Approximation Algorithms for Multidimensional Bin Packing

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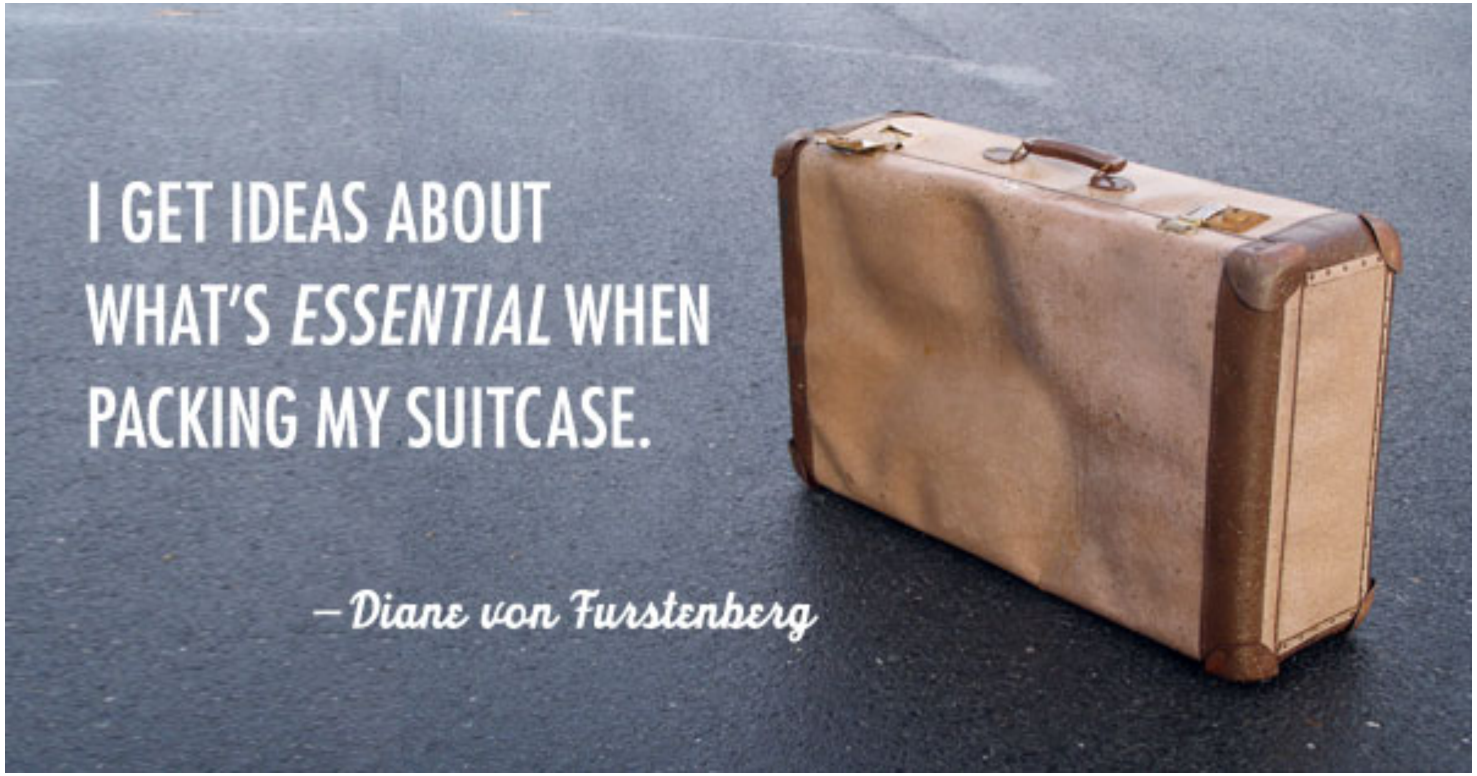
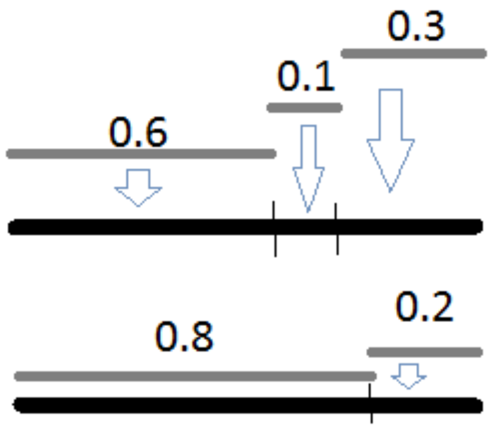
Packing Problems



Bin Packing Problem

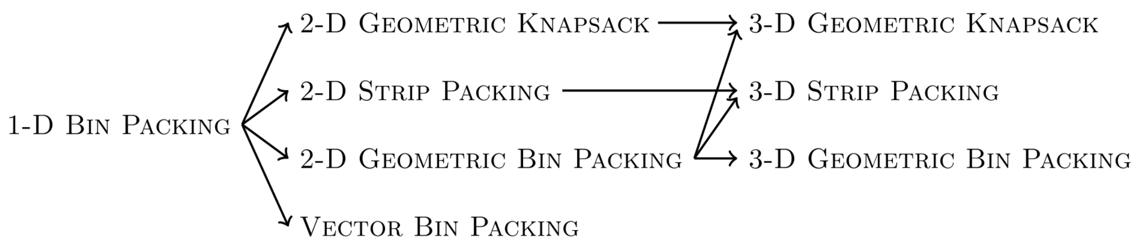
- Given : n items with sizes $s_1, s_2, \dots s_n$, s.t. $s_i \in (0,1]$,
- Goal: Pack all items into min # of unit bins.
- Example: items {0.8, 0.6, 0.3, 0.2, 0.1} can be packed in 2 unit bins: {0.8, 0.2} and {0.6, 0.3, 0.1}.
- 3/2 hardness of approximation (from *Partition*).
- This does not rule out OPT+1 guarantee.
- delaVega-Lueker, Combinatorica ’81: APTAS,
- Karp-Karmarkar, FOCS ’82: $\mathrm{OPT} + O(\log^2(OPT))$,
- Hoberg-Rothvoss, SODA ’17: $\mathrm{OPT} + O\big(\log(OPT)\big)$.



Talk Overview

- Five generalizations of Bin Packing:

1. Geometric Bin Packing (GBP),
2. Strip Packing (2SP),
3. Geometric Knapsack (2GK),
4. Vector Bin Packing (VBP),
5. Weighted Bipartite Edge Coloring (WBEC).



Pitas


PTAS

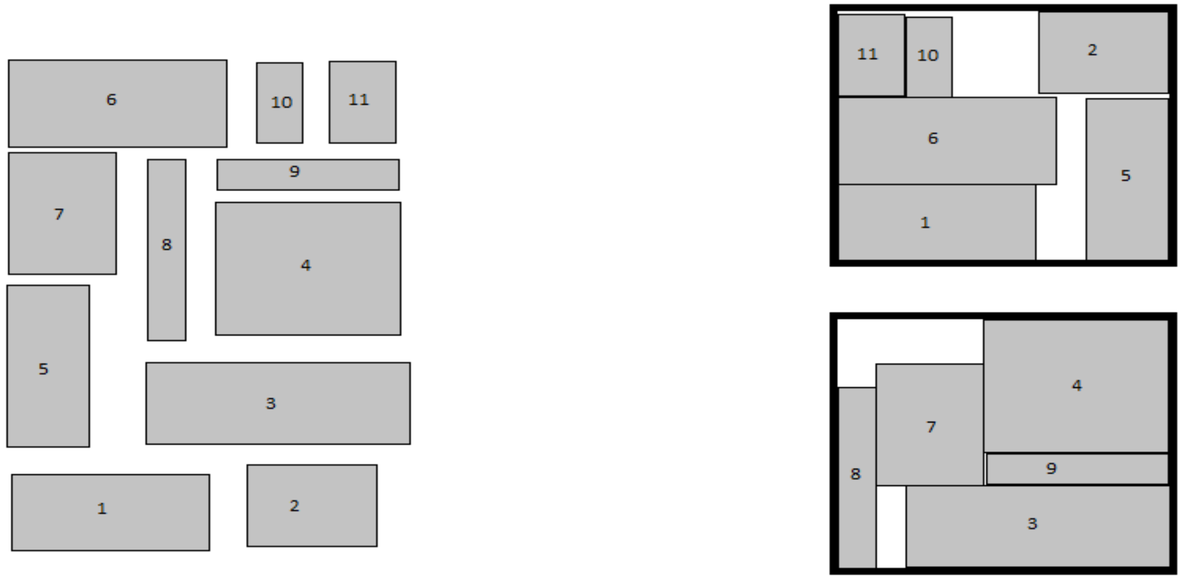
- Polynomial Time Approximation Schemes (PTAS):
  If for every $\varepsilon > 0$, there exists a poly-time ($O(n^{f(\varepsilon)})$-time) algorithm A_ε such that $A_\varepsilon(I) \ \leq \ (1+\varepsilon)\ OPT(I)$.
- Efficient PTAS (EPTAS): if running time is $O(f(\varepsilon).n^c)$.
- Fully PTAS (FPTAS): if running time is $O((n/\varepsilon)^c)$.
- Asymptotic PTAS (APTAS): $A_\varepsilon(I) \ \leq \ (1+\varepsilon)\ OPT(I) + O(1)$.
- QuasiPTAS (QPTAS): $(1+\varepsilon)$-approximation in $n^{(\log n)^{O(1)}}$-time.
- PseudoPTAS (PPTAS): $(1+\varepsilon)$-approximation in $n^{O(1)}$-time, where n is the number of items and the numeric data is polynomially bounded in n.

1. Geometric Bin Packing

2-D Geometric Bin Packing

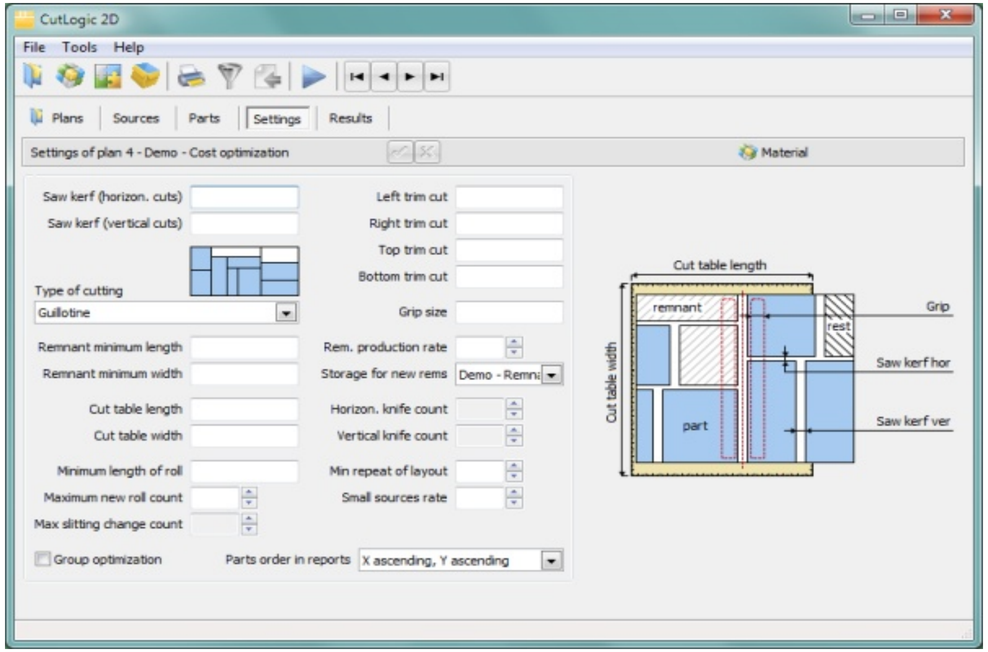
- Given: Collection of rectangles (by width, height),
- Goal: Pack them into minimum number of unit square bins.

- Orthogonal Packing: rectangles packed parallel to bin edges.
- With 90 degree *rotations* and *without rotations*.
- Reduces to 1-D bin packing,
  if all items have height = 1.
- For d-D GBP, we have d-D cuboids
  and bins instead of rectangles.



Applications:

- Cloth cutting, steel cutting, wood cutting
- Placing ads in newspapers
- Memory allocation in paging systems
- Truck Loading
- Palletization by robots



2D BP: Tale of approximability

Algorithm (Asymptotic)
2.125 [Chung Garey Johnson, JACM '82]
$2+\epsilon$ [Kenyon-Remilla, FOCS'96]
1.69 [Caprara, FOCS'02]
1.52 [Bansal-Caprara-Sviridenko, FOCS'06]
1.5 [Jansen-Praedel, SODA'13]
1.405 [Bansal-K., SODA'14] (with and w/o rotations)

Hardness
No APTAS (from 3D Matching) [Bansal-Sviridenko, SODA'04],
3793/3792 (with rotation), 2197/2196 (w/o rotation) [Chlebik-Chlebikova, CIAC'06]

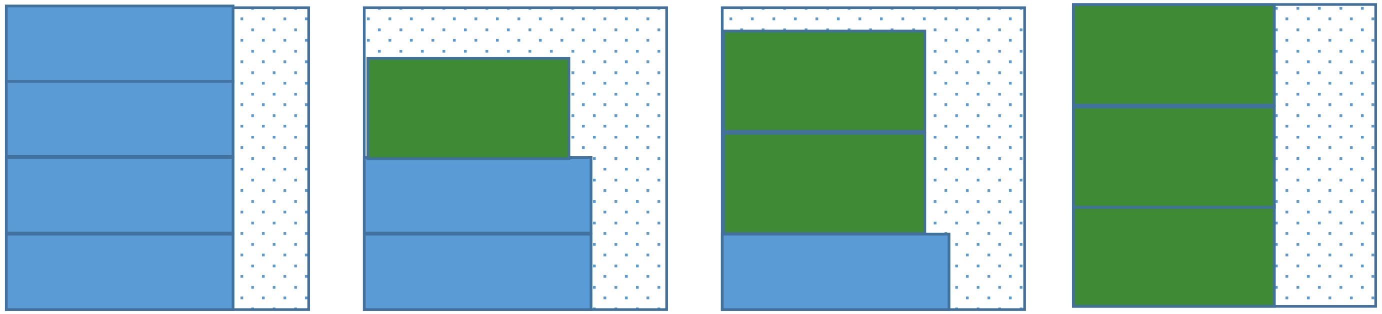
- d-dimensional (d>2) GBP (without rotations): $\mathbf{1.69^{d-1}}$ [Caprara, FOCS'02].
  (with rotations): 1.69^{d-1} [Sharma, '21].
- APTAS for d-dimensional squares: [Bansal-Sviridenko, SODA'04].

Configuration LP

- $\mathbb{C}$: set of configurations (possible way of feasibly packing a bin).

0.75×0.25 0.66×0.33

- Set of (maximal) configurations without rotations.



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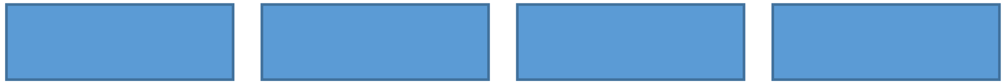

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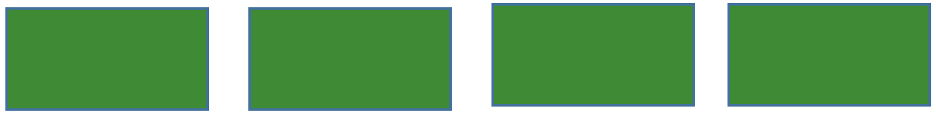
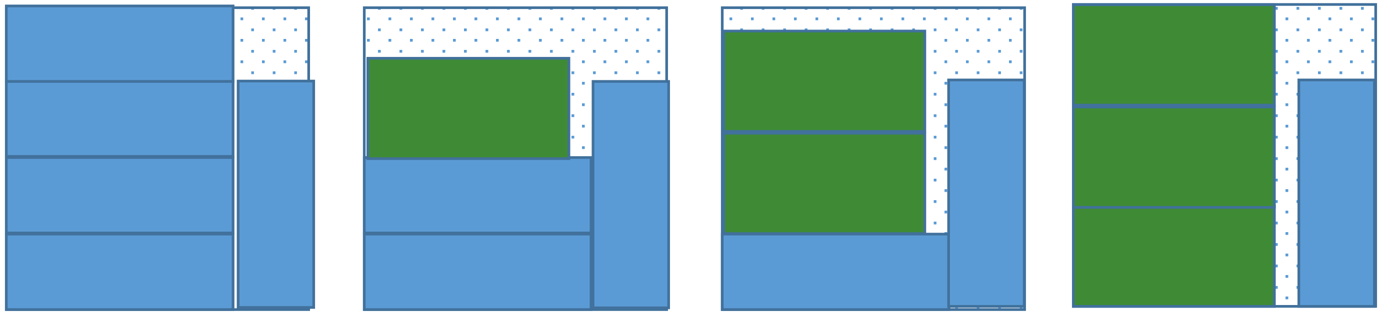


0.66×0.33

- Set of (maximal) configurations with 90 degree rotations.



Configuration LP

- $\mathbb{C}$: set of configurations (possible way of feasibly packing a bin).

Primal:

$$\min \{ \sum_{C} x_C : \sum_{C \ni i} x_C \geq 1 \ (i \in I), x_C \geq 0 \ (C \in \mathbb{C}) \ \}$$

Dual:

$$\max \{ \sum_{i \in I} v_i : \sum_{i \in \mathrm{C}} v_i \leq 1 \ (C \in \mathbb{C}), v_i \geq 0 (i \in I) \}$$

Separation problem of dual :
Given one bin, pack as much area as possible.
- PTAS [BCJPS, ISAAC 2009]

- Problem: Exponential number of configurations!
- Solution: Can be solved within $(1+\epsilon)$ accuracy using separation problem for the dual.

Round and Approx (R&A) Framework [Bansal-K. '14]

- Given a packing problem Π

1. If the configuration LP is solved within $(1 + \epsilon)$ factor

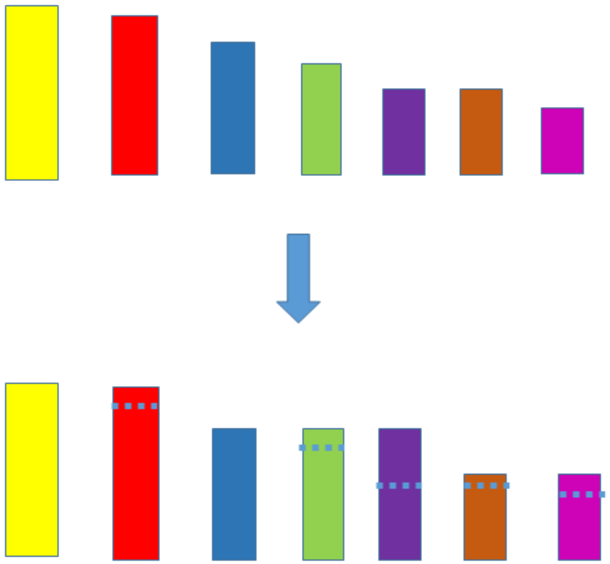
$$\min \left\{ \sum_{C} x_C : \sum_{C \ni i} x_C \geq 1 \; (\, i \in I), x_C \geq 0 \; (C \in \mathbb{C}) \right\}$$

2. There is a ρ approximation rounding-based algorithm.

- Then there is (1+ln ρ) approximation for Π.

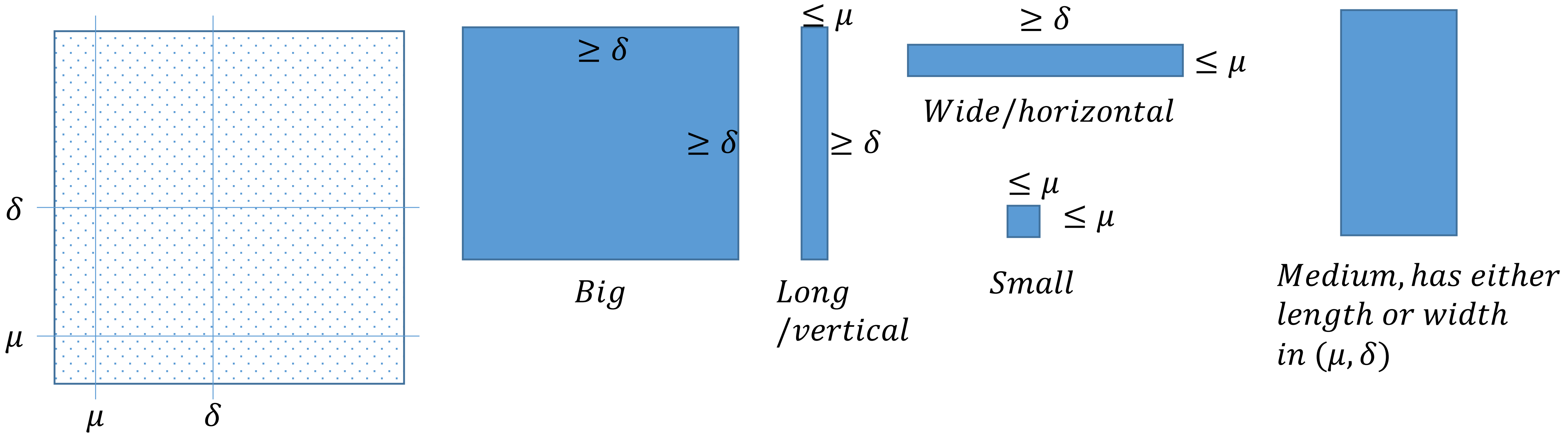
Rounding based Algorithms:

- Rounding based algorithms are ubiquitous in bin packing.
- In general, packing of small items is easy.
- Big items are problematic.
- Big Items are replaced by larger items from O(1) types.
- Loss: Due to larger items.
- Gain: Fewer configurations. O(1) types of large items imply rounded instance can be solved optimally.
- Example: Linear grouping [delaVega-Luker, Kenyon-Remilla], Geometric Grouping [Karp-Karmarkar], Harmonic Rounding [Lee-Lee, Caprara, Bansal et al.], JP rounding [JansenPradel].



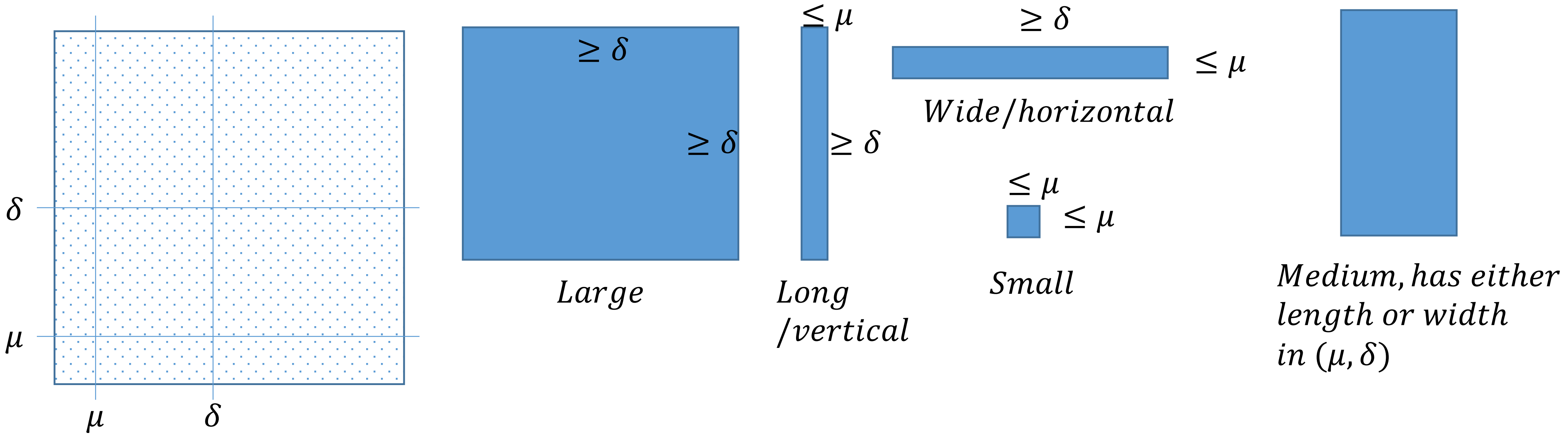
Rounding based Algorithms in 2D

- Classification of items into big, wide, long, medium and small by defining two parameters $\delta, \mu(\ll \delta)$ such that total area of medium rectangles is very small.



Rounding based Algorithms in 2D

- Small & medium rectangles are packed separately, not incurring much loss.
- The main difficulty is in packing large, long/vertical & wide/horizontal items.



Rounding in 2D : container-based packing.

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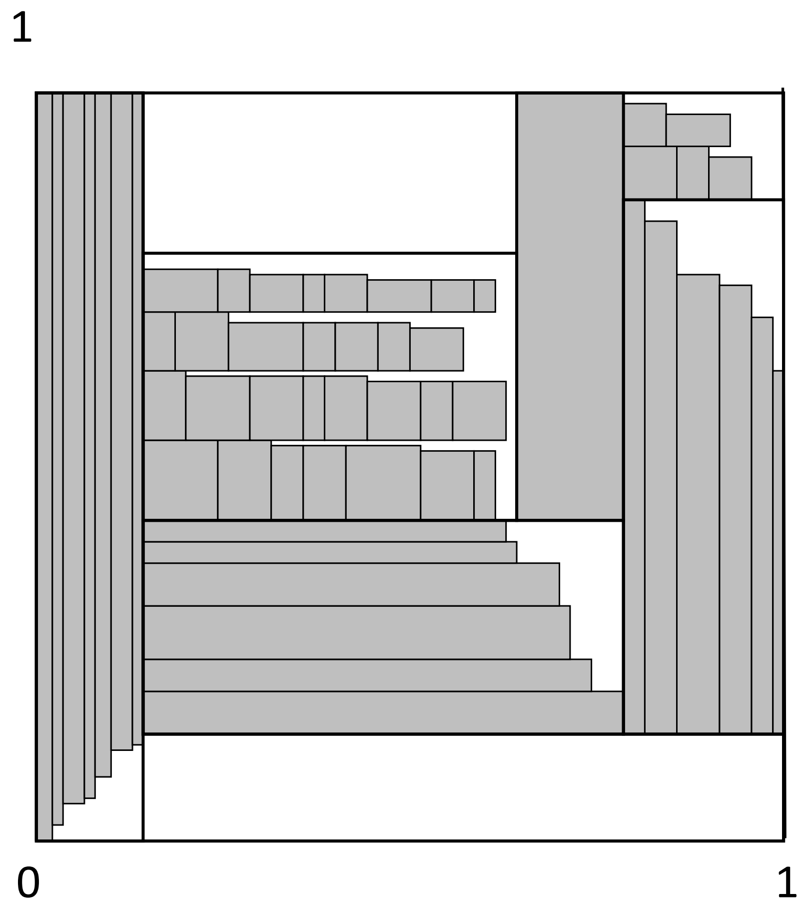
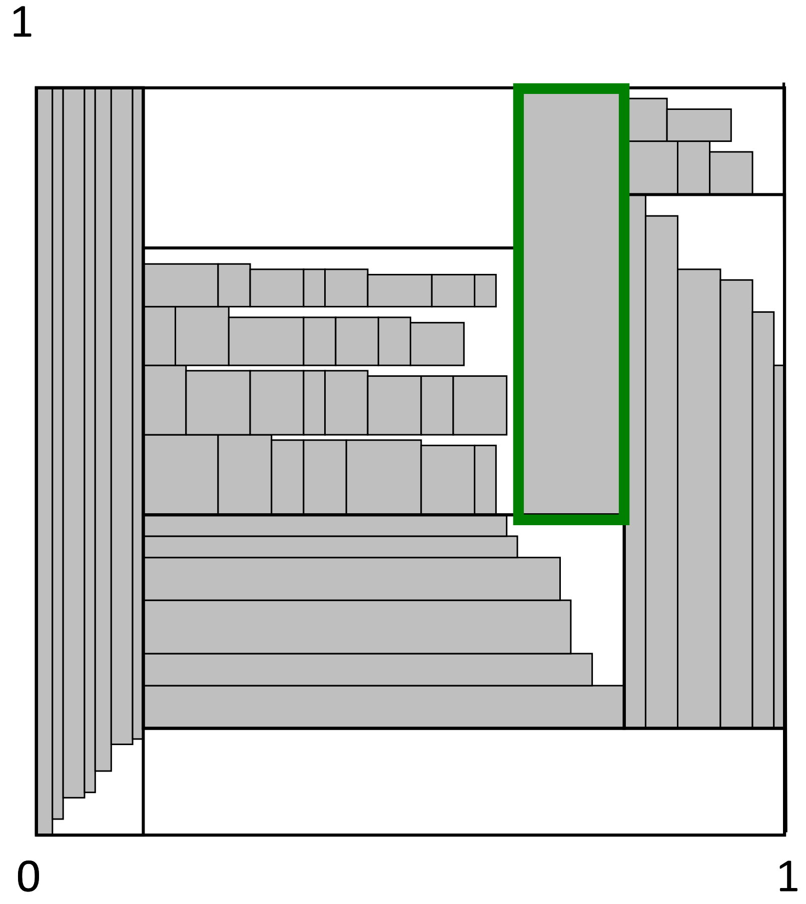
- Container is an axis-aligned rectangular region such that



Rounding in 2D : container-based packing.

- Container is an axis-aligned rectangular region such that
- either it contains one large item.



Rounding in 2D : container-based packing.

- Container is an axis-aligned rectangular region such that
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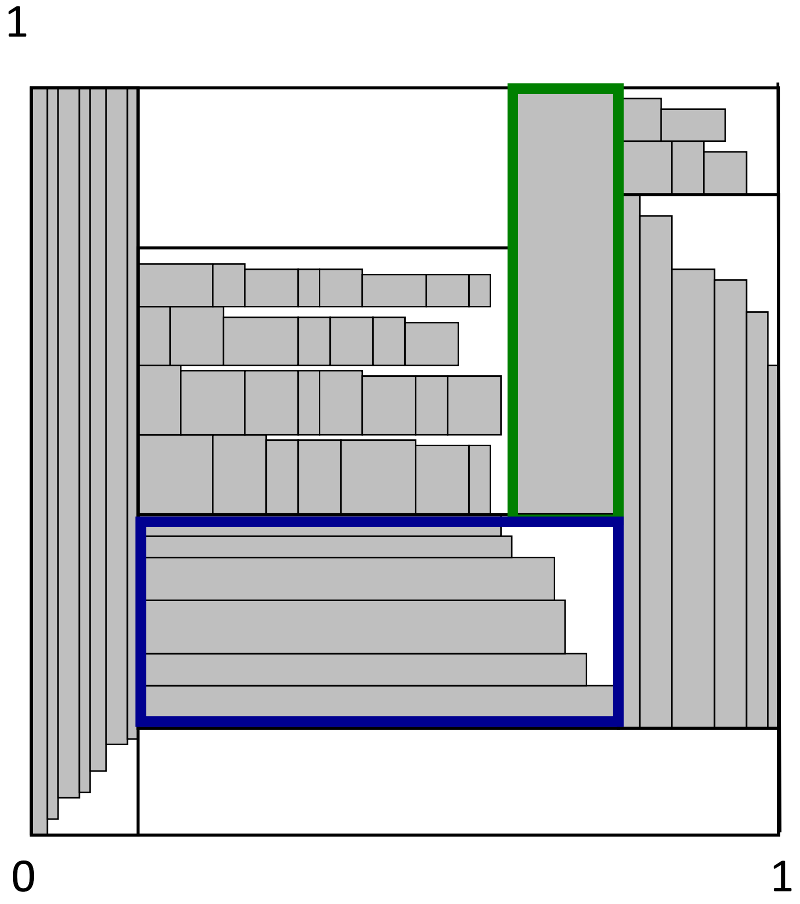
- or items are packed inside the containers either as a horizontal stack or vertical stack



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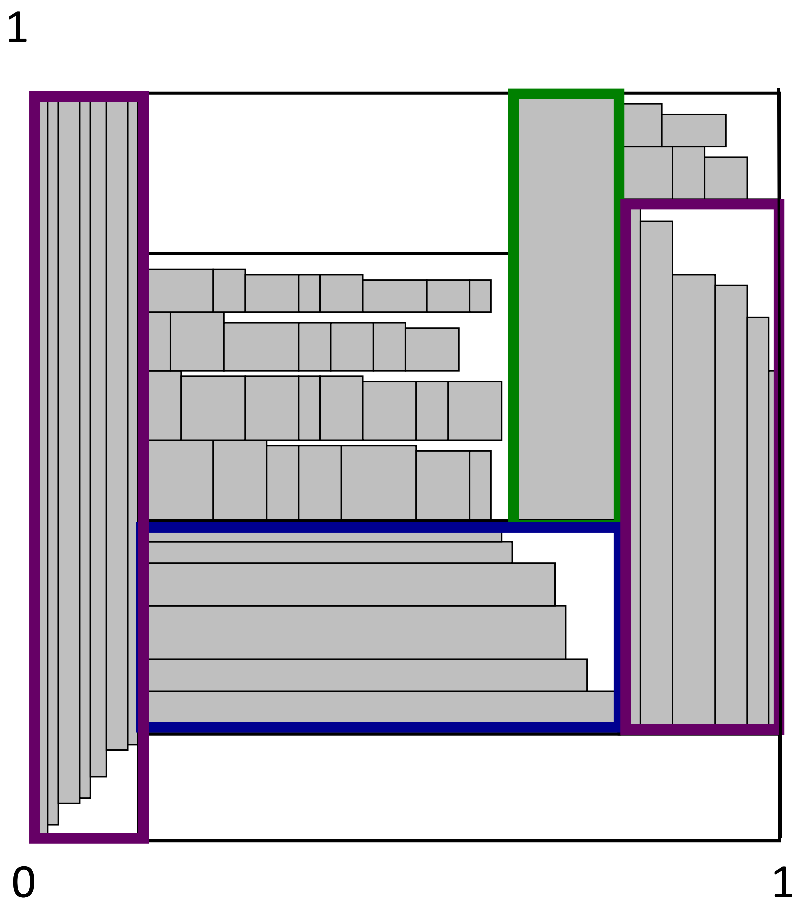
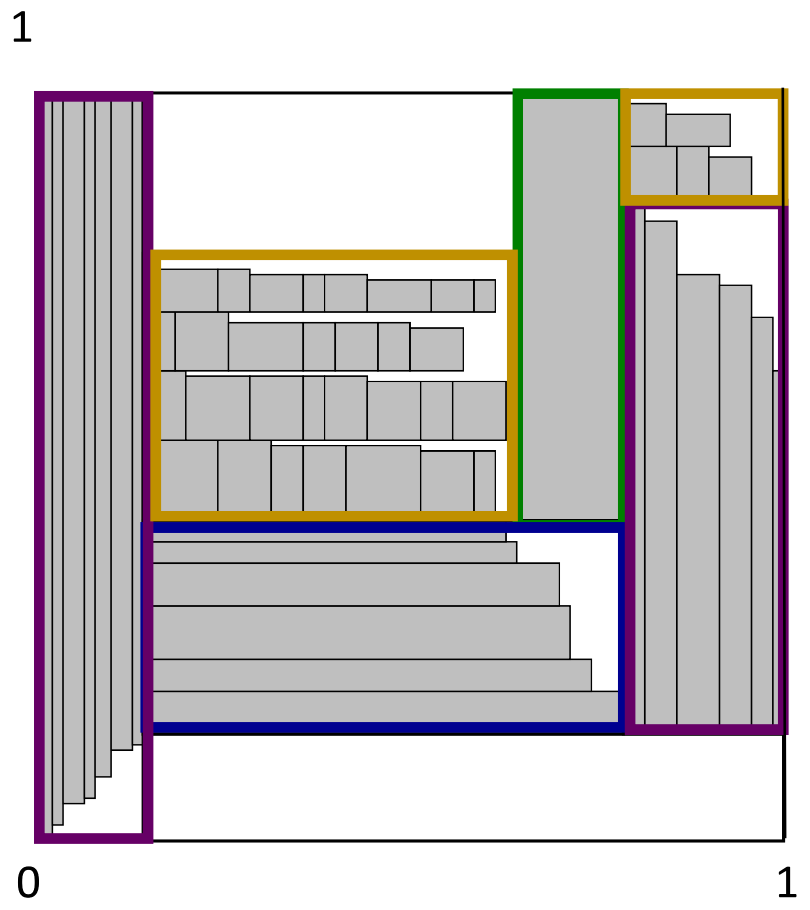


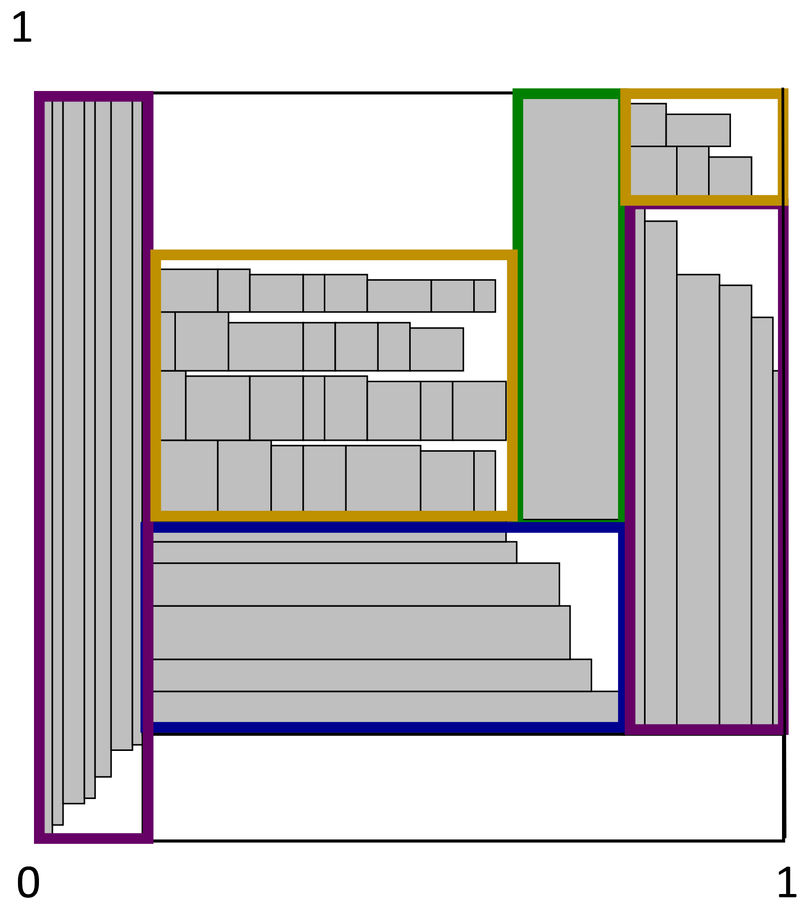
Rounding in 2D : container-based packing.

- Container is an axis-aligned rectangular region such that
- either it contains one large item.
- or items are packed inside the containers either as a horizontal stack or vertical stack
- or all items inside it are very small in both dimensions.

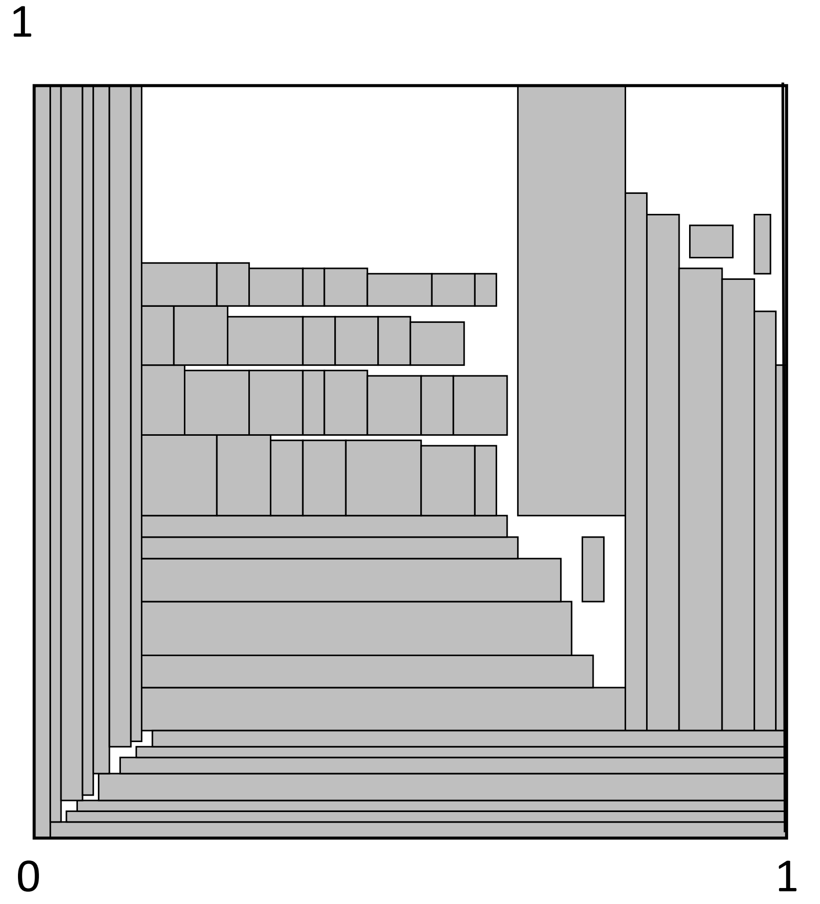


Rounding in 2D : container-based packing.

- If there are O(1) types of containers, then one can view that all large dimensions are rounded to O(1) number of values.
- In polynomial time we can guess the sizes of containers.
- The gap between δ and μ ensures fractional and integral packing of wide/long items are very close.

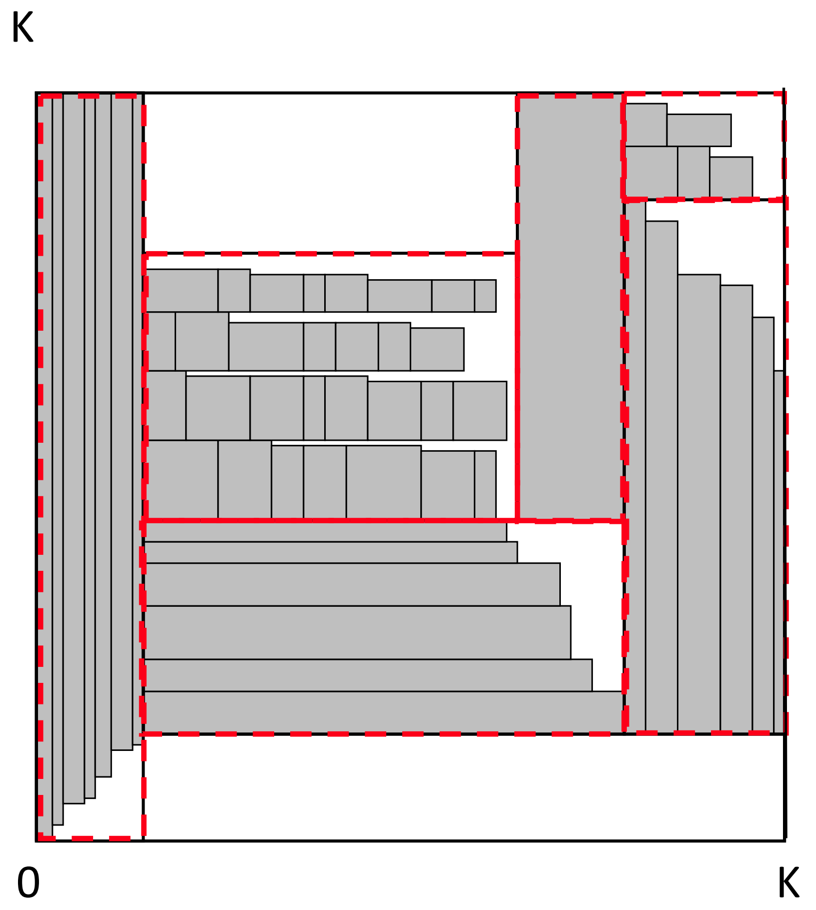


Rounding in 2D : α-approximation using container-based packing.



Rounding in 2D : α-approximation using container-based packing.

- Existence: For any arbitrary feasible packing in m bins, items can be packed in $\alpha m + O(1)$ bins of container-based packing with O(1) type of containers.
- Guess the packing: Guess the sizes and positions) of C containers in $n^{O(C)}$ time.
- Pack the items: Containers can be packed using a Dynamic Program based PTAS for multiple-knapsack problem.



Round and Approx Framework (R & A)

- 1. Solve configuration LP using APTAS. Let $z^* = \sum_{\{C \in \mathbb{C}\}} x_C^*$.

Primal:

$$\min \{ \sum_{C} x_C : \sum_{C \ni i} x_C \geq 1 \ (i \in I), x_C \geq 0 \ (C \in \mathbb{C}) \}$$

Round and Approx Framework (R & A)

- 1. Solve configuration LP using APTAS. Let $z^* = \sum_{\{C\in\mathbb{C}\}} x^*_C$.
- 2. Randomized Rounding: For q iterations :

  select a configuration C' at random with probability $\frac{x^*_{C'}}{z^*}$.

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  select a configuration C' at random with probability $\frac{x_{C'}^*}{z^*}$.
- 3. Approx: Apply a ρ approximation rounding based algorithm A on the residual instance S.
- 4. Combine: the solutions from step 2 and 3.

R & A Rounding Based Algorithms

- Probability item i left uncovered after rand. rounding

$= \left(1 - \sum_{\{C \ni i\}} \frac{x_C^*}{z^*}\right)^q \leq \frac{1}{\rho}$ by choosing $q = (\ln \rho) LP(I)$.

- Number of items of each type shrinks by a factor ρ

  e.g., $\mathbb{E}\left[|B_j \cap S|\right] = \frac{|B_j|}{\rho}$.for some item type B_j.

- Concentration using Independent Bounded Difference Inequality.

Proof Sketch

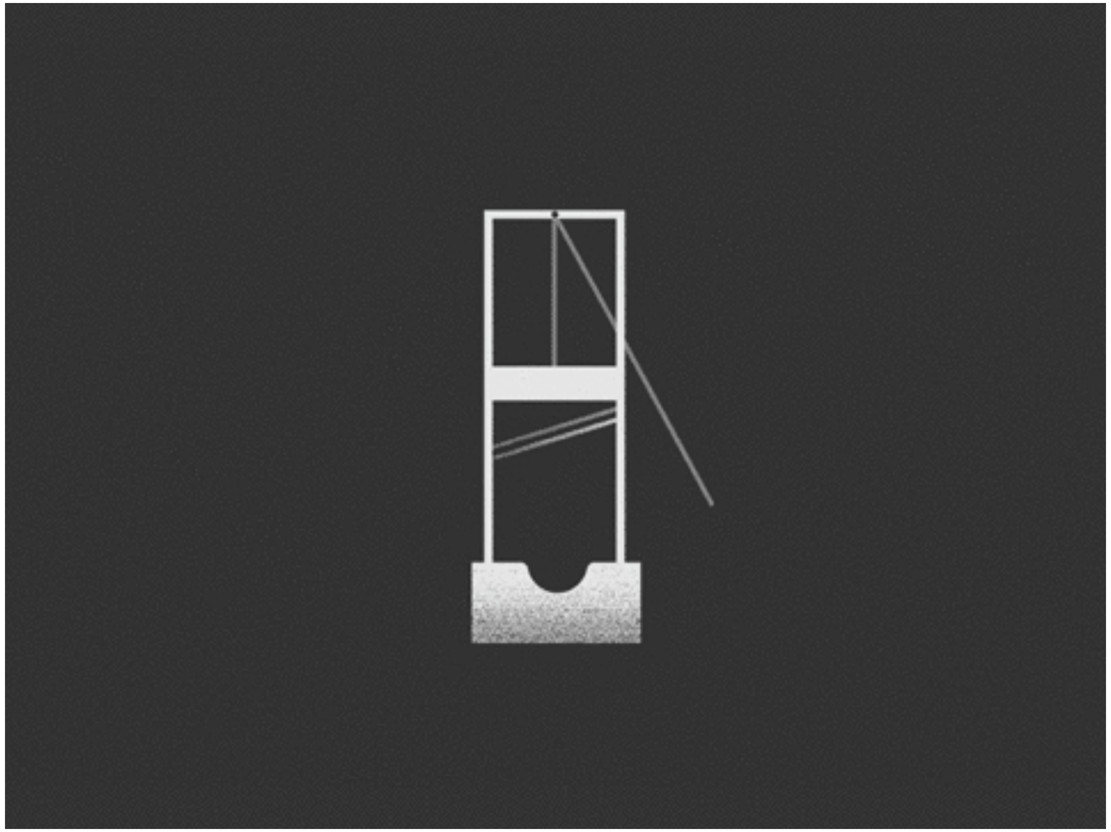
- Rounding based Algo : O(1) types of items $= O(1)$ number of constraints in configuration LP.
- $ALGO(S) \approx OPT(\tilde{S}) \approx LP(\tilde{S})$.
- As # items for each item type shrinks by ρ, $LP(\tilde{S}) \approx \frac{1+\epsilon}{\rho} LP(\tilde{I})$.
- $\rho -$ approximation: $ALGO(I) \approx LP(\tilde{I}) \leq \rho\, OPT(I) + O(1)$.
- $ALGO(S) \approx OPT(I)$.

Proof Sketch

- Thm: R&A gives a $(1 + \ln \rho + \epsilon)$ approximation.
- Proof:
- Randomized Rounding : $q = \ln \rho . LP(I)$
- Residual Instance S $= (1 + \epsilon) OPT(I) + O(1)$.

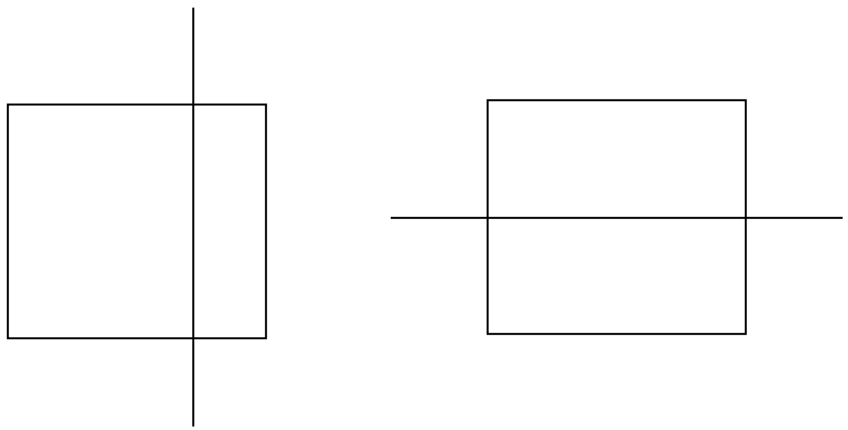
- Round + Approx => $(\ln \rho + 1 + \epsilon) OPT(I) + O(1)$.

Guillotine Packing



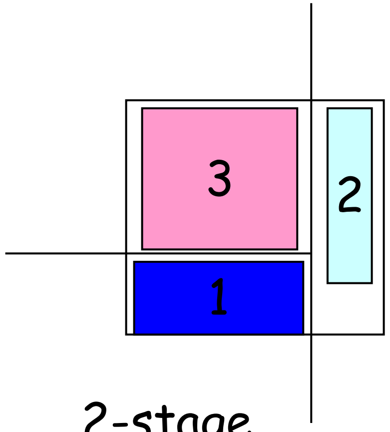
Guillotine Packing

Guillotine Cut: Edge to Edge cut across a bin

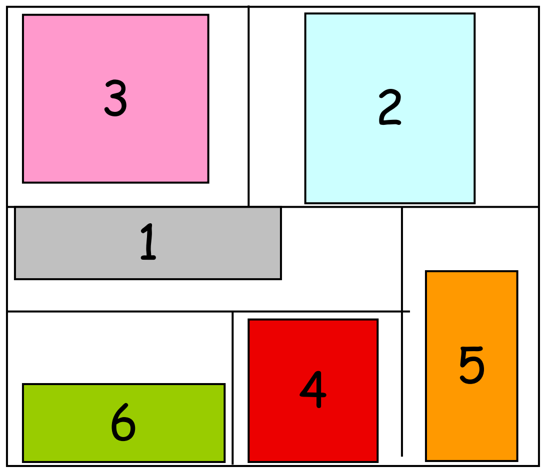


Guillotine packing: All items can be separated by a sequence of guillotine cuts.
Objective: Minimize number of bins such that packing in each bin is a guillotine packing.

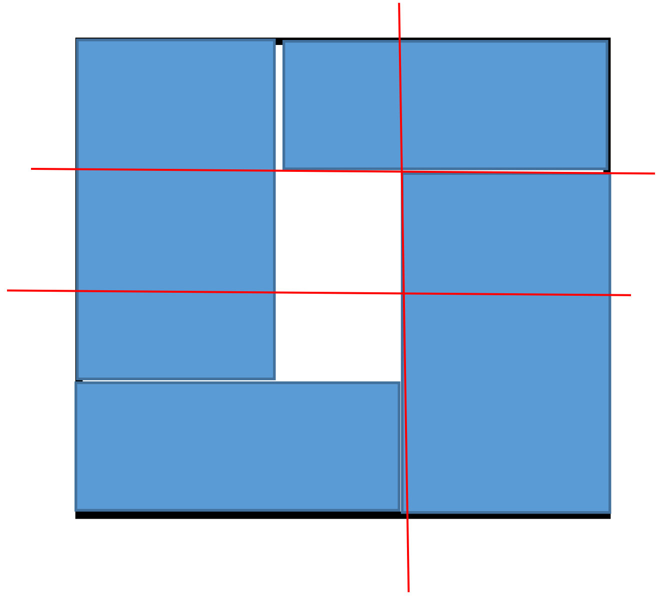
Connection between guillotine & general packing



2-stage



4-stage



- APTAS for guillotine 2-D bin packing [Bansal Lodi Sviridenko, FOCS'05].
- Conjecture: Given any packing of m bins, there is a guillotine packing in $4m/3 + O(1)$ bins. This will imply $(4/3 + \varepsilon)$-approximation for 2-D BP.
- Conjecture: Given any packing of m bins, there is a 2-stage packing in $3m/2 + O(1)$ bins.

2. Strip Packing

Strip Packing Problem: (2-D)

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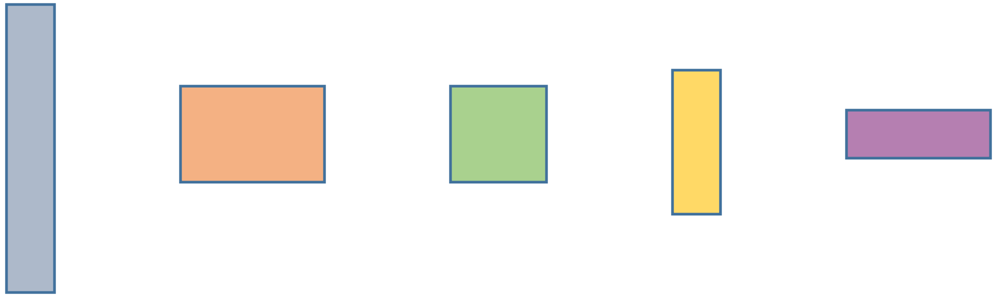
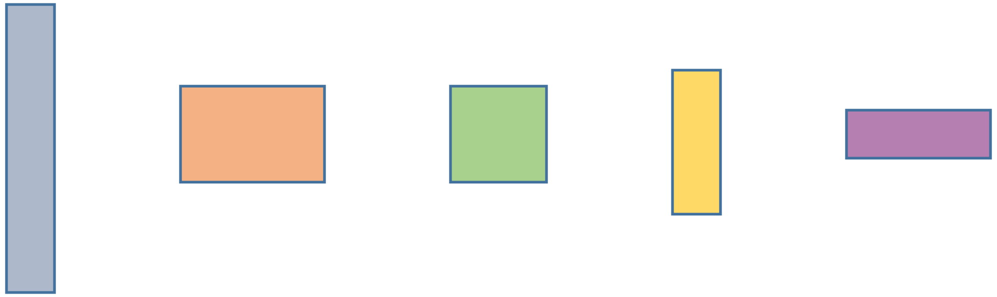


R_1 (1,6) R_2 (3,2) R_3 (2,2) R_4 (1,3) R_5 (3,1)

Strip Packing Problem: (2-D)

- Input :
  - Rectangles $R_1, R_2, \ldots, R_n$; Each R_i has integral width and height (w_i, h_i).
  - A strip of integral width W and infinite height.



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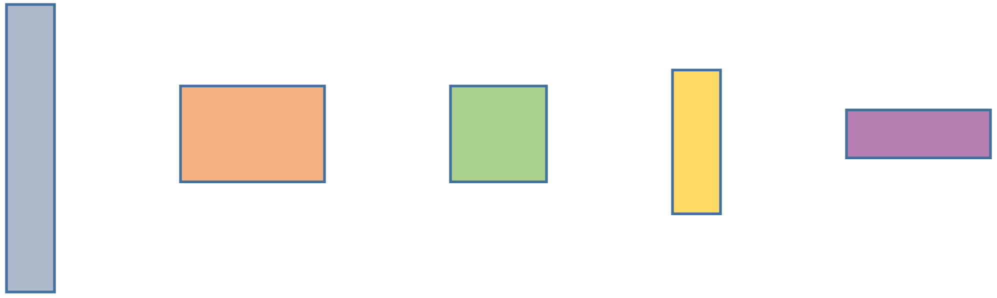

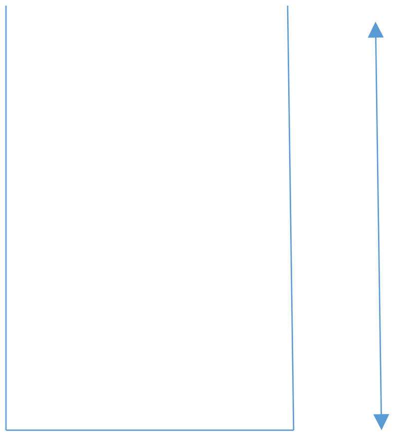

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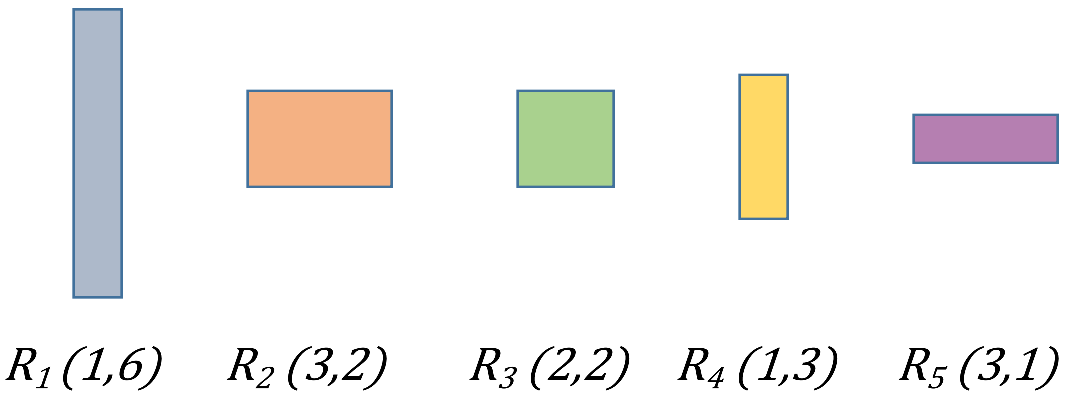

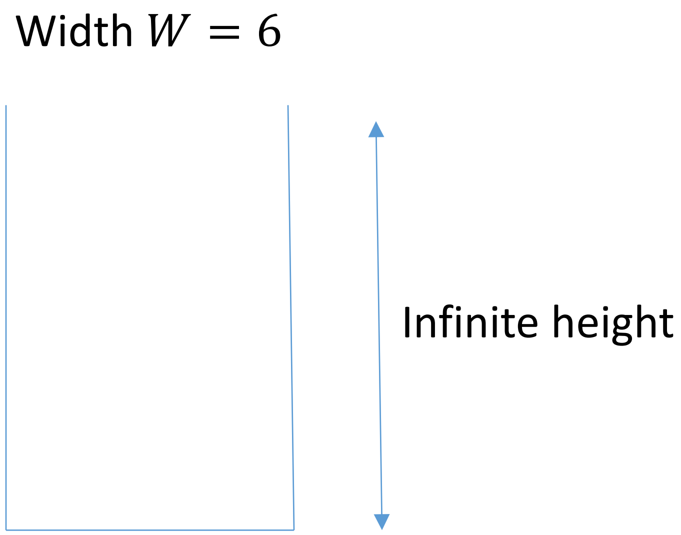

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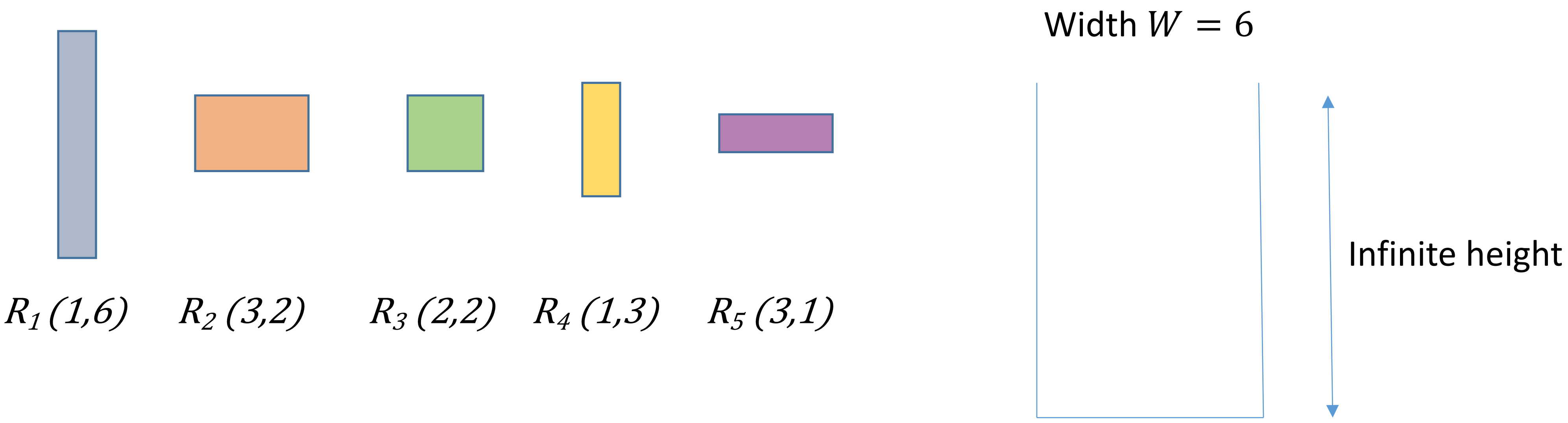
  - Axis-parallel non-overlapping packing.



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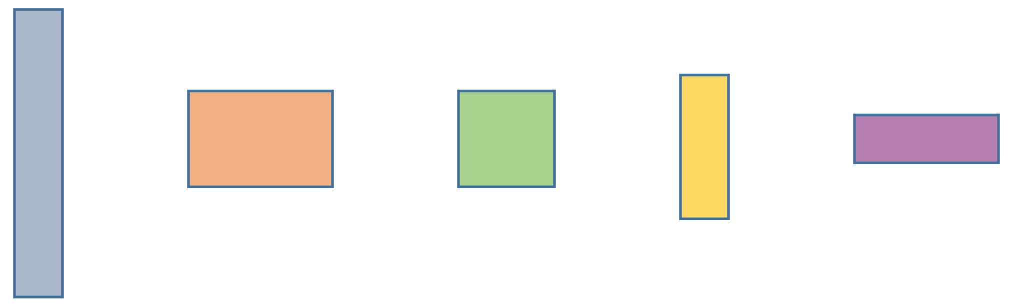

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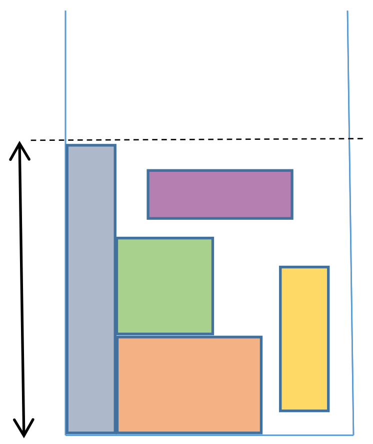

Variant 1:

No rotations are allowed!

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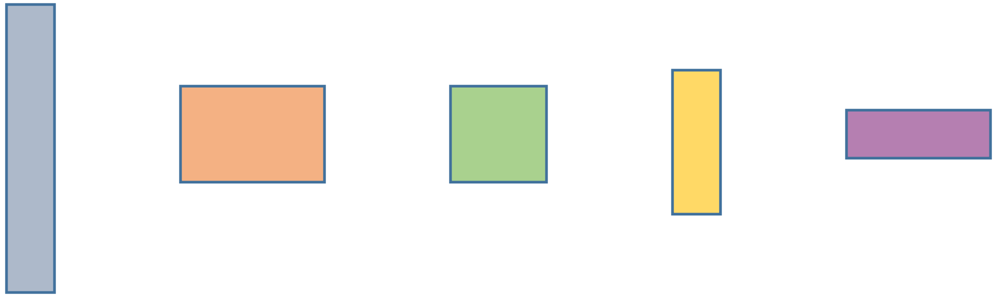

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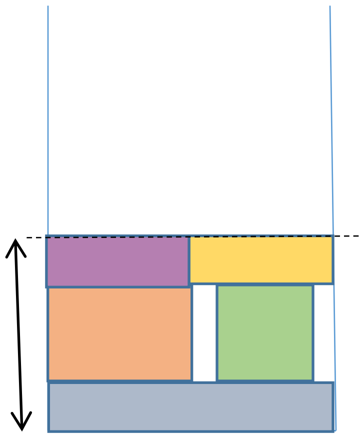
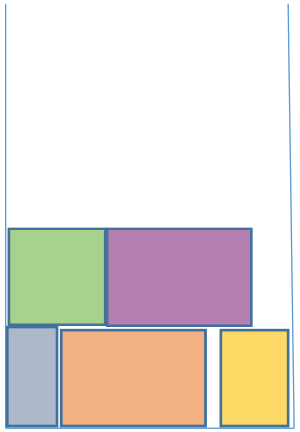


Variant 2:

90° rotations are allowed!

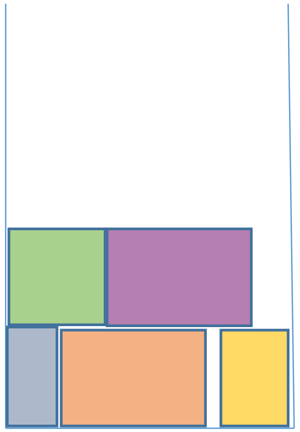
Strip Packing:

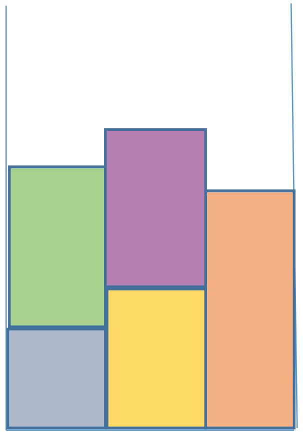
- Strip Packing generalizes
  - bin packing (when all rectangles have same height),



Strip Packing:

- Strip Packing generalizes
  - bin packing (when all rectangles have same height),
  - makespan minimization (when all rectangles have same width).





Tale of approximability.

- Asymptotic PTAS [Kenyon-Remila, FOCS'96] (Without rotations),
- Asymptotic PTAS [Jansen-vanStee, STOC'05] (With rotations).
- Absolute Approximation: (Polynomial time)
- 2.7-appx. [First-Fit-Decreasing-Height, Coffman-Garey-Johnson-Tarjan '80].
- 2-appx [Steinberg'97]
- 5/3+ε [Harren-Jansen-Pradel-vanStee, Comp.Geom.'14].
- Hardness of appx in poly-time: 3/2 (from Bin Packing).
- (3/2+ε)-appx for non-large rectangles [GGJJKR; APPROX'20]

Tale of approximability.

- Pseudopolynomial time ($O(nW)^c$):
- Algorithms:
- 1.5+ϵ [Jansen-Thole, SICOMP'10]
- 1.4 +ϵ [Nadiradze-Wiese, SODA'16]
- 4/3 +ϵ [Galvez, Grandoni, Ingala, K., FSTTCS'16; Jansen-Rau, WALCOM'17]
- 5/4 +ϵ [Jansen, Rau, ESA'19]
- Hardness:
- 12/11 Adamaszek, Kociumaka, Pilipczuk, and Pilipczuk, TOCT'17
- 5/4 Henning, Jansen, Rau, and Schmarje, CSR'18

3. Geometric Knapsack

Geometric Knapsack: (2-D)

- Input :

  - Rectangles $I := \{R_1, R_2, \ldots, R_n\}$; Each R_i has integral width and height (w_i, h_i) and profit p_i.

  - A Square $K \times K$ knapsack.

Geometric Knapsack: (2-D)

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  - Rectangles $I := \{R_1, R_2, \ldots, R_n\}$; Each R_i has integral width and height (w_i, h_i) and profit p_i.
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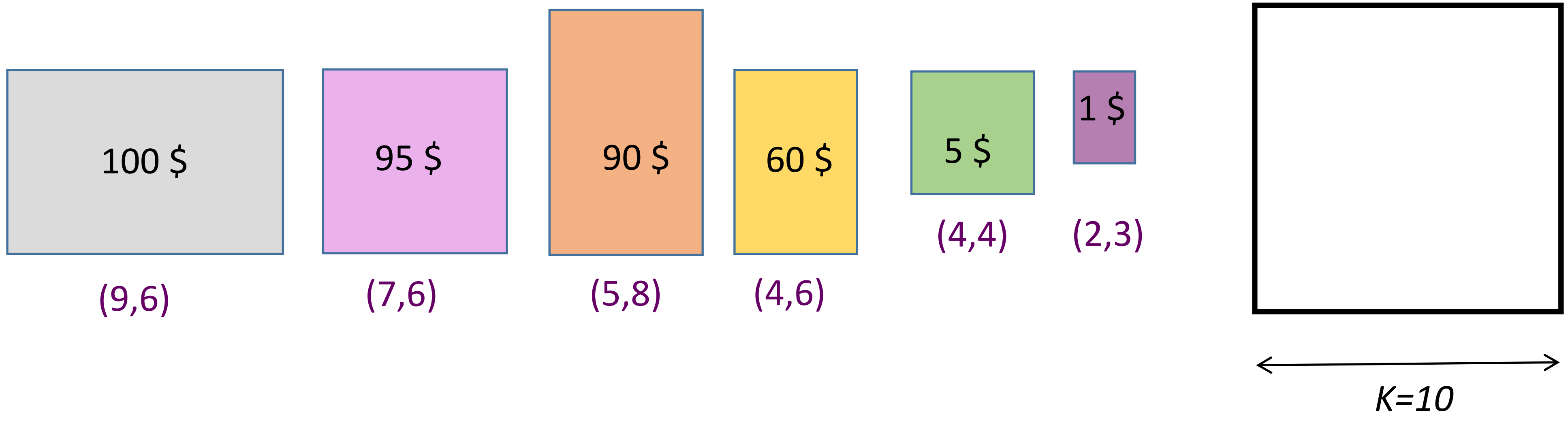

Geometric Knapsack: (2-D)

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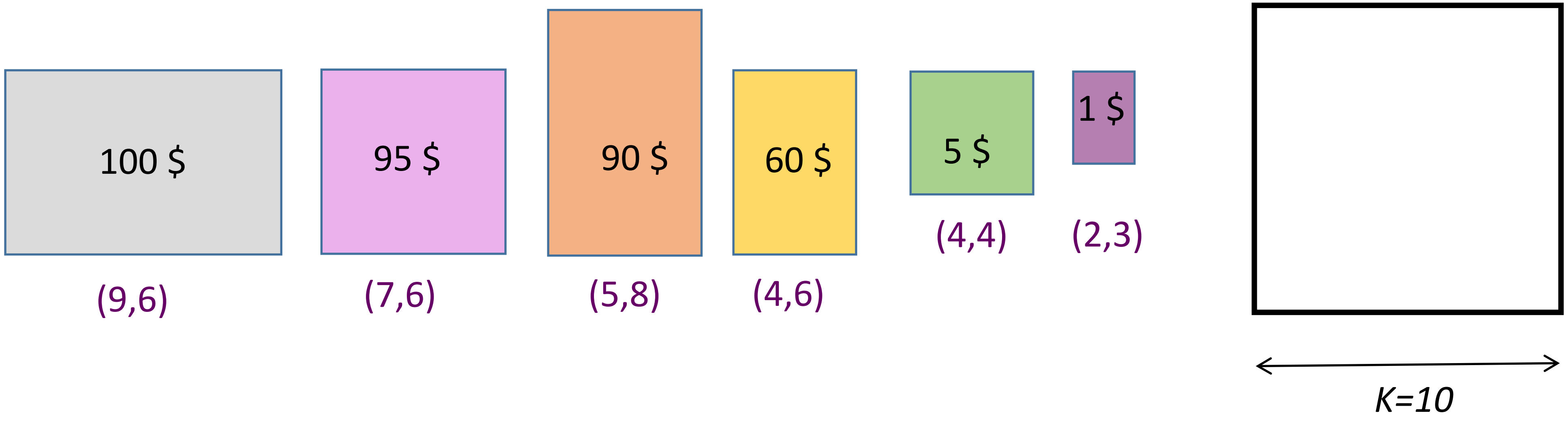
  - A Square $K \times K$ knapsack.

- **Goal :** Find an axis-parallel non-overlapping packing of a subset of input rectangles into the knapsack that maximizes the total profit.



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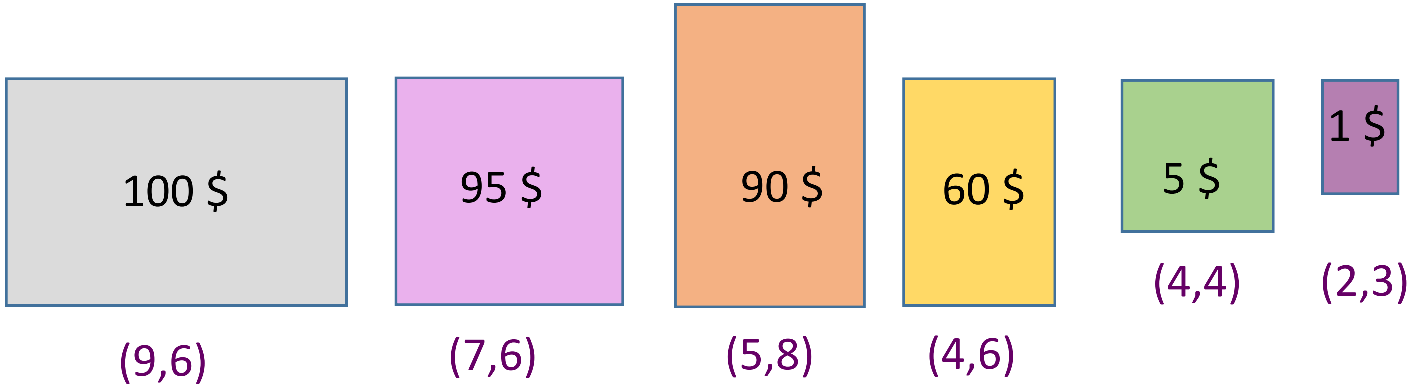
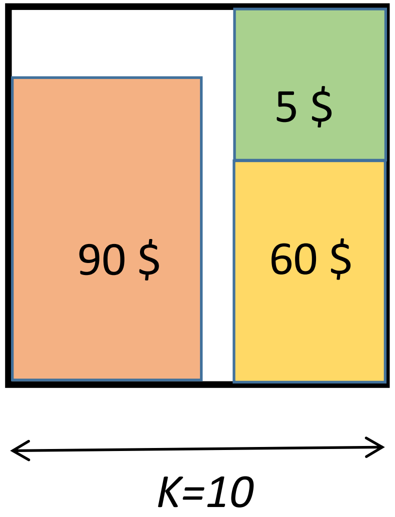




Variant 1: 2DK

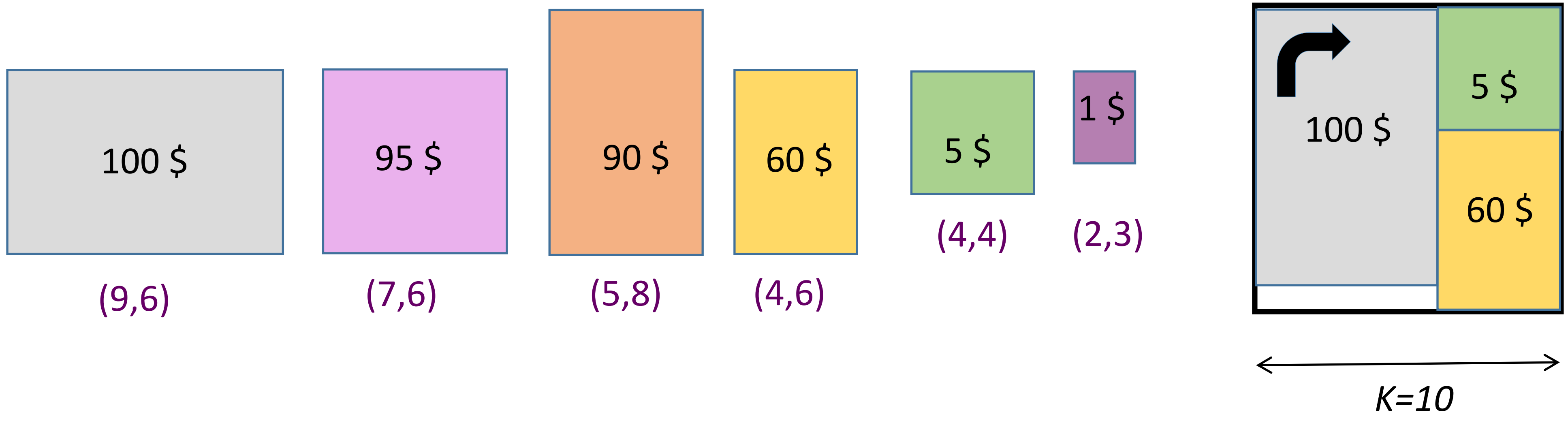
No rotations are allowed!

OPT=155

Geometric Knapsack: (2-D)

- Input :
  - Rectangles $I := \{R_1, R_2, \ldots, R_n\}$; Each R_i has integral width and height (w_i, h_i) and profit p_i.
  - A Square $K \times K$ knapsack.

- Goal : Find an axis-parallel non-overlapping packing of a subset of input rectangles into the knapsack that maximizes the total profit.



Variant 2: (2DKR)
90 degree rotations are allowed!

OPT=165

Geometric Knapsack: Complexity

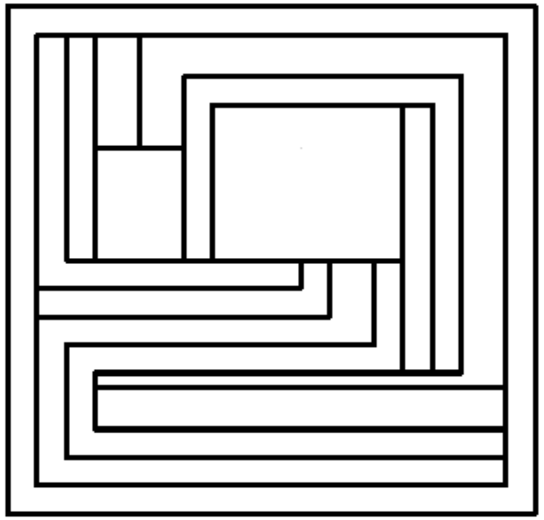
- Geometric Knapsack is Strongly NP-hard (even when all items are squares with profit 1), [Leung et al., 1990]
  - No exact algorithm even in pseudo-polynomial time (unless P=NP).
- W[1]-hard [Grandoni, Kratsch, Wiese, ESA'19], So no EPTAS.
- Not known whether the problem is APX-hard.
- The existence of a PTAS/QPTAS/PPTAS is still open!
- (1+ε)-approximation known if
  - profit of an item is equal to its area. [Bansal et al., ISAAC '09].
  - items are relatively small [Fishkin et al., MFCS '05].
  - items are squares [Wiese-Heydrich, SODA '17].

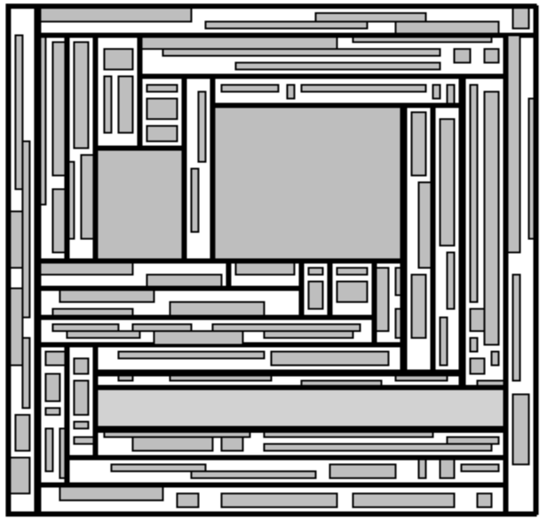
Geometric Knapsack:

- (2+ε)-approximation [Jansen-Zhang, SODA'04]
  - for both with and without rotations.
  - even in the cardinality case (when all profits are 1).
- Broke the barrier of 2 [Galvez-Grandoni-Ingala-K.-Wiese, FOCS'17]
  - Without rotations: $(17/9+\varepsilon)<1.89$-appx.
  - With rotations: $(1.5+\varepsilon)$-appx.
  - Cardinality case: 1.72, $(4/3+\varepsilon)$-appx., resp.
- Pseudopolynomial time $(4/3+\varepsilon)$-appx.
  [Galvez-Grandoni-K.-Romero-Wiese, SoCG'21]
- PPTAS for guillotine 2-D knapsack [K.-Maiti-Sharma-Wiese, SoCG '21]

Corridor decomposition

- Any feasible packing can be partitioned into O(1) corridors (rectilinear polygons) defined by O(1) number of line segments and intersecting only rectangles of profit $\leq \varepsilon p(OPT)$.
- Process the corridors so that we can retain a large fraction of profits in a packing that only have O(1) types of containers or corridors with two bends.





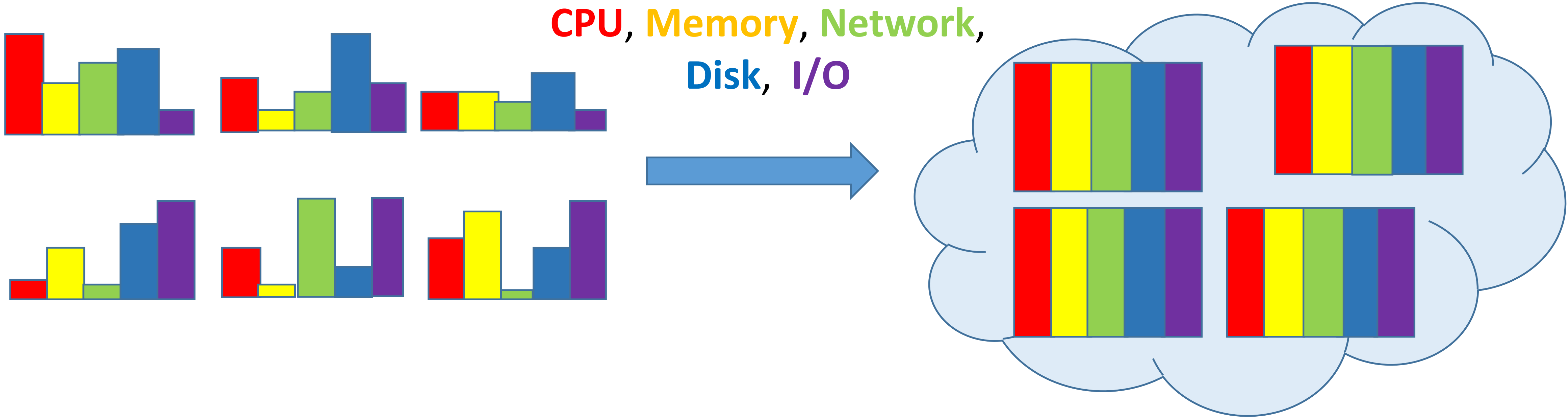
4. Vector packing

Vector Packing: Multidimensional Bin Packing

Jobs (items)

Servers (bins)

Resources (dimensions):
CPU, Memory, Network,
Disk, I/O



Goal: Assign all jobs to the servers s.t. min number of servers are needed.

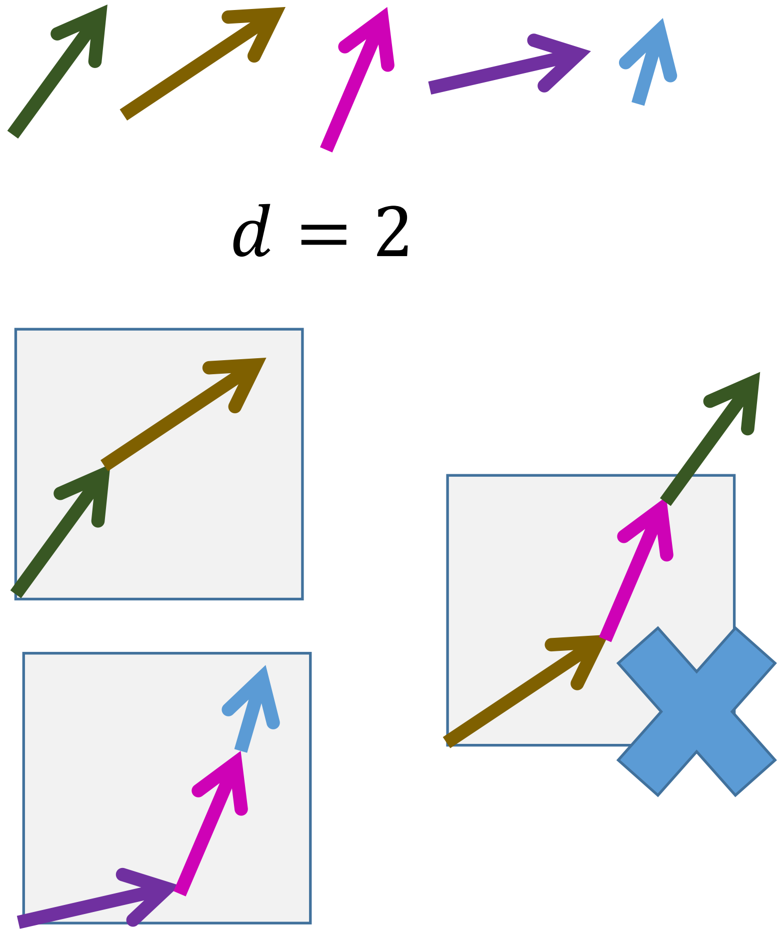
Vector packing

- Input:

Set of d-dimensional vectors: $[0,1]^d$

- Goal:

pack all vectors into minimum # of unit vector bins such that for each bin for each dimension the coordinate wise sum of packed vector in it is $\leq\ 1$.



A tale of approximability

- Dimension d is constant, otherwise approximation hardness $\Omega(d^{1-\epsilon})$.
- Asymptotic Approximation:
  - $d + \epsilon$ [Linear grouping: de la Vega-Lueker '81]
  - $2 + \ln(d) + \epsilon$ [Assignment LP: Chekuri-Khanna '99]
  - $1 + \ln(d) + \epsilon$ [Configuration LP: Bansal-Caprara-Sviridenko FOCS '06]
- Absolute/Nonasymptotic: 2 for $d = 2$ [Kellerer-Kotov 2003]
- Hardness:
- No APTAS (from 3D Matching)[Woeginger 1997].

Recent progress:

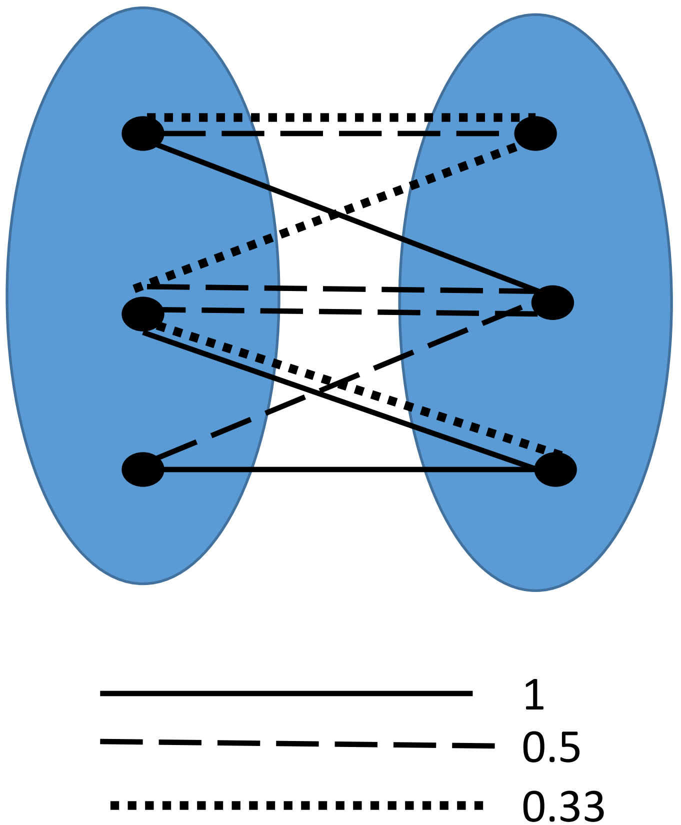
- Bansal, Elias, K., SODA' 16: [Multiobjective matching+R&A framework]
- Almost tight $(1.5+\epsilon)$ (Absolute) Approximation for 2-D.
  1.405 Asymptotic Approximation for 2-D.
- $0.807+\ln(d+1)$ Asymptotic Approximation for d dim.
- Hardness of d for constant rounding based algorithms.
- If we allow extra resource of ϵ in $(d-1)$ dimensions, we can find a packing in polynomial time in $(1+\epsilon)Opt+O(1)$ number of bins.
- Sai Sandeep, 2021: $\Omega(\log d)$-hardness, when d is large.

5. Weighted Bipartite Edge Coloring

Edge Coloring Meets Bin Packing

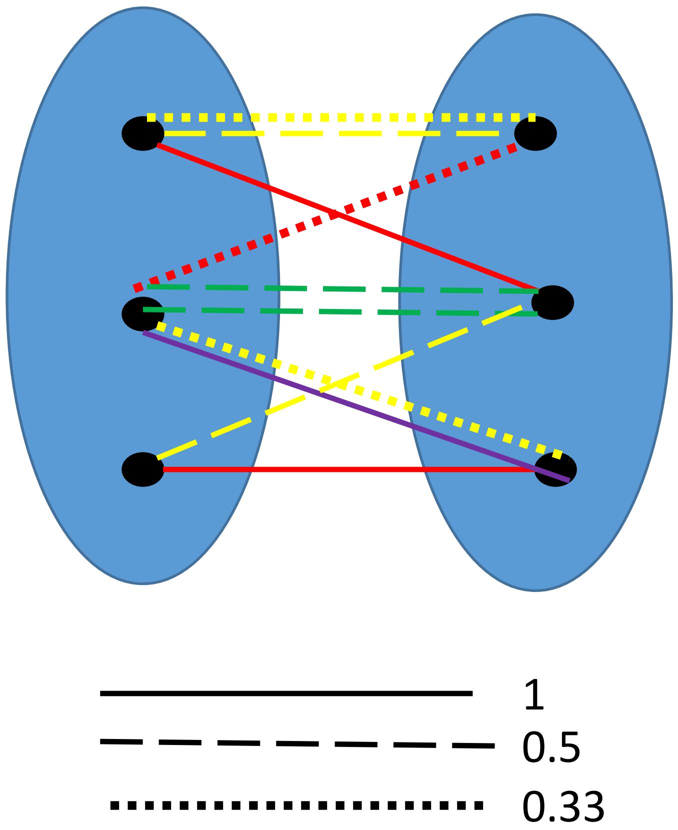
Weighted Bipartite Edge Coloring

- *Given:* An edge-weighted bipartite multi-graph $G := (L \cup R, E)$with edge-weights $w\colon\ E \rightarrow [0,1]$.
- $Goal$: Find a proper weighted coloring with minimum number of colors.
- *Proper weighted coloring:* Sum of the edges incident to any vertex of any color is ≤ 1.



Weighted Bipartite Edge Coloring

- *Given:* An edge-weighted bipartite multi-graph $G := (L \cup R, E)$with edge-weights $w\colon\ E \to [0,1]$.
- $Goal$: Find a proper weighted coloring with minimum number of colors.
- *Proper weighted coloring:* Sum of the edges incident to any vertex of any color is ≤ 1.
- Reduces to bin packing if |L|=|R|=1.



Weighted Bipartite Edge Coloring: Previous Works

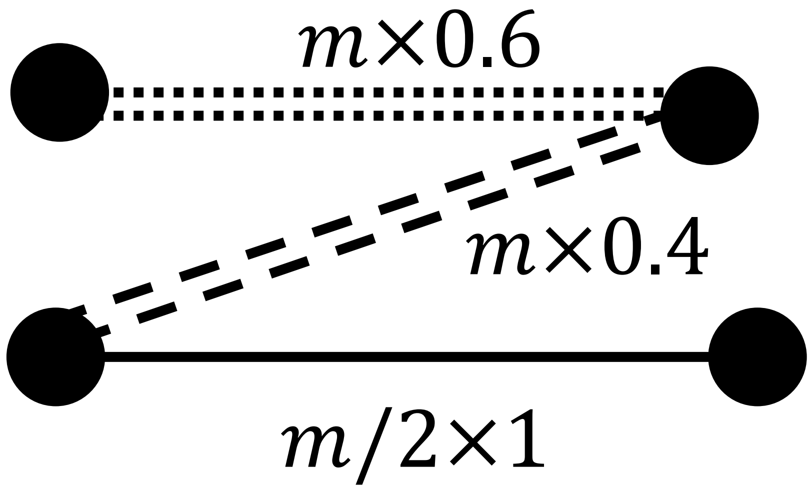
- Conjecture 1. [Chung & Ross, 1991]
  There is a proper weighted coloring with $2m$ -1 colors where
  $m = \max_{\{v \in V\}}\{\min \#\ bins\ to\ pack\ w_e's \mid e \in \delta(v)\}.$

Lower bound:

- Ngo -Vu SODA'03 : 1.25 m

Upper bound:

- Correa-Goemans *STOC '04*: 2.548 m
- Feige-Singh *ESA '08*: $9m/4 = 2.25\ m$
- K.-Singh, FSTTCS'15: 2.22m. [FF-based algorithm, Config LP-based analysis]



Other related problems.

- Approximation and Online Algorithms for Multidimensional Bin Packing: A Survey, Christensen-K.-Pokutta-Tetali, Computer Science Review 2017.

- Storage Allocation Problem [Momke, Wiese, ICALP'20].
- Unsplittable Flow on a Path [Grandoni, Momke, Wiese, Zhou, STOC'18].
- Dynamic Storage Allocation [Buchsbaum et al., STOC'03].
- Maximum Independent Set of Rectangles [Mitchell'21].
- Pach-Tardos Conjecture [K., Reddy, APPROX'20].

Summary of present status

Problem	Approximation Guarantee	Hardness
1-D Bin Packing	OPT+O(log OPT)	Strongly NP-hard
2-D Geometric BP (NR/R) d-D Geometric BP (NR/R) (d>2)	1.405 [BK, SODA'14] 1.69^{d-1} [C, MathOR'08]	No APTAS [BCKS, MathOR'06].
d-D Vector BP	0.81+$O(\log d)$ [BEK, SODA 2016]	$\Omega(\log d)$[Sai Sandeep, '21]
2-D Strip Packing (NR/R) 3-D Strip Packing (NR) d-D Strip Packing (NR)	PT (abs.): $5/3+\epsilon$. [HJPS, CompGeo'14] PPT(abs.): $5/4+\epsilon$ [JR, ESA'19] $3/2+\epsilon$ [JP, SOFSEM'14] 1.69^{d-1} [C, MathOR'08]	3/2 [Folklore] 5/4 [HJRD, CSR'18] No APTAS
2-D Geometric Knapsack (NR/R) d-D Geometric Knapsack (NR/R)	PT: 1.89, $1.5+\epsilon$ [GGHIKW FOCS'17] PPT: 4/3+ϵ [GGKRW SoCG'21] PT: $3^d(1+\epsilon)$ [Sharma '21]	W[1]-hard [GKW ESA'19] APX-hard (d>2)
Weighted Bipartite Edge Coloring	2.22 [KS FSTTCS'15]	Strongly NP-hard.

Top 10 open problems

1. Algorithm with OPT+O(1)-guarantee for bin packing.
2. Resolve integrality gap of configuration LP.
3. A poly(d)-approximation or hardness for d-dim geometric bin packing?
4. Improve 1.405- asymp approximation for 2D GBP.
5. Resolve guillotine conjecture (4/3) for 2-D BP.
6. Resolve 2-stage conjecture (3/2) for 2-D BP.
7. $(3/2+\varepsilon)$-(absolute) approximation for 2D strip packing?
8. PTAS (or PPTAS or QPTAS) for 2-D GK (even with rotations)?
9. Resolve Chung-Ross Conjecture.
10. Study parameterized approximation/exact/practical algorithms.

THANK

YOU