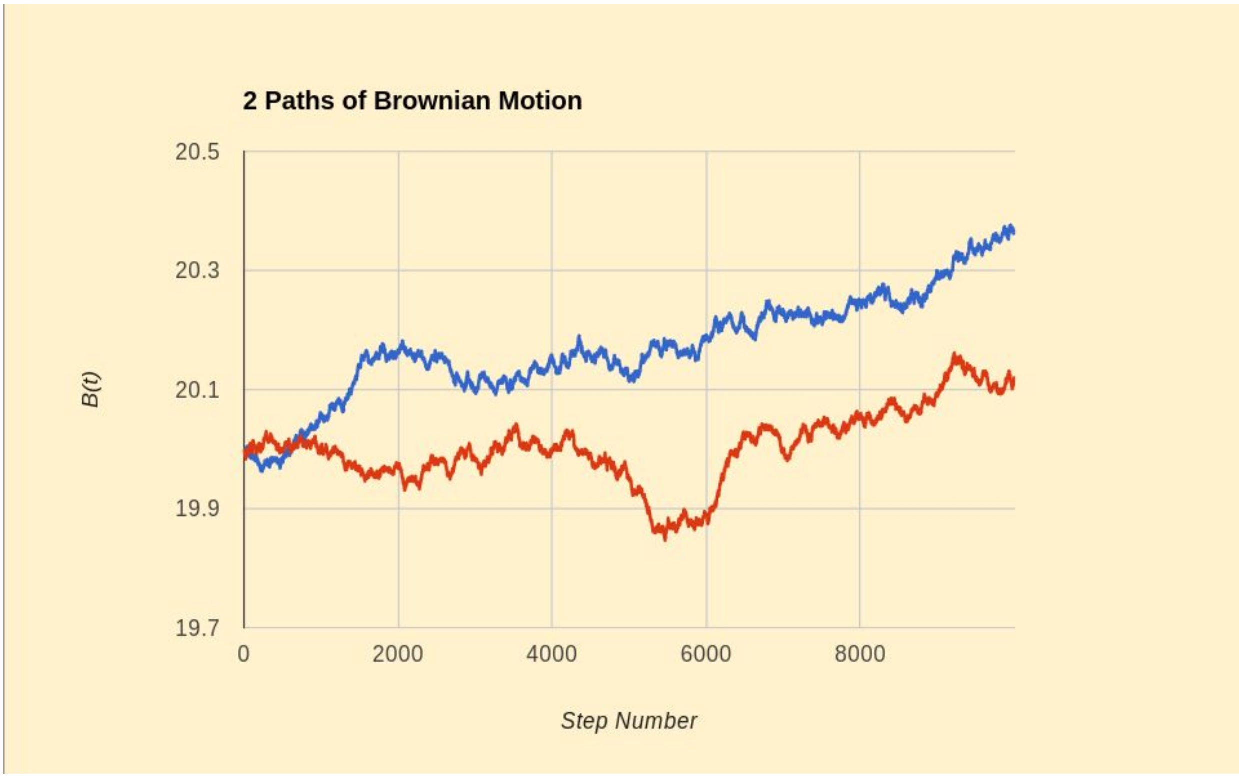
   We see that the empirical values are within 1% of the expected values. As we increase the the number of simulations and decrease size of time partitions, the empirical values will converge to the expected values almost surely
6. Checking properties of Brownian Motion with Drift
   For BM with drift, $X(t_i) = X(t_{i-1}) + (\mu(t_i - t_{i-1}) + \sigma\sqrt{t_i - t_{i-1}}U_i)$, each increment $X(t) - X(s)$ is normally distributed with expected value, $\mu(t-s)$ and variance $\sigma\sqrt{t-s}$. To check this, we run 100000 BM simulations for time 0 to 10 partitioned into 1000 steps. We choose $\sigma = 0.5$ and $\mu = 0.25$. This means $dt = t_i - t_{i-1} = 0.01$. The code is contained in *BrownianMotion#checkMeanAndVarianceWithDrift.* Here is a sample result:

```
Enter number of simulations: 100000
Enter number of steps : 1000
Enter time to check. Must be before or at T: 10
Enter time, T: 10
Enter Mean, mu: 0.25
Enter standard deviation, sigma: 0.5

---------------------------------------------------------------------------
B(t+dt) - B(t) is normally distributed with mean mu*t and Variance sigma^2*T
Theoretical E[B(t)] = 2.5; Empirical E[B(t)] = 2.48928
Theoretical Var[B(t)] = 2.5; Empirical Var[B(t)] = 2.50517
---------------------------------------------------------------------------
```

   Just like we saw in task 5, the empirical values are within 1% of the expected values and the empirical values converge to the theoretical values for smaller partitions and more simulations almost surely.

7. 2-D Brownian Motion: We generate two BMs and plot them in excel. See the attachment below for this simulation:



Thanks.