

Benchmark Optimization Functions Using Genetic Algorithms

1. Introduction

This project explores the optimization of benchmark multimodal functions using Genetic Algorithms (GAs). Two 2-dimensional multimodal functions were selected from the provided bibliographic reference, implemented in Python, and optimized using a configurable GA. The performance of different GA configurations was statistically analyzed to determine their effectiveness in finding function minima.

2. Function implementation and Visualization

For this assignment, two multimodal benchmark functions were chosen: the Ackley Function and the Rastrigin Function. These functions are commonly used in optimization because of their complex landscapes filled with many local minima, which pose a challenge for optimization algorithms. Each function was defined over a two-dimensional input space (x, y), resulting in a three-dimensional surface when visualized.

2.1. Ackley function

- Choose constants to be $a = 20$, $b = 0.2$ and $c = 2\pi$.
- $$f(x, y) = -20 \exp\left(-0.2 \sqrt{\frac{1}{2}(x^2 + y^2)}\right) - \exp\left(\frac{1}{2}(\cos(2\pi x) + \cos(2\pi y))\right) + 20 + e$$
- Domain: $x, y \in [-32.768, 32.768]$
- Global minimum: $f(0,0) = 0$
- The Ackley function features a nearly flat outer region and a large hole at the center, with many local minima. This makes it difficult for optimization algorithms to find the global minimum without getting trapped in one of the surrounding "valleys."

2.2. Rastrigin function

- $$f(x, y) = 20 + (x^2 - 10 \cos(2\pi x)) + (y^2 - 10 \cos(2\pi y))$$
- Domain: $[-5.12, 5.12]$
- Global minimum: $f(0,0) = 0$

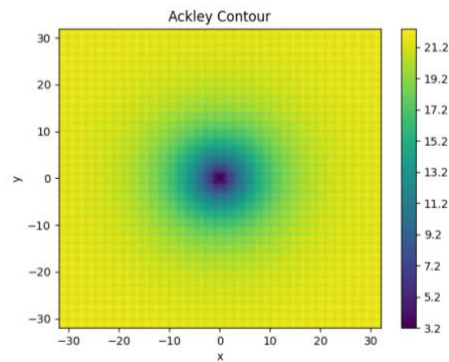
- The Rastrigin function has many local minima arranged in a grid pattern, making it very hard for algorithms to avoid getting stuck early in a local minimum instead of finding the global one.

3. Function visualization

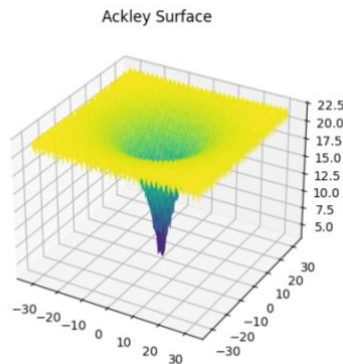
The functions were visualized using 2D contour plots and 3D surface plots to illustrate their multimodal nature and the locations of their minima.

3.1 Ackley Function

- 2D Contour Plot: Shows concentric circles around the global minimum at $(0, 0)$.

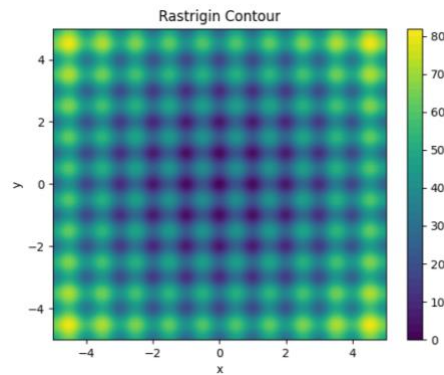


- 3D Surface Plot: Displays a flat surface with a sharp drop at the origin, surrounded by small oscillations.

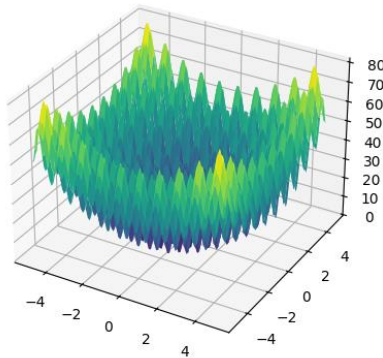


3.2. Rastrigin Function

- 2D Contour Plot: Reveals a grid of local minima with the global minimum at (0, 0).



- 3D Surface Plot: Shows a highly oscillatory surface with many peaks and valleys.



4. Genetic Algorithm (GA) Implementation

The GA was implemented with the following configurable components:

- Encodings:
 - Binary encoding.
 - Real-valued encoding.
- Crossover Methods:
 - Binary: 1-point and 2-point crossover.
 - Real-valued: Arithmetic crossover and BLX- α crossover.
- Other Adjustable Parameters:
 - Population size
 - Number of generations
 - Mutation rate

- Crossover rate
- Seeds
- Other specific parameters for certain encodings

5. Optimization experiments

For the initial run we will use the following config:

```
CONFIG = {
    "population_size": 50,
    "generations": 100,
    "mutation_rate": 0.05,
    "crossover_rate": 0.8,
    "encoding": "binary", # real, binary
    "crossover_method": "1point", # 1point, 2point, arithmetic, blx
    "blx_alpha": 0.5,
    "binary_length": 32,
    "seeds": [79, 17, 6, 67, 44, 71, 37, 38, 43, 24,
              21, 92, 63, 65, 89, 15, 36, 61, 30, 42,
              80, 8, 96, 9, 94, 87, 28, 51, 60, 98],
}
```

Summary of Results:

function	encoding	crossover_method	best	mean	std	median
Ackley	binary	1point	8.846665e-05	0.001513	0.000819	1.557974e-03
Ackley	binary	2point	4.731241e-04	0.002366	0.001678	1.848774e-03
Ackley	real	arithmetic	9.127846e-08	0.000801	0.000875	4.394637e-04
Ackley	real	blx	4.440892e-16	0.036913	0.202137	4.440892e-16
Rastrigin	binary	1point	1.043359e-06	0.037637	0.204786	5.263369e-05
Rastrigin	binary	2point	8.844685e-06	0.013330	0.058015	1.410783e-04
Rastrigin	real	arithmetic	2.070362e-10	0.718704	0.564030	9.949591e-01
Rastrigin	real	blx	0.000000e+00	0.727920	0.671579	7.227305e-01

For the Ackley Function:

BLX (real) finds the solution (a very close to 0 number), but the mean and std are terrible, sign of instability.

All methods perform bad.

Binary encodings have very high best, mean, and std which keeps them stuck far from global minimum.

For the Rastrigin Function:

All mean values are increased a lot → solutions are far from the global minimum on average.

Real encoding performed poorly overall despite hitting 0.0 once.

High std (e.g., 0.67 for BLX) → very inconsistent behavior.

After some parameter tuning, using the below config, we got to visible better results:

```
CONFIG = {
    "population_size": 100,
    "generations": 150,
    "mutation_rate": 0.02,
    "crossover_rate": 0.8,
    "encoding": "binary", # real, binary
    "crossover_method": "1point", # 1point, 2point, arithmetic, blx
    "blx_alpha": 0.8,
    "binary_length": 32,
    "seeds": [79, 17, 6, 67, 44, 71, 37, 38, 43, 24,
              21, 92, 63, 65, 89, 15, 36, 61, 30, 42,
              80, 8, 96, 9, 94, 87, 28, 51, 60, 98],
}
```

Summary of Results:

function	encoding	crossover_method	best	mean	std	median
Ackley	binary	1point	1.678966e-08	1.663279e-07	1.524532e-07	1.096148e-07
Ackley	binary	2point	2.328306e-08	1.987417e-07	1.407121e-07	1.785503e-07
Ackley	real	arithmetic	4.440892e-16	3.246815e-04	7.771053e-04	1.419030e-05
Ackley	real	blx	4.440892e-16	4.440892e-16	0.000000e+00	4.440892e-16
Rastrigin	binary	1point	0.000000e+00	4.641976e-12	1.109297e-11	5.702105e-13
Rastrigin	binary	2point	2.842171e-14	1.614762e-11	7.122505e-11	5.275780e-13
Rastrigin	real	arithmetic	0.000000e+00	3.628739e-01	5.294948e-01	1.883245e-02
Rastrigin	real	blx	0.000000e+00	2.933677e-01	4.451000e-01	0.000000e+00

For the Ackley Function:

Both real encodings found the global minimum (4.44e-16, which is very close to 0), which indicates high precision.

The BLX crossover in real encoding outperformed all others: it achieved zero variance, perfect mean, and perfect median, making it highly consistent.

Binary encodings performed well but showed higher mean and standard deviation, implying some degree of instability.

For the Rastrigin Function:

All configurations found the global optimum (0.0) at least once.

Binary encoding + 1point crossover had the lowest mean and standard deviation, showing it was the most consistent and stable performer.

Although real encodings also reached 0.0, they showed high mean and variability, indicating that they were less reliable across runs.

The arithmetic and BLX crossovers in real encoding likely struggled due to Rastrigin's rugged fitness landscape.

6. Conclusion

This project explored Genetic Algorithms (GAs) for optimizing the Ackley and Rastrigin functions, comparing binary and real-valued encodings with various crossover methods. Early experiments showed that binary encoding struggled, especially on Ackley, but improved significantly after adjusting the parameters.

Real-valued encoding, especially with the BLX- α crossover, achieved better results on both functions. It was the most stable and accurate for Ackley and performed strongly on Rastrigin, outperforming real-valued arithmetic crossover.

The mutation rate proved crucial: too low led to poor exploration, while a balanced rate improved consistency. Increasing the number of generations further enhanced performance.

Overall, the study highlighted the importance of problem representation, crossover choice, and parameter tuning in GA performance.

7. Bibliography

This project used the following Python libraries:

- numpy: <https://numpy.org/>
- matplotlib: <https://matplotlib.org/>
- pandas: <https://pandas.pydata.org/>

The functions used for this report were taken from Sonja Surjanovic's webpage on the website of Simon Fraser University: <https://www.sfu.ca/~ssurjano>