Assignment for CH2017/BG2211
(Please put all your computer scripts and results including tables and figures into a pdf file with your name and matric number uploaded into NTULearn by Nov 21st, 23:59)

1. (30 marks)

A culture of bacteria increases at a rate that is proportional to the number of bacteria present at that instant. Assuming that the number doubles every 5 hours, a biomedical engineer can estimate the number of bacteria present at a future time using the differential equation

$$\frac{dy}{dt} = ky$$

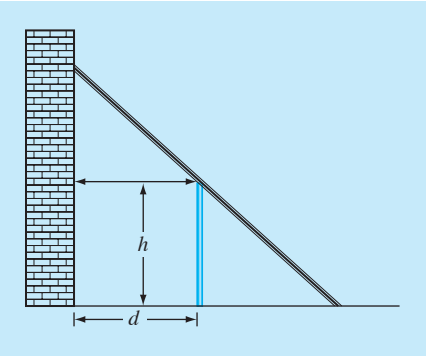
where y is the number of bacteria present at time t.

(a) Determine the value of k

(b) Assuming at time $t = 0$, $y = 1$, use Matlab toolbox to calculate and plot the number of bacteria present y as a function of time t from 0 to 12 hours.

2. (20 marks)

Determine the length of the shortest ladder that reaches from the ground over the fence to touch the building's wall (Figure below) with $h = d = 4$ m.



3. (30 marks)

Enzymatic reactions are used extensively to characterize biologically mediated reactions in environmental engineering. Proposed rate expressions for an enzymatic reaction are given below where $[S]$ is the substrate concentration and v_0 is the initial rate of reaction. Which formula best fits the experimental data? (Here k and K are fitting parameters.)

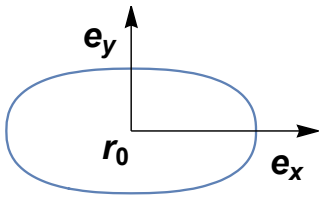
$$v_0 = k[S] \quad v_0 = \frac{k[S]}{K + [S]} \quad v_0 = \frac{k[S]^2}{K + [S]^2} \quad v_0 = \frac{k[S]^3}{K + [S]^3}$$

[S], M	Initial Rate, 10^{-6} M/s
0.01	6.3636×10^{-5}
0.05	7.9520×10^{-3}
0.1	6.3472×10^{-2}
0.5	6.0049
1	17.690
5	24.425
10	24.491
50	24.500
100	24.500

4. (20 marks)
A superellipse is defined by the inequality

$$\left|\frac{x}{a}\right|^n + \left|\frac{y}{b}\right|^n \leq 1,$$

where x and y are the Cartesian coordinates, and a and b are the length of long and short axes with $n > 2$ the deformation parameter. An example of a superellipse centered at $\boldsymbol{r}_0$ with $a = 2$, $b = 1$ and $n = 2.5$ is shown in the figure below, where $\boldsymbol{e}_x$ and $\boldsymbol{e}_y$ are the unit vectors pointing to the direction of long and short axes, respectively, and $\boldsymbol{e}_x \perp \boldsymbol{e}_y$. All vectors here are column vectors.



Use Matlab toolbox numerically to calculate the area S of a superellipse for given a and b. For $a = 2$, $b = 1$, plot S as a function of $n \in [2,10]$, and compare your result with the analytical formula

$$S = \frac{4^{1-\frac{1}{n}} ab\sqrt{\pi}\Gamma\left(1+\frac{1}{n}\right)}{\Gamma\left(\frac{1}{2}+\frac{1}{n}\right)},$$

where $\Gamma(\cdot)$ is the Gamma function.